Today's colloquium: Megan Donahue on clusters!

Date and time of final? Afternoon of Monday, Dec 18, time and place TBD.

Clusters: Observations

Last time we talked about some of the context of clusters, and why observations of them have importance to cosmological issues. Some of the reasons why clusters are useful probes of cosmology are (1) their formation happened relatively recently, and hence depends on a variety of cosmological parameters, (2) they can be observed in many different wavebands, and (3) the dominant components of their mass are relatively simple (dark matter and smoothly distributed hot gas; galaxies can be thought of as tracer particles to first order). Now we need to determine what the observations actually are, and what they say about cosmology, structure formation, and the composition of the universe. To do this we need to determine what we want to measure about clusters and how to measure those properties.

Measurement of important quantities

Ask class: what are some quantities that we want to measure? Mass and luminosity distribution as a function of radial location in the cluster and of the redshift; separate distribution of the gas and the dark matter, shape of the cluster, optical depth are examples. Redshift dependence of these is an especially important cosmological probe (because it reflects the evolution of clusters), and it is also important to measure these for a large sample of clusters, to understand the luminosity and mass distribution.

The next thing we need to know is how to measure these quantities. As always, our measurements are somewhat indirect, so we need to have a handle on how they're done. Let's take the mass as an example. **Ask class:** what are ways to measure the mass of an individual cluster? One way is by determining the velocity dispersion of the galaxies. That means that you take the spectrum of many separate galaxies, determine the redshift of each, then compare to the average redshift of the cluster. From these velocities, the mass is estimated assuming that the motion of galaxies has been virialized. Another method is measurement of the temperature of the gas. If the temperature of the gas is close to the virial temperature, this also indicates the mass. Finally, there is gravitational lensing. Measurement of the gravitational effect on light from background galaxies is a qualitatively different indicator of the mass.

Measurement biases

Let's consider each of these individually, to understand possible problems or selection biases. First, measurement of cluster mass by galaxy motions. **Ask class:** what effects might complicate mass determination by this method or bias the results? One point is that if the velocities are not virial (i.e., due to the gravity of the cluster alone), the mass measurements can be inaccurate. This is related to the report several years ago that there was a $10^{11}\,M_\odot$ black hole in the center of a galaxy; the velocities turned out to be due to a collision with another galaxy instead of just the orbital velocity. Another problem is cluster membership. The typical orbital velocities are around 1000 km s^{-1}, so that would be the velocity dispersion, but in redshift space this would correspond (using Hubble's law) to a distance of about 14 Mpc! This produces two effects. One is the "finger of God" effect. Suppose you have a large number of galaxies in a cluster, and that they move with a typical velocity of 1000 km s^{-1}, in random directions. The angular width of the cluster isn't changed by the velocities, so it looks 1 Mpc wide. However, the apparent length of the cluster along the line of sight is 20-30 Mpc, so the effect is of a giant finger in redshift space pointing at you! The other effect is one that seems to enhance the membership of clusters. Say you have a cluster, or for that matter a long filament oriented with the long axis perpendicular to the line of sight. Galaxies not associated with the cluster or filament tend to fall in towards it. If the galaxy is closer to us than the cluster is, this increases its recession velocity, making it seem closer to the cluster in redshift space. If the galaxy is farther away, the fallback velocity towards the cluster decreases its recession velocity, again making it seem closer to the cluster in redshift space. Therefore, there is an artificial enhancement of the density. These effects are all linked to the fact that many times only the redshift is an indicator of distance. **Ask class:** how might these problems be circumvented? If the true distance, rather than just the redshift, can be measured, this would provide an independent check. This is tricky, but measurements of standard candles (e.g., the Tully-Fisher relation for spirals or the dn-σ relation for ellipticals) can help.

The next method is gas temperature. **Ask class:** what effects might mess this up? If the temperature is not the virial temperature, then the mass estimate will be bad. **Ask class:** what are some things that can change the temperature? Cooling of a variety of types (bremsstrahlung, atomic recombination, molecular cooling, metal line cooling) can lower the temperature, whereas shock heating can increase the temperature. Thinking about our galaxy, for example, the virial temperature for hydrogen is about 10^7 K, but many regions of the ISM are much cooler. The advantage with clusters is that at 10^8 K and the observed densities, cooling is relatively slow, so the temperature is a reasonable indicator of the virial temperature. In particular, **Ask class:** what is the dominant emission process at such high temperatures? It's bremsstrahlung. The volume emissivity at an electron number density n_e cm^{-3} and a temperature $T = 10^8 T_8$ K is

$$j_{\rm bremss} = 1.5 \times 10^{-23} n_e^2 T_8^{1/2}\,\mathrm{erg\ cm^{-3} s^{-1}}\ . \tag{1}$$

At typical densities $n = 10^{-3}$, this is about 10^{-29} erg cm^{-3} s^{-1}, compared with an energy density $nkT \approx 10^{-11}$ erg cm^{-3}, so the cooling time is around 10^{18} s, which is longer than the current age of the universe. Note that temperature measurements are independent in some ways from the galaxy velocity method; redshift determinations aren't necessary, just a measurement of the spectrum.

It should also be mentioned that temperature or galaxy velocity measurements give the standardly inferred masses if Newtonian gravity operates on large scales, $\sim$1 Mpc, but if gravity is modified at low accelerations (as advocated by Stacy McGaugh), the masses could be less. This is a dark horse explanation, but should be kept in mind unless future observations definitively rule it out.

The last type of bias to mention pervades extragalactic astronomy. It is called Malmquist bias. Suppose you have a survey that is flux-limited, that is, the set of objects includes all those above some flux. You will therefore include close low-luminosity sources as well as distant high-luminosity sources. As a result, your inferences about the population of interest can be severely biased. A local example is that if you take all stars in the Milky Way, their numbers are dominated by M stars. But if you take all stars visible to the naked eye, their numbers are dominated by A stars because you can see them to much greater distances. One can try to deal with this in various ways, such as by doing volume-limited samples instead; for example, you could decide to include only objects out to a certain redshift, such that you can detect all objects to that distance. This means that you throw away a lot of data, namely, the brighter things farther away, but it makes your sample more uniform. Malmquist bias comes in many forms, one of which has a particularly severe effect on a type of gravitational lensing statistic.

Gravitational Lensing

Gravitational lensing is a tool that has been applied to clusters comparatively recently, and it provides information in a relatively clean way, so we'll go over it briefly. The basic physics, of course, is that light is deflected by the presence of mass, since light follows a null geodesic in curved spacetime. In clusters one essentially never has to deal with strong curvature, so we can use the lowest-order limit: a light ray with a nearest approach distance x relative to a mass M is deflected by the angle $\alpha = 4GM/xc^2$. This is twice the Newtonian value. One can relate this to the velocity dispersion σ of gas or galaxies; when the various factors are put in, assuming an isothermal sphere ($\rho \sim 1/r^2$), one gets $\alpha = 4\pi(\sigma/c)^2$. This deflection redistributes light, but does not create it. Specifically, that means that if you went to a large distance from a source of radiation, and measured the total luminosity from that source over 4π steradians, it would be unchanged by the presence of matter between you and the source, assuming absorption and scattering could be neglected. However, in a given direction, the flux could be increased or decreased. This is the effect of gravitational lensing. Two types of lensing can be usefully distinguished: strong lensing, in which multiple images are produced, and weak lensing, in which they are not. If the impact parameter of the light ray is small enough, multiple images can be produced. For example, consider light from a source that is deflected by a spherical object. There is a focal point beyond that object, so if you are on that optic axis you see a ring of radiation. More generally, suppose that the lens is at an angular diameter distance D_l from us; the source is at an angular diameter distance D_s from us; and the angular diameter distance between the two is D_{ls}. If the angular impact parameter β is less than the Einstein radius

$$\theta_E = \frac{4\pi\sigma^2 D_{ls}}{c^2 D_s} = 2.6'' \sigma_{300}^2 D_{ls}/D_s \ , \tag{2}$$

then there will be multiple images seen. Here $\sigma = 300$ km s$^{-1}\sigma_{300}$. The effective angular cross section for multiple imaging is $\pi\theta_E^2$. For weak lensing, the images are sheared and distorted.

Lensing provides a direct measure of the mass, and hence is free from many of the biases that affect mass estimates from gas temperature or galaxy motions. The only tricky thing is that usually the shearing is not obvious, and galaxies have a diversity of shapes anyway, so care must be taken to measure averaged shear and rotation. Once this is done, a mass map of the whole cluster is possible. Tony Tyson at Bell Labs has been particularly successful with this work. On a smaller scale, one can also do detailed reconstruction of multiple imaging in particular cases, and from it estimate the Hubble constant and other cosmological parameters (typically from single galaxy lenses, not clusters). Another possible use of lensing is statistical: for a given cosmological model one can calculate the fraction of background objects at a given redshift that will be multiply imaged. The caveat here is that Malmquist bias (called magnification bias in this case) can be really strong. That is, since lensed objects are brighter than unlensed objects, a flux-limited sample will have a *much* higher fraction of multiply lensed objects than is really representative.

Masses, luminosities, etc.

Here, then, is a summary of the observed properties of clusters. Note that some of this may change due to the much higher resolution observations available with Chandra and XMM-Newton (see Megan Donahue's talk). In particular, much of the detailed structure is currently inferred from a combination of observations and numerical simulations. Some recent references are: Schindler, S. 1999, A&A, 349, 435; Wu, X.-P. 2000, MNRAS, 316, 299; Allen, S. W. 2000, MNRAS, 315, 269.

Radial dependence of luminosity.—The luminosity as a function of radius is well-described by the so-called "beta model", in which the surface brightness $S(r)$ is

$$S(r) = S_0 \left[1 + (r/r_c)^2\right]^{-3\beta+1/2} \ , \tag{3}$$

where r_c is the core radius ($r_c \sim 0.05 - 0.3$ Mpc) and β is the slope ($\beta \sim 0.4 - 1$).

Radial distribution of dark matter.—Found first in numerical simulations, and confirmed by observations, the virialized dark matter halo density follows the Navarro, Frenk, and

White (NFW) profile

$$\rho = \rho_s / \left[(r/r_s)(1 + r/r_s)^2 \right] \ , \tag{4}$$

where ρ_s and r_s are respectively the characteristic density and radius. Therefore, at $r \ll r_s$, $\rho \propto r^{-1}$, whereas at $r \gg r_s$, $\rho \propto r^{-3}$. Note that this is a *volume* density, whereas the beta model is a *surface* brightness, so they can't be compared directly. Nonetheless, the dark matter and the gas do appear to have different distributions.

Mass-luminosity relation.—The gas mass within 500 kpc is fairly tightly correlated with the bolometric X-ray luminosity:

$$M_{\rm gas}(< 500 \text{ kpc})/10^{14}\, M_\odot \approx 0.3 L_x / 10^{45} \text{erg s}^{-1} \ . \tag{5}$$

Within 500 kpc, the gas mass is typically a fraction 0.1–0.3 of the total mass.

Cluster-cluster correlation function.—The spatial distribution of clusters is self-correlated, i.e., a cluster is more likely to be near another cluster than far away. This is often expressed in terms of a two-point correlation function. The probability that a randomly chosen cluster has a neighbor at a distance between r and $r + dr$ is

$$dP = 4\pi n r^2 dr \left[1 + \xi(r)\right] \ , \tag{6}$$

where n is the number density of clusters and a good fit to $\xi(r)$ appears to be

$$\xi(r) \approx \left[18 \text{ Mpc}/r\right]^{1.8} \ . \tag{7}$$

There are therefore many close clusters, and in fact there are many clusters in the process of merging.

Luminosity function.—The differential number distribution of cluster luminosities, like the differential number distribution of galaxies, is well-described by a Schechter function:

$$\frac{dn}{dL_x}(L_x) = A \exp(-L_x / L_x^*) L_x^{-\alpha} \ , \tag{8}$$

where for the brightest clusters in the ROSAT sample (Ebeling et al. 1997, ApJ, 479, L101), the bolometric parameters are $\alpha = 1.84^{+0.09}_{-0.04}$ and $L^*_{x,44} = 37.2^{+16.4}_{-3.8}$. Note that the clusters in this sample are at relatively low redshifts, $z < 0.3$, so the cosmological constant doesn't play much of a role. For higher redshift, it would, so the luminosity would have to be quoted for a specific Ω_m and Ω_Λ.

Evolution of luminosity function?—A major cosmological question is whether the luminosity function and mass function of clusters has evolved recently (for $z < 1$, say) or not. **Ask class:** with all else being equal, would they expect more evolution if there is more total mass in the universe, or less? More, because gravitational clustering drives the evolution. Therefore, if $\Omega_m = 1$, one would expect recent evolution in cluster masses and luminosities; that is, one would expect fewer bright clusters in the past. Whether this occurs is a matter of current debate. A note of caution: here's a case where you have to be careful about what you are really comparing. For example, Vikhlinin et al. (1998) said they found strong evidence of evolution in the ROSAT survey because there were significantly fewer bright clusters at $z > 0.3$ than one would expect for no evolution. However, they computed the expected number for no evolution in an $\Omega_m = 1$ universe. If instead $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, then the luminosity distance to a given redshift is higher, meaning that at a given flux one needs a greater luminosity, meaning that even for no evolution one then expects far fewer clusters! The evidence can then cut both ways. The right thing to do (and this is a matter of statistics) is to calculate consistently the flux number distribution with redshift in two different models: a strongly evolving $\Omega_m = 1$ universe and a weakly evolving $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ universe, and see which does the best at reproducing the data.

Cooling flows.—The final observation we'll mention today is cooling flows. This is an issue of great current interest, and is being studied extensively with high angular resolution X-ray satellites such as Chandra and, to a lesser extent, XMM-Newton. The point is that since cooling is dominated by bremsstrahlung, which has a volume emissivity $j \propto n^2 T^{1/2}$, then the cooling time goes as $nkT/j \propto n^{-1} T^{1/2}$. Therefore, in the inner, denser regions of gas, the cooling time can be much less than the age of the universe. That means that the temperature drops, which decreases the cooling time further. But a decreasing temperature decreases the pressure, so hydrostatic equilibrium no longer holds. Then gas from outer regions flows into the inner regions and cools in turn, producing therefore a cooling flow. Right now it isn't clear where the gas goes. It seems to disappear after cooling to about 1 keV. Does it form stars? Does it cool catastrophically? Stay tuned to Chandra results to find out!