The golden age and Hawking radiation

Reference webpages:

http://en.allexperts.com/e/n/no/no_hair_theorem.htm
http://en.wikipedia.org/wiki/Hawking_radiation
and Chapters 7 and 12 in Thorne.

Questions to keep in mind are:

1. What are the aspects of black holes that make them such simple macroscopic objects?
2. Is Hawking radiation important? We can ask this in terms of phenomena in the universe, or in terms of tests of theories or implications for fundamental physics.

Introduction

Even with Finkelstein's 1958 (re)discovery, which led to a reappraisal of the case for black holes, the 1960s began with far more questions than answers. The Schwarzschild solution (and a similar one for electrically charged black holes) assumed exact spherical symmetry. Surely this couldn't be realistic! All things rotate for one, and in addition one might expect various bumps on black holes that would violate this exact symmetry. In principle, it might seem that there would be an unlimited variety of black hole shapes in the same way that asteroids can appear to be everything from nearly spherical to oddly shaped lumps. What other properties might they have? Could they have magnetic fields or other properties? Finally, if black holes are eternal, is the fate of all the matter and energy in the universe that it ends up in a black hole?

In this lecture, we will discuss the mathematical golden age of black holes in the 1960s and 1970s, including the fascinating but perhaps unimportant subject of Hawking radiation.

The mathematical golden age

Even with a renewed acceptance of the possibility of black holes (still called "Schwarzschild singularities" at the time), it is interesting to note that initially John Archibald Wheeler (who would become one of the most visible advocates of the possibility that black holes exist, and would in fact coin the term "black hole") thought that their actual formation might be prevented by the yet-unknown effects of quantum gravity. From a modern standpoint we know that in fact the "curvature" of spacetime around an astrophysical black hole is not enough to make quantum gravity important. That is, since the distances over which spacetime changes significantly even at the event horizon are quite macroscopic (kilometers for a stellar-mass black holes), the quantum effects of the very small are not expected to play an important role. At this point you might be wondering about Hawking radiation, which is thought to exist based on quantum arguments. As we shall see later, Hawking radiation is not thought to have any influence at all on stellar-mass or larger black holes, so the statement stands.

Nonetheless, in the early 1960s not just Wheeler but many physicists (including Wheeler's most famous student, Richard Feynman) were trying to push for a quantum theory of gravity. They did so for good reasons, that are echoed today in the attempts of physicists to produce a "theory of everything". Quantum mechanics is the most quantitatively successful theory every produced. By the early 1960s, and much more so now, thousands of experiments had attested to its accuracy. And yet gravity was understood in a non-quantum theory. To a large fraction of prominent physicists of the time, the most important goal was not to explore consequences of Einstein's theory, but instead to incorporate gravity with the elecromagnetic, weak, and strong forces. It rapidly became clear, however, that this was going to be very difficult indeed. Such people thus turned their attention to other things and left the general relativists to do their work.

It is worth a moment to explore why it is that exact solutions to Einstein's field equations are so difficult to come by. The reason is that general relativistic gravity is *nonlinear*. This means that unlike in Newtonian gravity, where you can compute the total gravitational acceleration on an object by adding up the gravitational accelerations due to all the other objects around it, in general relativity this does not give the exact answer. Thus whereas the Newtonian gravity around any object at all (no matter how misshapen) can be computed (laboriously!) by adding up the gravity from all of the object's component parts, this will not give you the right answer in general relativity. As a result, exact solutions to Einstein's field equations (exact in the sense that they can be written down as an equation with a finite number of terms!) are only possible in specially symmetric situations. The first of these was discovered by Karl Schwarzschild way back in 1916: assume that the matter distribution is spherically symmetric, and (for his first solution) that you are measuring the gravity in a vacuum, outside all of the matter. But since everything in the universe rotates it is clear that the Schwarzschild solution could not be exactly right for anything.

At this point we need to step back to avert a possible misunderstanding. In this class we have encountered a number of cases in which our conceptual understanding has made a sharp turn (e.g., special relativity or quantum mechanics). In each case, however, we noted on closer examination that the new conception reduced to the old one in the previously understood domain of phenomena. Is that true here, given that nonlinear gravity seems fundamentally different from linear (Newtonian) gravity? Yes. The nonlinear components of the Einstein equations are very small compared to the linear (thus Newtonian) components unless the gravity is very strong. Therefore, in weak gravity environments such as the Solar System, we can perfectly well add accelerations together. It had to be like this; general relativity had better be able to reproduce the phenomena we know well! Given this property of weak gravity, people were able to make some progress by exploring *post-Newtonian* theory;

basically, working out the deviations from Newton one step at a time, instead of all together in their full nonlinear messiness.

Nonetheless, in conditions of special symmetry it might be possible to get the full solution. This was a task that the still very small community of general relativists set themselves.

The race was won by Roy Kerr, a young physicist from New Zealand who was working at the University of Texas. The assumptions that he made (in effect; his actual assumptions were mathematical) were that (1) the spacetime was rotationally symmetric, and (2) the spacetime was time-independent. After just a few months of work that built on previous technical advances, Kerr published his result in a paper that was a mere page and a half long and was accepted and published within a month. The world's general relativists understood the significance immediately; Chandrasekhar (whom we met in the context of white dwarfs) indicated that this revelation was the most shattering of his career. Two years later, Ted Newman generalized the solution to include electric charge (the form of the solution is almost identical to the form without charge). As we will discuss a bit later in this lecture, electric charge does not play any significant role for astrophysical black holes, so we will ignore it.

A remarkable aspect of the Kerr solution had been anticipated as early as 1918 for spinning weakly gravitating objects. The rotation of a gravitating body drags spacetime with it (less for more distant locations, of course). This property is called frame-dragging, and means that if you drop something radially towards a spinning object (say, a black hole, but it doesn't have to be), with no angular momentum, then as the thing you dropped gets closer to the object it deviates from a straight-line path. You can think of this as if you have a massive thing spinning on a rubber sheet. Near your object, the sheet is twisted in the direction of the spin.

But what is the Kerr solution? An important distinction between the Kerr solution and the Schwarzschild solution is that whereas the Schwarzschild solution applies outside of *any* spherical, nonrotating object, the Kerr solution is only applicable outside of rotating black holes. It turns out that for slowly rotating objects of mass M and angular momentum J the Kerr solution is pretty good, and far away from any spinning object it also works well. But near, e.g., a rapidly spinning neutron star the spacetime deviates from Kerr. At first glance you might think that this is a problem; it means that if you want to get close to rapidly spinning compact objects, you have to have new solutions. In fact, however, as we will revisit when we get to gravitational radiation, the uniqueness of the Kerr solution to black holes will yield very precise probes of strong gravity. If, for example, measurement of the orbit of a small black hole around a supermassive one yields the results expected from the Kerr solution, this will strongly corroborate both general relativity and the existence of black holes as described in general relativity (as opposed to bizarre exotic solutions).

The no-hair theorem

But merely adding rotation surely isn't enough! When we consider stars or planets we realize that these objects have many additional characteristics, including magnetic fields and (for planets) mountains or other lumpiness. It seemed, even after Kerr's discovery, that endless other solutions for black holes would be necessary, just in the same way that the Newtonian (and indeed general relativistic) gravitational fields around ordinary objects have a potentially unlimited variety.

Remarkably, however, general relativistic black holes can be described by only three parameters: mass, angular momentum, and electric charge. There could be some quantum fuzz and in higher dimensions there would be additional complexity, but effectively it is only those three parameters that matter. In his book, Kip Thorne describes one of his own contributions, which is to realize that as an object contracts to form a black hole its magnetic field becomes squashed more and more, until at the horizon the magnetic field does not communicate with the outside world any more (this assumes that the hole is uncharged; more about that soon).

We can get some insight about why the lumpiness goes away if we think about mountains on terrestrial planets (Mercury, Venus, Earth, and Mars). Earth is the biggest of those planets, so one's first thought might be that it should have the highest mountains of the bunch. It is the other way around: Earth's highest mountain (Mount Everest) is actually *smaller* than the highest mountains on the other planets. In fact, if we consider the three planets big enough to have undergone some form of plate tectonics (Earth, Venus, Mars), the smaller the planet the higher the highest mountain on that planet. In fact, we find that the height of the highest mountain on a terrestrial planet is inversely proportional to the surface gravity on that planet. On reflection this makes sense; above a certain pressure rock will flow and thus for a given surface gravity the maximum height is set by the flow condition.

If you imagined compressing the Earth more and more while keeping its mass fixed, then not only the *absolute* size of the maximum mountain, but also its *relative* size (height divided by the diameter of the planet) would drop. As the radius approached the black hole critical radius, the surface gravity would approach infinity and the maximum mountain size would approach zero. Therefore, for a nonrotating black hole, the event horizon is perfectly smooth: a sphere. For a rotating black hole the horizon (and exterior gravitational field) is also perfectly smooth, but can be different at the equator versus at the poles in the same way that a rotating planet such as Saturn bulges out at the equator.

This is amazing. It means that black holes are the simplest macroscopic objects in the universe. For ones with masses a few times that of our Sun or greater, just the mass and angular momentum describe everything about them. Why do we ignore electric charge? The electromagnetic force is much greater than the gravitational force for small objects. For example, the electromagnetic force between two electrons is nearly 10^{40}(!!) times larger than the gravitational force between them. To put it another way, suppose that all the electrons from one hair of an astronaut's head were removed and put on the space shuttle launching pad. The resultant force would be so great that the shuttle would be unable to take off! What would really happen, of course, is that the electric charges would stream to equalize. Since there are positive and negative electric charges, on large scales they are inevitably almost exactly balanced. In contrast, there is no such thing as negative mass, so bigger things become overwhelmingly dominated by gravity. Thus we only worry about mass and angular momentum for black holes.

The mathematical proof that black holes have no other fundamental properties was obtained by several people in the 1960s through early 1970s, and is collectively known as the "no-hair theorem" (as in "black holes have no hair", meaning no complexity, in the phrase of John Wheeler). This means that unlike for most objects, where you need to approximate to determine their properties, for black holes you can get exact mathematical expressions. Ironically, this makes the study of black holes *seem* complicated, because for most things we just give up and use simple approximations.

It is also essential to realize that although the holes themselves are simple, as are the properties of "test particles" (particles imagined to react to the hole but not affect them; a single photon or single particle around a black hole would play this role) around the holes, self-interacting gas does not have to be simple. Given that it is from this gas, which can have magnetic fields in it, that we receive information about most black holes, complexity does sneak in. Luckily, for many purposes we know that the total mass of the gas is tiny compared to that of the hole, hence we can ignore its contribution to the spacetime.

Incidentally, a key aspect to the simplicity of black holes is that, as the name suggests, the "event horizon" is a region separating what we can see (anything outside the horizon) from what we cannot see (anything inside the horizon). Getting closer to the horizon doesn't help; a centimeter outside the horizon, you still can't see anything from inside. This insulates us from any bizarre quantum stuff that might be going on in the inside. The formal mathematical solutions for black holes suggest that at a point at the very center (for nonrotating, hence Schwarzschild, black holes) or at a ring surrounding the center (for rotating, hence Kerr, black holes) various quantities including the density and tidal acceleration should approach infinity. This infinity point or region is called the singularity of the black hole. Thus if you fell into a black hole, even if the relative acceleration between your head and feet was only moderate at the horizon (as would be true for a supermassive but not a stellar-mass black hole), the relative acceleration would increase as you got closer to the singularity, and not only you but atoms and elementary particles would eventually be torn to bits.

Now, for most physicists infinities in your calculations mean that you are using a theory beyond its realm of applicability. They therefore suggest that in a full theory of quantum gravity, singularities such as in a black hole or at the beginning of the universe would disappear and be replaced by a quantum nugget or something. These might not be observable, though: the "cosmic censorship conjecture" suggests that there are no "naked singularities"; instead, singularities are always clothed by event horizons and thus cannot influence the outside universe. Counterexamples in artificial situations may have been found, but it seems possible or even likely that this conjecture is true for the real universe. If so, the observable effects of quantum gravity are very subtle rather than in your face!

Of the various properties of the spacetime around black holes, one of the most important is that not all circular orbits are stable. To see what this means, consider the case of Newtonian gravity, where all circular orbits are stable. That is, if we have an object on a circular orbit around something massive, and something pushes the object a little bit, the new orbit is just slightly elliptical, but nothing catastrophic happens. We know this from experience; when the Earth is hit by an asteroid, or by tiny little gas particles, it does not plunge into the Sun! In Newtonian gravity, if the massive thing at the center is a point, all circular orbits are stable in this way.

In contrast, in general relativity circular orbits are not stable inside of a certain radius. A slight perturbation to such an orbit will send it spiralling quickly to the center. Mind you, this has to be a pretty tight orbit. For a nonrotating object, the radius of this *innermost stable circular orbit*, or ISCO for short, is just three times the radius of the event horizon. This means that in practice only neutron stars and black holes can have unstable circular orbits around them. They matter a lot for black holes, though. Their existence means that rather than allowing gas to spiral down, in nearly circular orbits, all the way to the event horizon (where they would release a huge amount of energy), once they cross inside this radius they can spiral quickly in and release no additional energy. In addition, the radius of this orbit depends on the spin of the object. Gas orbiting in the same direction as a maximally spinning hole can spiral almost to the event horizon, for example. This spin-dependence is key to attempts to measure the spins of holes, which is still a very challenging enterprise.

Hawking radiation

The end of the mathematical golden age can somewhat arbitrarily be placed at the discovery that black holes are not eternal, but instead radiate very slowly. This discovery, which made Stephen Hawking a star, is called Hawking radiation.

The ideas that led to Hawking radiation emerged from what seemed to be a rather mathematical and technical exercise. Jacob Bekenstein, a graduate student of John Wheeler, was bothered by the appearance that black holes violate the second law of thermodynamics. This is the law that says that entropy (which can be loosely construed as a degree of disorder) never decreases for any closed system (i.e., one that does not interact with anything else).

If a complicated star collapses into a simple black hole, your disorder has decreased rather substantially, right? This wasn't a concern for most researchers at the time; all physical laws have limits, so black holes don't obey the second law, big deal. But research by Bekenstein and others ultimately suggested that it is the *loss of information* in the collapse that can be interpreted as the analog of black hole entropy. That is, for a given black hole mass M and angular momentum J there are a very large number of different ways to obtain those values, but no measurement of the hole can tell you exactly what went into it.

This line of thought lead to the postulate that the entropy of a black hole is proportional to the surface area of the horizon, and that it is *enormous*; the entropy of a black hole could be $\sim 10^{20}$ times the entropy of the star that produced it. Further investigations of this sort led people to the conclusion that there was an awfully good correspondence between "black hole thermodynamics" and ordinary thermodynamics. For example, in exact analogy with ordinary thermodynamics, if two black holes merge with each other the single black hole that is left over has to have a horizon area that is at least as large as the sum of the areas of the two that went into it.

The only problem with all this is that if you really wanted to push the analogy you would say that black holes have a nonzero temperature, and that they would therefore radiate if put in a vacuum. As black holes obviously can't radiate, this is absurd and we shouldn't push the analogy too far. As Thorne describes in his book, under the pressure of this argument Bekenstein backed off from this conclusion in the early 1970s.

A little while later, the Soviet astrophysicist Yakov Zel'dovich came up with an argument that in fact some black holes *could* radiate. He focused in particular on rotating black holes. As we mentioned earlier, spinning black holes (and other spinning objects) drag spacetime with them. Zel'dovich realized that this could be married, sort of, with yet another bizarre quantum mechanical concept, for which we will take a brief diversion.

The concept is the one of *virtual particles*. To understand this we can go back to the uncertainty principle. The most familiar formulation of this principle is that you cannot measure both the position and momentum of an object with infinite precision at one and the same time. It also follows, however, that you cannot measure the *energy* and the *time* of something with infinite precision. A bizarre consequence of this is that over a short time in a small region the universe can "borrow" energy, and particle-antiparticle pairs can exist for fleeting instants before they go back into the vacuum. Therefore the vacuum, instead of being nothing, is a seething foam of these virtual pairs popping into and out of existence. Mind you, just like with any of these bizarre consequences, we need to ask whether this really happens. The answer is yes, or at least that there are calculable predictions for measurements that would happen in reaction to these virtual pairs, and those predictions have been verified to high accuracy. Weird!

Taking a deep breath, what does all this mean for rotating black holes? The insight Zel'dovich had was to consider a virtual pair near a rotating black hole. If the pair pops up and one member of the pair is closer to the hole than the other, the closer one might get sucked in while the farther one is flung out with such speed that it becomes real. That is, the farther one could gain energy, so that it doesn't have to borrow based on the uncertainty principle. From the perspective of a distant observer, all they would see was the emission of a particle. Where did the energy come from? From the rotation. Zel'dovich thought this meant that rotating black holes would slow down, even in a vacuum, by radiating particles. However, he also thought that once the rotation was dealt with, the remaining nonrotating black hole would not radiate any more.

He told these ideas to Hawking when Hawking visited Moscow. After considerable thought, Hawking realized that in fact even nonrotating black holes would radiate. Going back to the virtual pair idea, we can conceive of such a pair born close to the event horizon of a black hole. The tidal gravitational field of the hole can split the pair apart, with one going into the hole and the other escaping to infinity. If this happens, then the sequence would be (1) the pair is born with zero total energy, but just borrowing some for a moment, (2) the tidal gravitational field separates the two, effectively adding enough energy that they become real, (3) one particle falls in, while the other escapes, meaning that (4) distant observers see the hole radiating particles. The energy to produce the real particles comes from the hole's gravitational field, but this comes directly from the hole's mass, so the mass of the hole decreases with time. By the way, as Thorne discusses, further investigations showed that observers close to but outside the horizon would actually see a sea of real particles, so even the concept of a virtual particle depends on the reference frame.

Hawking's insight was examined by the community after his publication, and fairly quickly all general relativists agreed with him. Hawking radiation is even considered to be useful for testing candidate theories such as string theory, and the greatest success thus far of string theory is its ability to reproduce the magnitude and spectrum of Hawking radiation in some special cases.

Hawking radiation is certainly exciting, and it has some deep implications. For example, almost all the stuff that falls into a black hole is ordinary matter. However, almost all of what is radiated is photons (photons are their own antiparticle, so you can have a photon-photon virtual pair), because the effective wavelength of what is emitted is roughly the radius of the black hole, and although particles such as electrons have their own effective wavelengths these are really short and thus can only be emitted when the hole is already tiny. When that does happen, it is equally probable to emit antiparticles as particles, so particle conservation laws such as the conservation of "lepton number" (e.g., the number of electrons minus the number of antielectrons [also called positrons]) go out the window with Hawking radiation. This has also been the focus of fundamental studies of information and quantum theory.

However, the rate of this radiation is utterly, absolutely, negligible for any black hole that has ever been seen, and possibly for any black holes. A black hole with ten times the mass of our Sun, which is standard for the evolution of a massive star, would take more than 10^{70} years to evaporate. Given that the universe is only about 1.4×10^{10} years old, this is ridiculously long! To put it another way, suppose that we emptied the entire universe of all matter and radiation except that we put in one ten solar mass black hole and, ten billion light years away, put the dimmest main sequence star in the universe. Over time the black hole would *gain* mass because of the few photons it would receive from the star! Some people hope that mini black holes exist, formed in the early universe, that might show significant radiation, or that particle accelerators might form them if some exotic theories (involving extra spatial dimensions beyond the three we see!) are correct. In the latter case, by the way, these holes would immediately evaporate, so we are in no danger. In fact, in retrospect it is obvious that we are not in danger; cosmic rays much more energetic than anything humanity will produce in accelerators hit the atmosphere and each other all the time, and if Earth-killing black holes could be produced in accelerators they already would have been millions of times per year. We're here, so they aren't produced.

This is therefore a point for discussion. Hawking radiation may never be observed, and it may not have any measureable effects on anything. Is it, therefore, worthy of deep study?