CARMA Memo #33
Tsys calculation from ambient load and sky measurements

David Woody, March 13, 2006

This intended to explain the use, measurement and calculation of system temperatures for CARMA. Similar treatments have been carried out for BIMA, OVRO and other radio observatories. Some of the basic background is included to aid in understanding the definitions and calculations. This memo has been done in MathCad so that rigorous math calculations can be done and then tested and verified with real examples. All of the various manipulation steps are included. Those interested in just the answers should skip to the summary of required formulas at the end of the memo.

The definition of "noise temperature" is the temperature of a load or signal that doubles the output power relative to the output power in the absence of any signal or a load of 0K. We will use Rayleigh Jeans requivalent temperature throughout to avoid having to use the full Planck black body flux formulas. The standard method for measuring the noise temperature is to place two different loads (hot and cold) in front of the system and apply the following formula:

$$\mathrm{Tnoise}(\mathrm{Phot},\mathrm{Thot},\mathrm{Pcold},\mathrm{Tcold}) := \frac{(\mathrm{Thot}-\mathrm{Tcold})}{(\mathrm{Phot}-\mathrm{Pcold})}\cdot \mathrm{Pcold}-\mathrm{Tcold}$$

The ideal place to measure the system temperature for a telescope is at the top of the atmosphere so that the effect of the atmospheric absorption and emission are included in the system temperature. We can use the CMB as the cold load and use a fictitious hot load at temperature Tcal above the atmosphere as the hot load. The system temperature is then given by:

$$\mathrm{Tcmb} := 2.73\cdot \mathrm{K}$$

$$\mathrm{Tsys}(\mathrm{Psky},\mathrm{Pcal},\mathrm{Tcal}) := \frac{\mathrm{Psky}}{\mathrm{Pcal}-\mathrm{Psky}}\cdot(\mathrm{Tcal}-\mathrm{Tcmb})-\mathrm{Tcmb}$$

The CMB is always in our beam and needs to be included in the system temperature for the purposes of calculating the telescopes sensitvity or the noise in a measurement. So we will add its contribution back in giving:

$$\mathrm{Tsys}(\mathrm{Psky},\mathrm{Pcal},\mathrm{Tcal}) := \frac{\mathrm{Psky}}{\mathrm{Pcal}-\mathrm{Psky}}\cdot(\mathrm{Tcal}-\mathrm{Tcmb})$$

$$\mathrm{Yf}(\mathrm{Pload},\mathrm{Psky}) := \frac{\mathrm{Pload}}{\mathrm{Psky}}$$

or in terms of the Y-factor = Pcal/Psky

$$\mathrm{Tsys}(\mathrm{Y},\mathrm{Tcal}) := \frac{1}{\mathrm{Y}-1}\cdot(\mathrm{Tcal}-\mathrm{Tcmb})$$

Example for Tcal=300K and Y-factor=2.0

$$\mathrm{Tsys}(2.0,300\cdot \mathrm{K}) = 297.27\,\mathrm{K}$$

The actual measurements use an absorbing load at a temperature Tload close to the receiver. The measured power for this load is Pload. The exercise is to find the temperature Tcal of an idealized load outside the atmosphere that gives the same output power Pcal=Pload and hence the same Y-factor. The above formula can then be used to calculate the system temperature. The Tcal will be calculated as incremental corrections to the Tload.

The obvious effects that need to be taken into consideration are the spillover losses from the ground, ohmic losses and some of the feed leg scattering plus the opacity of the atmosphere.

Trec = noise temperature of the receiver
Tload = temperature of the calibration load
Tgnd = temperature of the ground
τg = effective optical depth for the spillover losses, spillover fraction = (1-exp(-τg)
Tatmo = the effective temperature the atmospheric absorption
τa = atmospheric opacity

The detected power for the idealized load at the top of the atmosphere at temperature Tcal is:

$$\mathrm{Pcal}(\mathrm{Trec},\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tcal}) := \mathrm{Trec} + \left(1 - e^{-\tau g}\right)\cdot\mathrm{Tgnd} + e^{-\tau g}\cdot\left(1 - e^{-\tau a}\right)\cdot\mathrm{Tatmo} + e^{-\tau a}\cdot e^{-\tau g}\cdot\mathrm{Tcal}$$

Note that the detector power for looking at a blank sky is obtained by simply replacing Tcal with Tcmb.

$$\mathrm{Psky}(\mathrm{Trec},\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g) := \mathrm{Pcal}(\mathrm{Trec},\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tcmb})$$

The detected power for the actual absorbing load.

$$\mathrm{Pload}(\mathrm{Trec},\mathrm{Tload}) := \mathrm{Trec} + \mathrm{Tload}$$

The effective temperature for the idealized load at the top of the atmosphere is found equating Pcal=Pload and solving for Tcal.

Given

$$\mathrm{Pcal}(\mathrm{Trec},\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tcal}) = \mathrm{Pload}(\mathrm{Trec},\mathrm{Tload})$$

$$\mathrm{Find}(\mathrm{Tcal}) \rightarrow \frac{-\mathrm{Tgnd} + \mathrm{Tgnd}\cdot\exp(-\tau g) - \exp(-\tau g)\cdot\mathrm{Tatmo} + \exp(-\tau g)\cdot\mathrm{Tatmo}\cdot\exp(-\tau a) + \mathrm{Tload}}{\exp(-\tau a)\cdot\exp(-\tau g)}$$

$$\mathrm{Teff}(\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tload}) := \frac{-\mathrm{Tgnd} + \mathrm{Tgnd}\cdot\exp(-\tau g) - \mathrm{Tatmo}\cdot\exp(-\tau g) + \mathrm{Tatmo}\cdot\exp(-\tau g)\cdot\exp(-\tau a) + \mathrm{Tload}}{\exp(-\tau a)\cdot\exp(-\tau g)}$$

$$\mathrm{Teff}(\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tload}) \rightarrow \frac{-\mathrm{Tgnd} + \mathrm{Tgnd}\cdot\exp(-\tau g) - \exp(-\tau g)\cdot\mathrm{Tatmo} + \exp(-\tau g)\cdot\mathrm{Tatmo}\cdot\exp(-\tau a) + \mathrm{Tload}}{\exp(-\tau a)\cdot\exp(-\tau g)}$$

Regroup terms to reveal the dependence on temperature differences and make it more readable

$$\mathrm{Teff}(\mathrm{Tatmo},\tau a,\mathrm{Tgnd},\tau g,\mathrm{Tload}) := \mathrm{Tload} + e^{\tau a}\cdot\left(e^{\tau g} - 1\right)(\mathrm{Tload} - \mathrm{Tgnd}) + \left(e^{\tau a} - 1\right)(\mathrm{Tload} - \mathrm{Tatmo})$$

Try an example

$Trec2 := 100 \cdot K$

$Tgnd2 := 260 \cdot K$

$\tau g2 := 0.05$

$Tatmo2 := 0.94 \cdot Tgnd2$

$\tau a2 := 0.5$

$Tload2 := 300 \cdot K$

$Psky(Trec2, Tatmo2, \tau a2, Tgnd2, \tau g2) = 205.729\, K$

$Pload(Trec2, Tload2) = 400\, K$

$Tcal2 := Teff(Tatmo2, \tau a2, Tgnd2, \tau g2, Tload2)$

$Tcal2 = 339.45\, K$

$Pcal(Trec2, Tatmo2, \tau a2, Tgnd2, \tau g2, Tcal2) = 400\, K$

$$Y2 := \frac{Pload(Trec2, Tload2)}{Psky(Trec2, Tatmo2, \tau a2, Tgnd2, \tau g2)}$$

$Tsys(Y2, Tcal2) = 356.581\, K$

The expected result is the sky power divided by the net signal loss.

$$Tsys2 := \frac{Psky(Trec2, Tatmo2, \tau a2, Tgnd2, \tau g2)}{e^{-\tau g2} \cdot e^{-\tau a2}}$$

$Tsys2 = 356.581\, K$

Thus the Tsys calculated from the measured Y-factor is correct if you use the corrected Tcal in place of Tload. If you don't make this correction you get

$Tsys3 := Tsys(Y2, Tload2)$

$Tsys3 = 314.804\, K$

This results in a 12% under estimate of the actual Tsys

$$\frac{Tsys2 - Tsys3}{Tsys2} = 0.117$$

The above formulation works for single sideband receivers and for double sideband receivers with equal response equal atmospheric losses in both sidebands. Double sideband receivers are handled by tracking the two signals and noise throught the atmosphere and receiver separately and combining them at the final detector.

Trec = noise temperature of the receiver
sbr = USB / LSB receiver gain ratio
Tload = temperature of the calibration load
Tgnd = temperature of the ground
τg = effective optical depth for the spillover losses, spillover fraction = (1-exp(-τg)
Tatmo = the effective temperature the atmospheric absorption
τaLSB = atmospheric opacity for the lower sideband signal
dτa = difference in the USB and LSB optical depth, τaUSB - τaLSB

Define the receiver sideband gains such that the sum of the gains in the two sidebands is one.

$$\mathrm{gUSB}(\mathrm{sbr},a) := a\cdot \mathrm{sbr}$$

$$\mathrm{gLSB}(\mathrm{sbr},a) := a$$

Given

$$\mathrm{gUSB}(\mathrm{sbr},a) + \mathrm{gLSB}(\mathrm{sbr},a) = 1$$

$$\mathrm{Find}(a) \rightarrow \frac{1}{\mathrm{sbr}+1}$$

$$\mathrm{gLSB}(\mathrm{sbr}) := \frac{1}{1+\mathrm{sbr}}$$

$$\mathrm{gUSB}(\mathrm{sbr}) := \frac{1}{1+\frac{1}{\mathrm{sbr}}}$$

Calculate the power measured in the IF chain for each sideband. We can use the same formulas derived above for the single sideband case with appropriate atmospheric opacity and modified by receiver sideband gain.

$$\mathrm{PcalLSB}(\mathrm{TrecDSB},\mathrm{sbr},\mathrm{Tatmo},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal}) := \mathrm{gLSB}(\mathrm{sbr})\cdot \mathrm{Pcal}(\mathrm{TrecDSB},\mathrm{Tatmo},\tau\mathrm{aLSB},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal})$$

$$\mathrm{PcalUSB}(\mathrm{TrecDSB},\mathrm{sbr},\mathrm{Tatmo},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal}) := \mathrm{gUSB}(\mathrm{sbr})\cdot \mathrm{Pcal}(\mathrm{TrecDSB},\mathrm{Tatmo},\tau\mathrm{aLSB}+\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal})$$

Add the two sidebands together.

$$\mathrm{PcalDSB}(\mathrm{TrecDSB},\mathrm{sbr},\mathrm{Tatmo},\tau\mathrm{a},\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal}) := \left|\begin{array}{l}\mathrm{Pu} \leftarrow \mathrm{PcalUSB}(\mathrm{TrecDSB},\mathrm{sbr},\mathrm{Tatmo},\tau\mathrm{a},\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal}) \\ \mathrm{Pl} \leftarrow \mathrm{PcalLSB}(\mathrm{TrecDSB},\mathrm{sbr},\mathrm{Tatmo},\tau\mathrm{a},\mathrm{d}\tau\mathrm{a},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tcal}) \\ \mathrm{Pnet} \leftarrow \mathrm{Pu}+\mathrm{Pl}\end{array}\right.$$

$\text{PcalLSB}(\text{TrecDSB}, \text{sbr}, \text{Tatmo}, \tau\text{aLSB}, \text{d}\tau\text{a}, \text{Tgnd}, \tau\text{g}, \text{Tcal})$ becomes

$$\frac{1}{1+\text{sbr}}\cdot\left[\text{TrecDSB} + (1-\exp(-\tau g))\cdot\text{Tgnd} + \exp(-\tau g)\cdot(1-\exp(-\tau\text{aLSB}))\cdot\text{Tatmo} + \exp(-\tau\text{aLSB})\cdot\exp(-\tau g)\cdot\text{Tcal}\right]$$

$\text{PcalUSB}(\text{TrecDSB}, \text{sbr}, \text{Tatmo}, \tau\text{aLSB}, \text{d}\tau\text{a}, \text{Tgnd}, \tau\text{g}, \text{Tcal})$ becomes

$$\frac{1}{1+\frac{1}{\text{sbr}}}\cdot\left[\text{TrecDSB} + (1-\exp(-\tau g))\cdot\text{Tgnd} + \exp(-\tau g)\cdot(1-\exp(-\tau\text{aLSB}-\text{d}\tau\text{a}))\cdot\text{Tatmo} + \exp(-\tau\text{aLSB}-\text{d}\tau\text{a})\cdot\exp(-\tau g)\cdot\text{Tcal}\right]$$

Solve for the equivalent DSB loss in the atmosphere.

$$\text{gLSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) := \text{gLSB}(\text{sbr})\cdot e^{-\tau\text{lsb}}$$

$$\text{gLSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) \rightarrow \frac{1}{\text{sbr}+1}\cdot\exp(-\tau\text{lsb})$$

$$\text{gUSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) := \text{gUSB}(\text{sbr})\cdot e^{-\tau\text{lsb}}\cdot e^{-\text{d}\tau}$$

$$\text{gUSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) \rightarrow \frac{1}{1+\frac{1}{\text{sbr}}}\cdot\exp(-\tau\text{lsb})\cdot\exp(-\text{d}\tau)$$

Given

$$\text{gUSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) + \text{gLSBnet}(\text{sbr}, \tau\text{lsb}, \text{d}\tau) = \text{ga}$$

$$\text{Find}(\text{ga}) \rightarrow \exp(-\tau\text{lsb})\cdot\frac{\text{sbr}\cdot\exp(-\text{d}\tau)+1}{\text{sbr}+1}$$

The effective DSB atmospheric loss is given by

$$\text{gDSBa}(\text{sbr}, \tau\text{aLSB}, \text{d}\tau\text{a}) := \frac{1+\text{sbr}\cdot e^{-\text{d}\tau\text{a}}}{1+\text{sbr}}\cdot e^{-\tau\text{aLSB}}$$

Try an example

$Trec2 := 100 \cdot K$

$sbr2 := 2.0$

$Tgnd2 := 260 \cdot K$

$\tau g2 := 0.05$

$Tatmo2 := 0.94 \cdot Tgnd2$

$\tau aLSB2 := 0.5$

$d\tau a2 := 0.2$

$Tload2 := 300 \cdot K$

$gDSBa2 := gDSBa(sbr2, \tau aLSB2, d\tau a2)$

$gDSBa2 = 0.533$

$PcalDSB(Trec2, sbr2, Tatmo2, \tau aLSB2, d\tau a2, Tgnd2, \tau g2, Tload2) = 373.363\,K$

$Pcal(Trec2, Tatmo2, -\ln(gDSBa2), Tgnd2, \tau g2, Tload2) = 373.363\,K$

The double sideband power and effective idealized load temperature can be calculated using the single sideband formulas by replacing the atmospheric opacity, τa, with the effective DSB atmospheric opacity, ln(gDSBa(sbr,τaLSB,dτa)).

$PcalDSB(Trec, sbr, Tatmo, \tau aLSB, d\tau a, Tgnd, \tau g, Tcal) := Pcal(Trec, Tatmo, -\ln(gDSBa(sbr, \tau aLSB, d\tau a)), Tgnd, \tau g, Tcal)$

$PskyDSB(Trec, sbr, Tatmo, \tau aLSB, d\tau a, Tgnd, \tau g) := PcalDSB(Trec, sbr, Tatmo, \tau aLSB, d\tau a, Tgnd, \tau g, Tcmb)$

$TeffDSB(sbr, Tatmo, \tau aLSB, d\tau a, Tgnd, \tau g, Tload) := Teff(Tatmo, -\ln(gDSBa(sbr, \tau aLSB, d\tau a)), Tgnd, \tau g, Tload)$

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This can be rewritten as Tload plus two correction terms

$$gDSBa(sbr, \tau aLSB, d\tau a) := \frac{1 + sbr \cdot e^{-d\tau a}}{1 + sbr} \cdot e^{-\tau aLSB}$$

$$\Delta Tgnd(gDSB, Tgnd, \tau g, Tload) := \frac{1}{gDSB} \cdot \left(e^{\tau g} - 1\right)(Tload - Tgnd)$$

$$\Delta Tatmo(gDSB, Tatmo, Tload) := \left(\frac{1}{gDSB} - 1\right)(Tload - Tatmo)$$

$$TeffDSB(Tatmo, gDSB, Tgnd, \tau g, Tload) := Tload + \Delta Tgnd(gDSB, Tgnd, \tau g, Tload) + \Delta Tatmo(gDSB, Tatmo, Tload)$$

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Example calculation starting from the previous example parameter values

$$gDSB2 := gDSBa(sbr2, \tau aLSB2, d\tau a2)$$

$$gDSB2 = 0.533$$

$$\Delta Tgnd2 := \Delta Tgnd(gDSB2, Tgnd2, \tau g2, Tload2)$$

$$\Delta Tgnd2 = 3.846\ K$$

$$\Delta Tatmo2 := \Delta Tatmo(gDSB2, Tatmo2, Tload2)$$

$$\Delta Tatmo2 = 48.669\ K$$

$$Tcal2 := TeffDSB(Tatmo2, gDSB2, Tgnd2, \tau g2, Tload2)$$

$$Tcal2 = 352.516\ K$$

$$Y2 := \frac{Pload(Trec2, Tload2)}{PskyDSB(Trec2, sbr2, Tatmo2, \tau aLSB2, d\tau a2, Tgnd2, \tau g2)}$$

$$Y2 = 1.797$$

$$Tsys(Y2, Tcal2) = 438.815\ K$$

The expected result is the sky power divided by the net signal loss.

$$Tsys2 := \frac{PskyDSB(Trec2, sbr2, Tatmo2, \tau aLSB2, d\tau a2, Tgnd2, \tau g2)}{e^{-\tau g2} \cdot gDSB2}$$

$$Tsys2 = 438.815\ K$$

Thus the Tsys calculated from the measured Y-factor is correct if you use the corrected Tcal in place of Tload. If you don't make this correction you get

$$Tsys3 := Tsys(Y2, Tload2)$$

$$Tsys3 = 372.933\ K$$

This results in a 15% under estimate of the actual Tsys

$$\frac{Tsys2 - Tsys3}{Tsys2} = 0.15$$

The above formulas are for calculating the DSB system noise temperature appropriate to single dish observations. The correlator separates the sidebands and the single sideband noise temperature in each sideband is required to calibrate the visibilities.

You can calculate Tsys from Psky directly as Psky/net_gain, where the net gain is that appropriate for the desired sideband.

The noise temperatures for a system with gain gainUSB and gainLSB in the two sidebands are given by

$$\mathrm{TsysDSBc}(\mathrm{gainUSB},\mathrm{gainLSB},\mathrm{Psky}) := \frac{\mathrm{Psky}}{\mathrm{gainUSB}+\mathrm{gainLSB}}$$

$$\mathrm{TsysUSBc}(\mathrm{gainUSB},\mathrm{Psky}) := \frac{\mathrm{Psky}}{\mathrm{gainUSB}}$$

$$\mathrm{TsysLSBc}(\mathrm{gainLSB},\mathrm{Psky}) := \frac{\mathrm{Psky}}{\mathrm{gainLSB}}$$

define

$$\mathrm{SBRatio}(\mathrm{gainUSB},\mathrm{gainLSB}) := \frac{\mathrm{gainUSB}}{\mathrm{gainLSB}}$$

then we have

$$\mathrm{TsysUSB}(\mathrm{Tdsb},\mathrm{SBR}) := \left(1+\frac{1}{\mathrm{SBR}}\right)\mathrm{Tdsb}$$

$$\mathrm{TsysLSB}(\mathrm{Tdsb},\mathrm{SBR}) := (1+\mathrm{SBR})\mathrm{Tdsb}$$

Note that SBR is the net sidband gain ratio.

$$\mathrm{SBR}(\mathrm{sbr},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a}) := \frac{\mathrm{gUSBnet}(\mathrm{sbr},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a})}{\mathrm{gLSBnet}(\mathrm{sbr},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a})}$$

$$\mathrm{SBR}(\mathrm{sbr},\tau\mathrm{aLSB},\mathrm{d}\tau\mathrm{a})\ \mathrm{simplify}\ \rightarrow \mathrm{sbr}\cdot\exp(-\mathrm{d}\tau\mathrm{a})$$

$$\mathrm{SBR}(\mathrm{sbr},\mathrm{d}\tau\mathrm{a}) := \mathrm{sbr}\cdot\exp(-\mathrm{d}\tau\mathrm{a})$$

Try an example

$TrecDSB3 := 50 \cdot K$

$sbr3 := 2.0$

$Tatmo3 := 260 \cdot K$

$\tau aLSB3 := 0.5$

$d\tau a3 := 0.2$

$Tgnd3 := 260 \cdot K$

$\tau g3 := 0.05$

$Tload3 := 300 \cdot K$

First calculate the double sideband noise temperature

$gDSB3 := gDSBa(sbr3, \tau aLSB3, d\tau a3)$

$gDSB3 = 0.533$

$\Delta Tgnd3 := \Delta Tgnd(gDSB3, Tgnd3, \tau g3, Tload3)$

$\Delta Tgnd3 = 3.846\,K$

$\Delta Tatmo3 := \Delta Tatmo(gDSB3, Tatmo3, Tload3)$

$\Delta Tatmo3 = 35.014\,K$

$Tcal3 := TeffDSB(Tatmo3, gDSB3, Tgnd3, \tau g3, Tload3)$

$Tcal3 = 338.86\,K$

$$Y3 := \frac{Pload(TrecDSB3, Tload3)}{PskyDSB(TrecDSB3, sbr3, Tatmo3, \tau aLSB3, d\tau a3, Tgnd3, \tau g3)}$$

$Y3 = 1.95$

$\mathrm{TsysDSB3} := \mathrm{Tsys}(\mathrm{Y3}, \mathrm{Tcal3})$

$\mathrm{TsysDSB3} = 353.895\,\mathrm{K}$

Now calculate the USB and LSB noise temperatures.

$\mathrm{SBR3} := \mathrm{SBR}(\mathrm{sbr3}, \mathrm{d\tau a3})$

$\mathrm{SBR3} = 1.637$

$\mathrm{TsysUSB}(\mathrm{TsysDSB3}, \mathrm{SBR3}) = 570.02\,\mathrm{K}$

$\mathrm{TsysLSB}(\mathrm{TsysDSB3}, \mathrm{SBR3}) = 933.386\,\mathrm{K}$

Test against straight calculation of expected Tsys.

$$\mathrm{TsysDSBc}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g}) := \frac{\mathrm{PskyDSB}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g})}{\mathrm{gUSB}(\mathrm{sbr}) \cdot e^{-\mathrm{\tau g}} \cdot e^{-(\mathrm{\tau aLSB} + \mathrm{d\tau a})} + \mathrm{gLSB}(\mathrm{sbr}) \cdot e^{-\mathrm{\tau g}} \cdot e^{-(\mathrm{\tau aLSB})}}$$

$\mathrm{TsysDSBc}(\mathrm{TrecDSB3}, \mathrm{sbr3}, \mathrm{Tatmo3}, \mathrm{\tau aLSB3}, \mathrm{d\tau a3}, \mathrm{Tgnd3}, \mathrm{\tau g3}) = 353.895\,\mathrm{K}$

$$\mathrm{TsysUSBc}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g}) := \frac{\mathrm{PskyDSB}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g})}{\mathrm{gUSB}(\mathrm{sbr}) \cdot e^{-\mathrm{\tau g}} \cdot e^{-(\mathrm{\tau aLSB} + \mathrm{d\tau a})}}$$

$\mathrm{TsysUSBc}(\mathrm{TrecDSB3}, \mathrm{sbr3}, \mathrm{Tatmo3}, \mathrm{\tau aLSB3}, \mathrm{d\tau a3}, \mathrm{Tgnd3}, \mathrm{\tau g3}) = 570.02\,\mathrm{K}$

$$\mathrm{TsysLSBc}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g}) := \frac{\mathrm{PskyDSB}(\mathrm{TrecDSB}, \mathrm{sbr}, \mathrm{Tatmo}, \mathrm{\tau aLSB}, \mathrm{d\tau a}, \mathrm{Tgnd}, \mathrm{\tau g})}{\mathrm{gLSB}(\mathrm{sbr}) \cdot e^{-\mathrm{\tau g}} \cdot e^{-(\mathrm{\tau aLSB})}}$$

$\mathrm{TsysLSBc}(\mathrm{TrecDSB3}, \mathrm{sbr3}, \mathrm{Tatmo3}, \mathrm{\tau aLSB3}, \mathrm{d\tau a3}, \mathrm{Tgnd3}, \mathrm{\tau g3}) = 933.386\,\mathrm{K}$

The Tsys calculated from the measured Y-factor is correct if you use the corrected Tcal in place of Tload.

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Summary of required formulas.

Y-factor

$$\mathrm{Yf}(\mathrm{Pload},\mathrm{Psky}) := \frac{\mathrm{Pload}}{\mathrm{Psky}}$$

Pload = detected power when looking at the calibration load
Psky = detected power when looking at the blank sky

The receiver sideband ratio is

$$\mathrm{sbrREC}(\mathrm{SBR},\mathrm{d\tau a}) := \mathrm{SBR}\cdot\exp(\mathrm{d\tau a})$$

SBR = sideband gain ratio, gainUSB/gainLSB, measured on a flat continuum astronomical source
dτa = atmospheric sideband opacity difference, USB opacity minus LSB opacity, calculated from an atmospheric radiative transfer code. It is domninated by the O2 line at 118GHz.

The double sideband atmospheric loss is given by

$$\mathrm{gDSBa}(\mathrm{sbr},\tau\mathrm{aLSB},\mathrm{d\tau a}) := \frac{1+\mathrm{sbr}\cdot e^{-\mathrm{d\tau a}}}{1+\mathrm{sbr}}\cdot e^{-\tau\mathrm{aLSB}}$$

sbr = receiver sideband ratio determined above
τaLSB = atmospheric opacity in lower sideband

note $\mathrm{gDSBa}(\mathrm{sbr},\tau\mathrm{aLSB},0) \rightarrow \exp(-\tau\mathrm{aLSB})$

Correction to Tload from the ground spillover losses

$$\Delta\mathrm{Tgnd}(\mathrm{gDSB},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tload}) := \frac{1}{\mathrm{gDSB}}\cdot\left(e^{\tau\mathrm{g}}-1\right)(\mathrm{Tload}-\mathrm{Tgnd})$$

gDSB = effective DSB atmospheric loss calculated above
Tgnd = temperature of the ground, where the spillover is coming from
τg = effective opacity for the ground spillover, -ln(1-loss)
Tload = temperature of the actual calibration load used for measuring the Y-factor

note $\Delta\mathrm{Tgnd}(\mathrm{gDSB},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tgnd}) \rightarrow 0$

Correction to Tload from the atmospheric absorption

$$\Delta\mathrm{Tatmo}(\mathrm{gDSB},\mathrm{Tatmo},\mathrm{Tload}) := \left(\frac{1}{\mathrm{gDSB}}-1\right)(\mathrm{Tload}-\mathrm{Tatmo})$$

Tatmo = effective physical temperature of the absorbing atmosphere, commonly taken as 0.95*Tgnd

note $\Delta\mathrm{Tatmo}(\mathrm{gDSB},\mathrm{Tload},\mathrm{Tload}) \rightarrow 0$

The net effective ideal load at the top of the atmosphere that gives the same Y-factor is

$$\mathrm{TeffDSB}(\mathrm{Tatmo},\mathrm{gDSB},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tload}) := \mathrm{Tload}+\Delta\mathrm{Tgnd}(\mathrm{gDSB},\mathrm{Tgnd},\tau\mathrm{g},\mathrm{Tload})+\Delta\mathrm{Tatmo}(\mathrm{gDSB},\mathrm{Tatmo},\mathrm{Tload})$$

The double sideband system noise temperature appropriate for single dish observations is

$$\mathrm{TsysDSB}(Y, \mathrm{Tcal}) := \frac{1}{Y - 1} \cdot (\mathrm{Tcal} - \mathrm{Tcmb})$$

Tcal = effective temperature of a load above the atmosphere that gives the same Pload, TeffDSB
Tcmd = Cosmic Microware Background radiation, 2.73K
Y = Y-factor defined above

the single sideband noise temperatures required to calibrate interferometer visibilities are given by

$$\mathrm{TsysUSB}(\mathrm{Tdsb}, \mathrm{SBR}) := \left(1 + \frac{1}{\mathrm{SBR}}\right)\mathrm{Tdsb}$$

$$\mathrm{TsysLSB}(\mathrm{Tdsb}, \mathrm{SBR}) := (1 + \mathrm{SBR})\mathrm{Tdsb}$$

Tdsb = double sideband noise temperature calculated above
SBR = sideband gain ratio, gainUSB/gainLSB, measured on a flat continuum astronomical source

Note that this treatment includes the 2.73K CMB as part of the system noise.