Combined Array for Research in Millimeter-Wave Astronomy

Memo #35

Thermal Stabilization of SIS Mixers in BIMA Receivers

Dustin Madison, U.C. Berkeley

25 August 2006

Change Record

REVISION	DATE	AUTHOR	SECTIONS/PAGES AFFECTED
	REMARKS		
A	8/25/2006	Dustin Madison	original version

Abstract

We devised a thermal low pass filter for the BIMA dewars which reduces the cyclical temperature fluctuation on the SIS mixers to < 10 mK p-p. At the same time, we improved the thermal contact between the mixer and the cold stage of the cryogenic refrigerator, lowering the mixer's temperature to ~ 3.5 K.

In the course of this project we measured the thermal conductivity of neodymium at 4 K. It is approximately 0.013 W/(cm-K).

1. Introduction

Each BIMA receiver is cooled by a three stage Gifford-McMahon refrigerator to approximately 3.5 K. The refrigerator completes a full cycle every two seconds, and over the course of this cycle, the temperature at the third stage fluctuates by about 200 mK about the 3.5 K average temperature (Plambeck 1998). Any signal coming into the receiver must go through one of two SIS mixers. These mixers must be in thermal contact with the third stage of the refrigerator, but their performance is quite temperature dependent; temperature fluctuations at the third stage cause fluctuations in the mixer's conversion loss, degrading the receiver stability. We thus sought a method of reducing the size of the temperature swings at the mixer without dramatically raising the average temperature of the mixer.

To work in familiar territory, we build an analogy between these thermal fluctuations and the response of a linear circuit to an AC voltage source. Ohm's law tells us that the current density in a linear medium equals the electric conductivity of the material multiplied by the negative gradient of the voltage. This is identical in form to Fourier's law governing heat flow: the heat flux is equal to the thermal conductivity of the material multiplied by the negative gradient of the temperature. So, heat flux is like current and temperature is like voltage in this analogy.

In electronics, with just resistors and capacitors, it is possible to create a passive low-pass filter which displays higher impedance to high-frequency voltage fluctuations than low-frequency ones. Thermal materials display similar resistance and capacitance to heat, so it ought to be possible to create a low-pass thermal filter. A simple low-pass filter in electronics has a 3dB roll-off frequency given by $1/(2\pi RC)$; this is the frequency at which a signal going through the filter is halved in power. The frequency of the fluctuations that we want to damp out is 0.5 Hz, so we want $RC \gg 1/\pi$. However, if the thermal resistance between the mixers and the cold third refrigeration stage is too great, even a small heat load will cause the temperature at the mixer to be markedly higher than the temperature at the third stage. This is undesirable, so we must make the product RC large by having a very large thermal capacitance in contact with a fairly low thermal resistance.

As a starting point, the unmodified BIMA receivers have a thermal resistance between the third stage and the mixers of about 40 mK/mW and a heat capacity of about 0.05 J/K due to the thermal capacitance of the aluminum feed horn mount, the feed horns, and mixer blocks themselves. The temperature fluctuations at the mixers are typically 20 mK and the mixers generally have an average temperature which is about 0.5 to 1 K above that of the cold head.

2. Materials for thermal capacitors

The heat capacities, thermal conductivities, thermal diffusivities, and skin depths of several common materials at about 3.7 K are given in Table 1. The thermal diffusivity describes the rate at which heat propagates through a material and the skin depth is a measure of how deeply thermal fluctuations penetrate into a material (Appendix A). Bolting a chunk of material with large heat capacity to the mixer does not automatically reduce the temperature swing – the thermal conductivity must be high enough that heat can flow in and out of the chunk within a 2 sec refrigerator cycle.

We considered constructing thermal capacitors out of 3 materials: helium, neodymium, and stainless steel.

Material	Specific Heat [J/(g-K)]	Conductivity [W/(cm-K)]	Diffusivity [cm^2/sec]	Skin Depth [cm] at 0.5 Hz
Lead	5.27E-04	25.9	4.33E+03	52.5
Copper	7.10E-05	105.	1.65E+05	324.5
SS 304	1.74E-03	0.0025	0.2	0.35
Neodymium	1.67E-02	0.013	0.12	0.28
Helium (35 psi)	3.32 (Cp) 2.34 (Cv)	.00019	.0004	0.016

Table 1. Thermal properties of materials at about 3.7 K. Most data are from Touloukian (1970), except the thermal conductivity of Nd is from our own measurements (section 2b). Helium data were generated using program HEPAK.[1]

a. Helium

Commercially available Sumitomo refrigerators use a helium pot at the cold end to reduce thermal fluctuations. If the pot contains liquid helium in equilibrum with vapor, the heat of vaporization provides a very large heat capacity, but only a single temperature. Since the cold end temperature of the BIMA cryocoolers varies from 3 to 4.5 K, we consider only the case of a single phase fluid.

We first considered filling a length of narrow, stainless steel capillary tubing with high-pressure helium, coiling it, and affixing it between the cold third stage and the SIS mixers. It is necessary to use capillary tubing for two reasons. Firstly, the thermal diffusivity of helium near these temperatures is small (on the order of .0004 cm^2/sec) and makes the skin depth only about 0.16 mm so that a cylinder of helium with a cross sectional area greater than about 0.08 mm^2 would have much wasted heat capacity that was not contributing to the RC filter and was just wasting space in the receiver. Secondly, to get enough mass of helium into a reservoir small enough to fit inside our receiver, the helium must be at very high pressure at room temperature. If the reservoir had too big of an internal surface area, the high pressure would blow it apart. With a thin capillary tube, the outward force per unit length from the helium is low enough for the reservoir to hold together. At room temperature (295 K) and 3000 psi, the density of helium is only 0.03 g/cm^3. If a capillary tube with a cross sectional area of 0.08 mm^2 were used as the skin depth related limitations allow, we would still need a 17 m length of tube to get a mere 1 J/K total heat capacity. It would be difficult to coil up such a length of tubing and try to solder it to the cold head of the receivers.

The second potential solution involving helium would have a relatively large volume in an external reservoir which was attached to a capillary tube leading into the receiver. The large external volume would allow us to treat the helium as being at an essentially constant pressure. The long length of the capillary tubing and its small diameter would prevent much heat from the outside from being wicked into the third stage. Unfortunately, the specific heat of helium at a constant pressure at 3.7 K is only 3.3 J/(g-K); this is only marginally larger than the specific-heat at constant volume and the large external helium reservoir could not withstand very high pressure. We would still need a very long length of capillary tubing to get any sizeable net heat capacity. Further, having a system connecting the inside of the receiver to the outside would make periodic disassembly and reassembly of the receiver much more difficult.

b. Neodymium

A second possibility is neodymium (Nd). Nd has a magnetic phase transition that occurs at about 5 K, so it has a fairly large volumetric specific heat of 0.1167 J/(cm^3-K) in this temperature range (Touloukian 1970). Nd oxidizes very easily and must be handled in an argon bath. For long term use in a dewar that may be opened often, the Nd must be sealed in an airtight capsule. We very quickly ran into a problem in trying to devise a scheme involving Nd that would work; we could not find the thermal diffusivity of Nd at 3.5 K anywhere, so we

[1] HEPAK version 2.4, 1990. Cryodata, Niwot, CO.

had no idea what the skin depth would be. We set up a simple experiment to roughly measure the thermal conductivity (k) of Nd in this range of temperatures, and because we knew the density (ρ) and the specific heat at constant pressure (C) of Nd, we could use the relation K=k/(ρC) to get the diffusivity, where K is the diffusivity.

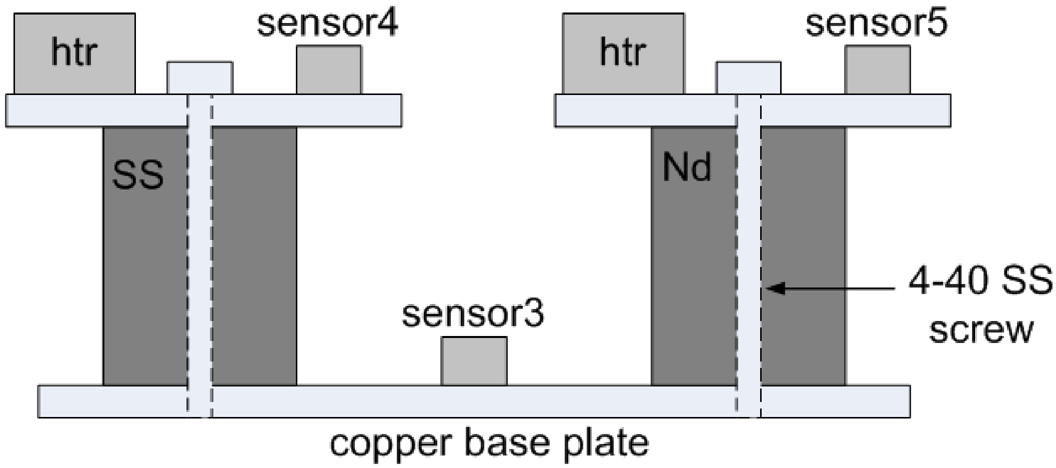

Figure 1. Test setup for measuring the thermal conductivity of neodymium and stainless steel at low temperature. The copper base plate was attached to the refrigerator 3rd stage. Indium pads were inserted at each end of each cylinder to minimize thermal resistance at the contact of the cylinders and the copper plates**.**

In the experiment (see Figure 1), we cut 0.591 inch long, 0.5 inch diameter cylinders of neodymium and of stainless steel #304 with #32 holes drilled axially. We wanted to measure the conductivity of the stainless steel because it is well documented and it would give us a sense of how well we were measuring what we hoped to be measuring. By using such short cylinders, we hoped that we could accurately assume that the temperature gradient along the length of the cylinders was linear and was given by the difference of the temperatures at the two ends. Atop each cylinder, we sandwiched a disk of indium under a plate of copper, and on each copper plate, we attached a silicon diode temperature sensor and a 200 ohm heater resistor. The two cylinders were attached on the other end to a common copper plate with another silicon diode temperature sensor. We could control the heat output of the heater resistors, and we could measure the temperature difference across the cylinders. Figure 2 shows the raw data. Then, the conductivity (k) was just given by k=Q/[(T_high)-(T_low)] where Q is the heat load at the top of each cylinder.

For stainless steel #304, we measured a thermal conductivity of about 0.003-0.005 W/(cm-K) over a temperature range from 3.5-5.5 K[2] (see figure 3). These values agree well (within about 3%!) with the data documented by Touloukian (1970). This agreement reassured us that the values arrived at for neodymium would be roughly correct. Our measured thermal conductivity for neodymium was between 0.012 and 0.015 W/(cm-K) over a temperature range of 3.5-4.5 K.

[2] These temperature values refer to the average of the temperatures measured at the top and bottom of the cylinder.

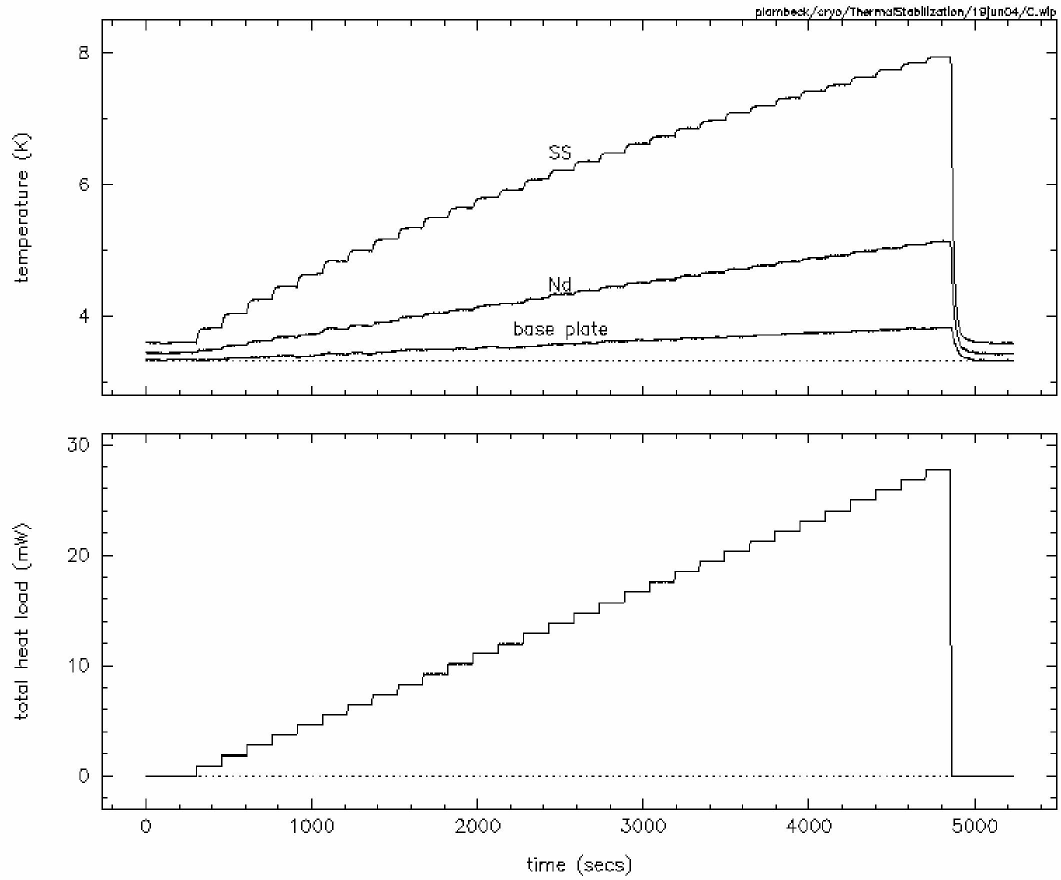

Figure 2. Raw data for the thermal conductivity measurement. The bottom panel shows the heater power vs. time; the top panel shows the temperatures recorded at the tops of the stainless steel and Nd cylinders, and on the base plate. Note that the heater power is divided equally between the Nd and stainless steel cylinders.

c. Stainless Steel

Although neodymium's conductivity is higher than that of stainless steel #304 by about a factor of 3, their densities are comparable and neodymium's specific heat capacity is about a factor of 10 greater than stainless steel #304; it turns out that stainless steel #304 has a diffusivity which is about two times that of neodymium's (see figure 4) and thus, because skin depth goes as the square root of diffusivity, has a skin depth almost one and a half times that of neodymium (the stainless skin depth is about 3.5mm). So, though Nd has a factor of 10 greater specific heat, if we decided to use stainless steel #304 in place of Nd, we could use pieces of metal with one and a half times the thickness of those we would have to use in a Nd scheme, we wouldn't have to work in an argon bath for fear of oxidation, we wouldn't have to devise a means of sealing the Nd in an airtight capsule, and we would save money since stainless steel #304 is considerably cheaper than Nd. We decided to use stainless steel to provide the thermal capacity in the RC filter.

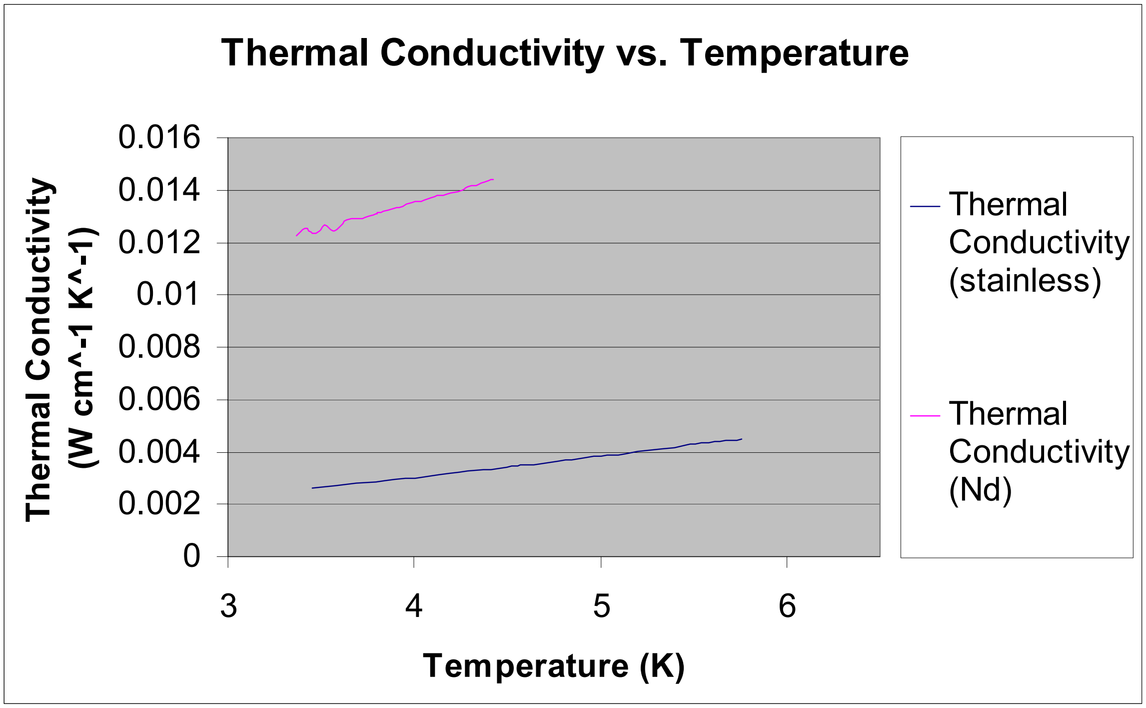

Figure 3

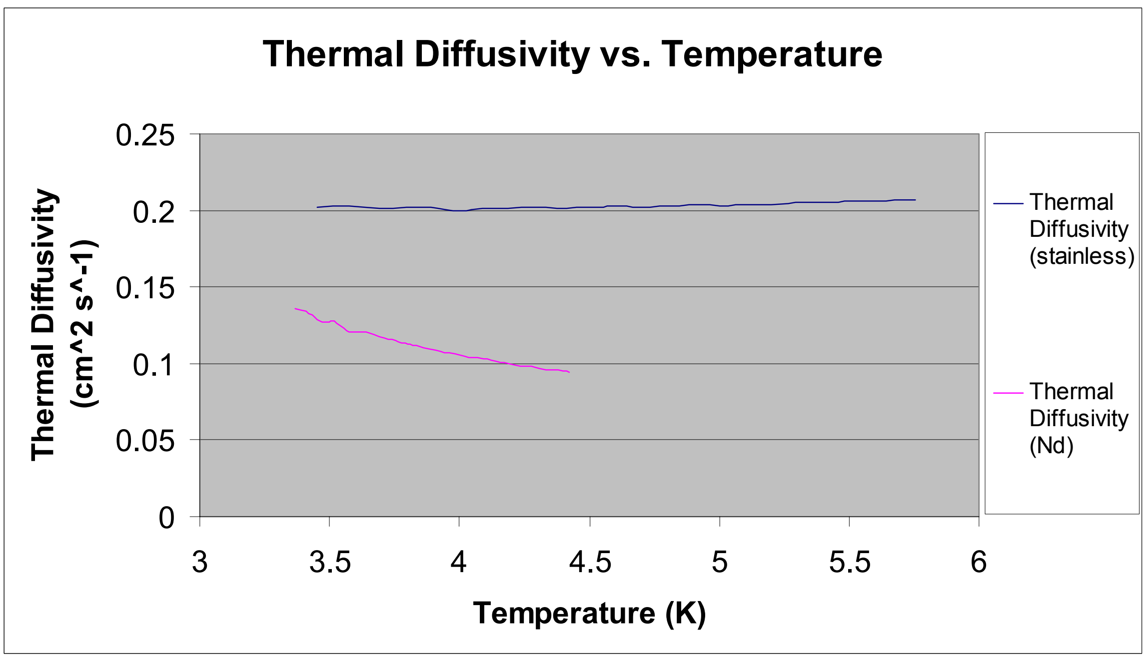

Figure 4

3. Final design

We wound up making what is essentially a two-pole passive RC filter with blocks of stainless steel #304 acting as capacitors and with copper heat straps acting as wires. Thermal resistance is in every element of the circuit, but is especially prevalent at the joints where different components meet. To minimize the thermal resistance, at every one of these joints we sandwiched a wafer of indium, a very malleable metal with good thermal conductivity. Here, the stainless steel had another edge on neodymium—stainless steel is much harder than Nd, aluminum, or copper, so any screws holding down heat straps could be tightened much harder in stainless than in alternative metals without stripping threads and the thermal resistance at the joints was decreased. Coming directly from the cold head, there are four copper heat straps attached to a copper plate. Soldered to the back of the copper plate is a stainless steel plate that is 0.12 in. thick (or about 3mm, just under one full skin depth). The copper plate is used here for its high thermal conductivity; because it is soldered to the stainless, they are in very good thermal contact and we can be sure that the heat is making it to the stainless steel plate. These two plates are then bolted onto the aluminum feed horn mount at the front of our receiver. More copper heat straps come off of this copper plate and go to one of two assemblies that the SIS mixers are mounted on (see Photos 1 and 2). We colloquially referred to these as single and double 'hotdogs'. They are essentially one skin depth thick stainless wrapped in highly conductive copper. Their design was inspired largely by Randy Doriese's 2002 Princeton thesis; alternating layers of connected copper and skin-depth thick stainless allows one to make a block with essentially whatever thermal capacity is desired. We put the single hotdog on the 1mm mixer and the double hotdog on the 3mm mixer, which has a greater temperature dependence.

4. Results

In the end, the thermal resistance between the cold head and either mixer was about 20 mK/mW (see Figure 5), a factor of two down from what it was originally, and the thermal capacitance was between 0.45 and 0.50 J/K, a factor of about 10 greater than before. So the final RC product is about 10, a factor of 5 better than before the modifications. This did indeed lead to smaller thermal fluctuations at the mixer: between 5 and 10 mK peak-to-peak swings (see figure 6). Also, since the thermal resistance was reduced, the mixers have an average temperature that is only about 0.1 K above that of the cold head; this is a factor of 5 to10 improvement since they used to rest at about 1K above the cold head.

References

Doriese, William Bertrand. "A 145-GHz Interferometer for Measuring the Anisotropy of the Cosmic Microwave Background." Princeton Dissertation (2002).

Plambeck, Richard L. "Long-Term Performance of 4K Gifford-McMahon Refrigerators on the BIMA Array." Advances in Cryogenic Engineering 43 (1998): 1815-1821.

Touloukian, Yeram Sarkis. *Thermal Conductivity: metallic elements and alloys*. New York: IFI/Plenum, 1970.

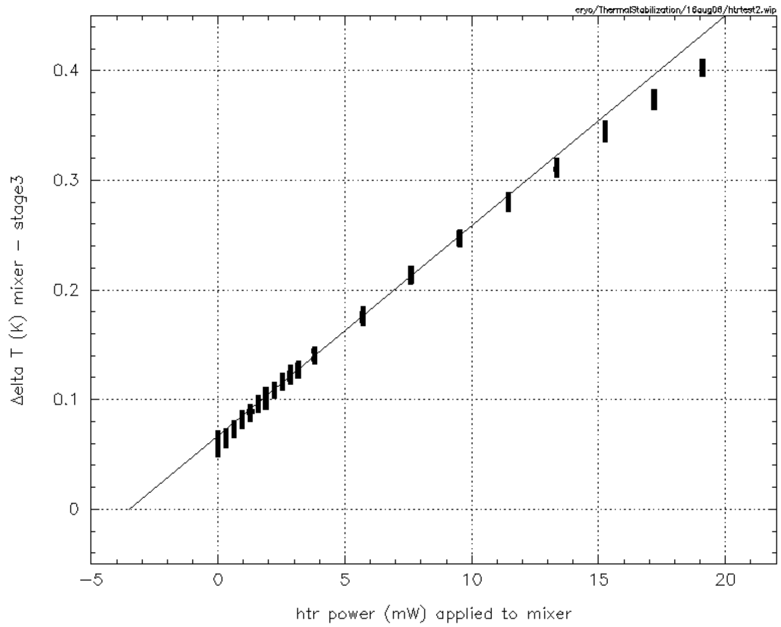

Figure 5. Temperature difference between the mixer block and refrigerator stage 3 as a function of heater power applied to the mixer. The straight line fit corresponds to a thermal resistance of 19 mW/mK. The ΔT =0 intercept implies that the base heat load on the mixer is 3.5 mW; this is, however, uncertain by several mW since it depends on accurate cross-calibration of the mixer and stage3 temperature sensors.

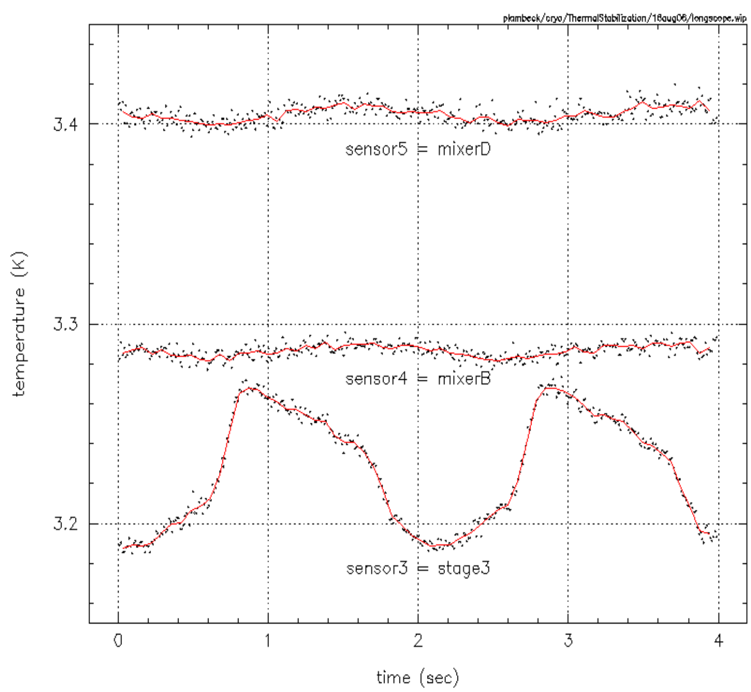

Figure 6. Time variation of the cold stage and mixer temperatures with thermal stabilization plates attached. Because of the tight thermal contact between stage 3 of the refrigerator and the 4 x 4" stainless steel plate, the cold stage temperature swing is reduced from its previous value of 200 mK to approximately 90 mK. The temperature swing of the 3mm mixer (mixerB) is < 10 mK p-p.

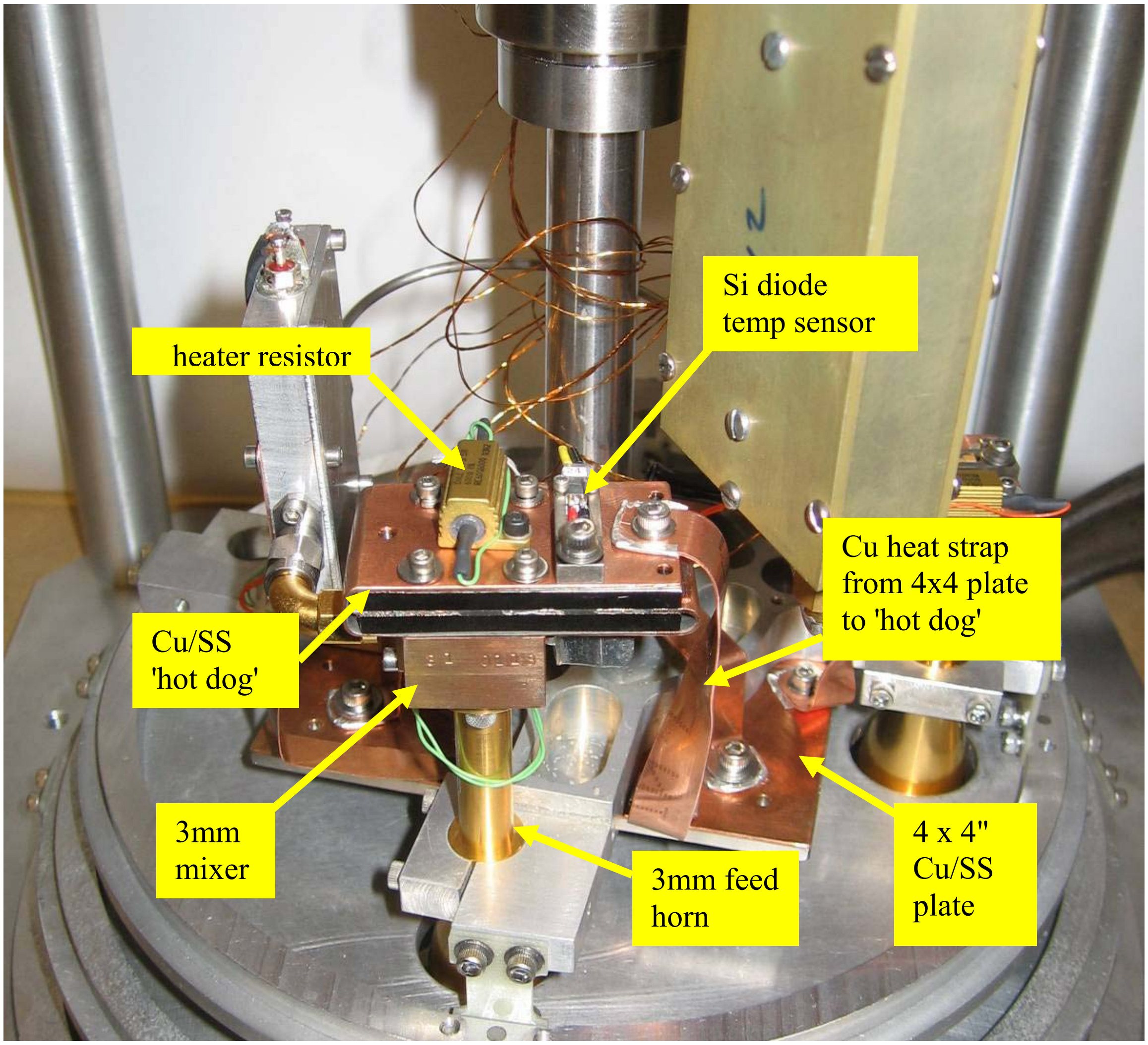
heater resistor
Si diode temp sensor
Cu heat strap from 4x4 plate to 'hot dog'
Cu/SS 'hot dog'
3mm mixer
3mm feed horn
4 x 4" Cu/SS plate

Photo 1.

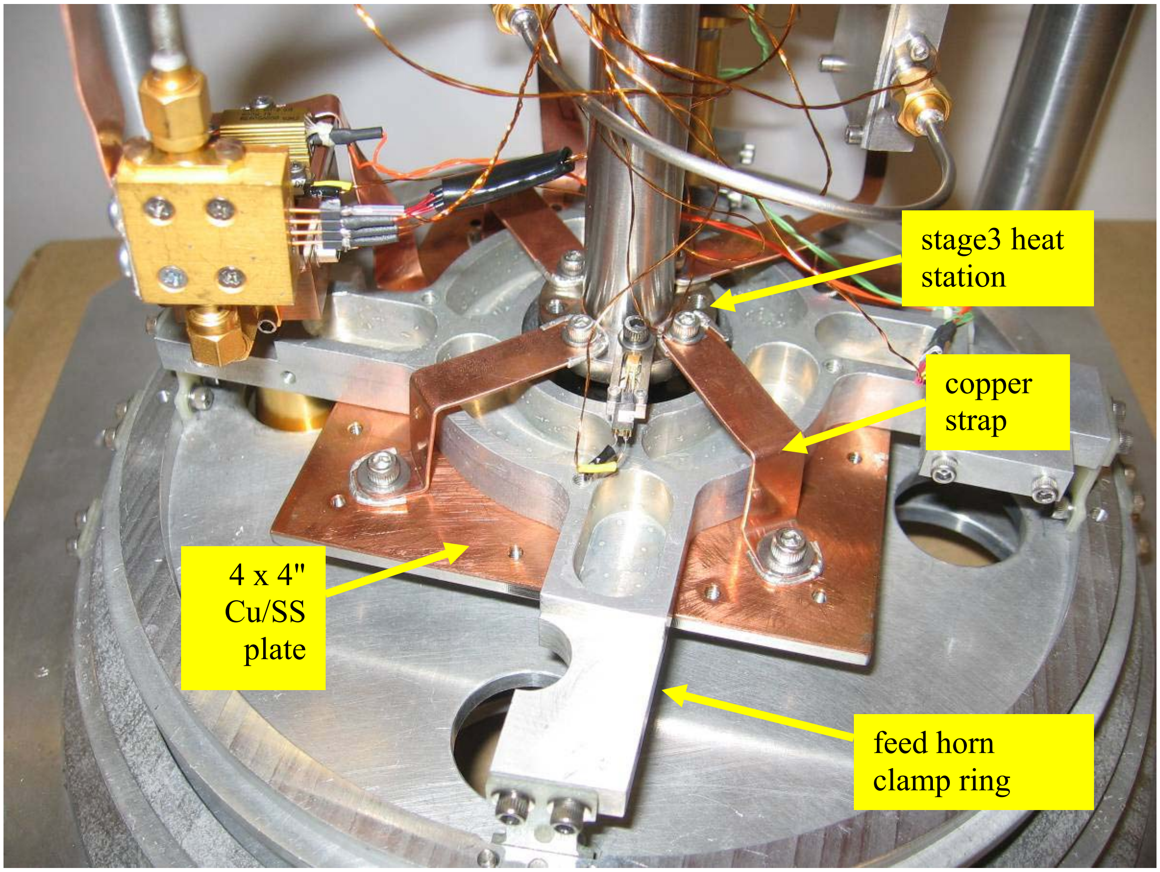
stage3 heat station
copper strap
4 x 4" Cu/SS plate
feed horn clamp ring

Photo 2.

AppendixA

Our task is to solve the heat equation with given initial and boundary conditions.

$$\partial_t \tau(x,t) = \kappa \partial^2_{xx} \tau(x,t)$$

$$\tau(x,0) = \tau_0$$

$$\tau(0,t) = \tau_0 + \frac{1}{2}\Delta\tau sin(\omega t)$$

$$\lim_{x\to\infty} \tau(x,t) = \tau_0$$

We must first assume that the solutions will be seperable:

$$\tau(x,t) = \chi(x)T(t)$$

Then, inserting this product of functions into our differential equation:

$$\chi T' = \kappa T \chi''$$

or

$$\frac{\chi''}{\chi} = \frac{T'}{\kappa T} = \mu$$

Where

$$T' = \kappa \mu T$$

which implies that

$$T = Aexp(\mu\kappa t)$$

However, the temperature should not grow or decay exponentially with time. This implies that μ is entirely imaginary and can henceforth be written as $\pm\imath\lambda$ where λ is some positive real number. So:

$$T(t) = Aexp(\pm\imath\lambda\kappa t)$$

Now, examining the spatial part of our solution:

$$\chi'' = \pm\imath\lambda\chi$$

which implies that

$$\chi = Bexp(x\sqrt{\pm\imath\lambda}) + Cexp(-x\sqrt{\pm\imath\lambda})$$

or

$$\chi = Bexp(x\sqrt{\frac{\lambda}{2}})exp(\pm\imath x\sqrt{\frac{\lambda}{2}}) + Cexp(-x\sqrt{\frac{\lambda}{2}})exp(\mp\imath x\sqrt{\frac{\lambda}{2}})$$

But, $\lim_{x\to\infty} \tau(x,t) = \tau_0$ forces $B = 0$.

Then, with different choices of sign, we have two solutions to the differential equation plus the trivial constant solution. However, none of them independently satisfy all initial and boundary conditions. We must correctly define our constants and use a linear combination of solutions to satisfy these conditions. The solutions are:

$$\tau_1 = Dexp(\imath\lambda\kappa t)exp(-x\sqrt{\frac{\lambda}{2}})exp(-\imath x\sqrt{\frac{\lambda}{2}})$$

$$\tau_2 = Dexp(-\imath\lambda\kappa t)exp(-x\sqrt{\frac{\lambda}{2}})exp(\imath x\sqrt{\frac{\lambda}{2}})$$

$$\tau_3 = \tau_0$$

To satisfy the boundary conditions we must identify

$$\lambda = \frac{\omega}{\kappa}$$

$$D = \frac{1}{2}\Delta\tau$$

Finally:

$$\tau_{final}(x,t) = \frac{1}{2\imath}(\tau_1 - \tau_2) + \tau_3$$

$$\tau_{final}(x,t) = \tau_0 + \frac{1}{2}\Delta\tau exp(-x\sqrt{\frac{\omega}{2\kappa}})(\frac{1}{2\imath})(exp(\imath(\omega t - x\sqrt{\frac{\omega}{2\kappa}})) - exp(-\imath(\omega t - x\sqrt{\frac{\omega}{2\kappa}})))$$

Then, identifying $\omega = 2\pi f$, we're left with the solution:

$$\tau_{final}(x,t) = \tau_0 + \frac{1}{2}\Delta\tau exp(-x\sqrt{\frac{\pi f}{\kappa}})\sin(2\pi f t - x\sqrt{\frac{\pi f}{\kappa}})$$

Now, there is a distance into the material at which the temperature fluctuations will be attenuated by a factor of $\frac{1}{e}$ relative to those at the surface (x=0). This value, L_d, is called the skin depth of the material, and it depends on the diffusivity κ of the material and the linear frequency f of the initial fluctuations. It occurs at the point where the argument of the exponential in our solution becomes equal to 1.

$$L_d = \sqrt{\frac{\kappa}{\pi f}}$$