The Interferometer Equation II

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July 7, 2003: Version B: typo corrections and clarifications

This memo repeats the content of "The Interferometer Equation" from Oct. 2001 and extends it to cover the explicit cases for OVRO and CARMA.

This math required to determine the effects of lobe rotation and delay on double down conversion of double sideband receivers is presented. It follows the approach given in TMS on page 152 (page 178 in the new edition). The interferometer configuration and nomenclature is defined in the figure below.

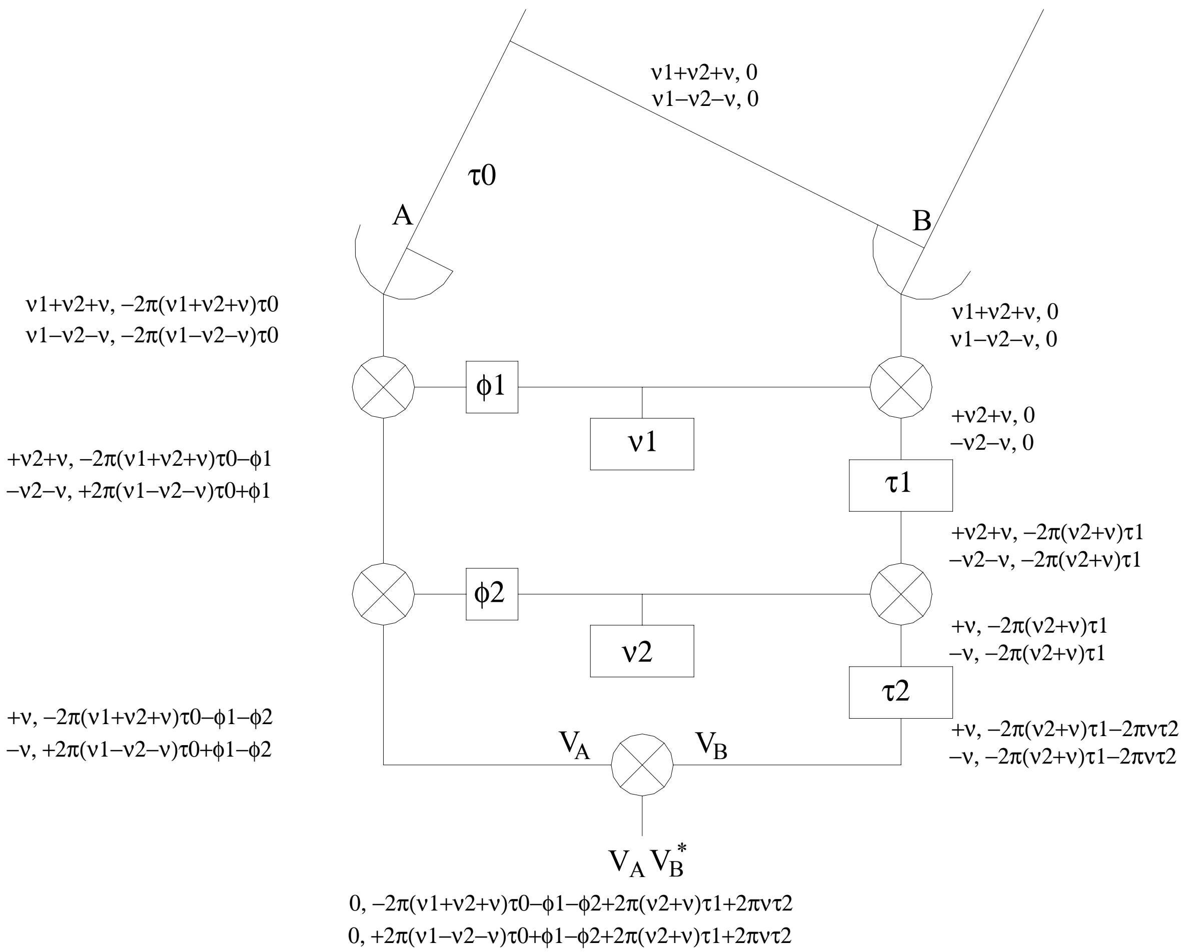

Fig. 1. Interferometer diagram. The frequencies and phases for the upper (top line) and lower (bottom line) sidebands are given at each point in the signal path. The second down conversion is assumed to be upper sideband.

The input signal phasors to the correlator from the two antennas are

$$V_A = V_U \exp\left(i[-2\pi(\nu_1+\nu_2+\nu)\tau_0 - \phi_1 - \phi_2]\right) + V_L \exp\left(i[+2\pi(\nu_1-\nu_2-\nu)\tau_0 + \phi_1 - \phi_2]\right)$$

$$V_B = V_U \exp\left(i[-2\pi(\nu_2+\nu)\tau_1 - 2\pi\nu\tau_2]\right) + V_L \exp\left(i[-2\pi(\nu_2+\nu)\tau_1 - 2\pi\nu\tau_2]\right)$$

The complex correlator produces the complex product of these phasors

$$
\begin{aligned}
r &= V_A V_B^{*} \\
&= \{ r_U \exp(i[-2\pi\nu_1\tau_0 - \phi_1]) + r_L \exp(i[2\pi\nu_1\tau_0 + \phi_1]) \} \\
&\quad \times \exp(-i[2\pi\nu_2(\tau_0 - \tau_1) + \phi_2 + 2\pi\nu(\tau_0 - \tau_1 - \tau_2)])
\end{aligned}
$$

r_U and r_L are the complex visibilities for the upper and lower sideband signals. Note that the effect of τ_0 and ϕ_1 have opposite phase effects for the two sidebands. It is necessary to set these phase terms to predetermined values to disentangle the two sidebands after correlation. The usual procedure is to track ϕ_1 such that

$$2\pi\nu_1\tau_0 + \phi_1 = N\frac{\pi}{2}$$

and the correlator output is

$$r_N = \left\{ r_U \exp\left(-iN\frac{\pi}{2}\right) + r_L \exp\left(iN\frac{\pi}{2}\right) \right\} \exp(-i\theta).$$

The interesting term is the last phase term θ .

$$\theta = 2\pi\nu_2(\tau_0 - \tau_1) + \phi_2 + 2\pi\nu(\tau_0 - \tau_1 - \tau_2)$$

Traditionally, the delay at the first IF is set to compensate for the geometric delay, $\tau_1 = \tau_0$, $\phi_2 = 0$ and $\tau_2 = 0$ giving $\theta = 0$. The precision with which the delay must be set is determined by the first IF frequency, $\nu_2 + \nu$. Sideband separation is quite simple then

$$r_U = \tfrac{1}{4}(r_0 - r_2 + ir_1 - ir_3)$$

$$r_L = \tfrac{1}{4}(r_0 - r_2 - ir_1 + ir_3)$$

Note that the geometric delay varies across the field of view and the first LO phase tracking only works perfectly at the field center and the ability to separate sidebands is compromised as you move away from field center. Also $\theta = 0$ only at field center and the phase change away from field center results in a finite IF delay beam on the sky.

The other common approach is to set $\tau_1 = 0$, $\tau_2 = \tau_0$ and track $\phi_2 = -2\pi\nu_2\tau_0$ again giving $\theta = 0$ and sideband separation is the same.

You can relax these requirements if you are willing to multiply the complex spectral output of the correlator by a predetermined phase correction factor on a short enough timescale. This works because both sidebands in a spectral channel have the same phase dependence for the IF delays when referred to the output of the correlator and you are working in the spectral domain and not the lag domain.

The delay precision only needs to be accurate enough to avoid decorrelation over the final channel bandwidth. Thus a digital correlator can use simple sample shifting after digitization for setting $\tau_2 \approx \tau_0$ as long as there are enough output channels and the delay phase correction is done often enough. A one sample shift between the two samplers produces a phase slope across the Nyquist bandwidth of π from the correlator which can be corrected by applying a linear phase ramp across the sideband separated spectrum, but the phase slope across a channel decreases its amplitude. It is assumed that the digital delay is applied in each antenna's digitizer and is always within ½ sample of the ideal, giving a maximum error of one sample on a correlator baseline. A phase ramp of amplitude β decreases the amplitude to $A = \frac{2}{\beta}\sin\left(\frac{\beta}{2}\right) \approx 1 - \frac{1}{24}\beta^2$. The worst-case phase ramp across a channel for an N channel spectrum is $\frac{\pi}{N}$. The distribution of delay error phase ramps has an rms value of $\frac{\pi}{\sqrt{6}N}$ and the average decrease in amplitude or loss in sensitivity is $A = \frac{2\sqrt{6}N}{\pi}\sin\left(\frac{\pi}{2\sqrt{6}N}\right) \approx 1 - \frac{1}{144}\left(\frac{\pi}{N}\right)^2$. Thus only a three channel spectrum is required to reduce the loss in sensitivity from the digitally tracked delay to less than 1%.

The delay phase correction needs to be done often enough to void phase errors and sensitivity loss in the highest correlator frequency channel. The delay is changing with time at a maximum rate of $2.4\text{x}10^{-10}$B[km]sec/sec, thus the rate of change of phase at a correlator base band frequency of ν is $86B[km]\nu[GHz]\deg/\sec$. The phase change is linear with time so that the average phase should be correct for the middle of the time range between phase corrections, but there can still be a loss of sensitivity associated with the changing phase between delay phase corrections. The effect is the same as for the loss in sensitivity for a phase slope across a channel and loss in sensitivity for a phase correction update interval of t is $A_{update} \approx 1 - .10(B[km]\nu[GHz]t[\sec])^2$. For COBRA used in the existing OVRO array, B_{max}=0.4km, ν_{max}=0.5[GHz] and a less than 1% loss in sensitivity requires a delay error phase correction update rate of t<1.7sec. CARMA will have the same ν_{max}=0.5[GHz] and B_{max} =2km. Thus less than 1% sensitivity loss for CARMA requires an update interval of t<0.34sec. The calculations will probably be done on a per antenna basis to high precision and then combined and applied in the correlator at the requisite time intervals.

An additional phase correction associated with the 2nd LO is required if we use $\tau_1 = 0$ and $\phi_2 = 0$. The phase correction is $\theta = 2\pi\nu_2\tau_0$. Note that ν_2 is the signed sum of all of the LOs after the first LO, including the digital sampler clock rate, and corresponds to the position of channel 0 from the correlator in the IF from the SIS mixer. This phase error is linear in time and so the average phase between phase corrections will be correct for the middle of the time interval between phase corrections. As with the delay error phase correction, there will be a sensitivity loss associated with the update time interval and the same formula applies with ν replaced by ν_2. At OVRO ν_2<4.5GHz which along with B_{max}=0.4km implies a 2nd LO phase correction update interval of t<0.18sec to keep the sensitivity loss to <1%. **Initially CARMA will have ν_2=4.5GHz and B_{max}=2km and hence require phase correction on timescales t<0.038sec to avoid a 1% loss in sensitivity.** If the CARMA IF band is extended to 10GHz, then t drops to <0.017sec.

Applying phase correction is straight forward if all four phase bins, r1, r2, r3 and r4 have the same number of samples, i.e. the Walsh sequence is complete. This is the case for COBRA at OVRO where the Walsh sequence completes in 0.1sec (16states x.006sec). The phase correction can be carried out on individual bins since the correlator produces + and – lags and the resulting spectrum from a single phase state is complex. Thus you can compute the complex spectra for the four bins separately and apply phase corrections to the four spectra without requiring a complete Walsh sequence. The sideband separated spectra would be obtained the same as before by appropriate complex addition of the spectra from the bins.

The difficulty comes when considering the purpose of the 0-180 and 90-270 phase switch states. This sign reversal is used primarily to remove the effects of offsets in the digitizers or analog correlator outputs and other artifacts or interference introduced in the IF chain, r0'=1/2(r0-r2) and r1'=1/2(r1-r3). **Ideally, any phase correction applied to the 0deg state should also be applied to the 180deg state to ensure good cancellation of artifacts, similarly for the 90deg and 270deg states.** The above time intervals are calculated to produce ~1% decrease in amplitude or equivalently a rotation of 0.5rad. If the 0 and 180 states have been rotated by this amount relative to each other, then only 88% of the artifacts would be removed, i.e. leave 12% as a false signal.

Although dc offsets are a large effect in analog correlators it is not clear how important they are in digital correlators. Typically the zero frequency spectral channel is explicitly removed by forcing the average of all of the lags to be zero. This is a linear operation and can be done for each Walsh state as it is collected. The more subtle problem is false fringes generated in the IF chain and correlator after the phase rotation has been applied at the 1st LO. These false fringes may have real frequency content and will not be removed by simply forcing the lag average to zero. The long term average delay and 2nd LO phase corrections are zero so that stable false fringes will average to zero. Thus the main question for CARMA is what is the magnitude and stability of the false fringes generated in the IF and correlator system. Unfortunately, this cannot be reliably answered until the full system is tested.

OVRO and other small arrays, either in terms of the number of antennas or longest baseline, can complete the full Walsh sequence before phase correction is required but this is clearly a problem for CARMA with its large number of antennas and long baselines. **The critical 0-180deg step sequence will complete quicker if we use a nested set of sequences, i.e. complete 0-180deg sequences inside each 0-90deg state.** Phase correction can be applied to the individual 0-90deg states without affecting the ability to separate sidebands. The accumulated r0' and r1' will be complex numbers that are combined in the usual way to give the two sidebands. The 0-180deg sequences are true Walsh sequences in that their product is also a Walsh sequence and it should be easier to find sets of sequences with whatever orthogonality requirements are needed. Limiting the number of antennas or stations in the sub-array correlator will also make it much easier to complete the sequences in the allotted time.

It may be necessary to apply phase rotation to the 2nd LO if there are unforeseen problems with the above scheme. This gains an order of magnitude in the phase correction update rate at the expense of controlling the phase of all of the 8x23=184 down converter LOs. Even then, the update time for the delay error correction is only 0.34sec. This can be circumvented by applying ¼ sampler clock delays at the digitizer and avoiding phase correction altogether.