1% Luminosity-Independent Distances to Nearby Galaxies with the Rotational Parallax Technique

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Hipparcos(Credit ESA)


SIM/Heavy (Credit JPL)


GAIA (Credit ESA)

Outline

- The Extra-galactic Distance Scale
- “Sanity in Errors”
  - Example: H_0, the CMB & Dark Energy
- Rotational Parallax
  - SIM & GAIA compared
- Conclusions
- Backup slides
  - More details: check Olling 2007 (MNRAS, 378, 1385) or http://www.astro.umd.edu/~olling/Papers/RP_H0_2007_Colloquium.pdf

The Extragalactic Distance Scale

- **"Standard Candle Methods:"**
  - Extinction & [Fe/H] may be greatest difficulties
    - For known Galactic Cepheids: $<A_V>$ ~ 1.7 mag
      - GAIA expects: ε(A_V) ~ 0.1 mag. Much better in NIR
- **BUT: Standard Candles will be calibrated much, much, much better by Gaia/SIM than the current state-of-the-art**
    - **GAIA:**
      - 17,000 binaries **(21 10^6 stars)** with masses **(distances)** <~ 1% and V <~15
      - Radii for ~360,000 stars in Eclipsing Binary systems with 1% distances
      - Uniform metallicity & extinction scale: photometric & spectroscopic
    - **SIM** will complement with the distant, very rare objects:
      old, metal poor, abundance peculiarities, uranium stars, PN central stars,
      stars of all stripes in instability strip, optical pulsars, ...

Extragalactic Distance Scale, cntd

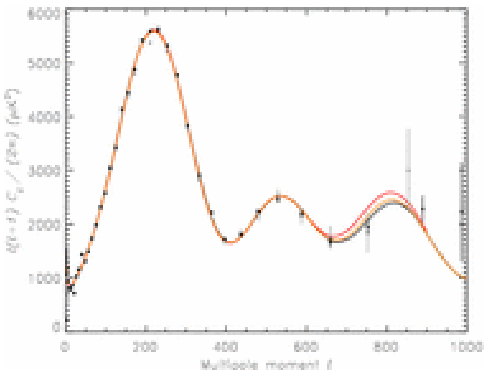
- "Geometric" Methods are still problematic
  - Baade-Wesselink-type methods for Cepheids [p-factor]
  - Velocity Gradient, [Applied to LMC by GAIA]
  - (H_2O) Masers in extra-galactic star formation regions [Few systems per galaxy: depends on external velocity-field data]
  - Extra-galactic (nuclear) Mega masers [Just 3 lines of sight: sensitive to systematics]
  - "Licht Echo" method; X-ray scattering of background sources; Expanding Photospheres of SNe (non=LTE) [Special events]
  - (Detached) Eclipsing Binaries; Gravitational Waves Close WDs [No calibrators in HIPPARCOS (fixed by GAIA?)]
- [summarized in Olling 2007; and see: Gould 2000; Argon *et al* 2004, Brunrhaler *et al* 2005, Braatz et al 2006; Panagia *etal* 1991, Gould 2000, Sparks 1994, Sugerman 2006; Draine & Bond 2004; Nugent *et al* 2006; Paczynski & Sasselov 1997, Fitzpatrick *et al* 2004, Stanek *et al* 1998; Cooray & Seto 2005; Freedman *et al.* 2008]

Need Sanity in Errors

- ***Need* independent cross-checks:**
  - **different methods & objects**
    to measure same parameter(s)
  - **Otherwise: results + errors can not be trusted**

- **Absolute distance (H_0) errors also important for cosmology & dark energy**
- **An absolute distance to a LG galaxy will eventually lead to an accurate H_0**

H_0, the CMB & Dark Energy



- From the shape of the power spectrum, **WMAP** “directly” [e.g., Hu 2005] **measures** the **physical densities** (ρ_{matter} and ρ_{baryon})
  - i.e., NOT the ρ_{crit}**-normalized densities**
  - **$\rho_{crit} = 3\ H_0^2 / (8\pi G)$ is the *critical density of Universe***
    - $\omega_b = \Omega_b\ h^2$ ∝ *the physical baryon density*
    - $\omega_m = (\omega_b + \omega_{DM}) = \Omega_m\ h^2$ ∝ *the physical matter density*
    - $h = H_0 / 100$

$$\Omega_m = \frac{\rho_b + \rho_{DM}}{\rho_{crit}} = \frac{\omega_m}{h^2} \quad \text{and} \quad \epsilon_{\Omega_m} = \sqrt{\left(\frac{\epsilon_{\omega_b}}{\omega_b}\right)^2 + \left(2\frac{\epsilon_h}{h}\right)^2}$$

H_0 , CMB and Dark Energy (cntd)

$$\Omega_m = \frac{\rho_b + \rho_{DM}}{\rho_{crit}} = \frac{\omega_m}{h^2} \quad \text{and} \quad \epsilon_{\Omega_m} = \sqrt{\left(\frac{\epsilon_{\omega_b}}{\omega_b}\right)^2 + \left(2\frac{\epsilon_h}{h}\right)^2} \quad \text{and similar for error}_{DE}$$

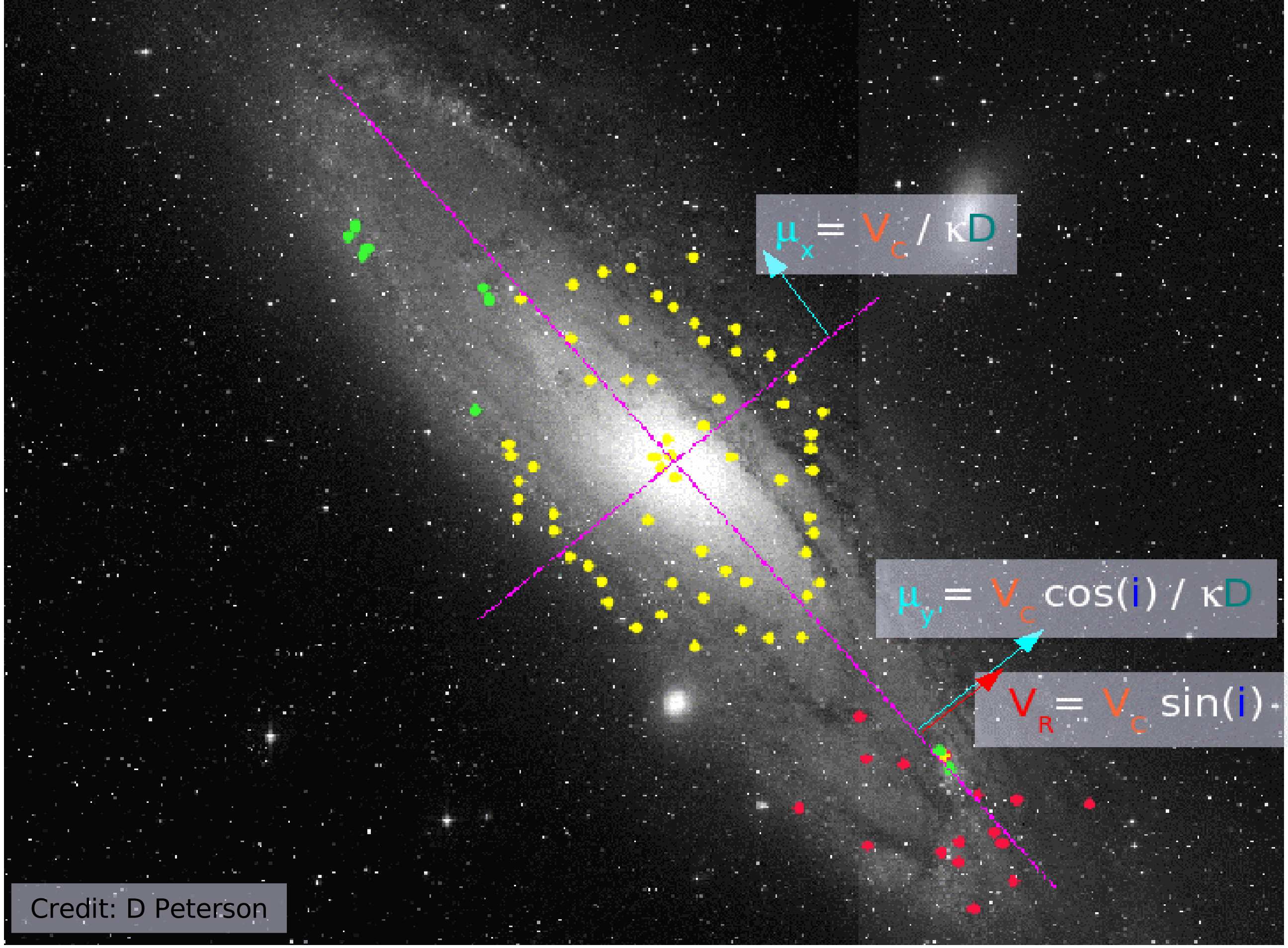
- IF one wants to determine w (or Ω_m), THEN need to know H_0 !!
  - Now: EOS of Dark Energy is known to +/- ***7%, and***
    ω_m and H_0 contribute about equally
- Decreasing error on ω_m (ω_b) leaves constant contribution from H_0 ===> hardly any decrease in ε_w
- Better (x 8) determination of ω_m with *PLANCK,*
- Need better (x10) determination of H_0 (e.g., with *SIM-Lite*)

Rotational Parallax Distances

- Distance (D) to *Local Group Spirals* can be determined via the *Rotational Parallax Method*
  [Peterson & Shao, 1997; Olling & Peterson, APH/0005484; Olling, 2007, MNRAS, 378, 1385]
- Principle ***very*** straightforward:
  - Measure circular rotation via radial-velocities (V_C)
  - Measure circular rotation via proper motions ($\mu_C \propto V_C / D$)
  - **Distance $\propto V_c / \mu_c$**
    - **EXPECT: Unbiased Distances**

Accuracy of several % out to ~1 Mpc

- Requires: - Large-scale ordered motions (e.g., rotation)
  - Ground-based radial velocities and
  - Space-based proper motions at the <~ 10 μas/yr level



Rotational Parallax Illustrated

M 31: $i \sim 77^o$

D~0.84 Mpc

$V_C \sim 270$ km/s

$\mu_C \sim 74$ μas/yr

- **For Circular Orbits:**
  - minor axis: $\mu_X = V_c/(\kappa D)$
  - Major axis: $\mu_{Y'} = V_c \cos(i) / (\kappa D)$
  - Major axis: $V_R = V_C \sin(i)$

- **Three equations,**
- **Three unknowns,**
- **Three solutions**
- **Several Approaches**

The Rotational Parallax Method *(cntd)*

- **How About?**
  - Space-motion of the galaxy
  - Warp
  - Non-circular motions
    - Spiral-arm streaming motions
    - Bar-induced motions
    - Tidal distortions
    - Et cetera
  - Rotation of astrometric grid
    - Any physical process that produces proper motion will have a corresponding radial velocity
    - Grid translation: don't care ==> V_{SYS}
    - Grid Rotation: no V_{RAD} equivalent ==> take out

!! The RP method is VERY ROBUST !!

General Rotational Parallaxes

- **Unknowns:**
  - **Total Space Velocity:**
    - $\mathbf{V}_{TOTAL} = \mathbf{V}_{SYS} + (V_{CIRC} + \mathbf{V}_{PEC}) + \mathbf{V}_{\sigma}$
    - = systemic + circular + peculiar + random

      $\Rightarrow$ **3 + 1 + 3 +3 = 10**

    Coordinate system:
    - Origin of coordinate system $\Rightarrow$ 2
    - Position angle of major axis (ϕ) $\Rightarrow$ 1
    - Distance and Inclination $\Rightarrow$ 2
  - **Star position in galaxy** $\Rightarrow$ 3
- **TOTAL:** **18** unknowns
- ***OBSERVABLES (per star):***

  2 positions + 2 proper motions + V_{RAD} **= 5** knowns

General Rotational Parallaxes *(cntd)*

- **However:**
  - Many unknowns are “shared” between test particles:
    - Center of galaxy + PA: 3 shared vars.
    - Systemic velocity: 3 shared vars.
    - Rotation Speed: 1 shared var.
    - Distance & inclination 2 shared vars.
    - Velocity dispersion: 3 shared vars.
    - **TOTAL** **12 shared variables**
  - **Left with:** 3 $\mathbf{V}_{\mathbf{PEC}}$'s & x,y,z: **6 star-dependent unknowns**
  - **No solution** because we have **5 observables per star**
  - $\Rightarrow$ **Eliminate 2 more variables**
    - e.g., assume $\langle V_{p;z} \rangle = 0$ and $\langle z \rangle = 0$
    - **4 star-dependent unknowns left**

General Rotational Parallaxes *(cntd)*

- **Then**: $(4\ N_* + N_{SV})$ unknowns
  $5\ N_*$ observables
- $\Rightarrow$ **Solution if:** $5\ N_* >= (4\ N_* + N_{SV})$
  $N_* >= N_{SV}$
- **In our example, if $N_* >= N_{SV} = 12$**
- **Get all parameters from 12 stars/galaxy**

- **Alternatively,** allow for **corrugations**
  - $z(\theta) = z_0 + {}_1\sum^{nz} A_n \cos(2n\pi\theta) + B_n \cos(2n\pi\theta)$
  - $V_{p;z}(\theta) = V_{p;z;0} + {}_1\sum^{nVpz} C_n \cos(2n\pi\theta) + D_n \cos(2n\pi\theta)$
  - $\Rightarrow$ **Increase N_{SV} & N_* by: $2 * (n_z + n_{Vpz} + 1)$**

General Rotational Parallaxes *(cntd)*

- *If peculiar motions have a "long-range" component that can be determined from a number of stars, a proper solution for the distance will be possible.*

- **Similar Procedures are/will-be employed for:**
  - Distance determination with maser-regions in galaxies
    - ~17 H_2O Masers in M31 & M33 at SKA sensitivity (Barely exceeds the minimum number of shared variables)
  - Velocity-field/Rotation Curve determination of Milky Way

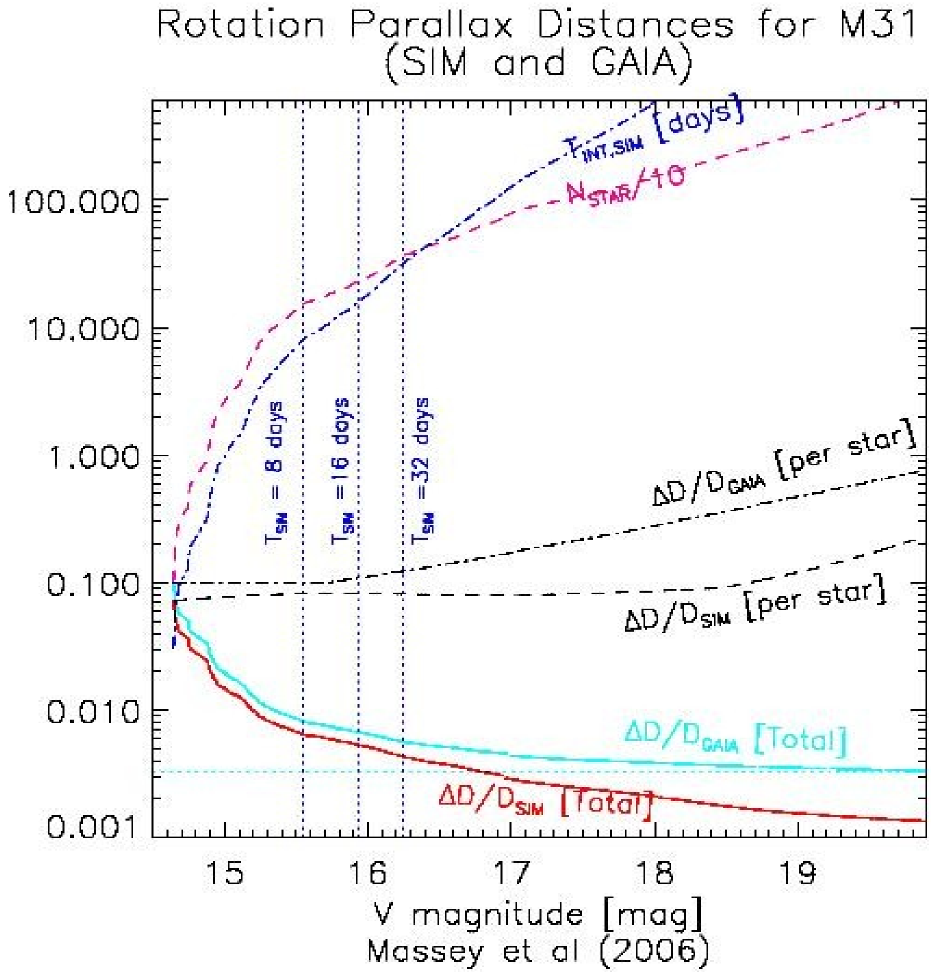
Rotational Parallax: Observability

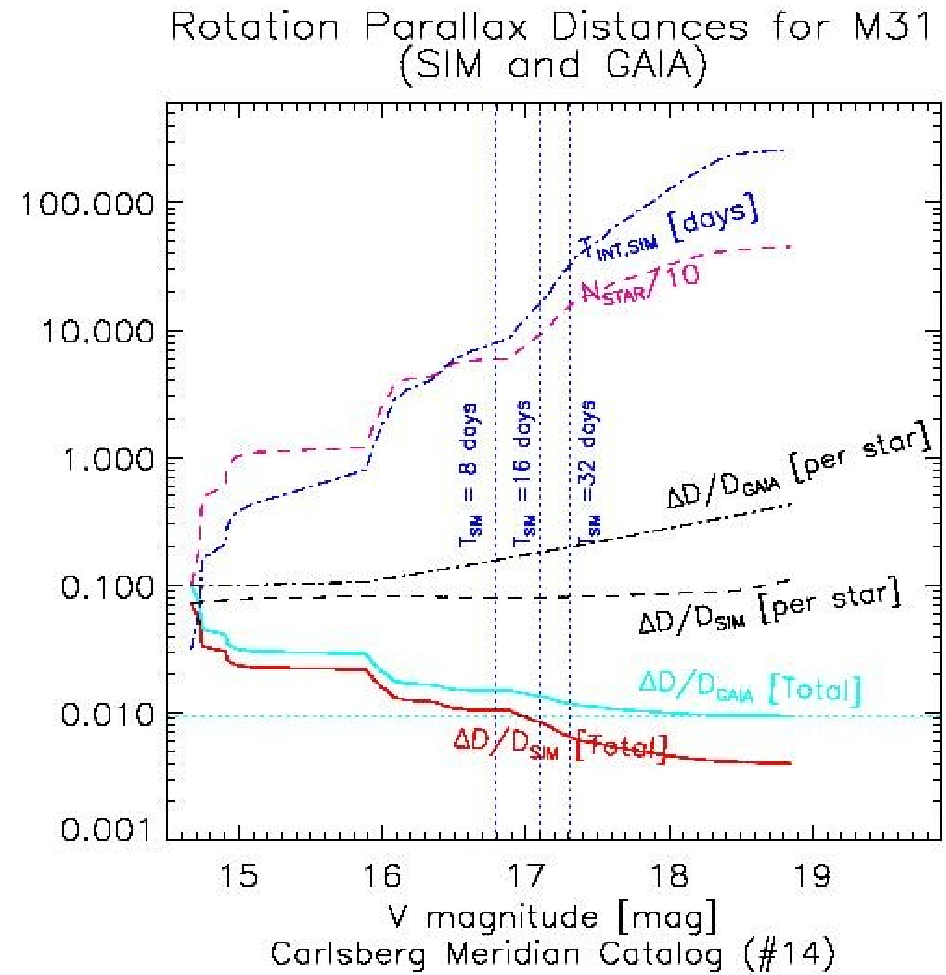
- **Need bright sources:**
  - Minimize confusion & Maximize observing speed
    - All stars share (almost) the same proper motion
  - Enough bright stars available
    - M 31's "ring of fire" (~0.6° - 1.5°)
      - ~300 GSC2 (V <= 17.5)
      - ~360 Massey et al (2006) (V <~ 16.5)
    - M 33: 300 2MASS stars (K_s <~ 15)
    - LMC: 23,000 UCAC stars (V <~ 16)
- Also: Need least disturbed galaxy
- Our Preference for SIM: M31, M33
- Low SIM "cost,"

  M31 in 8-32 days (1/8 - ½ Key Project)

  [uses wide-angle astrometry: can do better (intermediate-angle)]

GAIA & SIM: RP Performance, Graphically





SIM & SIM : RP Performance

Massey et al catalog:

t_SIM [days]	V_SIM [mag]	$\Delta D/D$ [%]	Number of Stars	$\Delta\mu_{EQUIV}$ μas/yr	
8	15.6	0.64	153	0.20	~3.5 SIM-NA accuracy?
16	15.9	0.53	232	0.17	~3.0 SIM floor?
32	16.3	0.42	365	0.13	
GAIA_400	16.5	0.50	400	0.16	~1/2 GAIA_GRID
GAIA_BEST	21.1	0.36	8,981	0.08	~1/4 GAIA_GRID

Carlsbad Meridian Catalog (#14)

t_SIM [days]	V_SIM [mag]	$\Delta D/D$ [%]	Number of Stars	$\Delta\mu_{EQUIV}$ μas/yr
8	16.8	1.05	59	0.34
16	17.1	0.84	91	0.27
32	17.3	0.65	153	0.21
GAIA_150	17.2	1.50	150	0.48
GAIA_BEST	18.9	1.00	463	0.32

typical projected orbital speed ~ 16 μas/yr

SIGA pros & cons

GAIA might do about as well as SIM for M31 because:

- there are many stars available
- BUT, ⇓
    - Can GAIA go a **factor of 10 - 20** beyond frame-rotation limit?
    - Can GAIA perform in crowded field such as M31 with ~2 10^6 star/deg^2 and a variable background?
      (varying scan orientations ==> varying contributions from faint companions)
- BUT, ⇑
    - GAIA can boost? accuracy via "local astrometry" on M31

SIM has clear advantage because:

- needs fewer (brighter) stars, less crowded conditions
- only goes **factor of 2?** beyond narrow-angle mode
- superior for smaller/more distant galaxies

In vein of my "Sanity in Errors" slogan, GAIA & SIM provide independent test at 0.5% distance level

Conclusions

- **1% Galaxy Distances will be possible**
  - **Luminosity-Independent**
    - SIM can do M31 & M33 in reasonable amount of time
    - Gaia will do LMC,   SMC?? & M31?
- **SIM & GAIA provide cross-check (V<=16.5)**

ΔD/D=0.5% level

ABSOLUTELY CRUCIAL

- RP-targets in LG galaxies: (4+2)D phase-space data
- 1% Distance to LG galaxies will calibrate 2^{ndary} calibrators (Cepheids, TRGB, EBs, ...) ==> $\mathbf{H_0}$
  - **Transfers Solar N.hood (<1%) stellar calibration to LG galaxies**
- **H_0 is important for Cosmology**

Backup Slides

The Rotational Parallax Method

- **GENERAL CASE, any position in galaxy**

$$\cos^2(i) = -(y'\,\mu_Y) \;/\; (x\,\mu_X)$$

$$D_G = V_R / \kappa \;*\; [\,-(y'/\mu_Y) \;/\; (x\,\mu_X + y'\,\mu_Y)\,]^{-1/2}$$

- **Flat Rotation Curve, Circular Orbits, HI Inclination**

$$D_{iHI} = V_R(\text{major axis}) \;/\; [\,\kappa * \sin(i) * \mu_x(\text{minor axis})\,]$$

$$\varepsilon(D_{iHI})^2 = D^2\,[\,(\varepsilon(V_R) / V_R)^2 + (\varepsilon(\mu_x) / \mu_x)^2\,]$$

- **Flat Rotation Curve, Circular Orbits, Unknown Inclination**

$$\cos(i) = |\mu_{y'}(\text{major axis})| \;/\; |\mu_x(\text{minor axis})|$$

$$D_{mM} = V_R \;*\; [\,(\mu_{y'}(\text{major axis}))^2 - (\mu_x(\text{minor axis}))^2\,]^{-1/2}$$

General Rotational Parallaxes *(cntd)*

- **Warp?**
  - $V_c(R) = V_c(R_0) + dV/dR * (R-R_0)$
    $i(R) = i\ (R_0) + di/dR * (R-R_0)$
  - Each relation adds 2 unknowns (zpt & slope)
    - Would require >(12+2+2)=16 stars

The Rotational Parallax Method (cntd)

- **Order of magnitude Estimates:**
  - **M 33:** $i \sim 56^o$, D~0.84 Mpc, $V_C \sim$ 97 km/s $\Rightarrow \mu_C \sim$ 24 μas/yr
  - **M 31:** $i \sim 77^o$, D~0.84 Mpc, $V_C \sim$270 km/s $\Rightarrow \mu_C \sim$ 74 μas/yr
  - **LMC:** $i \sim 35^o$, D~0.055 Mpc, $V_C \sim$ 50 km/s $\Rightarrow \mu_C \sim$ 192 μas/yr

- **Importance of Random Motions (σ) ~ "measurement errors"**
  - **M 33:** V_C/σ = 9.7 $\Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/$ 9.7 ~ 14.5 % (per star)
  - **M 31:** V_C/σ = 27.0 $\Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/$27.0 ~ **5.2 %** (per star)
  - **LMC:** V_C/σ = 2.5 $\Rightarrow \varepsilon_{D,HI} \sim (\sqrt{2})/$ 2.5 ~ 56.5 % (per star)

Rotational Parallaxes: Accuracy

The equations to solve

$$\kappa D\mu_x = V_{sys,x} + V_{\sigma,x} + V_{c,x} + V_{p,x}$$

$$\kappa D\mu_{y'} = V_{sys,ry'}\sin i_s - (V_{p,z} + V_{\sigma,z})\cos(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y})\sin(i)$$

$$V_r = V_{sys,ry'}\cos i_s + (V_{p,z} + V_{\sigma,z})\sin(i) + (V_{c,y} + V_{p,y} + V_{\sigma,y})\cos(i)$$

are **mildly non-linear** with reasonably **well-known initial conditions: Good solutions expected**

- Problem investigated by Olling & Peterson

[2000, aph/0005484]

- Solve V_r relation for $(V_{p,z}+V_{\sigma,z})$ and substitute in $\mu_{y'}$
- Or solve V_r relation for $(V_{c,y}+V_{p,y}+V_{\sigma,y})$ and substitute in $\mu_{y'}$
- Or solve $\mu_{y'}$ relation for $V_{c,y} = V_{c,x}$*x/y and substitute in μ_x

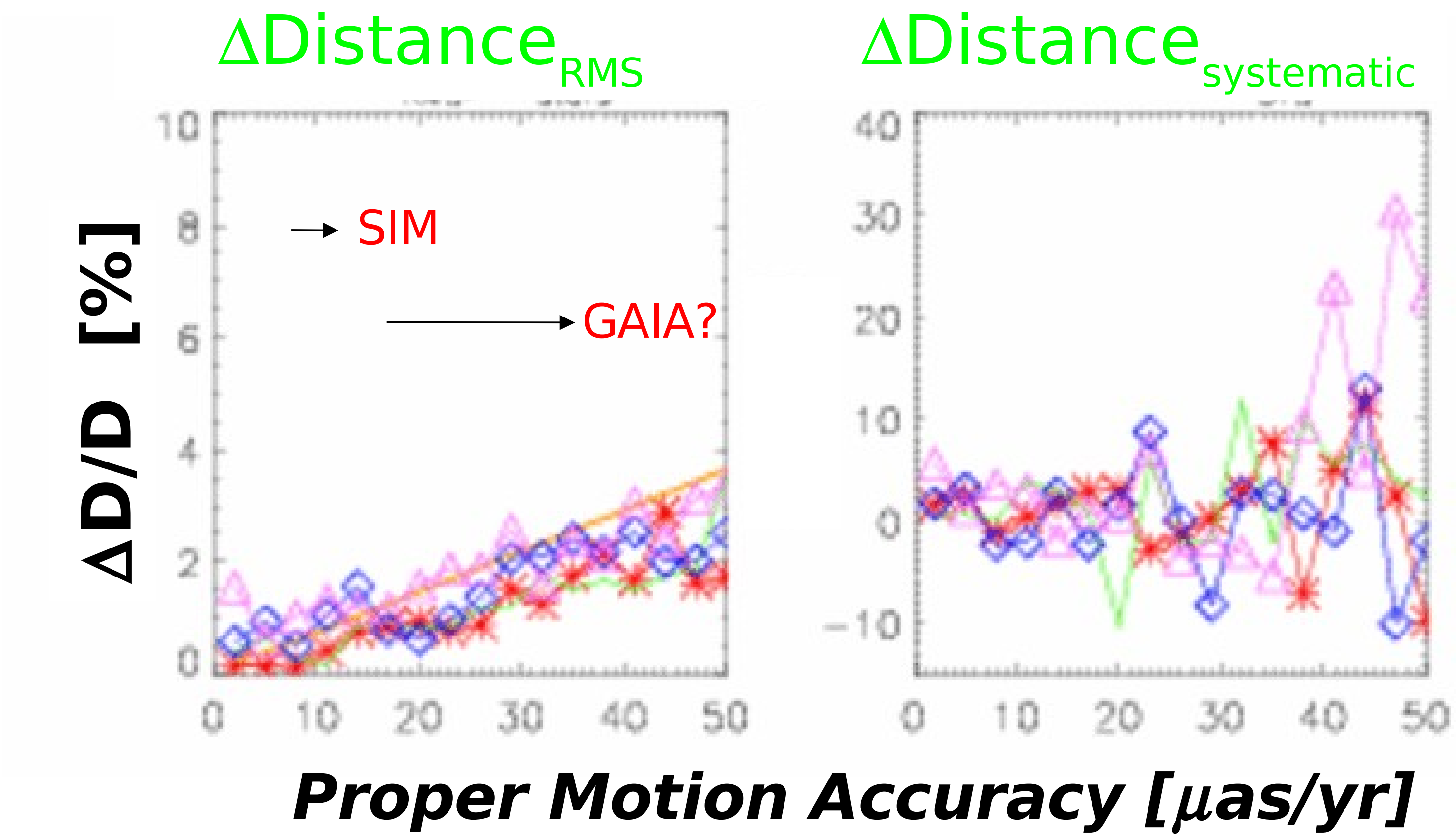
Rotational Parallaxes: Accuracy *(cntd)*

Rewrite equations employing observables x,y'

- $\mu_{y'}(V_R) = \alpha_{y'r} * V_R + \gamma_{y'r}$
- $\mu_x (V_R; y'/x) = \alpha_{xr} * V_R * y'/x + \beta_{xr} * y'/x + \gamma_x$
- $\mu_x (\mu_{y'}; y'/x) = \alpha_{xy'} * \mu_{y'} * y'/x + \beta_{xy'} * y'/x + \gamma_x$
- Solve for unknown α, β and γ coefficients

- The α and γ coefficients yield the desired parameters
  - $\cos^2(i) = -1 / \alpha_{xy'}$
  - $D = 1 / [\alpha_{y'r} \kappa \tan(i)]$
  - Non-circular motions and V_{SYS} appear only in $\gamma_{y'r}$ and βs
- Accuracy of fitted parameters follows from back-substitution and Fourier analysis of velocity field

Rotational Parallax: Expected Results.



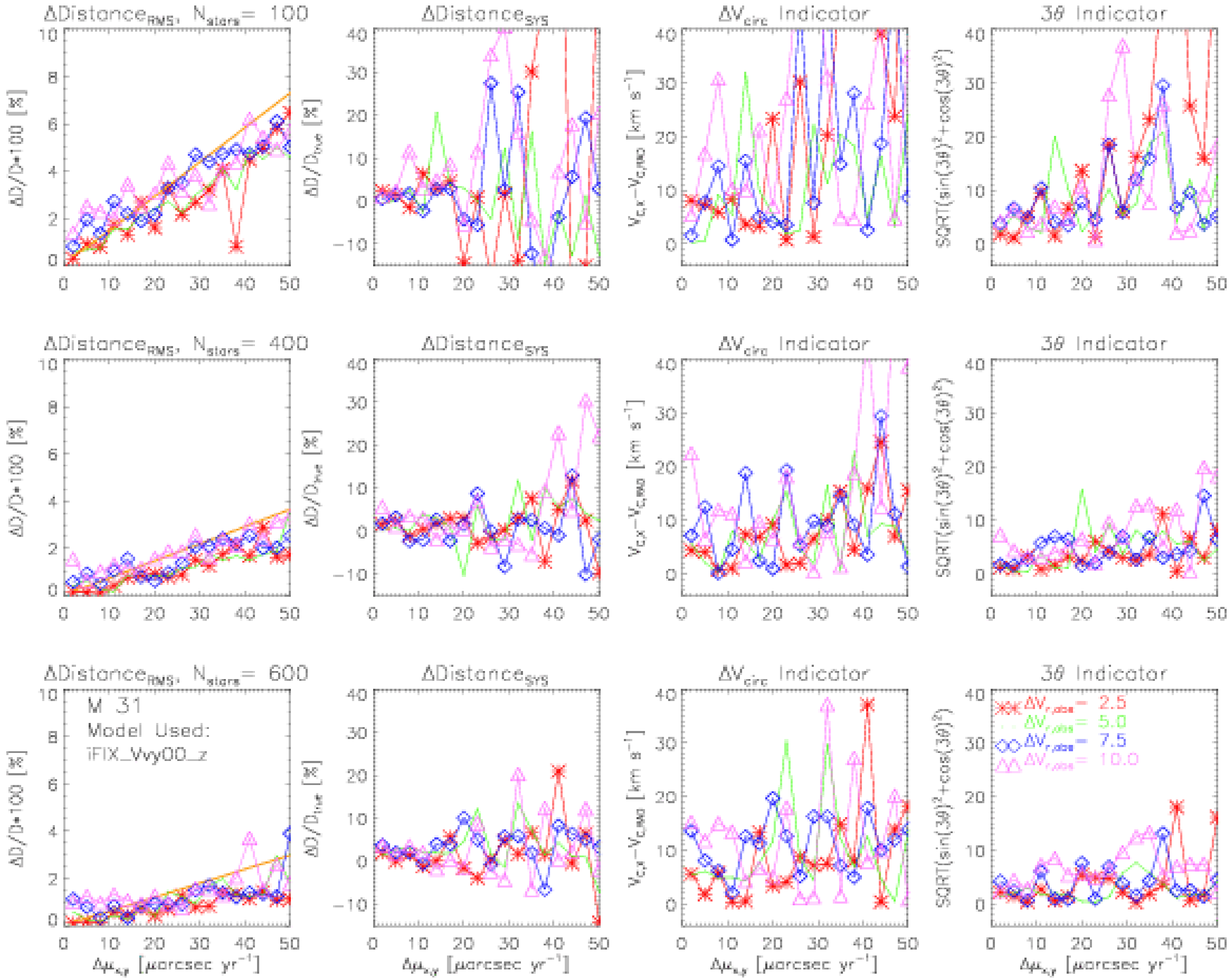
Achievable distance errors as a function of proper motion errors:

LEFT: random errors, RIGHT: systematic component

Symbols: accuracy of radial velocity data (2.5 – 10 km/s)

400 Stars used

from: Olling & Peterson (2000)



Distance accuracy (LEFT panel) and Systematic Effects (3 RIGHT panels) as a function of proper motion accuracy

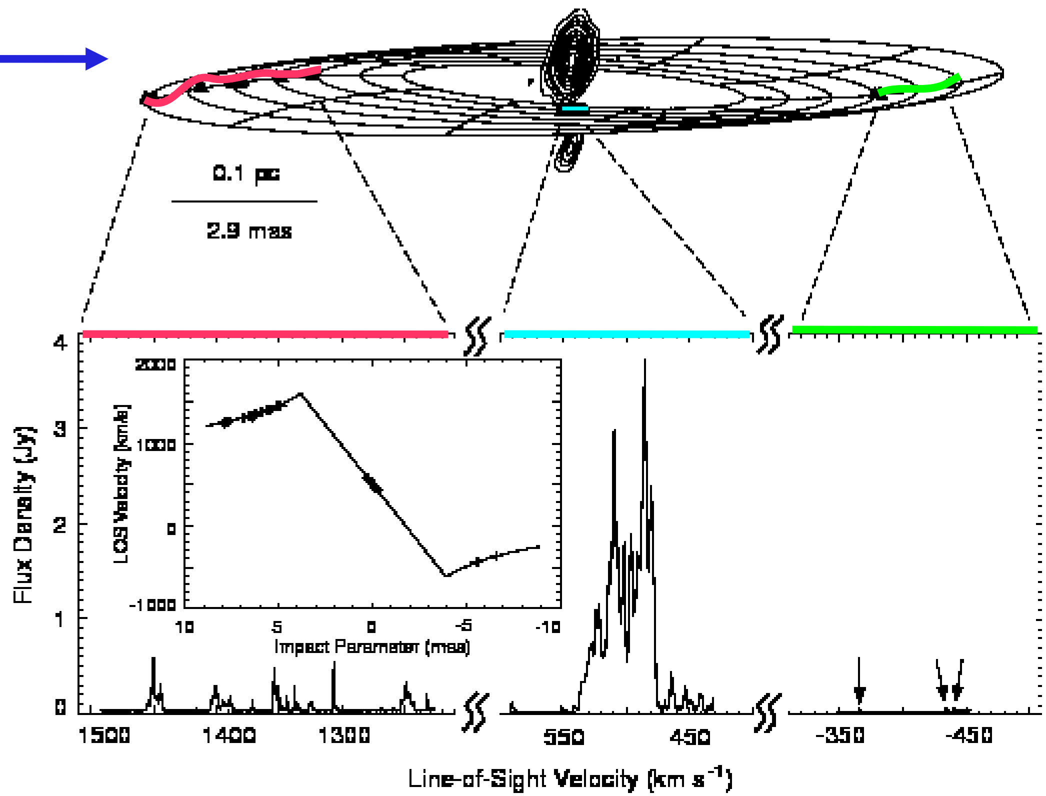
Probing the Hubble Flow:

- Need to go to >100 Mpc

  $\varepsilon(H_0) \sim V_{pec}/V_{Hubble} \sim 200$ km/s / (100 Mpc * 75 km/s/Mpc) ~ 2.6%

- The only known geometric method that probes that far:
  - **Extra-galactic H_2O Masers**

    Thin, edge-on disks



    - NGC 4258: D~ 7.3 Mpc

      ΔD/D~ 5%

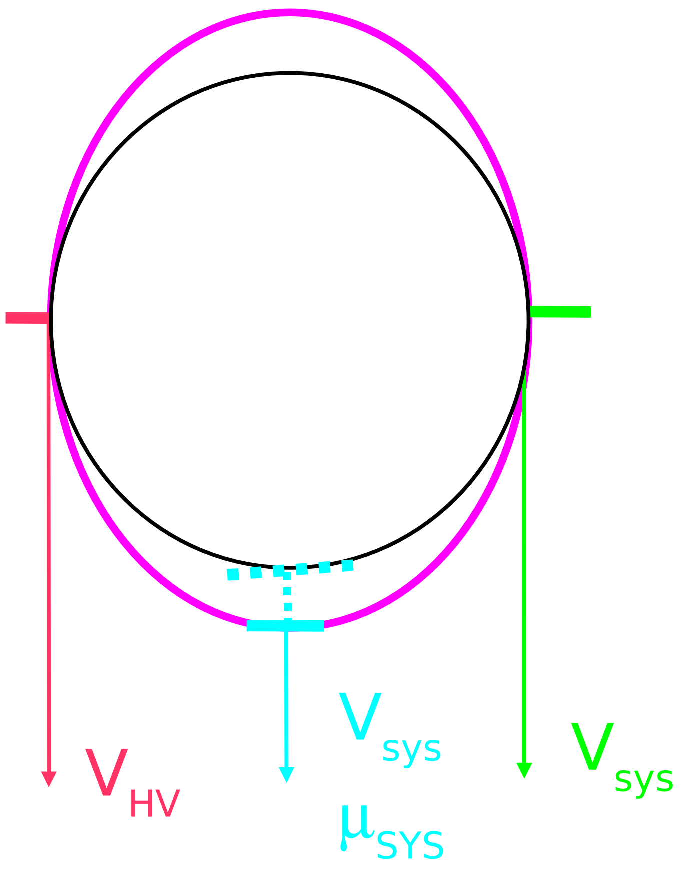
    - NGC 1068 D~ 14 Mpc
    - .... D~200 Mpc

      [e.g., Argon etal, 2007, ApJ, 659, 1040]

Mega Maser Distance Uncertainties:

- N 4258 Distance:
  7.2 ± 0.3 (random)
  ± 0.4 (systematic)
  - mostly due to orbital eccentricity [Argon 2007],
    - Up to e~0.3 due to, e.g., binary black holes

      [Eracleous, etal 1995]
    - But ruled out by monitoring
      [Gezari, Halpren, Eracleous, 2007]
    - Not clear that elliptical orbits exist, if not >60% has emissivity variations
      [Storchi-Bergmann etal, 2003]



- **Distance error in case of unmodeled eccentricity:**
  - $D_{CIRC} = D_{TRUE} [(1\mp e)^3 / (1\pm e)]^{1/2} \sim D_{TRUE} [1\mp 2e]$

Astrometry & Cosmology

- CMB, high-z galaxy data, Ly-α forest & BBN yield:

Hubble Constant	$= H_0$	= 71	± 2	± 7	[km/s/Mpc]
Age	$= t_0$	= 13.7	± 0.2		[Gyr]
Matter Density	$= \Omega_m$	= 0.27	± 0.02		$[\rho_{CRIT}]$
Total/Baryon Matter	$= \Omega_m / \Omega_b$	= 6.1	± 1.1		
Primordial Helium	$= Y_p$	= 0.2482	± 0.0004		

- Astrometry of *M31 (M33)* ⇒ strong limits on H_0
- Astrometry of *Galactic Objects* can set relevant limits on t_0, Y_p and Star Formation History

[Spergel *et al,* 2003, 2006; Freedman *et al* 2001; Mathews *etal*, 2005; Madau *etal,* 1996, this talk]

H_0, CMB and Dark Energy (cntd)

- WMAP yields: location (*l*A) of the acoustic peak:
- Cosmology yields:
  - 1) size of the acoustic oscillation
  - 2) the angular-size distance (DA) relation

$$D_A = a_* \int_{a_*}^{1} \frac{1}{a^2 H(a)}\, da \quad \text{with} \quad \frac{H(a)}{100} = \sqrt{\frac{\omega_m}{a^3} + \frac{h^2 - \omega_m}{a^{3(1+w)}}} \quad \text{and} \quad \Omega_{tot} = 1$$

- This is an integral equation with two unknowns:
  - H_0 and
  - "*w*" the Equation of State (EOS) of Dark Energy
- IF the Cosmological Constant is the Dark Energy, THEN w=-1 and: **WMAP determines H_0**

H_0 , CMB and Dark Energy (cntd)

- However, this may ***not very accurate.***
  The Assumptions were:
  - Flat Universe
  - Dark Energy has constant EOS
  - Dark Matter does not cluster, no tensor modes, no quintessence, no running spectral index, no strings, no domain walls, no non-Gaussian fluctuations, no deviations from GR, et cetera
- Allowing for a variable EOS of Dark Energy, Hu (2005) concludes that:
  - ``... the Hubble constant is the single most useful complement to CMB parameters for dark energy studies ... [if $H_0(z)$ is] ... accurate to the precent level ... .''

Alternatively, one can (try to) determine the **ages:**

- $\tau = {}_0\int^1 \, da \, / \, [a \, H(a)]$ ⇔ ages of oldest stars
- $\tau(z) = {}_0\int^{a(z)} \, da \, / \, [a \, H(a)]$ ⇔ ages of high-z galaxies

[e.g., Bothum etal 2006, Jimenez etal 2003, Simon 2005]

Summarized in Figure 4 of Spergel etal, 2004

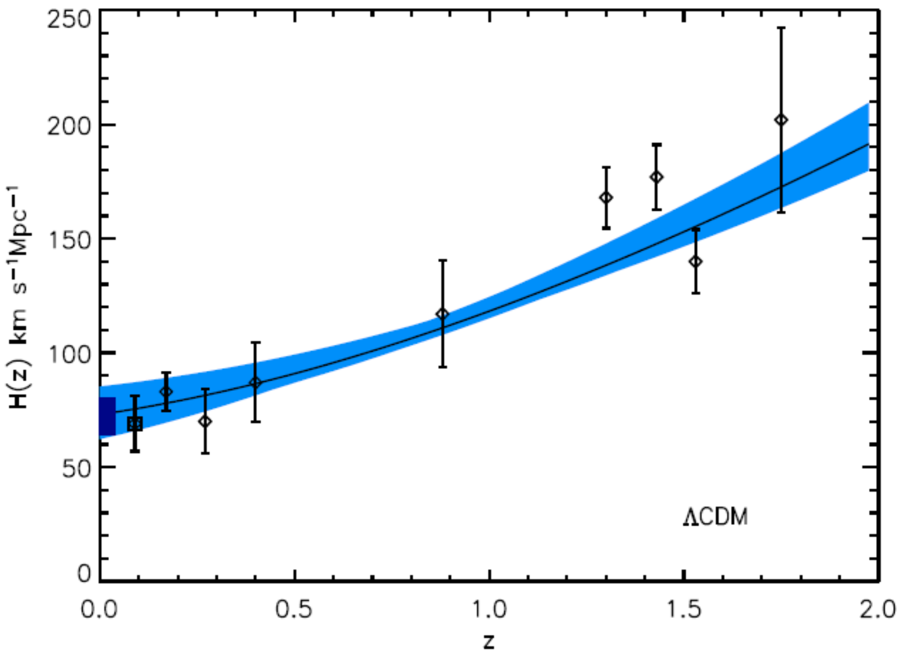
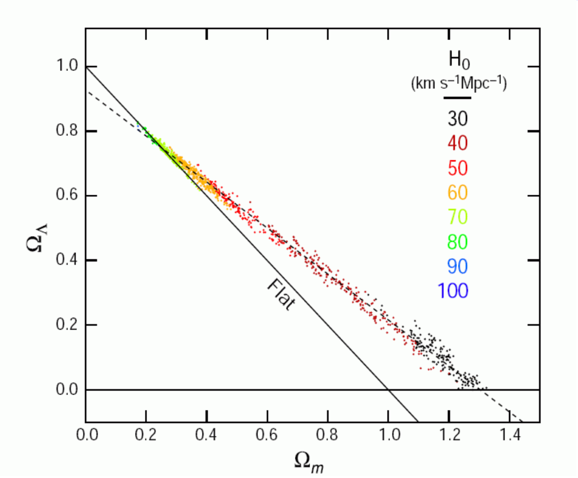


Fig. 4.— The ΛCDM model fit to the WMAP data predicts the Hubble parameter redshift relation. The blue band shows the 68% confidence interval for the Hubble parameter, H. The dark blue rectangle shows the HST key project estimate for H_0 and its uncertainties (Freedman et al. 2001). The other points are from measurements of the differential ages of galaxies, based on fits of synthetic stellar population models to galaxy spectroscopy. The squares show values from Jimenez et al. (2003) analyses of SDSS galaxies. The diamonds show values from Simon et al. (2005) analysis of a high redshift sample of red galaxies.

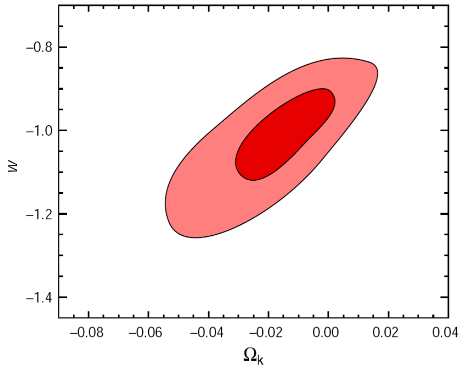
H_0 , CMB and Dark Energy (cntd)

- Many groups pursue other methods to determine some (combination of) parameter(s) that constrain the "integral"
  - **Luminosity-Distance relation from Supernovae Ia**
    - $D_L(z) = D_A(z) / a(z)^2$
  - **Baryon Oscillations (sensitive to local galaxy density)**
    - $Volume(z) = [\underline{D_A(z)} / a(z)]^2 / \underline{H(z)} * \Omega_{sky}\, \Delta z$
  - **Galaxy Cluster Abundance**
    - Depends on *Volume(z)* and non-linear structure growth
  - **Weak Lensing**
    - Depends on: $\underline{D_A(z)}$, $\underline{H(z)}$ and *structure growth*
- *[e.g., Albrecht et al 2006 = DETF]*

H_0 , CMB and Dark Energy (cntd)



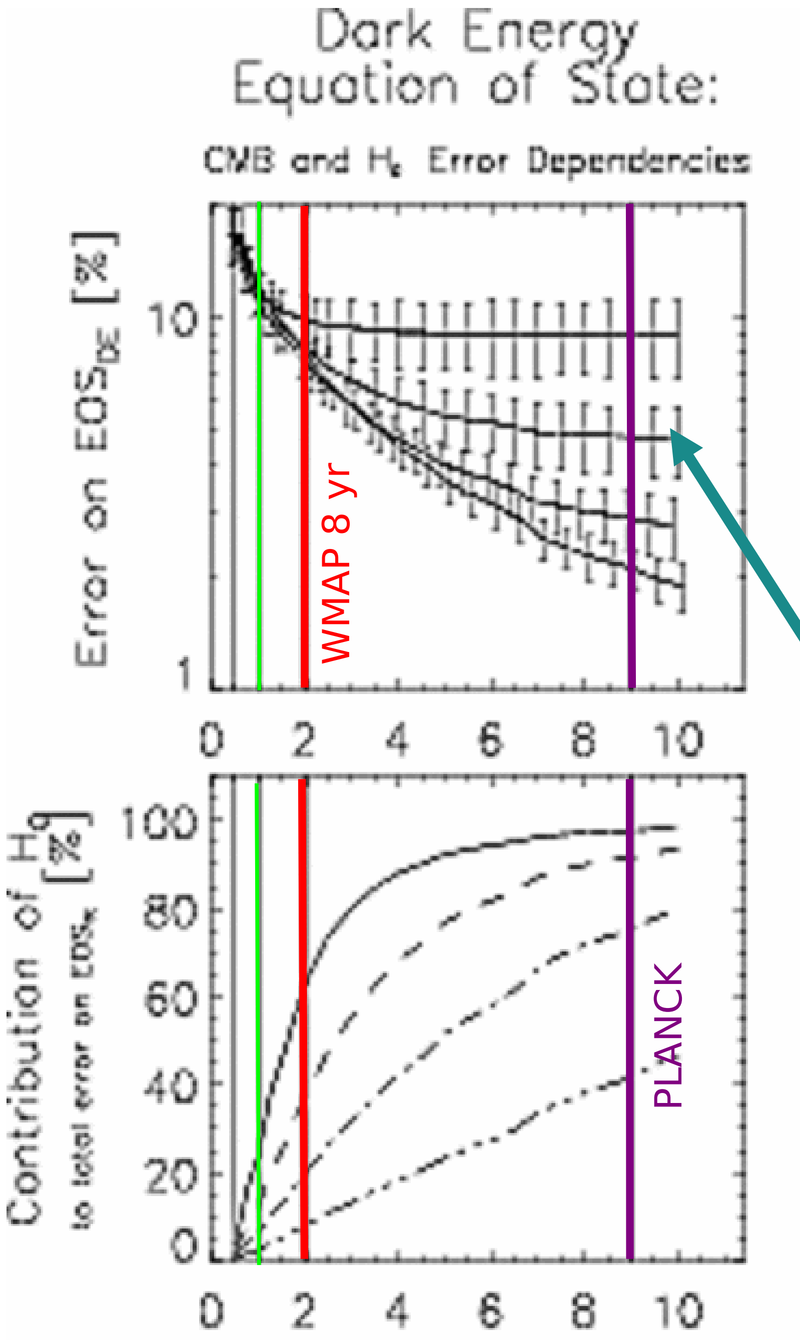
Data Set	Ω_K	Ω_Λ
WMAP + $h = 0.72 \pm 0.08$	$-0.003^{+0.013}_{-0.017}$	$0.758^{+0.035}_{-0.058}$
WMAP + SDSS	$-0.037^{+0.021}_{-0.015}$	$0.650^{+0.055}_{-0.048}$
WMAP + 2dFGRS	$-0.0057^{+0.0061}_{-0.0088}$	$0.739^{+0.026}_{-0.029}$
WMAP + SDSS LRG	$-0.010^{+0.011}_{-0.015}$	$0.728^{+0.020}_{-0.028}$
WMAP + SNLS	$-0.015^{+0.020}_{-0.016}$	$0.719^{+0.021}_{-0.029}$
WMAP + SNGold	$-0.017^{+0.022}_{-0.017}$	$0.703^{+0.030}_{-0.038}$



- We use the Spergel etal (2006/7) WMAP & “other data” to approximate the relations between the various parameters ($P_i = a_{ij} + b_{ij} P_j$) :

$$\Omega_\Lambda = a_{\Lambda m} + b_{\Lambda m}\,\Omega_m$$
$$\Omega_K = a_{K\Lambda} + b_{K\Lambda}\,\Omega_\Lambda$$
$$w = a_{wK} + b_{wK}\,\Omega_K$$

- **For a constant EOS, but a Universe of general curvature**



$\varepsilon(\omega_m;\text{NOW}) / \varepsilon(\omega_m) \longrightarrow$

To arrive at:

- $w = a_{wK} + b_{wK}\,(a_{K\Lambda} + a_{\Lambda m} b_{K\Lambda}) + b_{\Lambda m}\, b_{K\Lambda} b_{wK} * \omega_m / h^2$

  $= (-0.83 \pm 0.11) - (0.56 \pm 0.06)\ \omega_m / h^2$

- Error on EOS as a function of $\varepsilon(\omega_m)$:

$$\epsilon_w^2 = \ldots + b_{\Lambda m}\, b_{K\Lambda}\, b_{wK} \left[\left(\frac{\epsilon_{\omega_m}}{h^2} \right)^2 + \left(\frac{2\,\omega_m \epsilon_h}{h^3} \right)^2 \right]$$

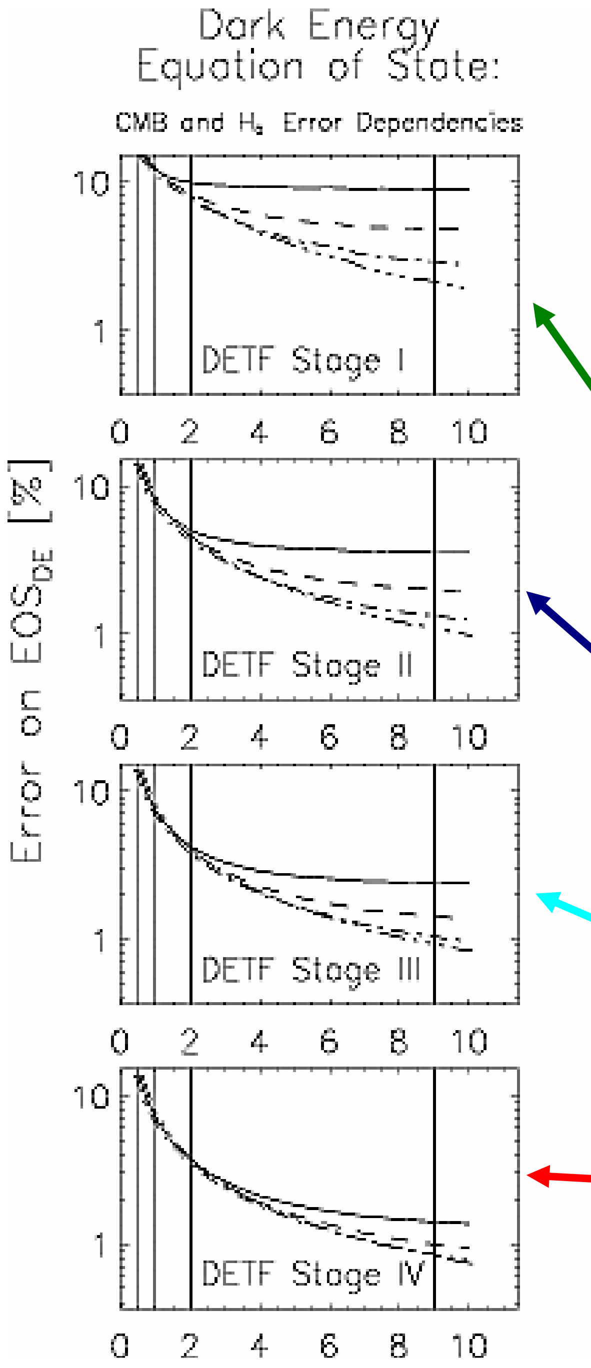
- In Figure: curves from top to bottom for $\varepsilon(H_0) = \varepsilon(H_0;\text{now}) * [1, 1/2, 1/4, 1/10]$

[Olling, 2007, MNRAS, 378, 1385]

H_0 , CMB and Dark Energy (cntd)

- The Dark Energy Task Force *[Albrecht et al 2006]* recommends several approaches to determine the "evolution" of the EOS:
  - Stage I: Current knowledge
  - STAGE II: Projects finishing soon (including *PLANCK)*
  - STAGE III: Photo- (spectro-) redshifts on 4^m (8^m) telescopes
  - STAGE IV: ***Large Synoptic Telescope,***
    Joint Dark Energy Mission,
    Square Kilometer Array
    - **At Stage IV, accurate H_0 knowledge matters <~50%**
- **Unpublished Minority Opinion (Freedman & Hu): Spend effort on determination of H0**

H_0 & Dark Energy in various stages of the DETF:



- At intermediate stages, small H_0 errors matter more:
  - Stage I: $\varepsilon(H_0)=10\% \Rightarrow \varepsilon_w \sim 8.9\%$
    $\varepsilon(H_0)= 1\% \Rightarrow \varepsilon_w \sim 2.3\%$
  - Stage II: $\varepsilon(H_0)=10\% \Rightarrow \varepsilon_w \sim 3.6\%$
    $\varepsilon(H_0)= 1\% \Rightarrow \varepsilon_w \sim 1.2\%$
  - Stage III: $\varepsilon(H_0)=10\% \Rightarrow \varepsilon_w \sim 2.4\%$
    $\varepsilon(H_0)= 1\% \Rightarrow \varepsilon_w \sim 1.0\%$
  - Stage IV: $\varepsilon(H_0)=10\% \Rightarrow \varepsilon_w \sim 1.5\%$
    $\varepsilon(H_0)= 1\% \Rightarrow \varepsilon_w \sim 0.9\%$