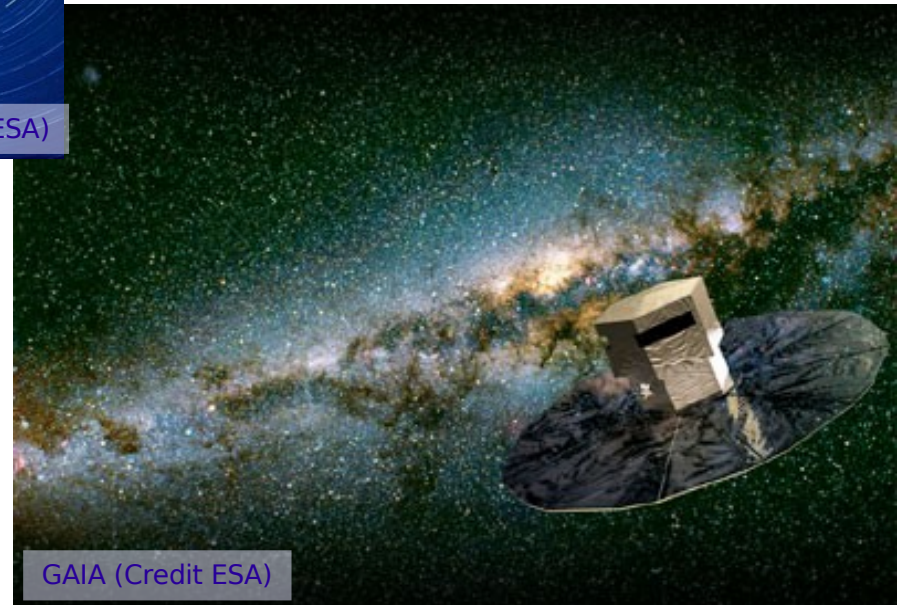
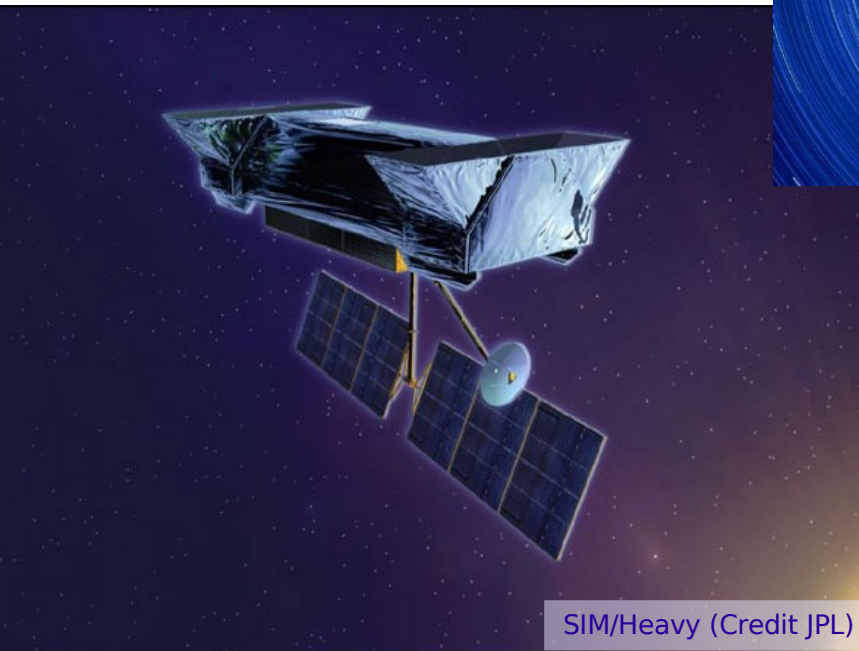


# Astrometric & Photometric Detection & Characterization of (massive) Extrasolar Giant Planets

Searching for Solar System Giant Analogs with SIM (and GAIA)  
Rob Olling (UMd)



# Outline

- **Dabblings** (2000-2006.5)

[http://www.astro.umd.edu/~olling/index\\_1.htm#My\\_Astrometry\\_USNO](http://www.astro.umd.edu/~olling/index_1.htm#My_Astrometry_USNO)

- **Astrometric Scales in Astronomy**
- **Astrometric Detections**
- **dFTS: Radial Velocities & TPF-C Characterization**
- **Transits w. scanning (Astrometric) Missions**
  - **LEAVITT: 10,000 Transiting planets down to  $R_{\text{EARTH}} * 4.6$**

- **Long Period Planets (Solar System Analogs)**

- **Observability: Where/Why?**
- **Traditional search methods**
  - **( $\mu_B$  problem)**
- **Position Differences**
  - **Hipparcos to the Rescue**
  - **Period & Mass determination**

- **Conclusions & Future work**

- **Backup slides**

- **Part of this talk is based on a contribution to the Extrasolar Planet Task Force**  
[Olling, 2007 [arXiv0704.30590](http://arxiv.org/abs/0704.30590) & <http://www.astro.umd.edu/~olling>]

# Astrometric Scales in Astronomy

## Parallaxes, in $\mu\text{as}$

- $\alpha$  Cen: 742,000
- RR Lyra: 4,380
- $\delta$  Cep: 3,320
- 
- 1 kpc: 1,000
- Gal. Center: 125
- LMC: 20
- M 31: 1.5

## Proper Motions, in $\mu\text{as/yr}$

- $\alpha$  Cen: 3,600,000
- RR Lyra: 200,000
- $\delta$  Cep: 16,500
- 
- 10 km/s @ 1 kpc: 2,110
- 200 km/s @ 8 kpc: 5,275
- 50 km/s @ LMC: 211
- 200 km/s @ M 31: 60

**USA @ 10 pc 2.9 ; 2  $M_{\text{EARTH}}$  @ 10 pc: 1  $\mu\text{as/yr}$**

**Sun's  $\oplus$  reflex motion 450 km**

# Astrometric Detections:

- Advantages of large numbers and/or high accuracy

- Large numbers:

- find rare objects (e.g., **old, high [Fe/H] stars**)
- accurate statistics/general properties of majority
- Identify ES planetary systems

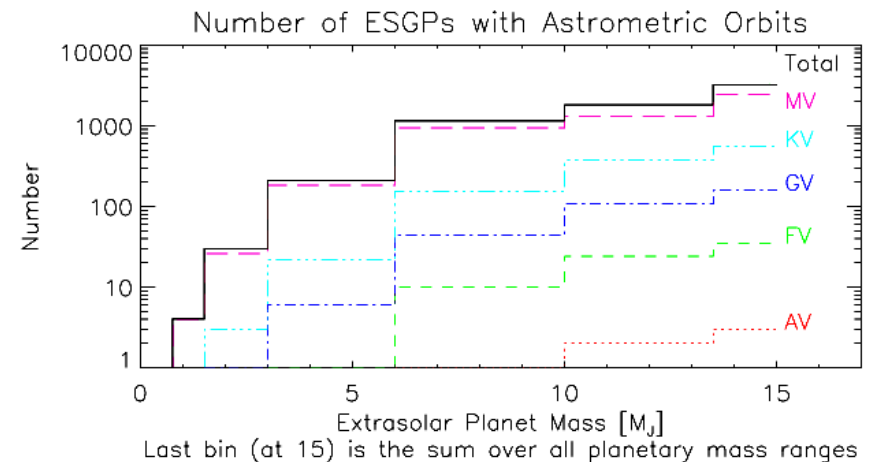
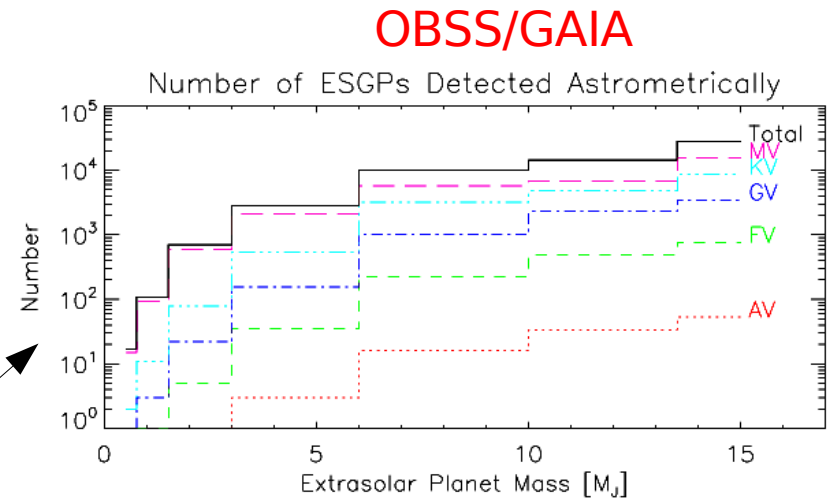
- High accuracy:  $\Rightarrow$  characterize individual objects

- Astrometry:

- FAME:  $4.5\sigma$ : 2,000;  $10\sigma$ : 1,500

- OBSS/GAIA

- $5\sigma$ : 28,000
- $15\sigma$ : 3,200



# Dispersed Fourier Transform Spectrometer:

- **dFTS @ USNO: PI: Arsen Hajian (now at Waterloo)**

<http://adsabs.harvard.edu/abs/2007ApJ...661..616H>

[http://www.astro.umd.edu/~olling/Papers/dFTS\\_white\\_paper\\_final.pdf](http://www.astro.umd.edu/~olling/Papers/dFTS_white_paper_final.pdf)

- Like conventional FTS, but dispersed by GRATING into many thousands of spectral channels
  - Much, much better sensitivity:  $S/N_{\text{dFTS}} = S/N_{\text{FTS}} * (R_{\text{GRATING}})^{1/2}$
  - Whole (optical) spectrum
    - Configurable spectral resolution (down to **TPF** needs)
  - Small size ( $\sim 1 \text{ m}^3$ )
  - Cheap (shoestring)
- Full-aperture metrology
  - Extreme wavelength sensitivity (“arbitrary” resolution)
    - $\sim 3 \text{ m/s}$  for our shoestring instrument
    - Many known improvements await funding
    - **cm/s** long-term stability expected ==> **Earth-mass planets**

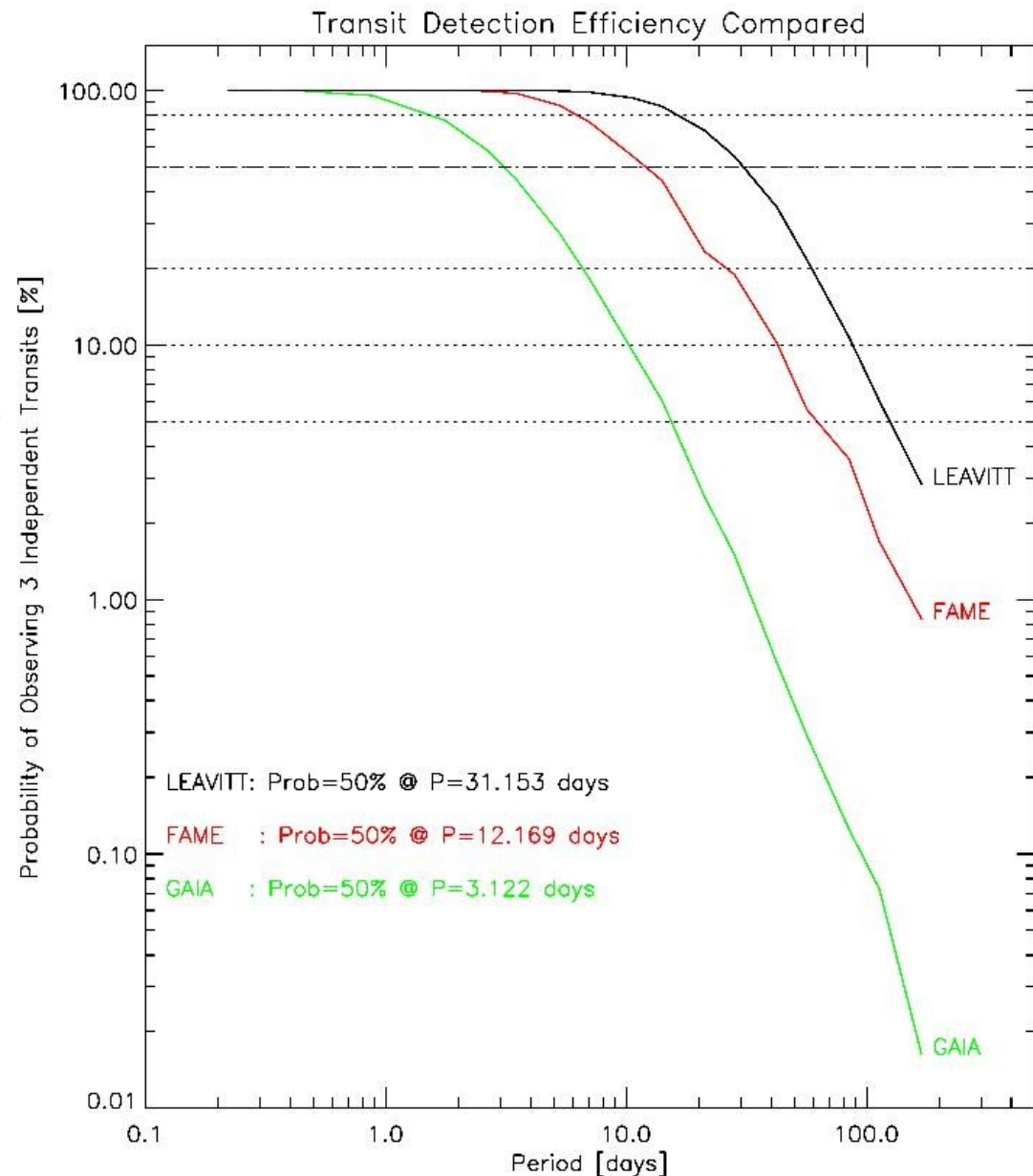
# Planetary Transits w. Astrometric Telescopes:

- HIPPARCOS/FAME/LEAVITT-like instruments are “good” for transit detections (GAIA spins too slowly)
  - Large number of (independent) observations
    - H  $100/3\text{yr} = 1 \text{ per } 7.7 \text{ “days”}$
    - F:  $129/5\text{yr} = 1 \text{ per } 9.9 \text{ “days”}$
    - G:  $60/5\text{yr} = 1 \text{ per } 21.3 \text{ “days”}$
    - L:  $183/5\text{yr} = 1 \text{ per } 7.0 \text{ “days”}$
  - **SHOW sky coverage**

# PTs w. Astrometric Telescopes: Detection Efficiency

Resulting detection efficiency depends critically on cadence

- **LEAVITT:**      **good**
- **FAME:**        **intermediate**
- **GAIA:**        **poor**





# Planetary Transits w. Astrometric Telescopes:

- Many observations (CCD transits) per “epoch” ==> good sensitivity for 2 mmag accuracy per 2.5 hour & not saturated & GV primary  
“Maximum” possible number of Extrasolar Giant Transiting Planets (1/2000)
  - H:  $V = [0.0, 9.5]$  mag ==>  $N'_{EGP} = 95$  ;  $R_{PLANET} = [0.9, 1.5] R_{NEPTUNE}$
  - F:  $V = [5.2, 10.8]$  mag ==>  $N'_{EGP} = 341$  ;  $R_{PLANET} = [0.5, 2.0] R_{NEPTUNE}$
  - G:  $V = [11.0, 15.9]$  mag ==>  $N'_{EGP} = 26,800$  ;  $R_{PLANET} = [0.7, 2.1] R_{NEPTUNE}$
  - **L:  $V = [5.8, 14.8]$  mag ==>  $N'_{EGP} = 12,000$  ;  $R_{PLANET} = [0.3, 2.4] R_{NEPTUNE}$**
- **Values need to be convolved with:**
  - **detection efficiencies**
  - **edge-on probability**
  - **Planetary PDF**



Quantity	unit	Symbol	FAME	GAIA	LEAVITT	TESS
Mission Type			<i>MIDEX</i>	<i>"PROBE"</i>	<i>MIDEX</i>	<i>SMEX</i>
Mission Duration	Years	$t_{MIS}$	5	5	5	2
In-scan Mirror Size	cm	$D_I$	40	<b>140</b>	55	13.3
X-scan Mirror Size	cm	$D_X$	9	<b>50</b>	14	13.3
Photon-collecting power			1	<b>19.4</b>	2.1	2.9
Time to cover accessible sky = Median re-visit Time	days	$t_{SKY,70\%}$	28	35	<b>7.5</b>	
In-scan Field of View	degrees	$FOV_I$	1.1	0.74	<b>3.5</b>	
Total Number of broad-band observations		$N_{BB}$	2,684	1,057	<b>10,253</b>	
Epoch Duration	hours	$t_{EPO}$	2.73	3.69	<b>6.60</b>	
Average # Broad-Band Observations per Epoch		$N_{BB/EPO}$	22.4	26.8	<b>61.0</b>	
# of Independent Epochs		$N_{EPO}$	120.1	39.4	<b>168.0</b>	
# of Photometric Observations per band (R=3; R=2 for FAME)		$N_{RS}$	244.0	96.1	<b>10,253.4</b>	
Average # Photometric Observations per Epoch		$N_{PHO/EPO}$	2.0	2.4	<b>61.0</b>	
Photometric Saturation Level [mag]		$V_{SAT}$	5.21	10.69	<b>5.76</b>	4.5
V magnitude for 2 mmag photometry in 0.83 hr	magnitude	$V_{2mmag}$	10.81	<b>15.59</b>	14.83	13.5
Number of Stars Surveyed	$10^6$	$N_{S,TR}$	1	<b>73</b>	36	2.5
Minimum Planetary Radius (GV)	$R_{NEPTUNE}$	$R_{PL,MIN}$	0.51	0.68	<b>0.30</b>	<b>1.00</b>
Number of <u>Planetary Transits</u> (AV, FV, GV, KV & MV stars)		$N_{EXOP,BB}$	115	<u>2,279</u>	<u>10,451</u>	<u>1,687</u>
Number of <u>Planetary Transits</u> (AV, FV, GV, KV & MV stars) & PHOTOMETRIC CHARACTERIZATION		$N_{EXOP,PHOT}$	10	400	<b>5,777</b>	
Number of <u>Eclipsing Binaries</u> (AV...MV stars) & PHOTOMETRIC CHARACTERIZATION		$N_{EB,PHOT}$	1,091	9,246	<b>79,572</b>	<b>2,111</b>
Orbital Period with $P_{DET} = 50\%$ for 5 Transits, FROM SCANNING LAW	days	$P_{50\%,D=3,S,CAN}$	6.24	1.48	<b>16.13</b>	2.5

# Mission Parameters & Abilities Compared

(More detailed talk available)

# Long Period Objects (Planets, BDs, Stars)

- **For astrometry & velocimetry:**  
**need:  $P_{\text{ORBIT}} < \sim$  twice observing span**  
**to determine  $P_{\text{ORBIT}}$**

- **Most of Solar System's angular momentum is in Jupiter & Saturn:**

- **Solar System Analog:**  
system that has a “Jupiter” and/or “Saturn”  
and/or Uranus/Neptune

- **All outer planets**  
**have  $P_{\text{ORB}} > 2 T_{\text{MISSION}}$**

Planet	AU	Period	Mass
Jupiter	5.2	11.9	318
Saturn	9.5	29.4	95
Uranus	19.2	84.0	15
Neptune	30.1	164.0	17

# How Many Long-Period Planets?

- **Which long-period planets:**
  - **SOSAs:**  $P \in [11.9, 165]$  yr  
 $M \in [0.05, 1] M_{JUP}$
  - **HOSAs:**  $P \in [11.9, 165]$  yr  
 $M \in [1, 13] M_{JUP}$
- **Fraction of Planetary Systems:**  
[Tabachnik & Tremaine (2002) or Cumming et al (2008)]
  - **SOSAs:** **13** % of planetary systems
  - **HOSAs:** **(17 +/- 3)%** of planetary systems
- **HOSAs:** **8** % of **Sun-like stars**

# Long Period Planets: Where/Why?

- Some Planetary Migration Theories predict
  - Inward migration (known “RV” planets)
  - Outward migration (Uranus & Neptune)
    - Outer edge: 50-100 AU (350 – 1,000 yr) [Ida & Lin, 2004]
  - Predict massive long-period planets
  - Would require more massive disk
- Without migration: 30-40 AU (165-250 yr)
- MUCH, MUCH, MUCH, MUCH longer than  $2T_{\text{MISSION}}$ 
  - **How to measure this?**

# Some Scales

$$\begin{aligned}
 a_0 &= 95/d_{10\text{pc}} (P^{+2} M_{\text{TOT}}^{-2})^{1/3} M_{\text{C;J}} \quad [\mu\text{as}] \\
 |\mu| &= 600/d_{10\text{pc}} (P^{-1} M_{\text{TOT}}^{-2})^{1/3} M_{\text{C;J}} \quad [\mu\text{as/yr}] \\
 |d\mu/dt| &= 3800/d_{10\text{pc}} (P^{-4} M_{\text{TOT}}^{-2})^{1/3} M_{\text{C;J}} \quad [\mu\text{as/yr}^2]
 \end{aligned}$$

<b>5</b>	$M_{\text{JUPITER}} @$	<b>20</b> pc		
Period	$a_0$	$ \mu $	$ d\mu/dt $	<b>Acceleration accuracies</b>
[yr]	[ $\mu\text{as}$ ]	[ $\mu\text{as/yr}$ ]	[ $\mu\text{as/yr}^2$ ]	
<b>10</b>	<b>1,099</b>	<b>690</b>	<b>433.8</b>	
<b>20</b>	<b>1,744</b>	<b>548</b>	<b>172.2</b>	
<b>40</b>	<b>2,769</b>	<b>435</b>	<b>68.3</b>	3- $\sigma$ ; Tycho-2
<b>80</b>	<b>4,396</b>	<b>345</b>	<b>27.1</b>	
<b>160</b>	<b>6,977</b>	<b>274</b>	<b>10.8</b>	3- $\sigma$ ; GAIA 5yr
<b>320</b>	<b>11,076</b>	<b>217</b>	<b>4.3</b>	3- $\sigma$ ; SIM 5yr
<b>640</b>	<b>17,582</b>	<b>173</b>	<b>1.7</b>	3- $\sigma$ ; GAIA+SIM

GAIA+SIM accuracy  $\sim$  2 x smaller than SIM  
 $\sim$ 10 x smaller than GAIA

# Only around Nearby/Bright Stars

- **Sun-like stars are really bright**
  - **GAIA saturates at V~12, but usable to V~6**

MS Star	F5	G0	G5	K0	K5		F5	G0	G5	K0	K5			
MV(abs)	3.5	4.4	5.1	5.9	7.4		2.35	4.13	5.9	7.63	13.1		[*/pc <sup>3</sup> ] / 1000	
Distance [pc]	apparent magnitude						Number of Stars out to D <sub>pc</sub>					Total # Stars	Total # OSAs	
5	2.0	2.9	3.6	4.4	5.9		1	2	3	4	7		17	1.4
10	3.5	4.4	5.1	5.9	7.4		10	17	25	32	55		139	11.1
20	5.0	5.9	6.6	7.4	8.9		79	138	198	256	439		1,109	88.7
<b>30</b>	<b>5.9</b>	<b>6.8</b>	<b>7.5</b>	<b>8.3</b>	<b>9.8</b>		<b>266</b>	<b>467</b>	<b>667</b>	<b>862</b>	<b>1,482</b>		<b>3,744</b>	<b>299.5</b>
40	6.5	7.4	8.1	8.9	10.4		630	1,106	1,582	2,044	3,512		8,874	709.9
60	7.4	8.3	9.0	9.8	11.3		2,126	3,732	5,338	6,899	11,853		29,948	2,395.9
80	8.0	8.9	9.6	10.4	11.9		5,040	8,847	12,653	16,353	28,095		70,988	5,679.1
100	8.5	9.4	10.1	10.9	12.4		9,844	17,279	24,714	31,940	54,873		138,649	11,091.9

Out to 30 pc,  
 must survey ~3,700 stars  
 expect to find ~ 300 HOSAs

# Old-Fashioned Way of finding long-period systems: w. Hipparcos & Tycho-2

- Use information from other astrometric catalogs
  - e.g., **Tycho-2** catalog comprises data from 144 catalogs going back to ~1907
    - Astrographic catalog (1907 @ 220 mas)
    - USNO's AGK2 (1930 @ 70 mas)
    - USNO's TAC (1980 @ 50 mas)
    - **Hipparcos** (1991 @ 1 mas)
    - ...
- **Compare proper motions:**
  - long-period cat (e.g., Tycho-2)
    - “mostly” center of mass motion if  $P < \sim 40$  years
  - short-period cat (e.g., Hipparcos)
    - Binary + center of mass motion if  $P < \sim 12$  years
  - **Difference is due to binary motion**
    - **FAILS if  $P > \sim 4 \times T_{\text{MISSION}}$**



# Eliminating $\mu_B$ : Backtrapolates

- Total motion (face-on; circular):

$$z_{\text{TOT}}(t) = z_0 + \mu_B t + z_{\text{ORBIT}}(t)$$

$$z_{\text{ORBIT}}(t) = a_0 \cos(2\pi t/P + \varphi)$$

- Expand  $Z_{\text{ORBIT}}(t)$

- $\zeta(t)/a_0 = \cos(\varphi) - (2\pi/P) \sin(2\pi t/P + \varphi)t - \frac{1}{2} (2\pi/P)^2 \cos(2\pi t/P + \varphi)t^2 + \dots$

- $Z_{\text{TOT}}'(t) = Z_0 + \mu_B t + \zeta(t)$   
=  $n^{\text{th}}$  order polynomial fit to SIM data

- Position Difference at Hipparcos epoch ( $\tau$ )

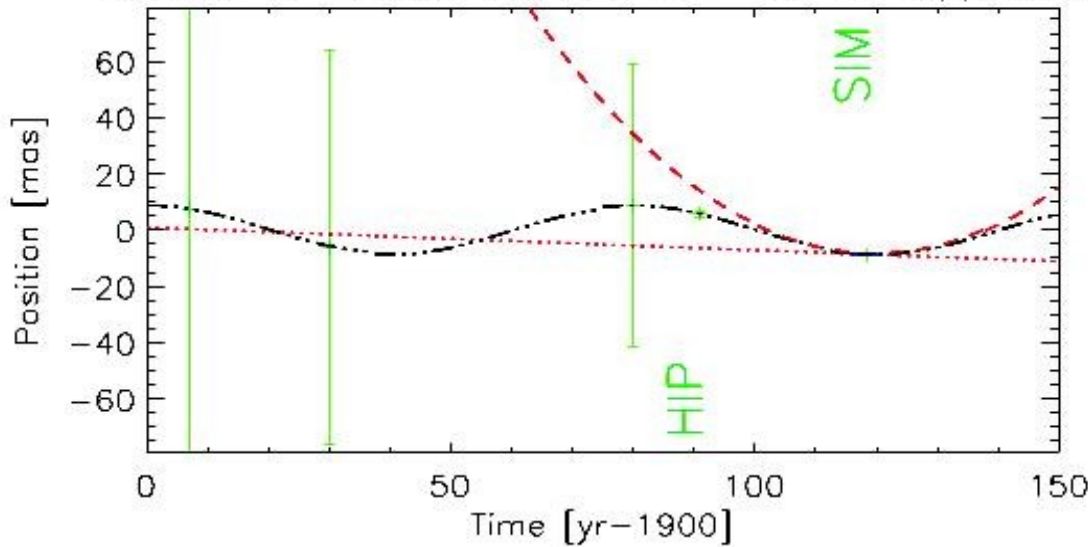
- $\Delta_z(\tau) = z_{\text{TOT}}(\tau) - z_{\text{TOT}}'(\tau) = z_{\text{ORBIT}}(t) - \zeta(\tau)$

- **INDEPENDANT of Barycentric motion**

- Only depends on orbit & its expansion

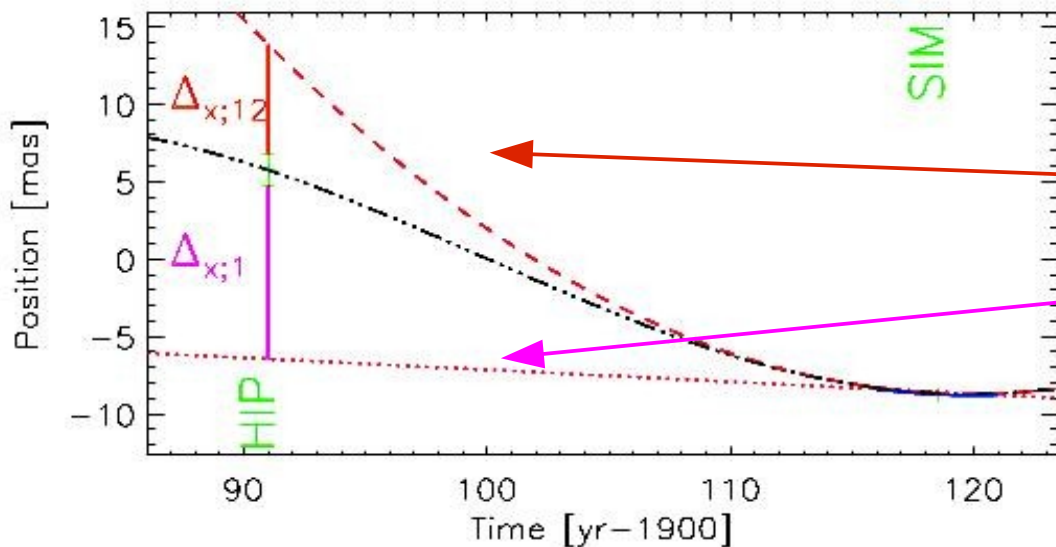
# Future Method of Finding long-period systems w. SIM & Hipparcos

Effects of Orbital Motion: SIM-Lite & Hipparcos



$M = 10 M_{JUP}$   
 $P = 80 \text{ yr}$   
 $D = 20 \text{ pc}$   
 $a_0 = 8.8 \text{ mas}$   
 $\mu_{ORBIT} = 0.69 \text{ mas/yr}$

Difference between:  
backtrapolations:

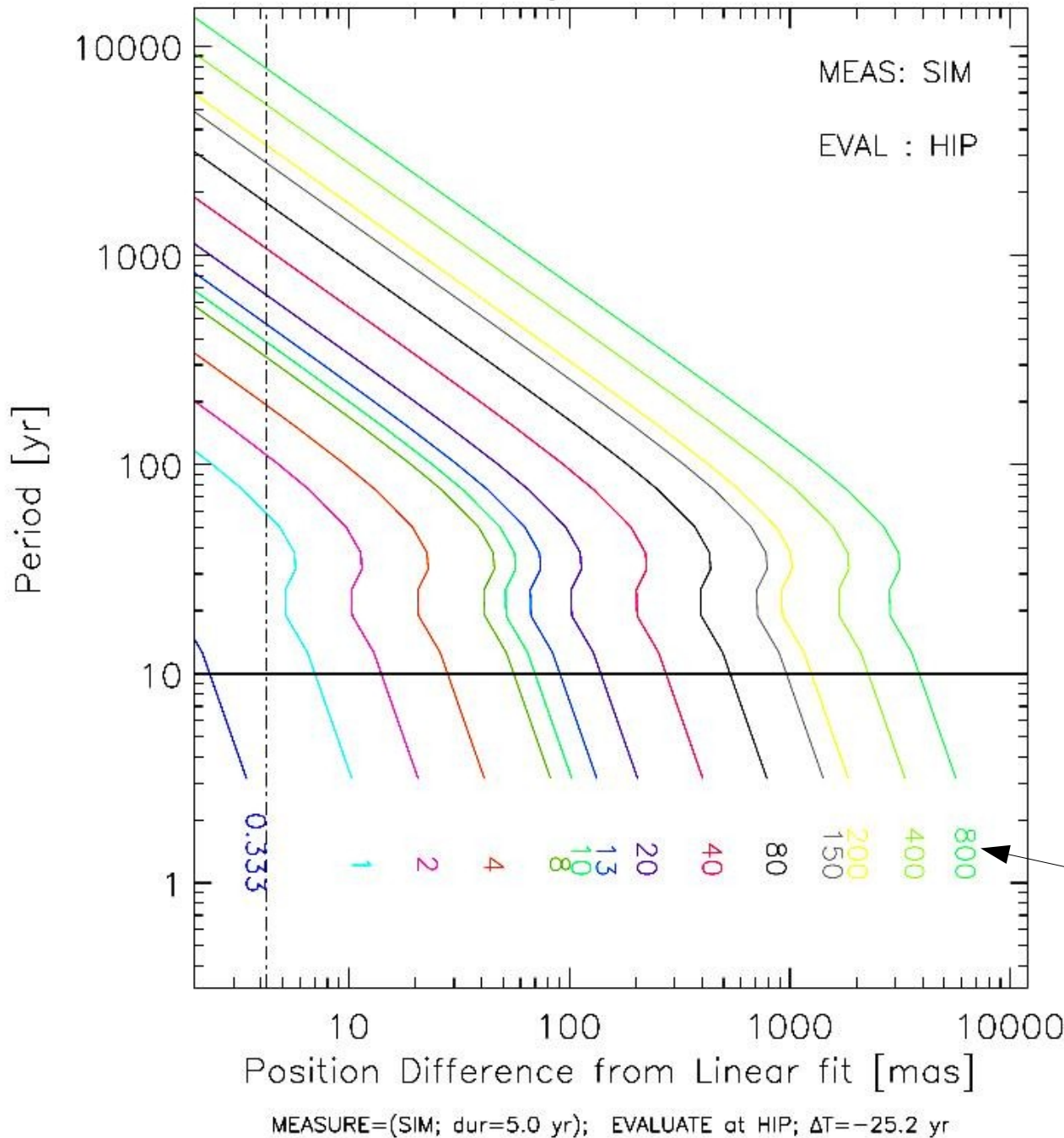


Quadratic:  $\Delta_{x;12}$

Linear:  $\Delta_{x;1}$

**Period/Mass dependent?**

# Motion of Primary: Position Differences



**YES !**

MEASURE: SIM  
B.TRAPOLATE: HIP

Position Differences from linear fit are degenerate:

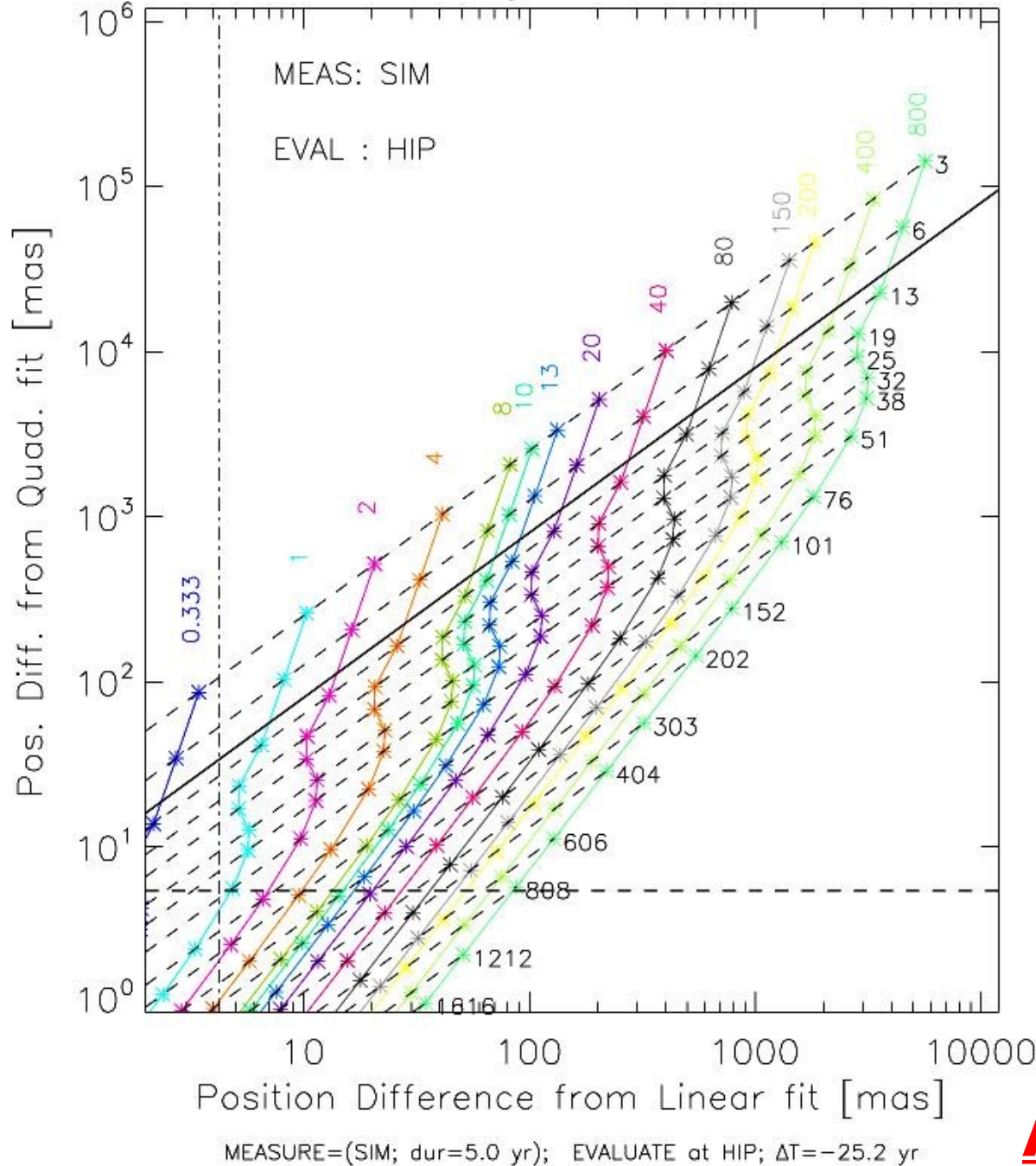
Multiple Masses & Periods

at given pos.dif

# Backtrapolates: Sensitive to Mass & Period

- **Order-dependent:**  $\Delta_{z;n}(\tau) = z_{\text{ORBIT}} - \zeta^n(\tau)$ 
  - Can be calculated analytically
- **No phase dependence for TOTAL pos. dif.**
  - Face-on & circular:  $\Delta_{XY;n} = (\Delta_{X;n}^2 + \Delta_{Y;n}^2)^{1/2}$
- **Periods** can be estimated from  $\Delta_{XY;n}$  values
  - $\mathcal{P}_{1,2} = 2/3 \pi \tau \Delta_{XY;1} / \Delta_{XY;2} \sim P$  for  $P \geq 2\tau$
  - $\mathcal{P}_{2,3} = 1/2 \pi \tau \Delta_{XY;2} / \Delta_{XY;3} \sim P$  for  $P \geq 2\tau$ 
    - $\mathcal{P} \sim P$  for  $P \ll \tau$
    - $\mathcal{P}$  oscillates strongly for  $P \sim [0.5, 1] \times \tau$
    - $\mathcal{P}$  decays (exponentially) towards  $P$  for  $P \sim [1, 2] \times \tau$
- **Masses** follow immediately once  $P$  is known

# Motion of Primary: Position Differences

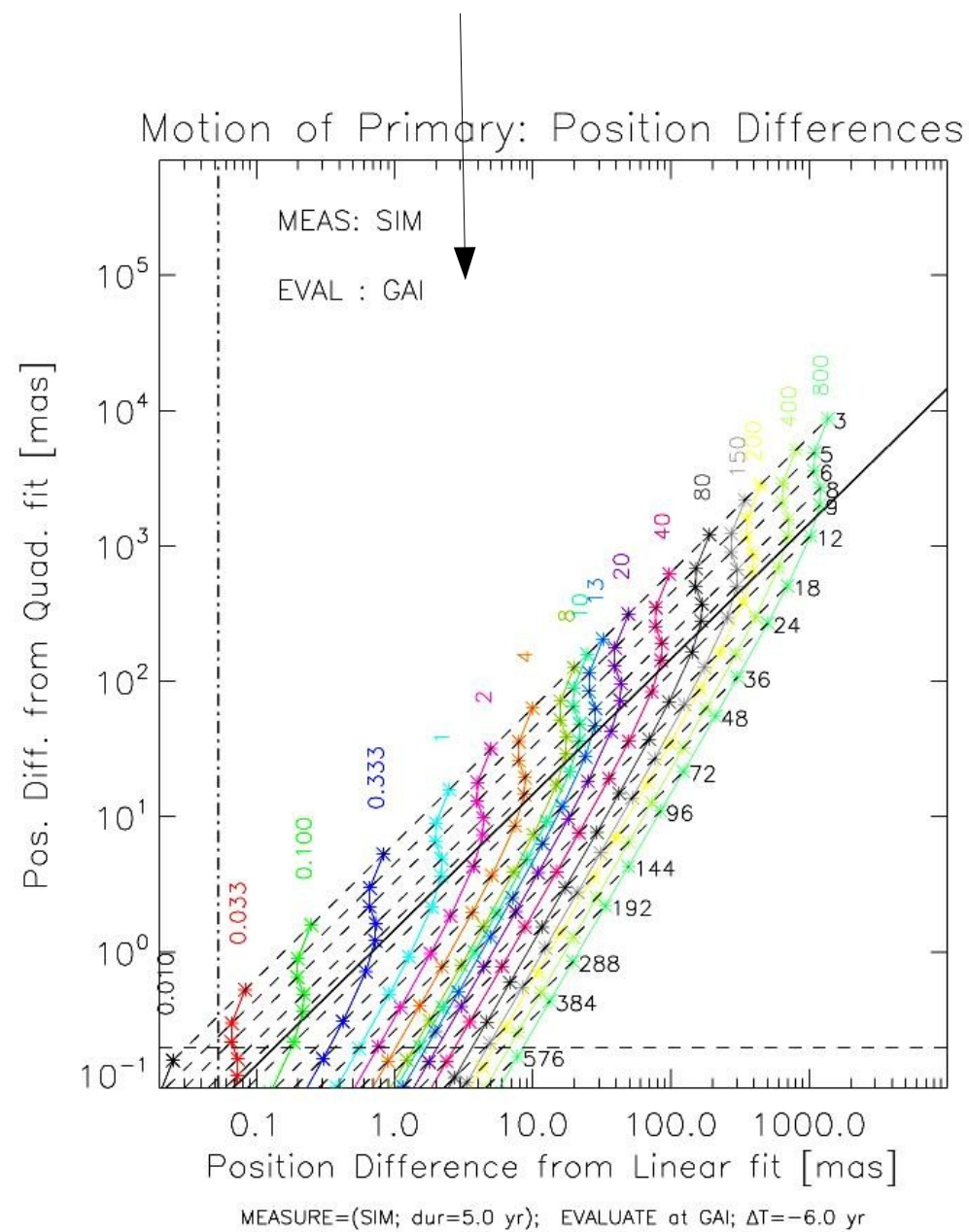
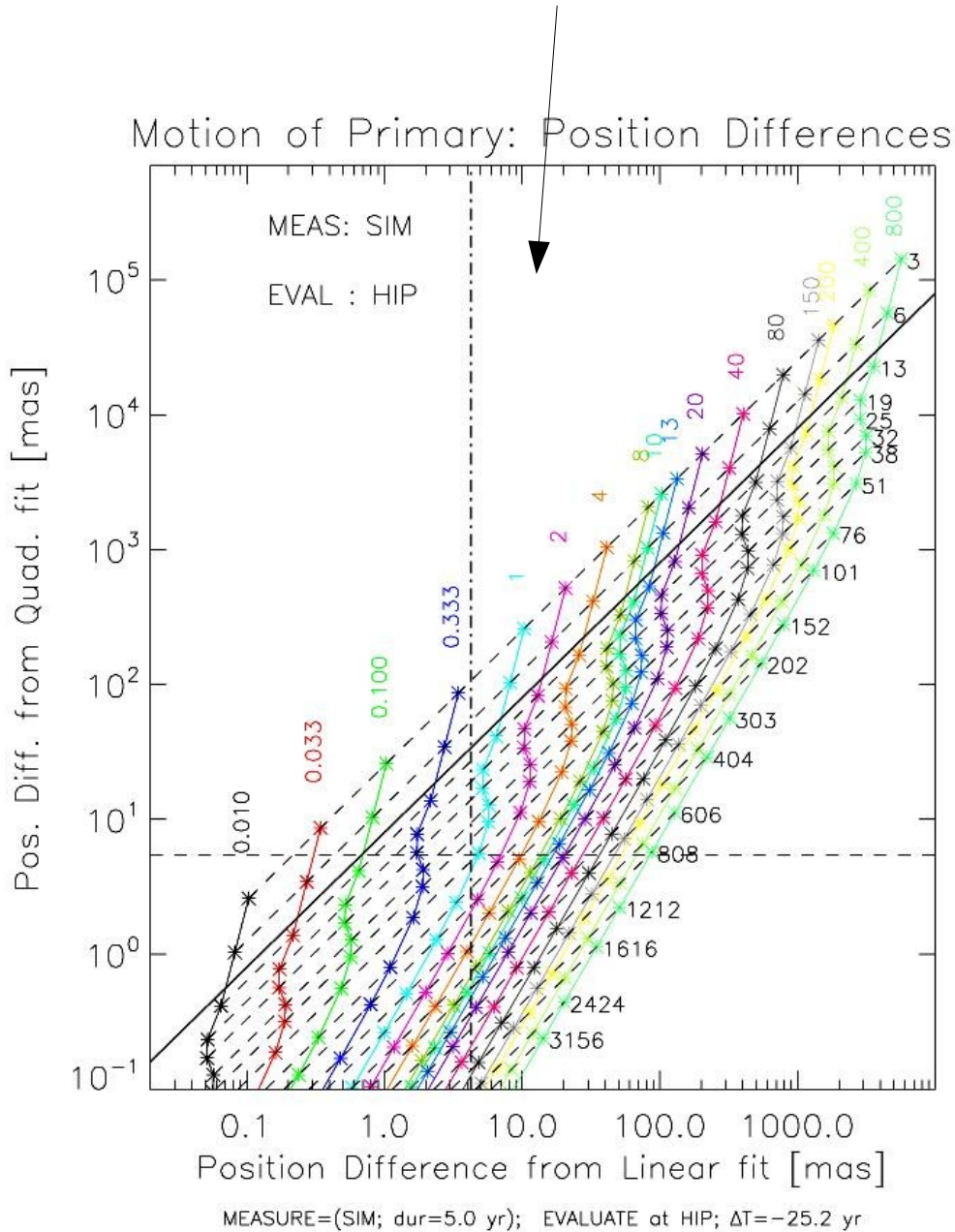


**Lift Degeneracy**  
when considering  
**quadratic fit**

**Analytically proven**



# SIM --> HIP vs. SIM --> GAIA



# • SIM & HIPPARCOS

- 1  $M_J$  and up;  $P < \sim 80$  yr
- 13  $M_J$  and up;  $P < \sim 160$  yr
  - **Improved 2<sup>nd</sup> generation Hipparcos @ 1/3 mas**
  - **GAIA data can be used to re-reduce Hipparcos to eliminate any residual systematic errors**
  - twice better Period Limits
    - **“Detection”** w.  $\Delta_{XY;1}(13M_J)$ :  $P < \sim 800$  yr

# • SIM & GAIA

- Characterization:  $\frac{1}{2}$  period range
- Detection: 50% larger period range

- **SIGNIFICANCE:** **x5 – x15 better**

- **Lower-mass range extended by x5 to 0.2  $M_{JUPITER}$**



# Conclusions & Future Work

- **Position Difference Powerful New Tool**
- **To find long-period objects** (initial estimates: P & M)
  - Samples the migration-cutoff regime (100s of years)
- **Need to develop method for generalized orbits**
  - **Expectations are:**
    - **Inclination not too important**
    - **Eccentric orbits: manageable** [MK2005]
    - **Orbit fitting employing historical data?**
- **Realistic observing time estimates**
  - **Local reference frames?**

# Backup Slides

# Dabblings

- I've been working on:
  - Astrometric detections (**FAME, AMEX, OBSS**)
  - Transit detections (**FAME, AMEX, OBSS**)
    - **FAME**: now-cancelled astrometric MDEX (USNO-led)
    - **AMEX**: proposed Germany/NASA/USNO SMEX
    - **OBSS**: proposed “Origins Probe” mission
      - Capable of duplicating GAIA, if necessary
  - Radial velocity work & TPF-C characterizations
    - Dispersed Fourier Transform Spectrometer
      - P.I., Arsen Hajian (USNO; now U. Waterloo)
  - **LEAVITT**: my MDEX-class planetary-transit finder
- **Solar System Analogs (SOSAs; 2008-present)**  
[http://www.astro.umd.edu/~olling/index\\_1.htm#My\\_Astrometry\\_Latest](http://www.astro.umd.edu/~olling/index_1.htm#My_Astrometry_Latest)

# Astrometry: Number Estimates

[http://www.astro.umd.edu/~olling/FAME/otm\\_plas\\_rpo\\_2004\\_01.pdf](http://www.astro.umd.edu/~olling/FAME/otm_plas_rpo_2004_01.pdf)

## Procedure:

Semi-major axis:

$$a = 95/d_{10\text{pc}} (P_{\text{YR}}/M_{\text{TOT,SUN}})^{2/3} M_{\text{PLANET,JUP}} \quad [\mu\text{as}]$$

x/y coordinates for face-on orbit

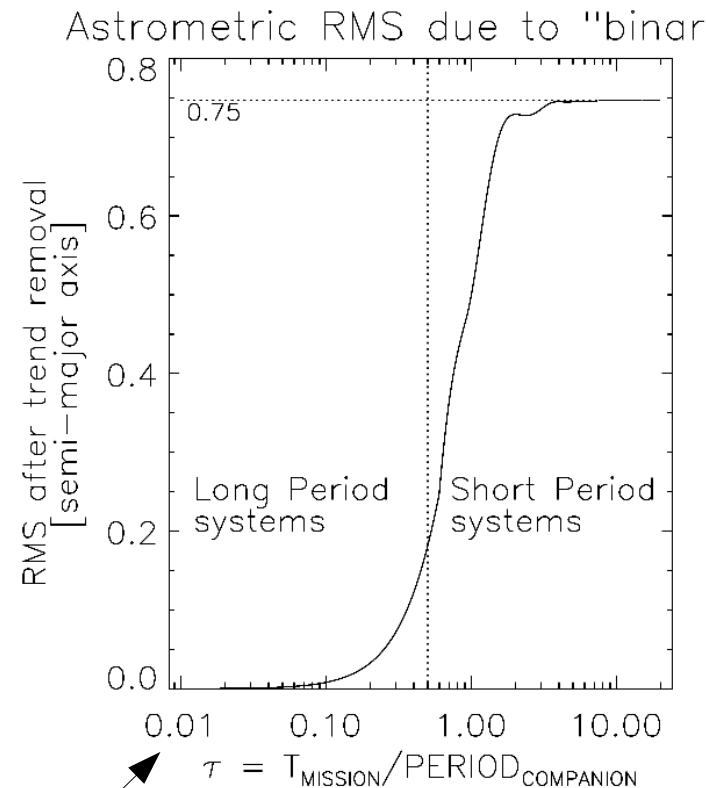
$$x = a \cos(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \quad [\mu\text{as}]$$

$$y = a \sin(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \cos(i) \quad [\mu\text{as}]$$

$$dx/dt = -2\pi/P_{\text{YR}} a \sin(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \quad [\mu\text{as/yr}]$$

$$dy/dt = +2\pi/P_{\text{YR}} a \cos(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \cos(i) \quad [\mu\text{as/yr}]$$

- For each model, compute x,y and fit linear proper m. model
- Compute RMS w.r.t. best fit
  - Depends **only** on  $\tau = T_{\text{MISSION}} / \text{Period}_{\text{COMPANION}}$
  - Turns out: "RMS"/a = Function(  $\tau$  )



# Astrometry: Number Estimates

[http://www.astro.umd.edu/~olling/FAME/otm\\_plas\\_rpo\\_2004\\_01.pdf](http://www.astro.umd.edu/~olling/FAME/otm_plas_rpo_2004_01.pdf)

## Procedure, cntd:

For a given astrometric error ( $\delta_{AE}$ ) per observation

- In Short Period Regime: “RMS<sub>SPR</sub>”  $\sim 0.75 * a$
- Detection: “RMS<sub>SPR</sub>” =  $\frac{3}{4} * a \geq N_{\sigma} * \delta_{AE}$
- $d'_{MAX;SPR} \sim 7.5 (P_{YR}/M_{TOT})^{2/3} M_{PL,JUP} (10/N_{\sigma}) * (10 \mu\text{as}/\delta_{AE})$  [pc]
- **In Long Period Regime: “RMS<sub>LPR</sub>”  $\geq \sim N_{\sigma} * \delta_{AE}$**
- $d'_{MAX;LPR} \sim 8.3 \tau^2 (P_{YR}/M_{TOT})^{2/3} M_{PL,JUP} (10/N_{\sigma}) * (10 \mu\text{as}/\delta_{AE})$  [pc]
- Photometric distance & magnitude-dependent  $\delta_{AE}$  introduces  
 $d_{MV} = 10^{(V_F+5-M_V)/5}$  and:
  - $d_{MAX;SPR;MV} = \text{sqrt}( d'_{MAX;SPR} * d_{MV} )$
  - $d_{MAX;LPR;MV} = \text{sqrt}( d'_{MAX;LPR} * d_{MV} )$

$d_{SPR}$  INcreased with P

$d_{LPR}$  DEcreased with P

Primary HAS to be closer

than  $d_{SPR}$  &  $d_{LPR}$

Maximum distance @

$$P(d_{MAX}) \sim 0.82 T_{MISSION}$$

$$\text{Volume}(P) \sim \frac{4}{3} \pi d(P)^3$$

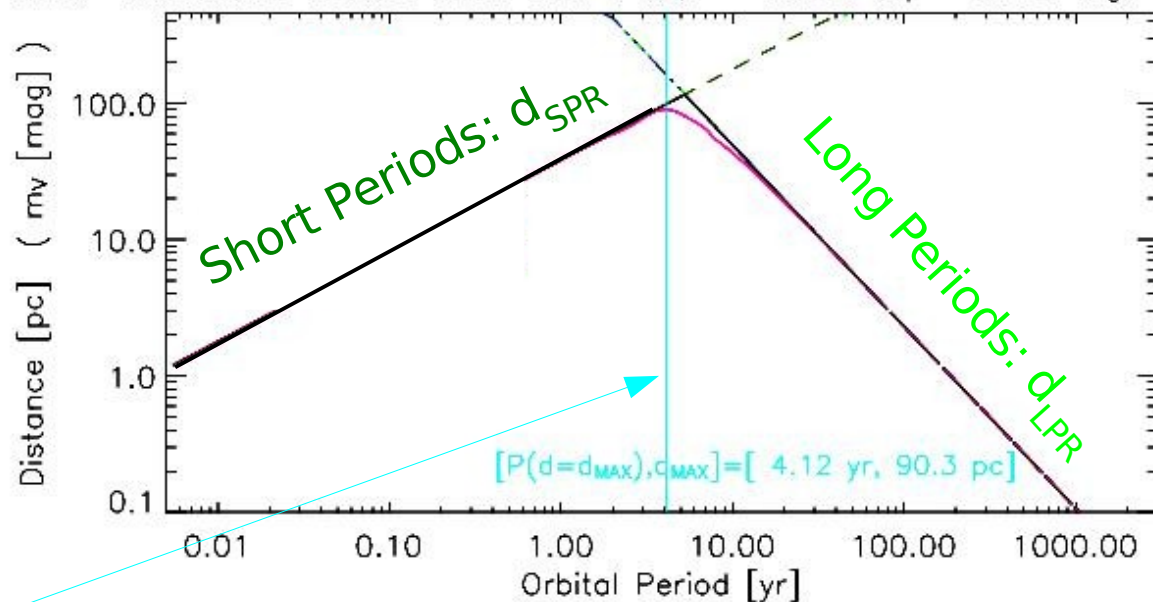
drops of quickly

either side of  $P(d_{MAX})$

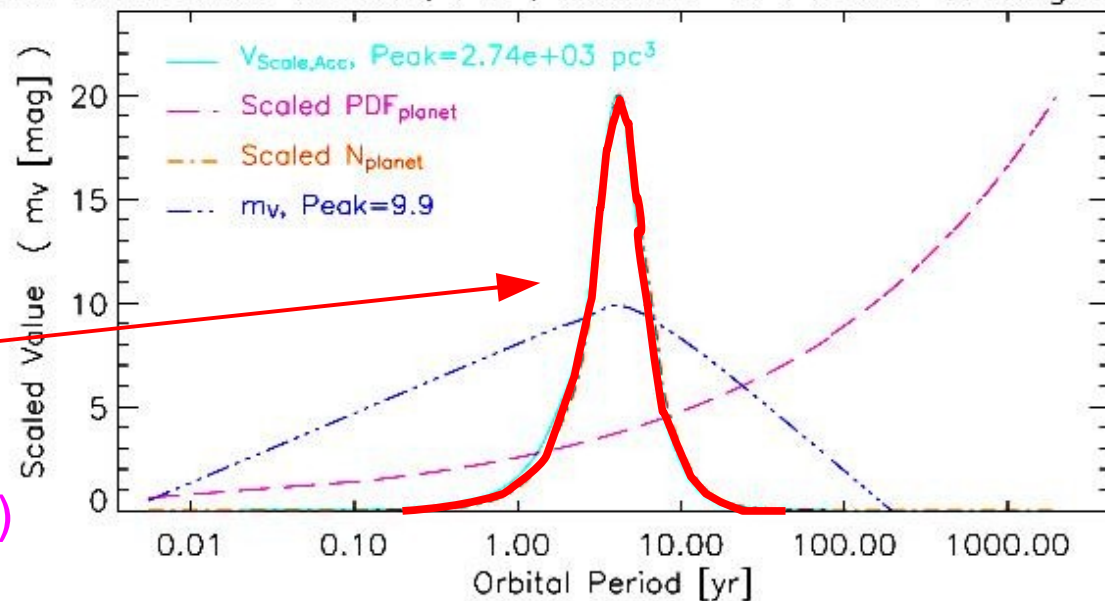
=> Most companions @  $P(d_{MAX})$

=> ... @ small range in  $m_v$

ESGP Distance Limits for: G5V :  $M_* = 0.95$ :  $M_p = 6.00 M_J$ :  $N_\sigma = 1$

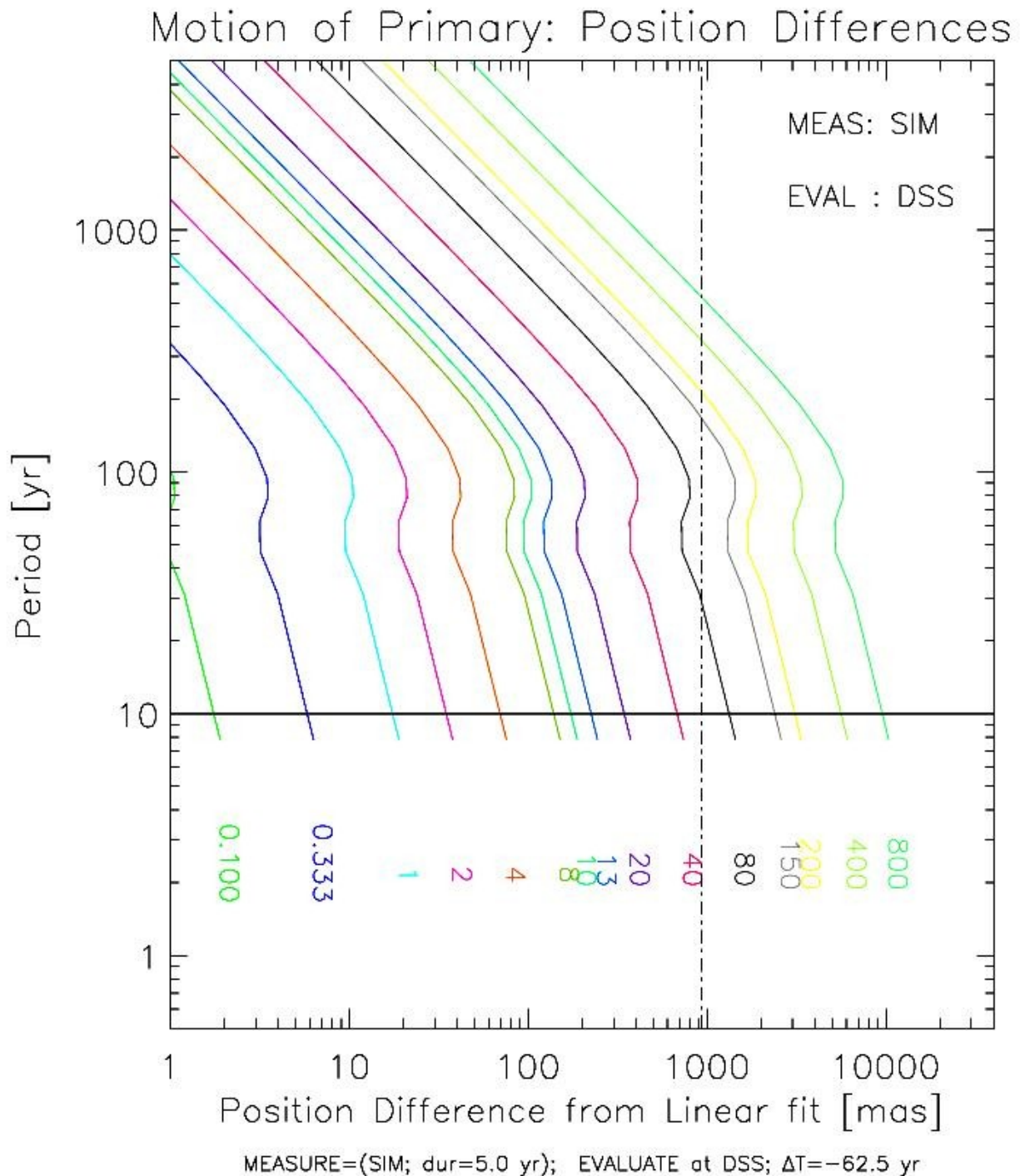


Scaled Accessible Volume, PDF, Number of Planets & Magnitude



# SIM & Ground-based Surveys

**DSS (1957)**

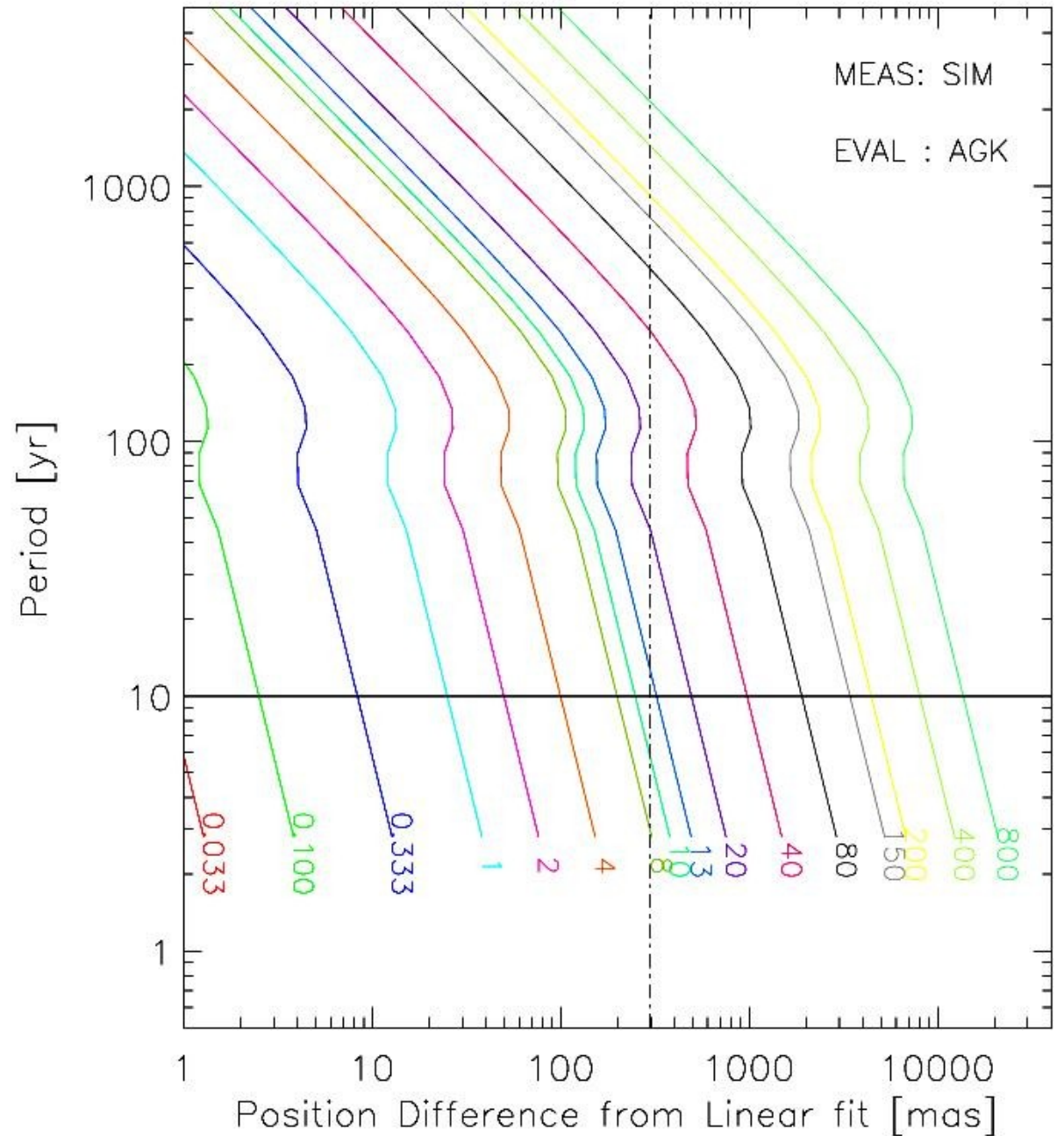




# SIM & Ground-based Surveys

**AGK (1930)**

Motion of Primary: Position Differences

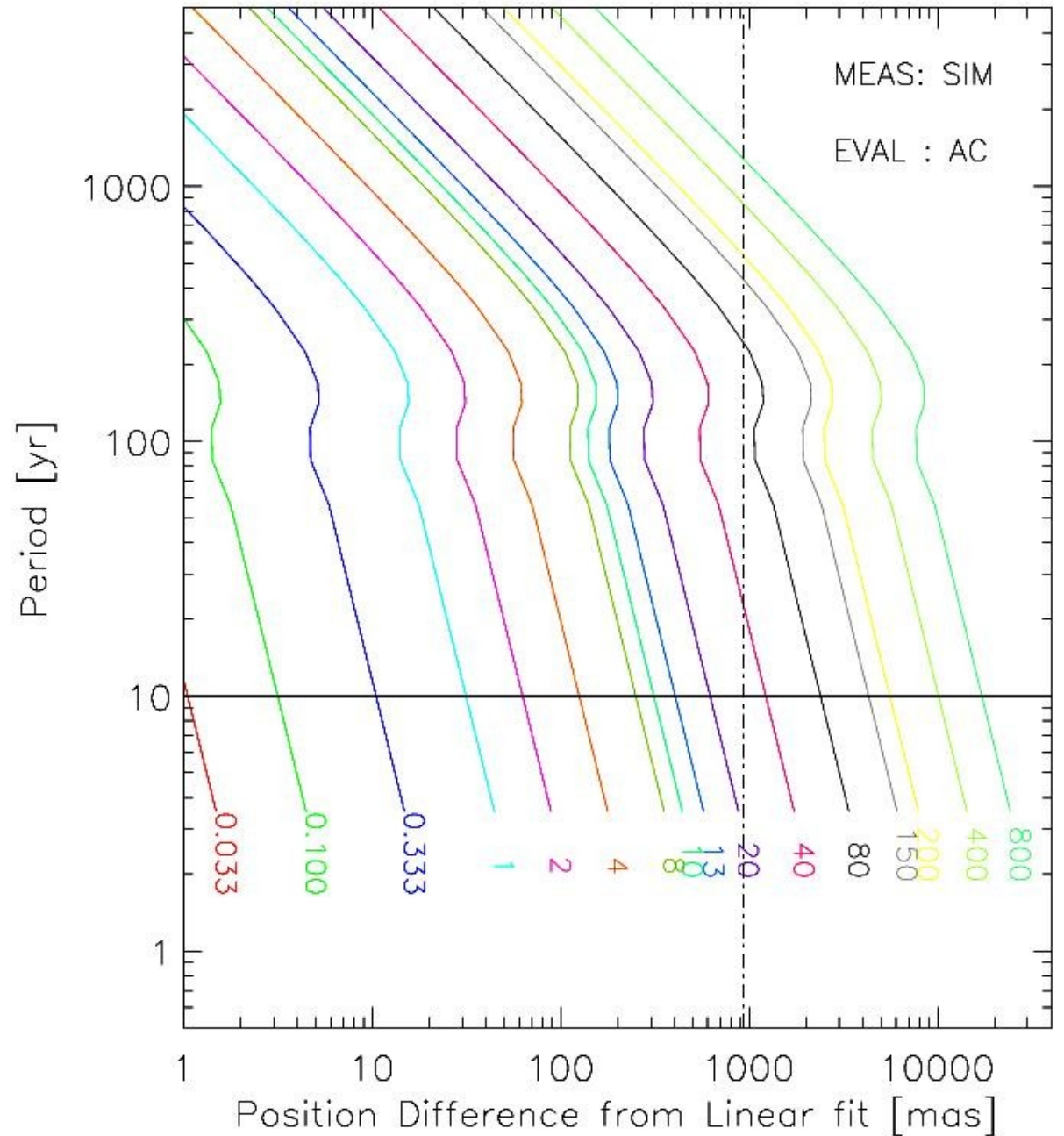


MEASURE=(SIM; dur=5.0 yr); EVALUATE at AGK;  $\Delta T = -89.5$  yr

# SIM & Ground-based Surveys

AGC (1907)

Motion of Primary: Position Differences



MEASURE=(SIM; dur=5.0 yr); EVALUATE at AC;  $\Delta T = -112.5$  yr