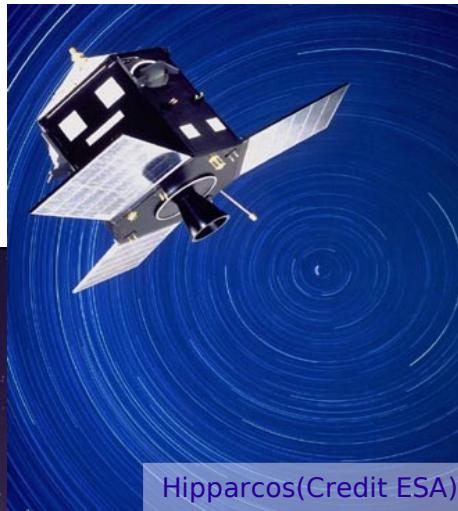


Connecting Stars (their planets), Galaxies and the Universe in the Decade of Astrometry

Rob Olling (UMd)



SIM/Heavy (Credit JPL)



Hipparcos(Credit ESA)

Hipparcos: 3 years,
early 1990s: **mas accuracy**



GAIA (Credit ESA)

SIM-Lite: 5-10 yrs; 201?+; **1/1000 mas**

GAIA: 5-7 years, 2012+; **1/100 mas**

Outline

Initiated at USNO while working on various astrometric missions: FAME, AMEX & OBSS

(2000-2006.5: http://www.astro.umd.edu/~olling/index_1.htm#My_Astrometry_USNO

Transits w. scanning (Astrometric) Missions

LEAVITT: 10,000 Transiting planets down to $R_{\text{EARTH}} * 4.6$

Astrometric Scales in Astronomy

Astrometric Detections of Planets

OBS/GAIA & **SIM**

Long Period Planets (Solar System Analogs)

Observability: Where/Why?

Traditional search method

Position Differences

Hipparcos to the Rescue

Period & Mass determination

Stars, their Planets, Galaxies & Universe

See also: <http://www.astro.umd.edu/~olling>
<http://adsabs.harvard.edu/abs/2007arXiv0704.3072O>
<http://adsabs.harvard.edu/abs/2009arXiv0902.3197O>

Planetary Transits

- Planetary Transits and stellar eclipses are “*episodic*” and are much harder to discover than *variable stars*
- **The probability of observing a transit:**

$$Pr_{TRANS}^{OBS}(P) dP \sim \int dP' Pr_{TRAN}(P'; R_{STR}) Pr_{EO}(P'; R_{PL}) Pr_{DET}^N(P')$$

- Pr_{TRAN} = % of time spent in transit
= “duration of transit”/ “orbital period”
~ “Diameter of star” / “ $2 \times \pi \times$ semi-maj-ax”
- Pr_{EO} = Probability that the system is edge-on
= angle subtended by PL as seen from star
 \propto “Diameter of Planet” / “semi-major axis”
- Pr_{DET}^N = Probability that N transits are observed with some observing strategy

Planetary Transits w. Scanning Astrometric Telescopes:

- To improve changes of finding transits:
 - Can only try to ↑ Pr_{DET}
- Need: Large number of observations
 - Covering ranges from hours (transit duration) to days to weeks (orbital period)
 - Latter one set by “repetition rate”
 - Hipparcos $100/3\text{yr} = 1 \text{ per } 7.7 \text{ “days”}$
 - FAME: $129/5\text{yr} = 1 \text{ per } 9.9 \text{ “days”}$
 - GAIA: $60/5\text{yr} = 1 \text{ per } 21.3 \text{ “days”}$
 - LEAVITT: $183/5\text{yr} = 1 \text{ per } 7.0 \text{ “days”}$
 - High photometric fidelity during time of transit say $>\sim 4$ obs/transit(s)

HIPPARCOS/FAME/LEAVITT-like instruments are “good” for transit detections (GAIA spins too slowly)

PTs w. Astrometric Telescopes: Detection Efficiency

Resulting detection efficiency depends critically on cadence

Efficiency from:

Cadence

Edge-on probability
from period distribution (PDF_{PLAN})

Duration of transit
==> $T_{\text{TRANS}}/P_{\text{ORBIT}} = R_{\text{PLAN}}/R_{\text{STAR}}$

Number of Stars surveyed

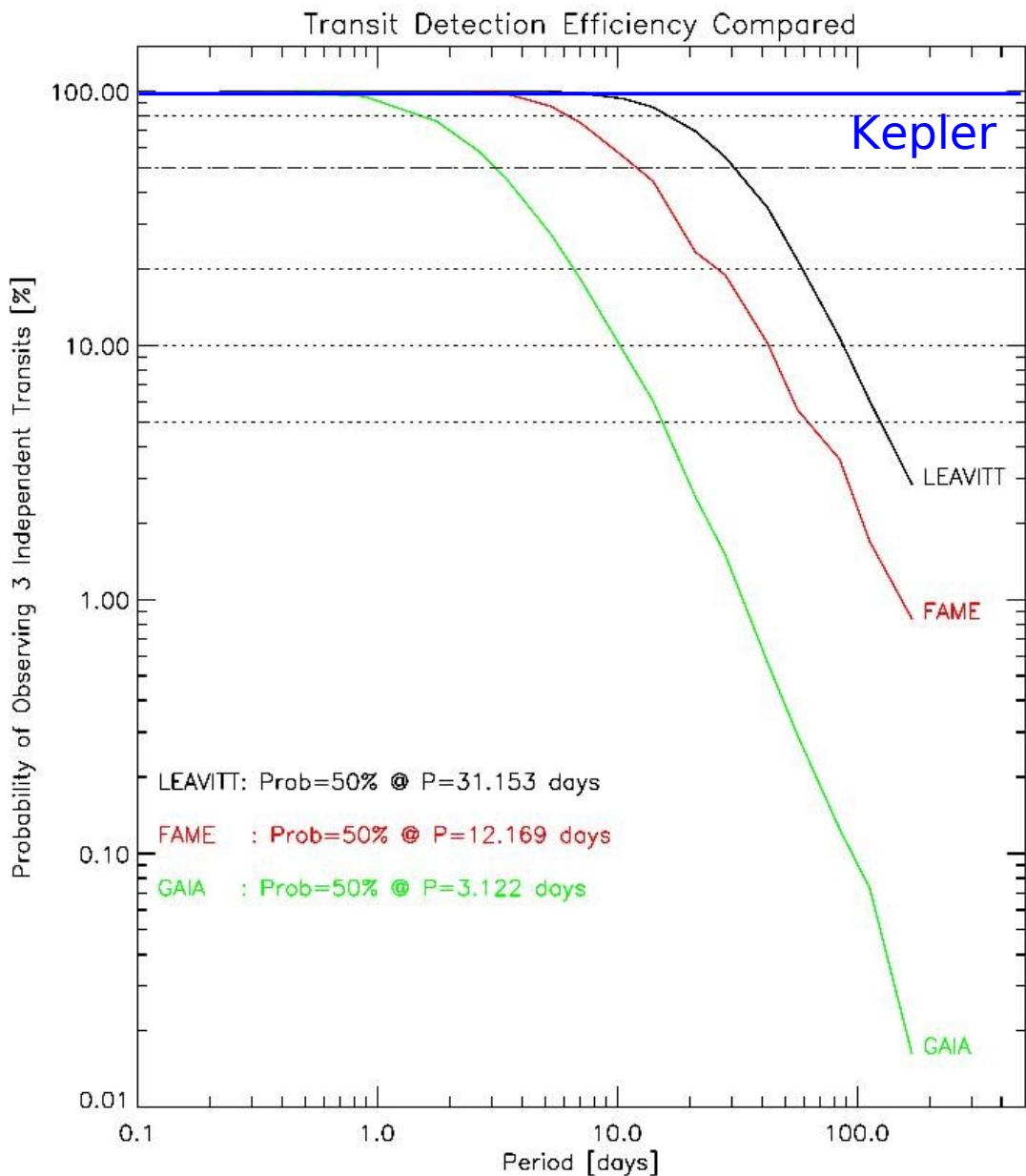
Bottom Line:

LEAVITT: good

Kepler: int./good

FAME: intermediate

GAIA: poor



Planetary Transits w. Astrometric Telescopes:

Many observations (CCD transits) per “epoch” ==> good sensitivity
for 2 mmag accuracy per 2.5 hour & not saturated & GV primary
“Maximum” possible number of Extrasolar Giant Transiting Planets (1/2,000)

NOT Corrected for observing efficiency

H: $V = [0.0, 9.5]$ mag => $N'_{EGP} = 95$; $R_{PLANET} = [0.9, 1.5] R_{NEPTUNE}$

F: $V = [5.2, 10.8]$ mag => $N'_{EGP} = 341$; $R_{PLANET} = [0.5, 2.0] R_{NEPTUNE}$

G: $V = [11.0, 15.9]$ mag => $N'_{EGP} = 26,800$; $R_{PLANET} = [0.7, 2.1] R_{NEPTUNE}$

L: $V = [5.8, 14.8]$ mag => $N'_{EGP} = 12,000$; $R_{PLANET} = [0.3, 2.4] R_{NEPTUNE}$

K: $V = [10.0, 14.0]$ mag => $N'_{EGP} = 1,000$; $R_{PLANET} = [0.3, 2.4] R_{NEPTUNE}$
mostly from reflection effects (phase variations)

T: $V = [4.5, 13.5]$ mag => $N'_{EGP} = 1,700$; $R_{PLANET} = [1.0, 2.4] R_{NEPTUNE}$

H =Hipparcos

F =FAME

G =GAIA

L =LEAVITT

K =Kepler

T =TESS (NASA/SMEX)

Quantity	unit	Symbol	FAME	GAIA	LEAVITT	TESS
Mission Type			<i>MIDEX</i>	<i>"PROBE"</i>	<i>MIDEX</i>	<i>SMEX</i>
Mission Duration	Years	t_{MIS}	5	5	5	2
In-scan Mirror Size	cm	D_I	40	140	55	13.3
X-scan Mirror Size	cm	D_X	9	50	14	13.3
Photon-collecting power			1	19.4	2.1	2.9
Time to cover accessible sky = Median re-visit Time	days	$t_{\text{SKY},70\%}$	28	35	7.5	
In-scan Field of View	degrees	FOV_I	1.1	0.74	3.5	
Total Number of broad-band observations		N_{BB}	2,684	1,057	10,253	
Epoch Duration	hours	t_{EPO}	2.73	3.69	6.60	
Average # Broad-Band Observations per Epoch		$N_{\text{BB/EPO}}$	22.4	26.8	61.0	
# of Independent Epochs		N_{EPO}	120.1	39.4	168.0	
# of Photometric Observations per band (R=3; R=2 for FAME)		N_{RS}	244.0	96.1	10,253.4	
Average # Photometric Observations per Epoch		$N_{\text{PHO/EPO}}$	2.0	2.4	61.0	
Photometric Saturation Level [mag]		V_{SAT}	5.21	10.69	5.76	4.5
V magnitude for 2 mmag photometry in 0.83 hr	magnitude	V_{2mmag}	10.81	15.59	14.83	13.5
Number of Stars Surveyed	10^6	$N_{\text{s,TR}}$	1	73	36	2.5
Minimum Planetary Radius (GV)	R_{NEPTUNE}	$R_{\text{PL,MIN}}$	0.51	0.68	0.30	1.00
Number of <u>Planetary Transits</u> (AV, FV, GV, KV & MV stars)		$N_{\text{EXOP,BB}}$	115	2,279	10,451	1,687
Number of <u>Planetary Transits</u> (AV, FV, GV, KV & MV stars) & PHOTOMETRIC CHARACTERIZATION		$N_{\text{EXOP,PHOT}}$	10	400	5,777	
Number of <u>Eclipsing Binaries</u> (AV..MV stars) & PHOTOMETRIC CHARACTERIZATION		$N_{\text{EB,PHOT}}$	1,091	9,246	79,572	2,111
Orbital Period with $P_{\text{DET}} = 50\%$ for 5 Transits, FROM SCANNING LAW	days	$P_{\text{50\%,D=3,S,CAN}}$	6.24	1.48	16.13	2.5

Mission Parameters & Abilities Compared

(More detailed talk available)

Astrometric Scales in Astronomy

Parallaxes, in μas

α Cen: 742,000

RR Lyra: 4,380

δ Cep: 3,320

1 kpc: 1,000

Gal. Center: 125

LMC: 20

M 31: 1.5

Proper Motions, in $\mu\text{as/yr}$

α Cen: 3,600,000

RR Lyra: 200,000

δ Cep: 16,500

10 km/s @ 1 kpc: 2,110

200 km/s @ 8 kpc 5,275

50 km/s @ LMC: 211

200 km/s @ M 31: 60

USA @ 10 pc 2.9 ; 2 M_{EARTH} @ 10 pc: 1 $\mu\text{as/yr}$

Astrometric Detections:

- Solar wobble (reflex motion) due to Solar System planets as “seen” from 10 pc
 - Period: ~ 30 years
 - Amplitude: ~ 1 mas ~ 1σ for Hipparcos

Show: Solar System Reflex Motion Animation

Astrometric Detections:

- Earth-induced reflex motion:
 - Size of Orbit: 1 AU
 - 93 million miles = 150 million km
 - $M_{\text{SUN}} / M_{\text{EARTH}} \sim 333,000$
 - Size of reflex-orbit: $1/333,000$ AU
 - $\sim 3 \mu\text{AU} \sim 280 \text{ mi} \sim 450 \text{ km}$
[Washington, DC <---> New York, NY]
 - From 1 pc $\rightarrow 3.0 \mu\text{as}$
 - From 10 pc $\rightarrow 0.3 \mu\text{as}$

Astrometric Detections:

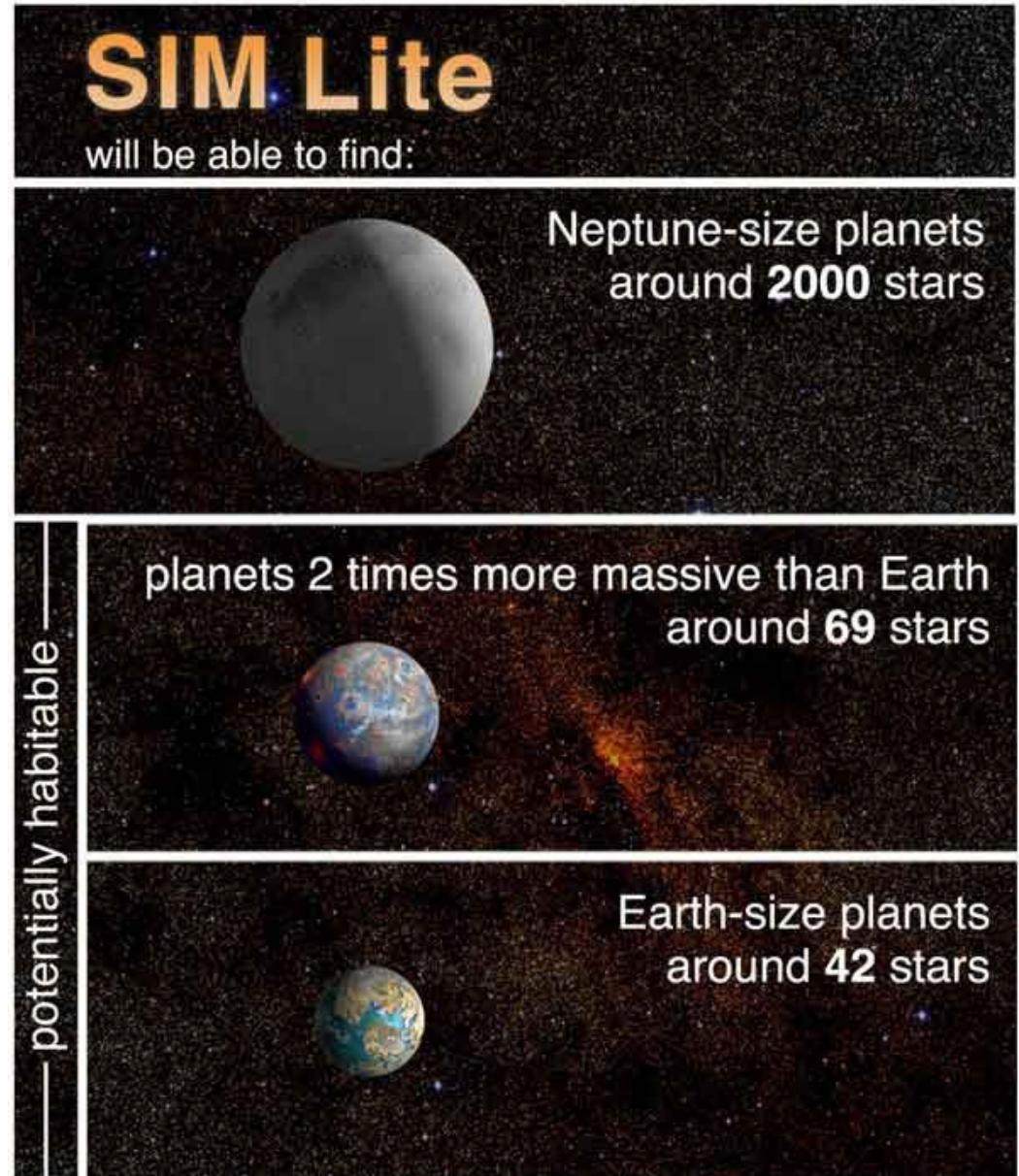
- How big is 1 μ as?
- Show JPL Movie

Astrometric Detections:

- How big is 1 μ as?
A Dime at the Moon, really?
 - Dime: ~2 mm
 - Earth-Moon ~ 376,000 km
 - \angle Dime@Moon = $2.0 \cdot 10^{-3} \text{ m} / 3.76 \cdot 10^8 \text{ m}$
~ $5.3 \cdot 10^{-12} \text{ rad}$
~ $3.0 \cdot 10^{-10} \text{ deg}$
~ $1.1 \mu\text{as}$

SIM-Lite Astrometric Observatory:

Planet Finding Capabilities



Astrometric Detections: Surveys

Advantages of large numbers and/or high accuracy

Large numbers: find rare objects
(e.g., **old, high [Fe/H] stars**)

accurate statistics/general properties of majority

Identify ES planetary systems

High accuracy: \Rightarrow
characterize individual objects

Astrometry for $300-\infty \times M_{EARTH}$

OBSS/GAIA

Detections: 5σ : 28,000 = 28*Kepler

Orbits: 15σ : 3,200 = 3*Kepler

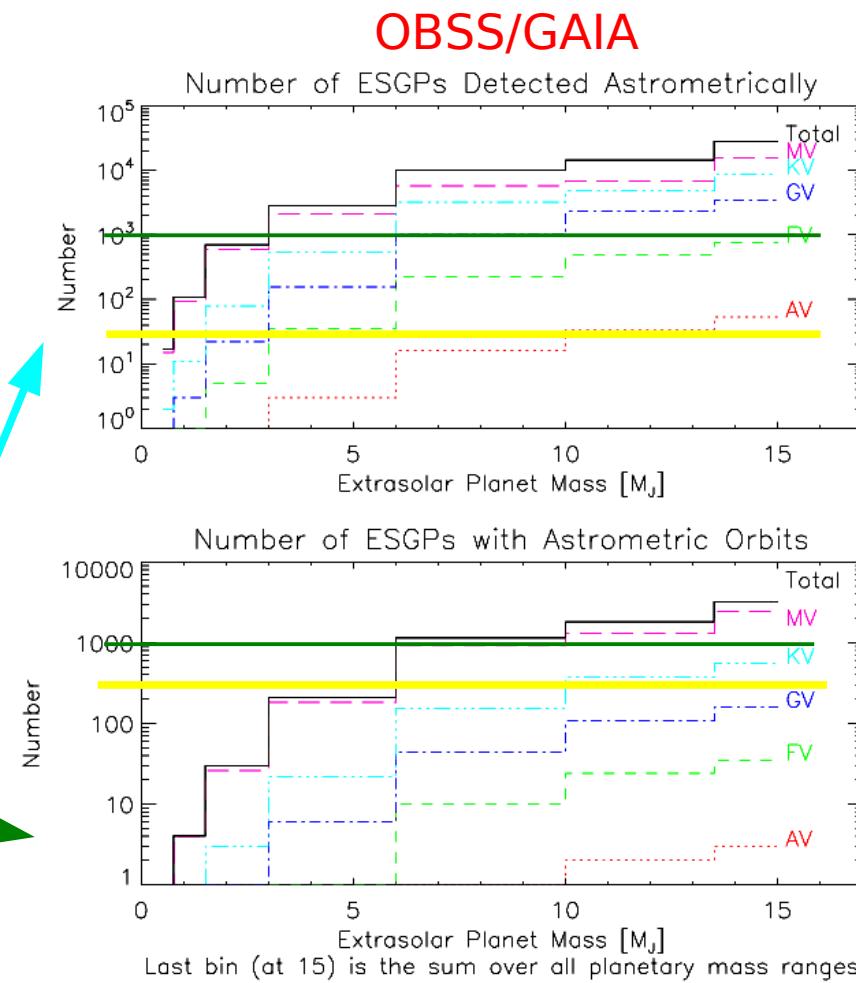
SIM-Lite: 2,000; 69; and yes, 42

$M_{EARTH} \times$

17

2

1

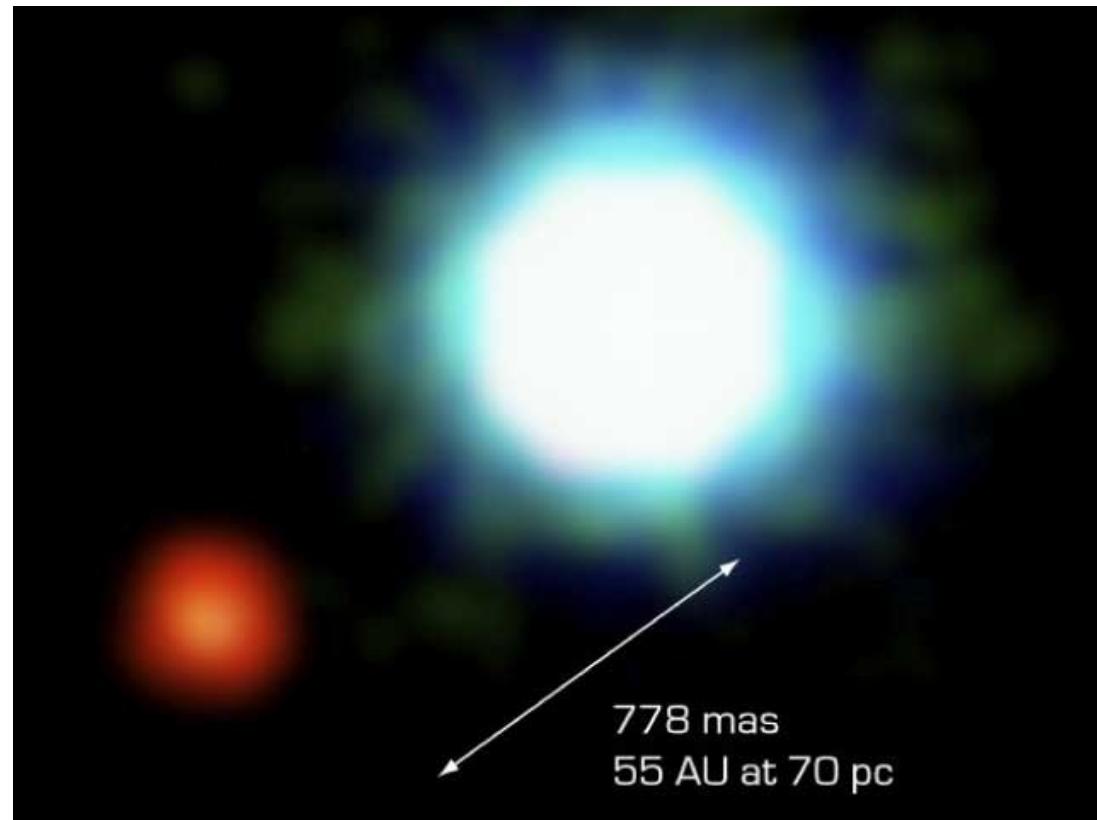


Long Period Objects (Planets, BDs, Stars)

w. Ed Shaya (UMd)

Young objects are still bright: can be detected with direct imaging, today!

Ironically, the first $5 M_{JUPITER}$
extrasolar *object* was
discovered by Chauvin et al,
(2004) around a
brown dwarf,
====> not a planet
1st brownplanet



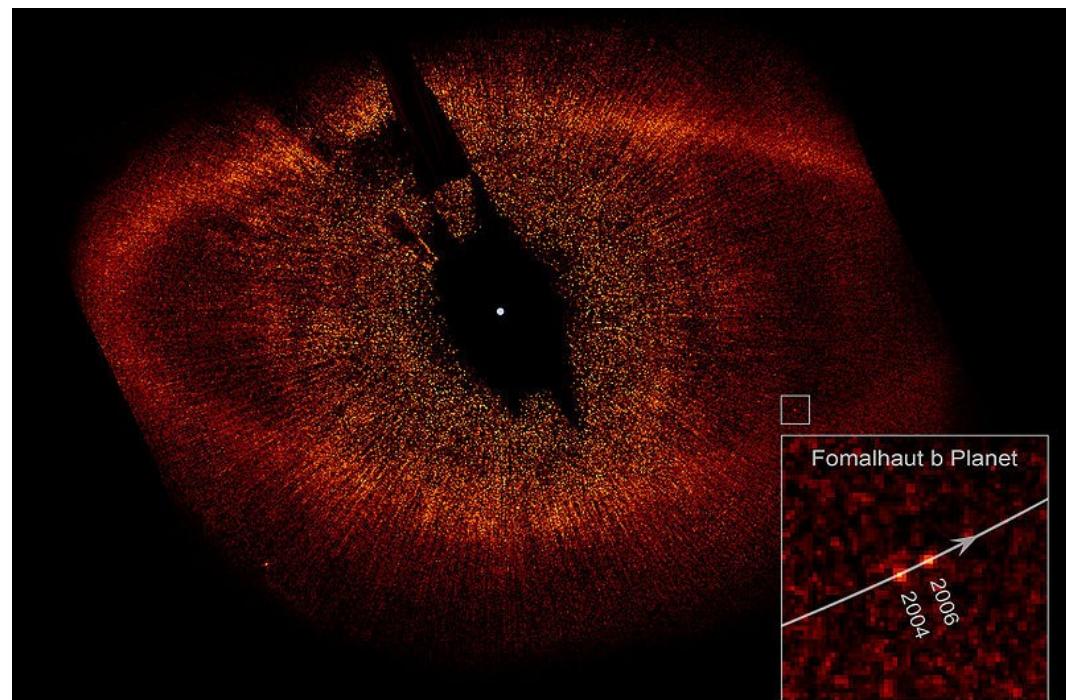
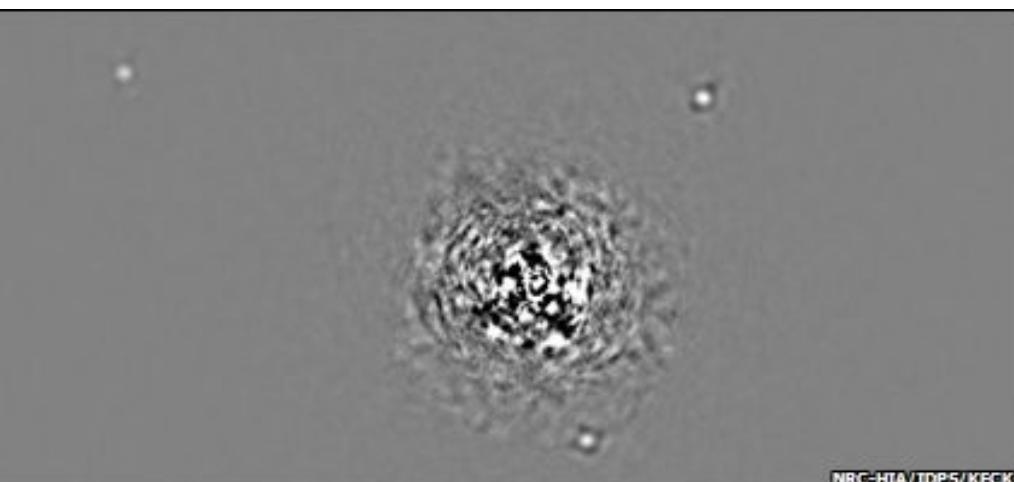
Distance actually 52 pc --> ~39 AU (Pluto)

Long Period Objects (Planets, BDs, Stars)

First planetary mass object orbiting a STAR discovered 4 years later:

Fomalaut [Kalas et al., 2008]

HR 8799 [Marois et al 2008]



Long Period Objects (Planets, BDs, Stars)

However, these objects cool down steadily

After a while, they are all but invisible.

Only census possible via RV or astrometry

Long Period Planets: Where/Why?

Some Planetary Migration Theories predict

Inward migration (known “RV” planets)

Outward migration (Uranus & Neptune)

Outer edge: 50-100 AU (350 – 1,000 yr) [Ida & Lin, 2004]

As per currently known systems (3)

Predict massive long-period planets

Would require more massive disk

Without migration: 30-40 AU (165-250 yr)

MUCH, MUCH, MUCH, much longer than $2T_{\text{MISSION}}$

How to measure this?

Long Period Objects (Planets, BDs, Stars)

**For astrometry & velocimetry:
need: $P_{\text{ORBIT}} < \sim \text{twice observing span}$
to determine P_{ORBIT}**

Most of Solar System's angular momentum is in Jupiter & Saturn:

Solar System Analog:

“Jupiter” and/or “Saturn” and/or Uranus/Neptune

All outer planets
have $P_{\text{ORB}} > 2 T_{\text{MISSION}}$

Planet	AU	Period	Mass
Jupiter	5.2	11.9	318
Saturn	9.5	29.4	95
Uranus	19.2	84.0	15
Neptune	30.1	164.0	17

How Many Long-Period Planets?

Which long-period planets:

SOSAs: $P \in [11.9, 165]$ yr
 $M \in [0.05, 1] M_{JUP}$

HOSAs: $P \in [11.9, 165]$ yr
 $M \in [1, 13] M_{JUP}$

Fraction of Planetary Systems:

[Tabachnik & Tremaine (2002) or Cumming et al (2008)]

SOSAs: 13 % of planetary systems

HOSAs: (17 +/- 3)% of planetary systems

HOSAs: 8 % of Sun-like stars

Only around Nearby/Bright Stars

Sun-like stars are really bright

GAIA saturates at V~12, but usable to V~6

MS Star	F5	G0	G5	K0	K5	F5	G0	G5	K0	K5	[*/pc ³] / 1000	
MV(abs)	3.5	4.4	5.1	5.9	7.4	2.35	4.13	5.9	7.63	13.1		
Distance [pc]	apparent magnitude					Number of Stars out to D _{pc}					Total # Stars	Total # HOSAs
5	2.0	2.9	3.6	4.4	5.9	1	2	3	4	7	17	1.4
10	3.5	4.4	5.1	5.9	7.4	10	17	25	32	55	139	11.1
20	5.0	5.9	6.6	7.4	8.9	79	138	198	256	439	1,109	88.7
30	5.9	6.8	7.5	8.3	9.8	266	467	667	862	1,482	3,744	299.5
40	6.5	7.4	8.1	8.9	10.4	630	1,106	1,582	2,044	3,512	8,874	709.9
60	7.4	8.3	9.0	9.8	11.3	2,126	3,732	5,338	6,899	11,853	29,948	2,395.9
80	8.0	8.9	9.6	10.4	11.9	5,040	8,847	12,653	16,353	28,095	70,988	5,679.1
100	8.5	9.4	10.1	10.9	12.4	9,844	17,279	24,714	31,940	54,873	138,649	11,091.9

Out to 30 pc, after surveying ~3,700 stars
expect to find ~ 300 HOSAs

Some Scales

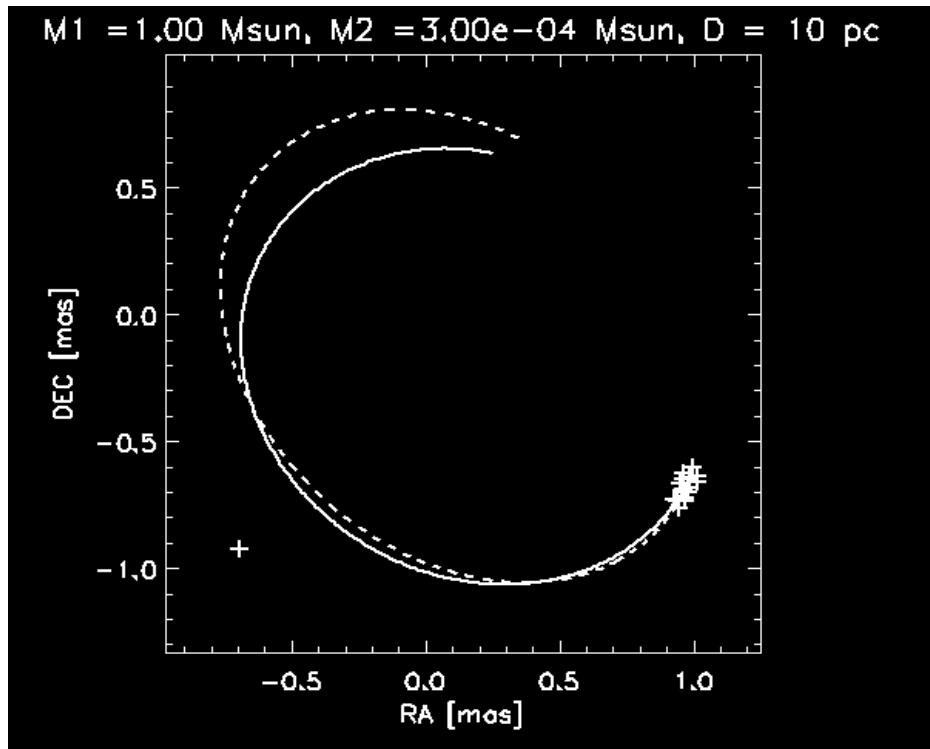
$$\begin{aligned}
 a_0 &= 95/d_{10\text{pc}} (P^{+2} M_{\text{TOT}}^{-2})^{1/3} M_{C;J} & [\mu\text{as}] \\
 |\mu| &= 600/d_{10\text{pc}} (P^{-1} M_{\text{TOT}}^{-2})^{1/3} M_{C;J} & [\mu\text{as}/\text{yr}] \\
 |d\mu/dt| &= 3800/d_{10\text{pc}} (P^{-4} M_{\text{TOT}}^{-2})^{1/3} M_{C;J} & [\mu\text{as}/\text{yr}^2]
 \end{aligned}$$

5	M_{JUPITER} @	20 pc		
Period	a_0	$ \mu $	$ d\mu/dt $	Acceleration accuracies
[yr]	[\mu\text{as}]	[\mu\text{as}/\text{yr}]	[\mu\text{as}/\text{yr}^2]	
10	1,099	690	433.8	
20	1,744	548	172.2	
40	2,769	435	68.3	3- σ ; Tycho-2
80	4,396	345	27.1	
160	6,977	274	10.8	3- σ ; GAIA 5yr
320	11,076	217	4.3	3- σ ; SIM 5yr
640	17,582	173	1.7	3- σ ; GAIA+SIM

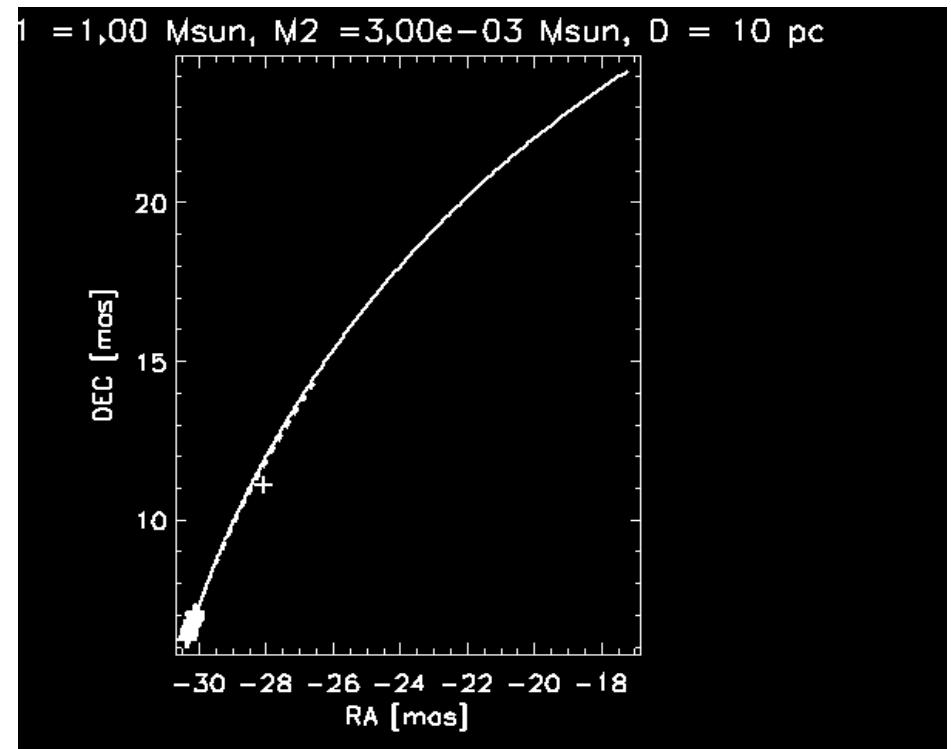
GAIA+SIM accuracy $\sim 2 \times$ smaller than SIM
 $\sim 10 \times$ smaller than GAIA

Long Period Planets have HUGE orbits

Reflex Motion from 10 pc due to 1/3rd Jupiter:



160 year orbit



1,700 year orbit

Simulated SIM-Lite Data (2017) + Hipparcos (1991)

- - - True Orbit

----- Fitted Orbit

Old-Fashioned Way of finding long-period systems: w. Hipparcos & Tycho-2

Use information from other astrometric catalogs

e.g., **Tycho-2** catalog comprises data from 144 catalogs going back to ~ 1907

Astrographic catalog (1907 @ 220 mas)

USNO's AGK3 (1930 @ 70 mas)

USNO's TAC (1980 @ 50 mas)

Hipparcos (1991 @ 1 mas)

...

Compare proper motions:

long-period cat (e.g., Tycho-2)

“mostly” center of mass motion if $P < \sim 40$ years

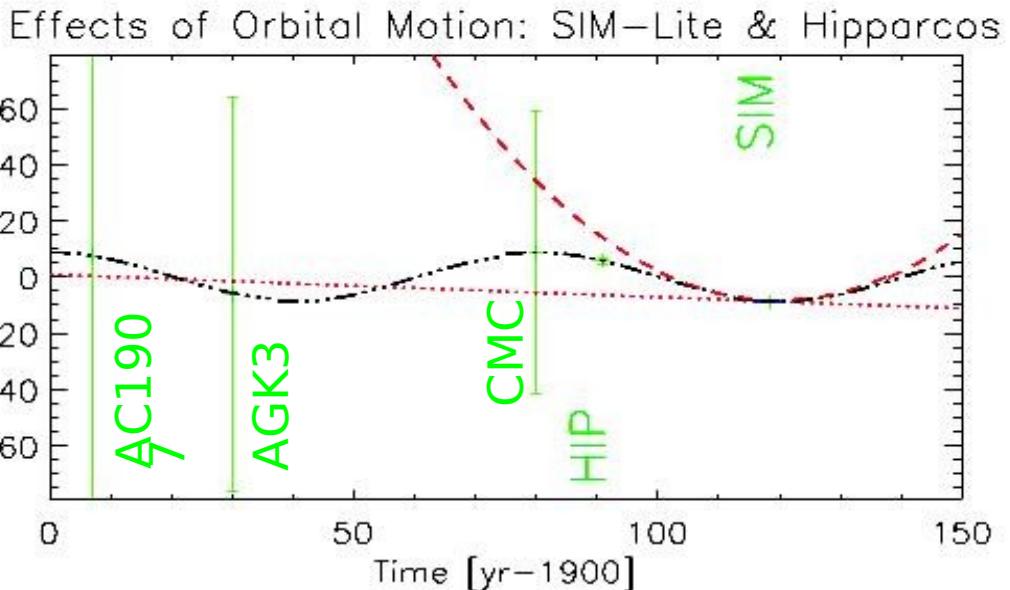
short-period cat (e.g., Hipparcos)

Binary + center of mass motion if $P < \sim 12$ years

Difference is due to binary motion

FAILS utterly if $P > \sim 4 \times T_{MISSION}$

Future Method of Finding long-period systems w. SIM & Hipparcos



$$\begin{aligned}
 M &= 10 & M_{JUP} \\
 P &= 80 & \text{yr} \\
 D &= 20 & \text{pc} \\
 a_0 &= 8.8 & \text{mas} \\
 \mu_{\text{ORBIT}} &= 0.69 & \text{mas/yr}
 \end{aligned}$$

Difference between:
backtrapolarations:

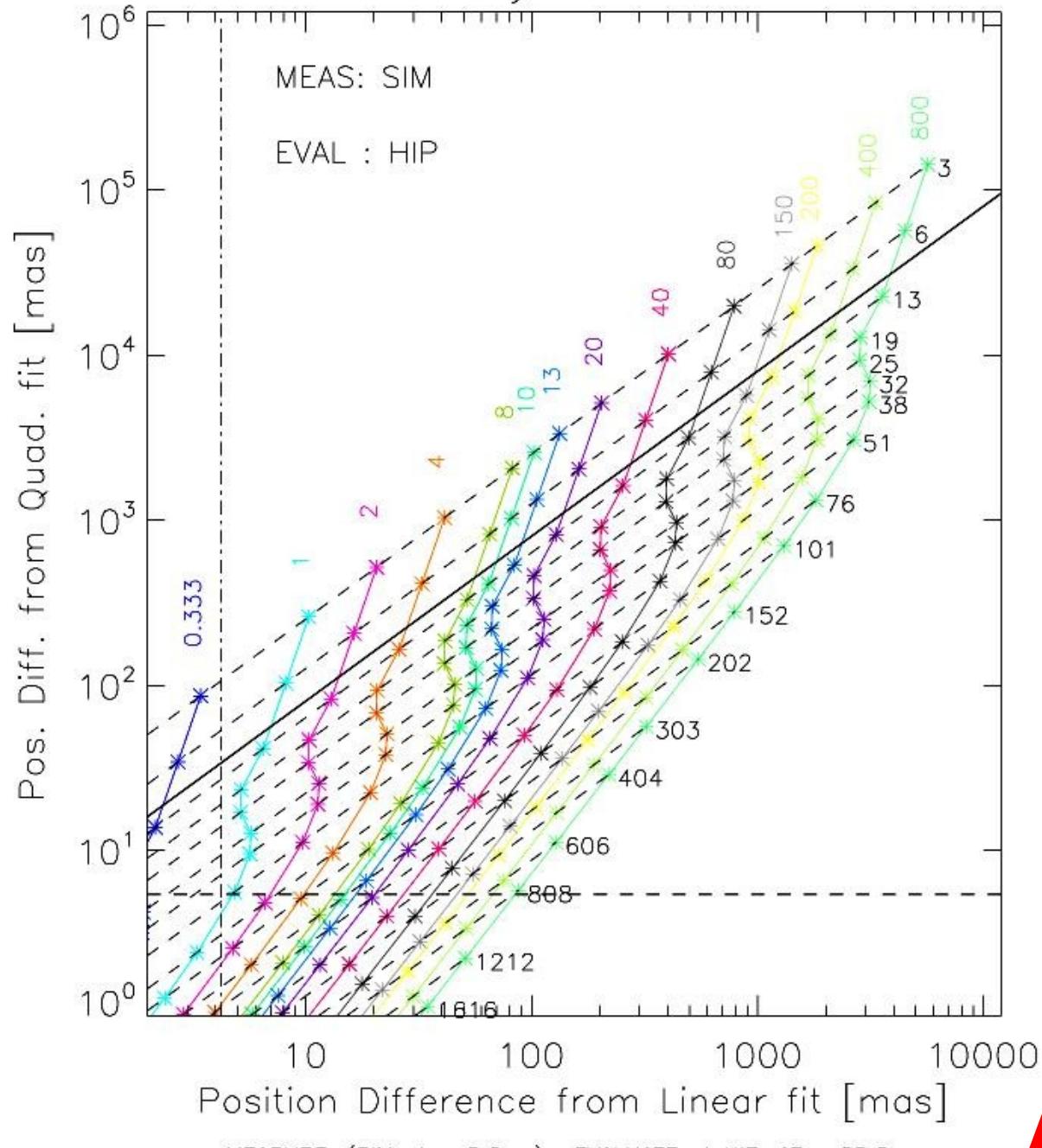
Quadratic: $\Delta_{x;12}$

Linear:

$\Delta_{x;1}$

Period/Mass dependent?

Motion of Primary: Position Differences



Lift Degeneracy
when considering
quadratic fit

Analytically proven

SIM & HIPPARCOS

$1 M_J$ and up; $P < \sim 80$ yr

$13 M_J$ and up; $P < \sim 160$ yr

Improved 2nd generation Hipparcos @ 1/3 mas

GAIA data can be used to re-reduce Hipparcos to eliminate any residual systematic errors

twice better Period Limits

“Detection” w. $\Delta_{XY;1}(13M_J)$: $P < \sim 800$ yr

SIM & GAIA

Characterization: $\frac{1}{2}$ period range

Detection: 50% larger period range

SIGNIFICANCE: x5 - x15 better

**Lower-mass range extended
by x5 to $0.2 M_{JUPITER}$**

An Era of Precision Astrophysics: Connecting Stars, Galaxies and the Universe an Astro2010 Science White Paper



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Donald Terndrup^{OS}, Patrick A. Young^{ASU}

^{AMNH} American Museum of Natural History, NY

^{ASU} Arizona State University

^{CMU} Caltech

^{CO} Carnegie Observatory

^{DC} Dartmouth College

^{MC} NexSci, Caltech

^{OS} Ohio State University

ST Space Telescope Science Institute

^{UCSC} UC, Santa Cruz

^{UR} University of Florida

^{UMD} University of Maryland

^{UR} University of Rochester

^{UNM} U.S. Naval Observatory, Washington D.C.

Connecting Stars, Galaxies and the Universe

The previous decadal report stresses that: “**the fundamental goal of ... astrophysics is to understand how the universe ... galaxies [and] stars ... formed, how they evolved, and what their destiny will be**” (McKee & Taylor, 2001). These age-old questions can be answered ... by ... μ as astrometry:

- 1) **Galactic archeology**: reconstructing the formation history of the Milky Way and other Local Group galaxies,
- 2) **The oldest stars in the Milky Way** and the age of the Universe, and
- 3) **H_0 and concordance cosmology**

In question form we can summarize these goals as: 1) What is the construction history of the Milky Way and other nearby galaxies? 2) what is the age, density and curvature of the Universe?

Connecting Stars, Galaxies and the Universe

These goals are achievable in the near future by:

- Survey of **eclipsing binaries** down to $V \sim 15$
- modest ground-based spectroscopic observing campaign
- μas astrometry from the proposed SIM-Lite mission

*The high-quality data like we are advocating for in this white paper will force the **biggest reassessment of stellar astrophysics in more than 50 years**, and its effects will be very beneficial for many disciplines of astrophysics*

Eclipsing Binaries Gold Standard for Stellar Evolution

Measure all fundamental parameters:

Mass

Radius

Temperature

To sub-percent accuracy, like the Sun

**Will finally allow for decent calibration of stellar models
Much less need for scaled-solar models**

The two+ stars share:

Metallicity

Helium abundance

Birthdate

Need to lie on same isochrone

Ground-based, GAIA & SIM Synergy

- Ground+GAIA: survey & identification
- Ground: spectroscopy/metallicity
- GAIA: distances (extinction of most cases)
- SIM: distances “most interesting” (oldest)
- SIM: 1% distance to M31 (Rotational Parallax)

Need:

- Wide range in metallicity/ages/types
- Wide range in separations (measure tidal effects)
- Preferably high-mass case on (sub)giant branch
- Supergiants in M31 & M33

Thank You



SIM/Heavy (Credit JPL)



Hipparcos (Credit ESA)



GAIA (Credit ESA)

SIM-Lite: 5-10 yrs; 201?+; **1/1000 mas**

GAIA: 5-7 years, 2012+; **1/100 mas**

Backup Slides

Connecting Stars, Galaxies and the Universe

- The **golden age of astrophysics** is upon us with both grand discoveries (**extra-solar planets, dark matter, dark energy**)
- **Fundamental understanding of the working of stars and galaxies is within reach**, from **precision measurements**
- **Micro-arcsecond astrometry** forms the basis of **model independent distances and masses**.
- **Stellar ages** can be ascertained **IF** their **luminosities/distances are accurately known**
- **The age of the universe** is the **inverse of Hubble's constant (H_0)**, + corrections from: the fate of the universe and the amount and nature of dark energy
- Some of the **strongest motivations** to vigorously pursue **accurate distance measurements are related to the history and fate of the universe.**

Dabblings

I've been working on:

Astrometric detections (FAME, AMEX, OBSS)

Transit detections (FAME, AMEX, OBSS)

FAME: now-cancelled astrometric MIDEX (USNO-led)

AMEX: proposed Germany/NASA/USNO SMEX

OBSS: proposed “Origins Probe” mission

Capable of duplicating GAIA, if necessary

Radial velocity work & TPF-C characterizations

Dispersed Fourier Transform Spectrometer

P.I., Arsen Hajian (USNO; now U. Waterloo)

LEAVITT: my MIDEX-class planetary-transit finder

Solar System Analogs (SOSAs; 2008-present)

http://www.astro.umd.edu/~olling/index_1.htm#My_Astrometry_Latest

Dispersed Fourier Transform Spectrometer:

dFTS @ USNO: PI: Arsen Hajian (now at Waterloo)

<http://adsabs.harvard.edu/abs/2007ApJ...661..616H>

http://www.astro.umd.edu/~olling/Papers/dFTS_white_paper_final.pdf

Like conventional FTS, but dispersed by GRATING into many thousands of spectral channels

Much, much better sensitivity: $S/N_{dFTS} = S/N_{FTS} * (R_{GRATING})^{1/2}$

Whole (optical) spectrum

Configurable spectral resolution (down to **TPF** needs)

Small size ($\sim 1 \text{ m}^3$)

Cheap (shoestring)

Full-aperture metrology

Extreme wavelength sensitivity (“arbitrary” resolution)

$\sim 3 \text{ m/s}$ for our shoestring instrument

Many known improvements await funding

cm/s long-term stability expected ==> **Earth-mass planets**

Astrometry: Number Estimates

http://www.astro.umd.edu/~olling/FAME/otm_plas_rpo_2004_01.pdf

Procedure:

Semi-major axis:

$$a = 95/d_{10\text{pc}} (P_{\text{YR}}/M_{\text{TOT,SUN}})^{2/3} M_{\text{PLANET,JUP}} \quad [\mu\text{as}]$$

x/y coordinates for face-on orbit

$$x = a \cos(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \quad [\mu\text{as}]$$

$$y = a \sin(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \cos(i) \quad [\mu\text{as}]$$

$$dx/dt = -2\pi/P_{\text{YR}} a \sin(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \quad [\mu\text{as}/\text{yr}]$$

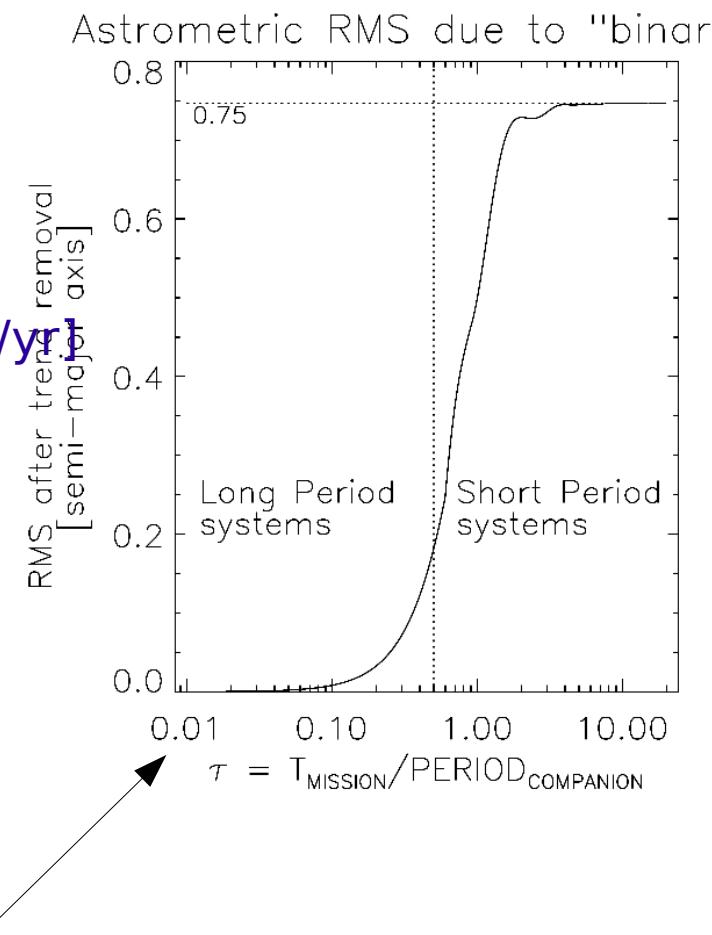
$$dy/dt = +2\pi/P_{\text{YR}} a \cos(2\pi t_{\text{YR}}/P_{\text{YR}} + \phi) \cos(i) \quad [\mu\text{as}/\text{yr}]$$

For each model, compute x,y
and fit linear proper m. model

Compute RMS w.r.t. best fit

Depends **only** on $\tau = T_{\text{MISSION}} / \text{Period}_{\text{COMPANION}}$

Turns out: "RMS"/a = Function(τ)



Astrometry: Number Estimates

http://www.astro.umd.edu/~olling/FAME/otm_plas_rpo_2004_01.pdf

Procedure, cntd:

For a given astrometric error (δ_{AE}) per observation

In Short Period Regime: “RMS_{SPR}” $\sim 0.75 * a$

Detection: “RMS_{SPR}” = $\frac{3}{4} * a \geq N_\sigma * \delta_{AE}$

$$d'_{MAX;SPR} \sim 7.5 \quad (P_{YR}/M_{TOT})^{2/3} \quad M_{PL,JUP} \quad (10/N_\sigma) * (10 \mu\text{as}/\delta_{AE}) \quad [\text{pc}]$$

In Long Period Regime: “RMS_{LPR}” $>\sim N_\sigma * \delta_{AE}$

$$d'_{MAX;LPR} \sim 8.3 \quad \tau^2 \quad (P_{YR}/M_{TOT})^{2/3} \quad M_{PL,JUP} \quad (10/N_\sigma) * (10 \mu\text{as}/\delta_{AE}) \quad [\text{pc}]$$

Photometric distance & magnitude-dependent δ_{AE} introduces

$$d_{MV} = 10^{(V_F + 5 - M_V)/5} \quad \text{and:}$$

$$d_{MAX;SPR;MV} = \sqrt{d'_{MAX;SPR} * d_{MV}}$$

$$d_{MAX;LPR;MV} = \sqrt{d'_{MAX;LPR} * d_{MV}}$$

d_{SPR} INcreased with P

d_{LPR} DEcreased with P

Primary HAS to be closer
than d_{SPR} & d_{LPR}

Maximum distance @

$$P(d_{MAX}) \sim 0.82 T_{MISSION}$$

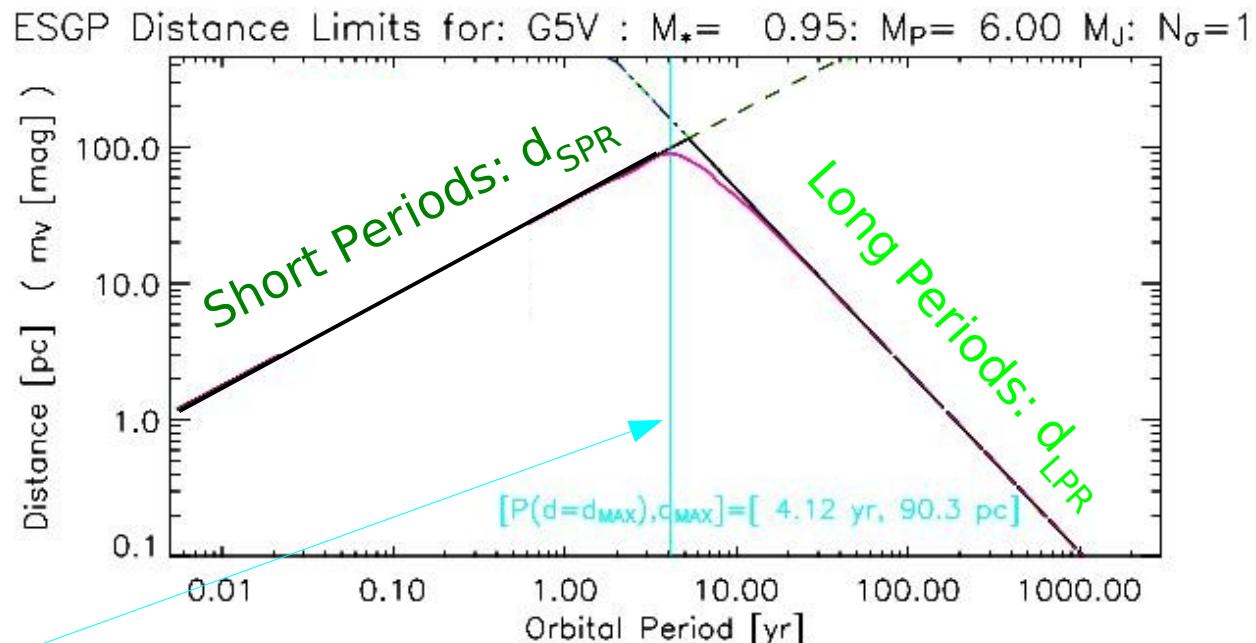
$$\text{Volume}(P) \sim 4/3 \pi d(P)^3$$

drops off quickly

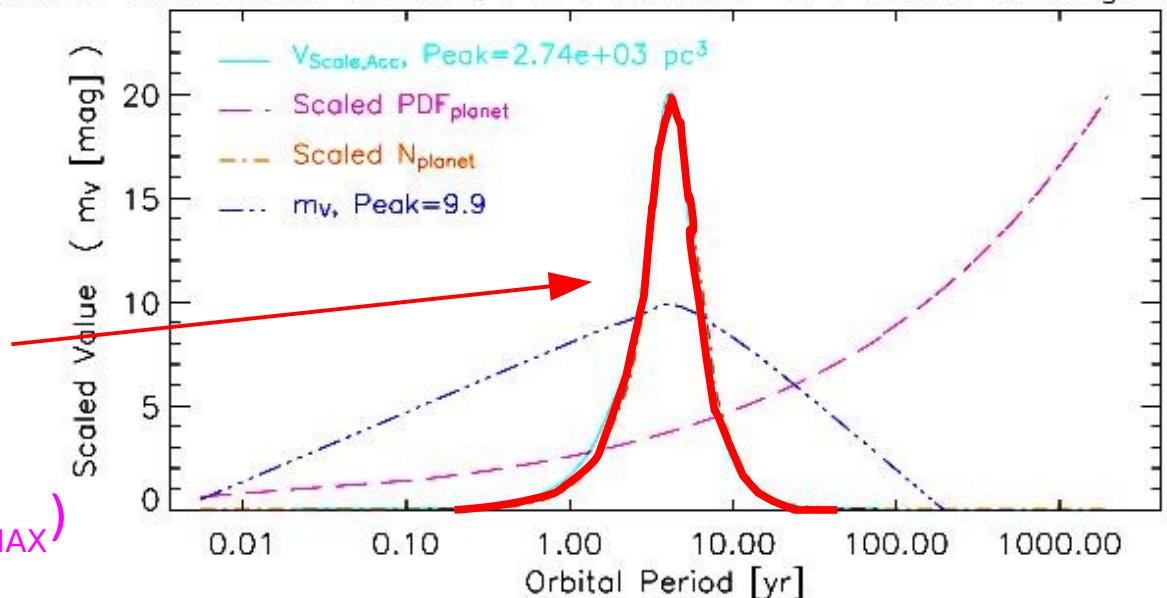
either side of $P(d_{MAX})$

=> Most companions @ $P(d_{MAX})$

=> ... @ small range in m_v



Scaled Accessible Volume, PDF, Number of Planets & Magnitude



Eliminating μ_B : Backtrapolates

Total motion (face-on; circular):

$$z_{\text{TOT}}(t) = z_0 + \mu_B t + z_{\text{ORBIT}}(t)$$

$$z_{\text{ORBIT}}(t) = a_0 \cos(2\pi t/P + \varphi)$$

Expand $Z_{\text{ORBIT}}(t)$

$$\zeta(t)/a_0 = \cos(\varphi) - (2\pi/P) \sin(2\pi t/P + \varphi) t - \frac{1}{2} (2\pi/P)^2 \cos(2\pi t/P + \varphi) t^2 + \dots$$

$$\begin{aligned} Z_{\text{TOT}}'(t) &= Z_0 + \mu_B t + \zeta(t) \\ &= n^{\text{th}} \text{ order } \underline{\text{polynomial fit to SIM data}} \end{aligned}$$

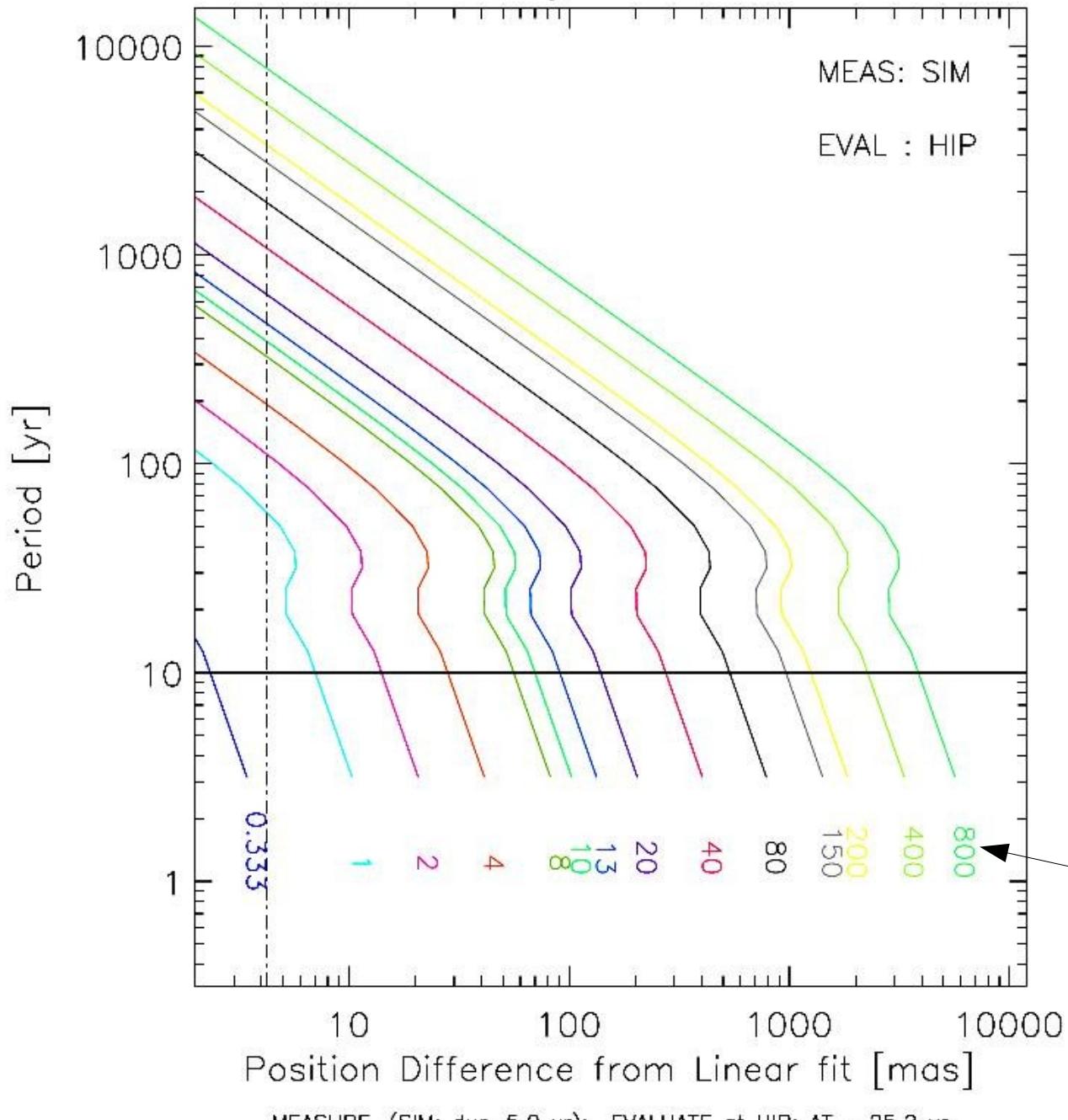
Position Difference at Hipparcos epoch (τ)

$$\Delta_z(\tau) = z_{\text{TOT}}(\tau) - Z_{\text{TOT}}'(\tau) = z_{\text{ORBIT}}(t) - \zeta(\tau)$$

INDEPENDANT of Barycentric motion

Only depends on orbit & its expansion

Motion of Primary: Position Differences



YES !

MEASURE: SIM
B.TRAPOLATE: HIP

Position Differences
from linear fit are
degenerate:

Multiple
Masses &
Periods

at given pos.dif

Backtrapolates: Sensitive to Mass & Period

Order-dependent: $\Delta_{z;n}(\tau) = z_{\text{ORBIT}} - \zeta^n(\tau)$

Can be calculated analytically

No phase dependence for TOTAL pos. dif.

Face-on & circular: $\Delta_{XY;n} = (\Delta_{X;n}^2 + \Delta_{Y;n}^2)^{1/2}$

Periods can be estimated from $\Delta_{XY;n}$ values

$$\mathcal{P}_{1,2} = 2/3 \pi \tau \Delta_{XY;1} / \Delta_{XY;2} \sim P \quad \text{for } P \geq 2\tau$$

$$\mathcal{P}_{2,3} = 1/2 \pi \tau \Delta_{XY;2} / \Delta_{XY;3} \sim P \quad \text{for } P \geq 2\tau$$

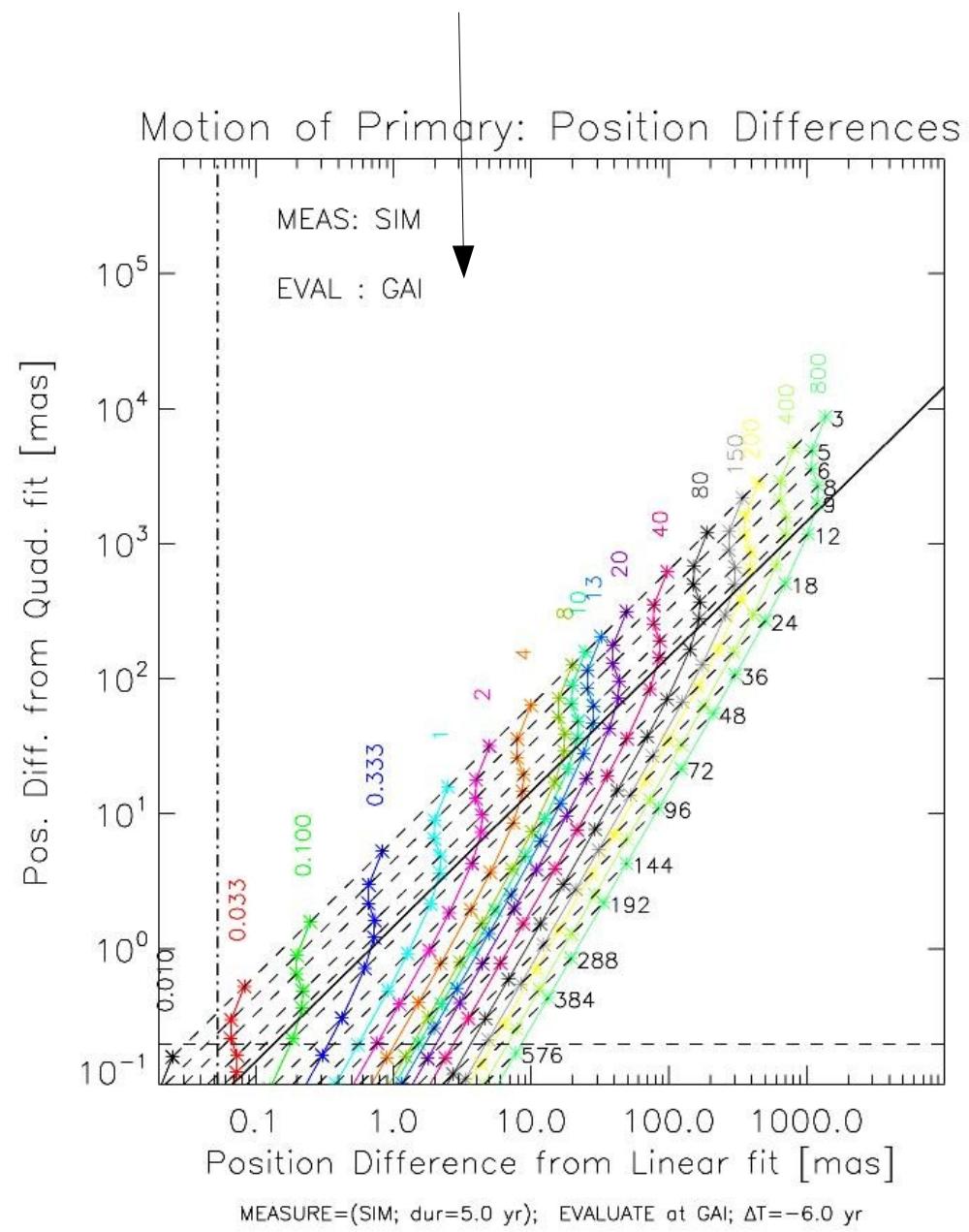
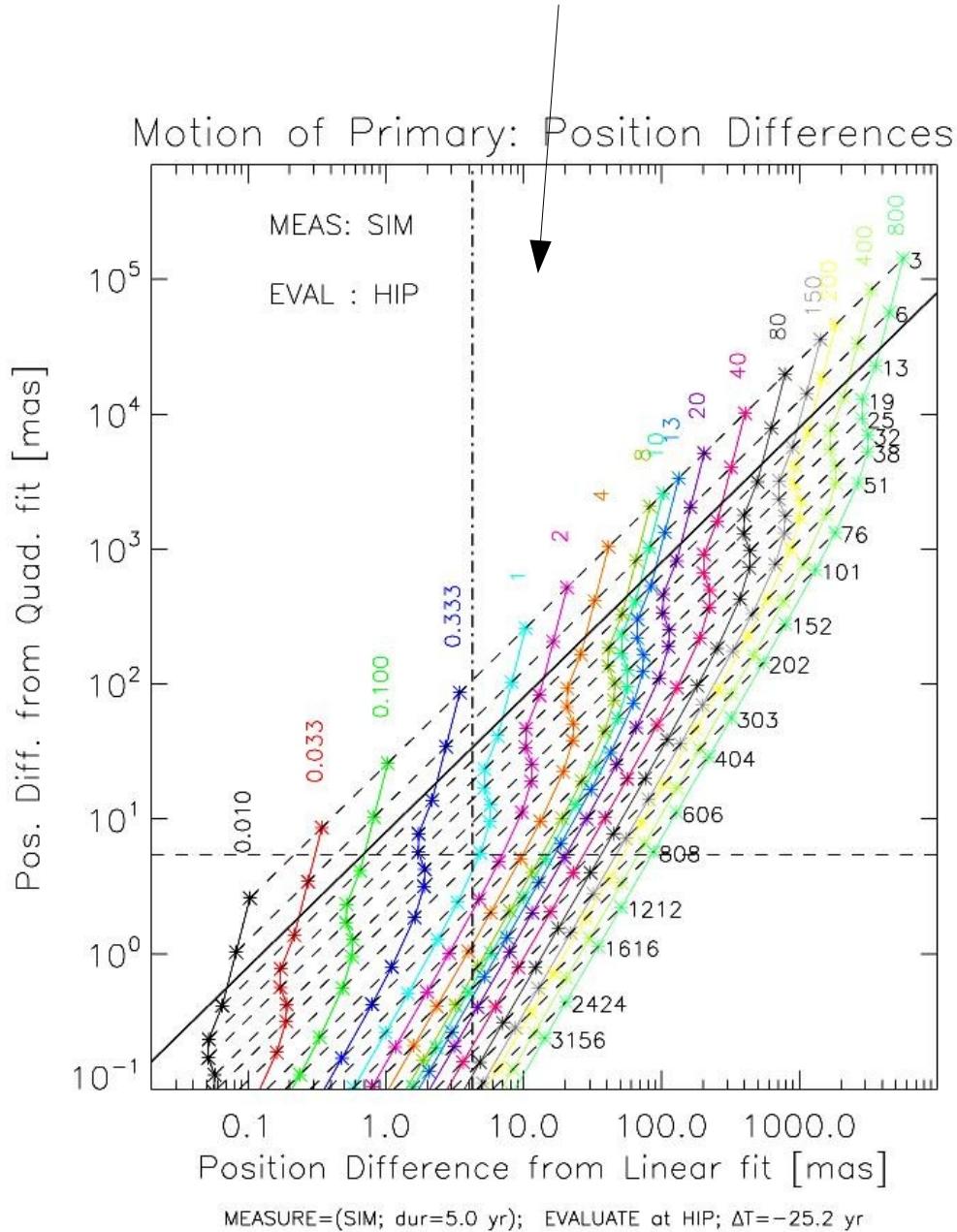
$$\mathcal{P} \sim P \quad \text{for } P \ll \tau$$

$$\mathcal{P} \text{ oscillates strongly} \quad \text{for } P \sim [0.5, 1] \times \tau$$

$$\mathcal{P} \text{ decays (exponentially) towards } P \quad \text{for } P \sim [1, 2] \times \tau$$

Masses follow immediately once P is known

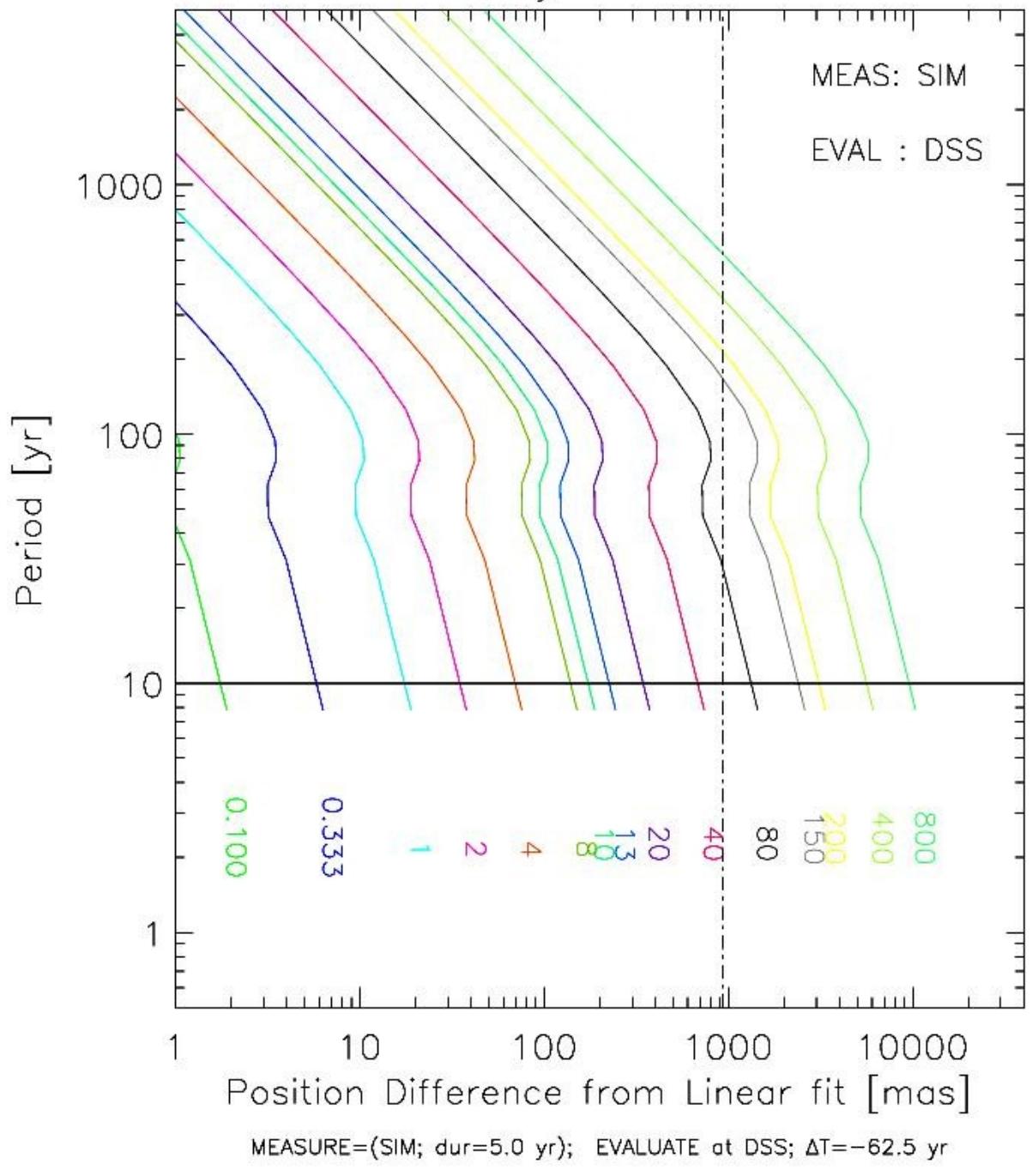
SIM --> HIP vs. SIM --> GAIA



SIM & Ground- based Surveys

DSS (1957)

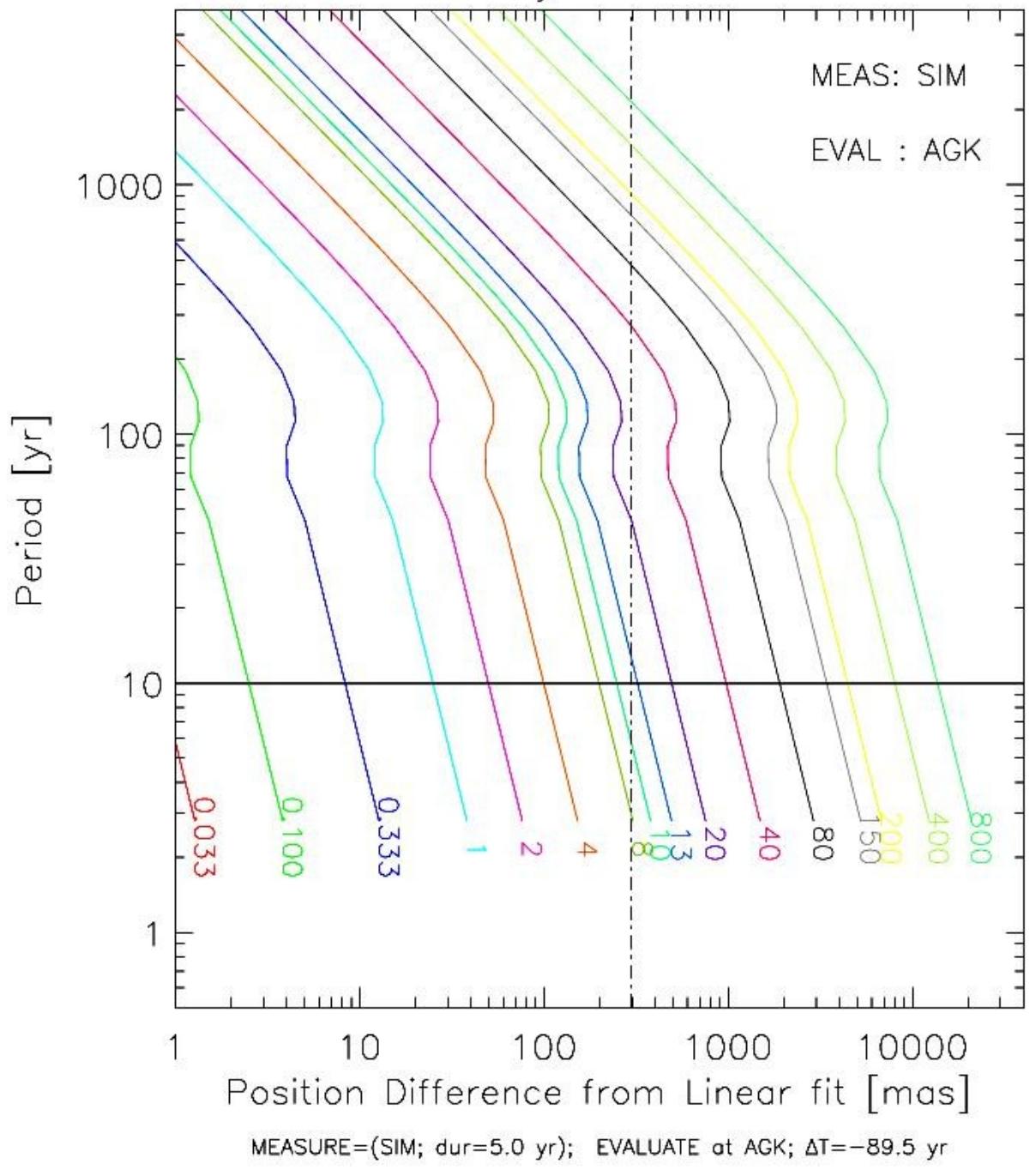
Motion of Primary: Position Differences



SIM & Ground- based Surveys

AGK (1930)

Motion of Primary: Position Differences



SIM & Ground- based Surveys

AGC (1907)

