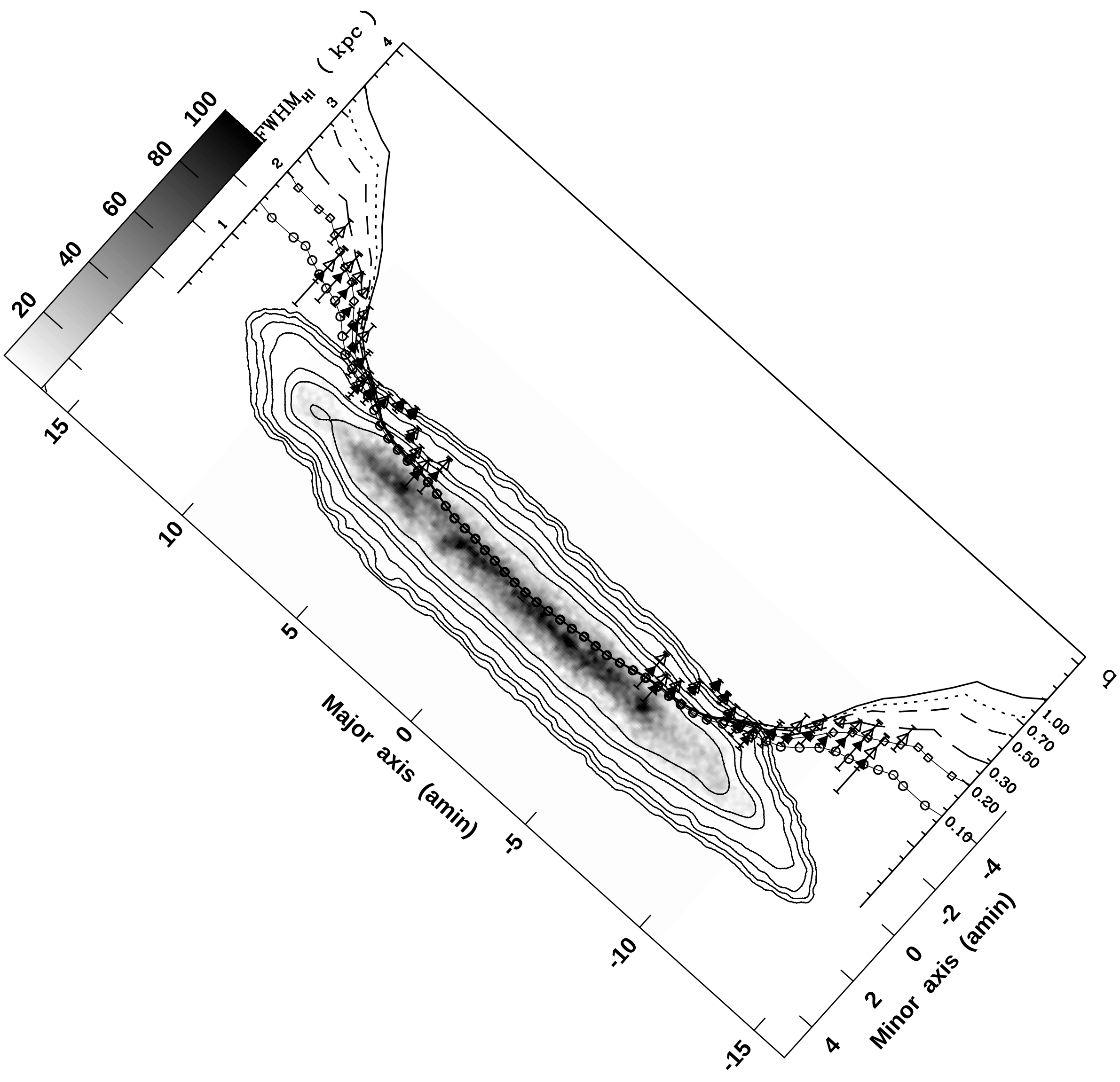
THE SHAPE OF DARK MATTER HALOS



Rob Olling

THE SHAPE OF DARK MATTER HALOS

Robert Paul Olling

DISSERTATION

Submitted in Partial Fulfillment
for the Degree of
DOCTOR of PHILOSOPHY
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY
OCTOBER 1995

Commitee members:

J.H. van Gorkom
K.H. Prendergast
A.P.S. Crotts
M. Kamionkowski
M.P. Rupen

ERRATA

The numerical changes reported below are a combined result of the usage of Efstathiou *et al.* 's (1988) determination for $M/\mathcal{L}_{\rm Bar}$ (1500^{+700}_{-400}) rather than the value (1350) used by Bahcall *et al.* (1995), and the inclusion an error in the mass gas fraction of rich clusters (see page 4, footnote 1)

Chapter 3 has been split, and somewhat modified, in 2 papers (both accepted for publication by AJ): Chapter 3.1 - 3.5, 3.7, 3.10, 3.11, 3.12, and 3.13 will be published as: "NGC 4244, a Low Mass Galaxy with a Falling Rotation Curve and a Flaring Gas Layer". Chapter 3.6, 3.8, and 3.10 will be published as: "The Highly Flattened Dark Matter Halo of NGC 4244".

page 4: $10h^{-1.5}\%$ has to be replaced by $(10 \pm 5)h^{-1.5}\%$

page 4: $0.015 - 0.02h^{-1.5}$ has to be replaced by $(0.0075 - 0.03)h^{-1.5}$

page 4: $h \geq 0.4$ has to be replaced by $h \geq 0.5$

page 4: $(0.2 \leq h \leq 0.8)$ has to be replaced by $(h \geq 0.2)$

page 4: $20h^{-0.74} \pm 7$ has to be replaced by $(35 \pm 6)h^{-0.64}$

page 5: $10h^{-1.5}\%$ has to be replaced by $(10 \pm 5)h^{-1.5}\%$

page 5: $20h^{-0.74} \pm 7$ has to be replaced by $(35 \pm 6)h^{-0.64}$

page 80: $10h^{-1.5}\%$ has to be replaced by $(10 \pm 5)h^{-1.5}\%$

page 80: $20 - 30h^{-0.5}\%$ has to be replaced by $(32 - 48)h^{-0.5}\%$

page 80: $h \geq 0.4$ has to be replaced by $h \geq 0.5$

page 80: $12 - 19h^{-1}\%$ has to be replaced by $(11 - 35)h^{-1}\%$

page 80: $(0.2 \leq h \leq 0.8)$ has to be replaced by $(h \geq 0.2)$

page 80: $20h^{-0.74} \pm 7$ has to be replaced by $(35 \pm 6)h^{-0.64}$

page 92: The correct reference for Braun 1995 has been inserted

Abstract

THE SHAPE OF DARK MATTER HALOS

Robert Paul Olling

After reviewing our current knowledge of dark matter (DM) in spiral galaxies (Chapter 1), I present a new method of deriving the shape of these dark halos (Chapter 2). Such information, if obtained for a large number of systems, can provide important boundary conditions for theories of the formation of galaxies (Chapter 5).

The halo-shape determination method relies on the comparison of model predictions of the thickness of the gas layer with observations of this flaring. Calculating the model gas layer widths from the observed gaseous velocity dispersion and the potential due to the *total* mass distribution of the galaxy we learn the following: a) beyond the optical disk the thickness of the gas layer is sensitive to the shape of the DM halo, b) the thickness of the gas layer is proportional to the ratio of the gaseous velocity dispersion and the rotation speed, c) the self-gravity of the gas contributes significantly to the vertical force, d) the derived shape of the DM halo is independent of the dark matter's radial density distribution, and is independent of the mass-to-light ratio of the stellar disk (f).

In Chapter 3 I present a new method (usable for inclinations larger than 60°) to determine the thickness of the gas layer of spiral galaxies from high resolution H I observations. I use VLA H I observations of the almost edge-on Scd galaxy NGC 4244 to determine the gaseous velocity dispersion, and the flaring and rotation curves. From the Keplerian decline of the rotation curve beyond the stellar disks it follows that the dark-to-luminous mass ratio is at most two and a half.

Combining the model predictions for the radial variation of the thickness of the gas layer with the measured flaring curve I find that the dark matter halo of NGC 4244 is highly flattened. The best fit occurs for a halo with an E8 shape (with a mass one-eight of an E0 halo), while the uncertainty (E5 - E9) is dominated by the errors in the gaseous velocity dispersion: a round halo is inferred if the vertical dispersion equals 70% of the planar dispersion.

Contents

List of Figures

List of Tables

Acknowledgements

Although this thesis is presented as the product of one person, it is build upon knowledge obtained by humanity over millions of years. Many events of astronomical importance must have been witnessed by our ancestors. Like for many of my generation, my interest in science was enormously stimulated by the dawn of the space age: watching the Apollo 13 mission to the Moon with my parents is one of my oldest memories. I am forever grateful to my parents for their love, dedication, encouragement throughout the years, and their continuing trust.

As a student at the Kapteyn Astronomical Institute in Groningen I benefitted greatly from the presence of many friends and fine scholars. In particular I would like to thank Penny Sackett, Frank Briggs and Paul Coleman for encouraging me to go to the United States to pursue a graduate education.

At Columbia University my foremost thanks go to Jacqueline van Gorkom, for taking me under her wings. Over the last five and a half years I have enjoyed her friendship and professional guidance. In our many conversations and discussions she has taught me a lot about astronomy and life. This dissertation would, quite likely, not have been finished had she not insisted that I wear a bicycle helmet on my rides through "potholy" New York. She has generously provided help and advice at every stage of my dissertation: image processing, galactic dynamics, writing VLA observing proposals, the VLA observations themselves, presentation of results, and last but certainly not least, writing it all down in a the most concise, sensible manner possible.

My special thanks also go to Michael Rupen. His work sparked my interest in flaring gas layers. I am very grateful for his insights and thoughtful comments in the early stages of my dissertation. During hikes, at dinner, and at work we had many enjoyable discussions. I am looking forward to our collaborations.

I am grateful to the entire faculty, staff, and fellow students in the Department of Astronomy at Columbia University for providing an intellectually stimulating and friendly environment. At colloquia, lectures, coffee breaks and dinner parties I have greatly enjoyed the professional expertise and humor of David Helfand, Norm Baker, Lloyd Motz, Ken Mighell, Wil van der Veen, Austin Tomaney, HongSheng Zhao, Gino Thomas, Millie García, Nelson Riverra, and many others. My particular thanks go to Kevin Prendergast for his valuable advice on the theoretical aspects of this thesis and careful reading of Chapter 2. Andy Rasmussen, John Hibbard, Pauline McMahon, Michiel Kolman, Eric Gothelf are a few of the many people without whom it would have been very difficult to adjust to a new life in the USA.

NRAO provided financial support for several observing trips to the VLA near Socorro, New Mexico. The hospitality of its director, Miller Goss, and the contagious enthusiasm and friendliness of its staff made the summers of 1991-1994 unforgettable. Thanks also go to Juan

Uson for discussing various aspects of calibrating VLA data and providing the astronomical community with the wonderful UV-editing tasks UVLIN and UVMLN. My many “friends in the desert”: Kevin Marvel, Chris dePree, Angelos Vourlidas, and Tas Philp made my stays at the AOC enjoyable.

Finally, a word of thanks to all my friends back home and here in New York: Otto & Gerry, Joost & Paula, Michiel, Frits, Henk & Marja, Polleke & Jan, vdB, Sip, René, Johan, Ron, Matthew, Susan and most especially Joanna and my brother Jan-Frits.

This work was supported in part through an NFS grant (AST-90-23254 to J. van Gorkom) to Columbia University.

Chapter 1

INTRODUCTION

1. Introduction

1.1. A BIT OF HISTORY

When I embarked on the topic of this dissertation (spring 1992) the shape of the polar ring galaxy NGC 4650A had just been determined: q_ρ= 0.3 - 0.7 (Sackett & Sparke 1990). At that time we realized that the thickness of the gas layer could be used as a probe of the shape of the dark halo as well (Chapter 1). With high resolution H I observations of the edge-on galaxy NGC 4244 available, we thus started the project summarized in this thesis.

Inside the stellar disk, where the potential is dominated by the stars, the thickness of the gas layer can be easily related to the mass and scale height of the stellar disk (e.g., van der Kruit 1981; Chapter 2, Appendix 2.8.). Beyond the optical disk, the thickness of the gas layer relates in a simple manner to the properties of the dark halo (Chapter 2, Appendix 2.8.). Van der Kruit (1981) also noted that the self-gravity of the gas can not be neglected in the outer parts of the stellar disk: the transition region between disk and halo dominated approximations. We thus followed Bahcall's (1984) method and calculated the vertical distribution of the gas layer in the combined potential of the stellar and gaseous disks, the dark matter halo and the contribution of the gradient in the rotation curve. A special analytic solution was found for the case the stellar velocity dispersion and the dark halo density do not vary with height above the plane (Chapter 2, Appendix 2.7.). In this method the potential is calculated from the local column of matter: the *local* approach. As a result, sharp features in the density distribution (spiral arms, the edge of the optical disk) may cause strong features in the thickness of the gas layer. However, because the potential is smoother than the density distribution, such effects are not expected in real galaxies. This would be a serious problem for the interpretation of the flaring measurements of NGC 4244 as its gas layer does not extend much beyond the truncation of the stellar disk. Furthermore, in the local approach one assumes a plane parallel density distribution of constant surface density. Following Kevin Prendergast's advise, we thus decided that a *global* approach should be followed, where the potential is calculated from the global mass distribution of the galaxy. The gaseous self-gravity is hard to handle in this method. Under pressure of my advisors and the referee (Phil Maloney) of Paper I (Chapter 2) this nut was finally cracked however.

On the observational side, we measured the thickness of NGC 4244's the gas layer from the edge channels early on, and used the local approach (Chapter 2, Appendix 2.7.) to determine the radial DM distribution. The combination of "irregularities" in the thickness measurements and the sensitivity of the local approach to small scale irregularities lead to strange dark matter distributions. Our sense of uneasiness was further amplified by the fact that Michael Rupen found large, rapid variations (a factor of two within 500 pc) in the thickness of the gas layer of NGC 891 (1981a). After realizing that the thickness and inclination could be determined from measurements in two channels, we concluded that we needed information from as many channels as possible to reduce the effects of local irregularities in the gas layer. After all, as Jacqueline van Gorkom kept on insisting, real galaxies do not look like models at all. Being a good graduate student I took her advise, and thus the simultaneous inclination and thickness determination method came to life (Chapter 3, Sec. 3.5.1.). Determining the flaring of the gas layer beyond the optical disk is practical only because radio telescopes like the VLA and the WSRT are now so sensitive that excellent data can be obtained in a reasonable observing time.

Being able to determine the thickness of the gas layer of edge-on systems, and with the theoretical tools in hand it thus became possible to go out into the real Universe and determine the shape of dark matter halos of edge-on galaxies with extended H I envelopes. One small problem arose however: such systems are not very common. It seemed that this method, like polar-ring method, would work for a very limited number of galaxies only. Fortunately however, I had followed Kevin Prendergast's advise as well and tried to explain everything in terms of simple analytic models. That brought the "Eureka" moment: the realization that the inclination and thickness determination method, developed for edge-on systems only, could work at inclinations as low as 60° .

This opens the possibility for determining the flaring of the gas layer, and hence the halo shape, for moderately inclined systems with extended H I layers: beasts a little less rare than the edge-on subset. In the future it will be possible to determine the shape of dark matter surrounding many galaxies of different Hubble types, hopefully providing constraints on the nature of dark matter and the process of galaxy formation.

1.2. DARK MATTER IN THE UNIVERSE

The luminous mass in gas, stars and stellar remnants amount to only $\sim 0.7\%$ of the closure density of the Universe (Pagel 1990). Thus, if the density of the Universe is close to the critical density ($\Omega = 1$), about 99% of the Universe is made up of Dark Matter: certainly a huge field of study. Numerous studies of clusters of galaxies, the largest bound entities in the Universe, indicate that the total mass associated with these clusters amount to $\Omega = 0.15$ - 0.2, corresponding to an overall mass-to-light ratio ($M/\mathcal{L}$) of 200 - 300 h (with $h = H_0/(100 \text{ km s}^{-1}\text{Mpc}^{-1})$, the dimensionless Hubble constant). The velocity dispersion of the member galaxies of clusters (e.g., Zwicky 1933, 1957; Peebles 1980) and X-ray emission from hot intracluster gas (e.g., Jones & Forman 1984) indicate that both the galaxies and the hot gas are reasonable tracers of the underlying potential (Mushotzky 1984; Lubin & Bahcall 1993). Gravitational lensing of background galaxies is a third independent means of determining the mass distribution in clusters of galaxies (e.g., Tyson *et al.* 1990), and produces mass estimates similar to the other two methods (Bartelmann & Narayan 1995). From the comparison of these three methods important information about the degree of subclustering can be obtained in the future (Bartelmann & Narayan 1995, and references therein). Bahcall *et al.* (1995) find that the mass-to-light ratio increases linearly with size till about 200 kpc to remain approximately constant up to the 10 Mpc scale, and suggest that all mass in galaxy clusters consist of the (stripped off) halos of the constituting galaxies. Gas in these clusters of galaxies amount to 10 $h^{-1.5}$% of the total mass (e.g., Briel *et al.* 1992; Mushotzky *et al.* 1995) so that the baryon density in rich clusters ($\Omega_{\rm B,cl}$) equals 0.015 - 0.02 $h^{-1.5}$, about twice as large as Pagel's estimate of the luminous mass. Primordial Big Bang Nucleosynthesis (BBN) models (Walker *et al.* 1991; Smith, Kawano & Malaney 1993) predict $0.01h^{-2} \leq \Omega_{\rm B,BBN} \leq 0.016h^{-2}$. From the cluster and BBN limits it follows that for $h \geq 0.4$ essentially all baryons in the universe are located in rich clusters. The average mass-to-light ratio[1] ($M/\mathcal{L}_{\rm Bar}$) of the cluster baryons can be determined in the region where the cluster and BBN estimates overlap ($0.2 \leq h \leq 0.8$): $M/\mathcal{L}_{\rm Bar,cl+BBN} \sim 20h^{-0.74} \pm 7$.

1.3. DARK MATTER ON GALACTIC SCALES

Dark matter (DM) on galactic scales ($\sim$ 30 kpc) has been inferred from optical and H I rotation curves (e.g., Bosma 1981; Rubin *et al.* 1980; Bahcall & Casertano 1985; Persic & Salucci 1988; Begeman 1989). The exact amount of dark matter, and how it is distributed within spiral galaxies is still debated. A lower limit on the dark mass is obtained from the "maximum-disk" hypothesis (van Albada & Sancisi 1986). From this, dynamical estimates of the dark-to-luminous mass ratios ($M_{\rm D}/M_{\rm L}$) between 0.5 and 10 are obtained (Broeils 1992). Models of galaxy formation which include simple estimates of the star formation history (e.g., Larson & Tinsley 1978) predict mass-to-light ratios which are in the range of values derived from observed rotation curves (Rubin *et al.* 1985; Athanassoula *et al.* 1987; Broeils 1992, Chap. 12). However, different star formation histories can easily result in mass-to-light ratios which differ by a factor of a few (Tinsley 1981; Worthey 1994; de Jong 1995, Chap. 4). As we have no reliable prior knowledge of the stellar mass-to-light ratio ($M/\mathcal{L}_{\rm str}$), stellar disks less massive than the maximum-disk value are allowed. Varying $M/\mathcal{L}_{\rm str}$, many disk-halo combinations produce acceptable fits to a given observed rotation curve (van Albada *et al.* 1985; Lake & Feinswog 1989; This thesis, Chapter 2): less massive stellar disks require more massive dark halos. Other means, besides the maximum-disk hypothesis, of constraining $M/\mathcal{L}_{\rm str}$ exist: van der Kruit (1981) calculates $M/\mathcal{L}_{\rm str}$ from the thickness of the gas layer, Efstathiou *et al.* (1982) invoke disk stability arguments, Athanassoula *et al.* (1987) apply spiral instability criteria, and Bottema (1993) uses stellar velocity dispersion measurements. The mass-to-light ratios found by these authors range from 50 to 100 % of the maximum-disk value. The dark-to-luminous matter ratio, corresponding to the possible range in $M/\mathcal{L}_{\rm str}$, is hence uncertain by a factor of two as well (Chapter 2, Fig. 3).

---

[1]From the estimated average luminosity density (2 10^8 $L_\odot$ Mpc^{-3}, Efstathiou *et al.* 1988) and the closure density of the Universe, one calculates that $\Omega = 1$ corresponds to $M/\mathcal{L}_{\rm Bar}$ =1350 h (see also Bahcall *et al.* 1995).

The dynamical mass-to-light ratios ($M/\mathcal{L}_{\rm dyn} \equiv \left(RV^2_{rot,l}/G\right)/L_{\rm tot}$) as determined from the rotation speed at the last measured point on the rotation curve ($V_{rot,l}$) is not affected by the uncertainty in $M/\mathcal{L}_{\rm str}$. Typical values for $M/\mathcal{L}_{\rm dyn}$ range from 4 to 100 h, with 80% of the systems between 6 and 20 h (Broeils 1992, 1995). The total amount of mass can be obtained from $M_{\rm dyn}$ only if the true three dimensional distribution of the matter is known. For example, for a flattened dark matter halo the total mass of the system is reduced somewhat.

The linear increase of $M/\mathcal{L}$ with radius up to the size of rich clusters suggests that the dark matter in clusters is made of the (stripped) halos of member galaxies (Bahcall *et al.* 1995). With non-baryonic dark matter amounting to $(100 - 10h^{-1.5})$% of the total mass it is possible that the dark halos of individual galaxies consist mainly of non-baryonic dark matter. The remaining $10h^{-1.5}$% of baryonic dark matter with $M/\mathcal{L}_{\rm Bar,cl+BBN} \sim (20h^{-0.74} \pm 7)$ allow for 100% baryonic galactic dark halo in gaseous form (e.g., Gott *et al.* 1974; Bahcall 1995; Sackett 1995). It is not clear however where, and in what form, these baryons reside in the galactic halos since all plausible forms of baryonic dark matter seem to be excluded (Hegyi & Olive 1986). One possibility is cold, dark, molecular material, located mostly beyond the optical disk (Pfenniger *et al.* 1994; Pfenniger & Combes 1994). In Chapter 3 I will use the flaring measurements of the H I layer of NGC 4244 to test this possibility.

1.4. THE SHAPE OF DARK MATTER HALOS

Motivated by the fact that massive spherical halos stabilize galactic disk against the formation of bars (Ostriker & Peebles 1973), the standard assumption has been that this dark mass distribution is spherical. The observed "flat" rotation curves suggested and isothermal density profile, characterized by the core radius, $R_{\rm c}$, and central density, ρ_0[e.g., Eqn. (15), Chapter 2]. Now we know that many spiral galaxies are barred and that "hot" stellar disks (Lewis & Freeman 1989; Bottema 1993) limit the formation of bars as well (Athanassoula & Sellwood 1986). The need for nearly spherical halos has weakened.

Dissipationless galaxy formation simulations tend to produce triaxial DM halos (e.g., Frenk *et al.* 1988; Warren *et al.* 1992), with an intrinsic flattening distribution peaked at $q_\rho = 0.5$ ($\pm$ 0.15). Adding 10% baryonic matter (gas) and including gasdynamics to these simulations produces more oblate halos (Katz & Gunn 1991; Udry & Martinet 1994; Dubinski 1994) while q_ρ remains unchanged. Those 10% baryons are then converted to the stars we observe today. Observationally, all but the flattest shapes have been reported (for a summary, see below). Significantly flattened halos suggest that the process of galaxy formation is more dissipational, which might occur if a larger fraction of the initial density perturbation is baryonic (Sackett *et al.* 1994). Considering the gas fraction inferred in Sec. 1.2. (22% for h = 0.6), it seems warranted to include simulations with larger gas fractions in future galaxy formation simulations.

Until recently, little was known observationally about the shape of dark halos: measurements of the equatorial rotation curve provide a one dimensional probe to the potential only. It is possible to construct both spherical and flat mass models which generate the same rotation curve. Therefore, the shape of the dark halo can not be inferred from galactic rotation curves alone (Chapter 2, Appendix 2.5.).

Existing methods to estimate the shape of dark matter halos have given mixed results. The shape of the dark halo of the Milky Way has been estimated (E0 - E7[2]) from the kinematics of extreme Population II stars (Binney & Ostriker 1987; Sommer-Larsen & Zhen 1990; van der Marel 1991; Amendt & Cuddeford 1994). Steiman-Cameron *et al.* (1992) used the dynamics of the precessing dusty disk of the S0 galaxy NGC 4753 to infer the shape of the DM halo (E1). It has been suggested (Dekel & Shlosman 1983; Toomre 1983; Sparke & Casertano 1988) that galactic warps, frequently observed in the outer parts of gaseous disks, result if the galactic disk is tilted with respect to the plane of a flattened dark halo. Hoffner & Sparke (1994) analyzed the warping behavior of H I disks and concluded that only one (NGC 2903) of the five systems studied requires a DM halo as flattened as E4. On the other hand, in studies of polar ring galaxies (Sackett & Sparke 1990; Sackett *et al.* 1994; Sackett & Pogge 1995) substantially flattened DM halos are found (E6-E7 for NGC 4650A, E5 for A0136-0801). Sackett *et al.* (1994), finding a similar shape for the stellar and dark distribution suggested a "conspiracy" of shapes. The results presented in Chapter 3 indicate that this conspiracy breaks down for the flattest systems.

1.5. THE SHAPE OF DARK MATTER HALOS, FROM FLARING GAS LAYERS

Like the polar rotation curve, the thickness of the gas layer depends on the vertical force, K_z, and hence on the shape of the DM halo (Maloney 1992; Olling & van Gorkom 1992; Kundić *et al.* 1993; Maloney 1993). That the shape of the dark halo influences the vertical distribution of the gas can be easily understood. Consider a round halo, with a certain density distribution, then decrease the vertical scale-height while maintaining the total mass. Consequently, the densities as well as the exerted gravitational forces will increase, resulting in a thinner H I disk and higher rotation speeds. In order to keep the same equatorial rotation curve, one has to deform the DM halo in a very specific way (Chapter 2, Appendix 2.5.) as a result of which the DM-halo densities (at large distances) will be roughly inversely proportional to the flattening q_ρ. The thickness of the gas layer beyond the optical disk is roughly proportional to the square root of the halo flattening, q_ρ(Maloney 1993; Chapter 2). Several authors have employed measurements of the gas layer thickness to infer the total mass density in the plane of spiral galaxies. Van der Kruit (1981) showed that, within the

[2] A spheroidal system with shortest-to-longest axis ratio c/a (q_ρ) of the density contours has a shape En, with n such that $q_\rho = 1 - n/10$

stellar disk, the width of a galaxy's gaseous disk should increase radially in an exponential fashion. From the thickness measurements of the gaseous disk of the edge-on galaxy NGC 891 by Sancisi & Allen (1979) van der Kruit concluded that the $M/\mathcal{L}$ ratio of the stellar disk does not change significantly with radius and that "the dark halo can not be as flat as the stellar disk". Rupen (1991a), using more sensitive and much higher resolution data, found that the width of the gaseous disks of NGC 891 and NGC 4565 increase exponentially, but for NGC 891 with large scatter. Our galaxy has been studied in much greater detail by Knapp (1987) and Merrifield (1992) in H I, and by Malhotra (1994, 1995) in CO. These authors find that the radial scale-length of the midplane mass density compares well with the radial scale-length of the stellar disk, implying a constancy of the mass-to-light ratio for the stellar disk.

Van der Kruit (1988) and Rupen (1991a) conclude that in the outer parts of stellar disks the local mass density is no longer dominated by the stellar disk so that the self-gravity of the gas will become important. In Chapter 2, I extend van der Kruit's pioneering work by incorporating the stellar and gaseous disks as well as the DM halo in a self consistent way: rather than making a *local* approximation to the vertical force, K_z is calculated from the *global* mass distribution of the galaxy. I also show that the self-gravity of the gas can play an important role and must be included in a self-consistent manner. The flattened potential of the stellar disk results in the existence of a preferred plane for the gas to settle in, even beyond the edge of the optical disk. Furthermore I will concentrate on the flaring behaviour in the region beyond the optical disk where the vertical force is dominated by the DM halo, and is hence sensitive to its *shape*. In combination with the newly achievable large sensitivities and the high resolution of H I synthesis observations (Rupen, 1991), these improved modelling methods allow for the determination of the shape of dark halos of spiral galaxies.

In a study similar to the one presented in Chapter 3, Rupen (1991a) analyzed high resolution VLA[3] H I observations of two edge-on galaxies, NGC 4565 and NGC 891. For these galaxies he finds that the flaring data imply stellar disks with masses $\geq \frac{2}{3}$ and $\geq \frac{1}{2}$ as massive as "maximum-disk", consistent with van der Kruit's (1981, 1988) results for NGC 891. Due to the increased sensitivity of the VLA, sensitive high resolution H I observations, as pioneered by Rupen (1991b), can now be routinely obtained and processed.

The high resolution H I data, taken on NGC 4244, presented in Chapter 3 are sensitive enough to determine the gaseous velocity dispersion and probe the region beyond the optical disk where the thickness of the gas layer is most sensitive to the shape of the DM halo. NGC 4244 is an intermediate mass system with a rotation speed of about 100 km/s. Its proximity (3.6 Mpc) results in the high linear spatial resolution (17.5 pc/arcsec) which is necessary

[3]The VLA of the National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

to successfully determine both the vertical structure of the gaseous disk and the rotation curve in the inner parts of the galaxy. The large inclination of NGC 4244 with respect to the line of sight, about $84\overset{\circ}{.}5$ degrees, allows for an accurate determination of the rotation curve. NGC 4244 is one of the few galaxies to show, on both sides, the characteristic drop in rotation velocity beyond the optical disk.

Chapter 2

Paper I, 1995, AJ, 110, 591

2. On the Usage of Flaring Gas Layers to determine the Shape of Dark Matter Halos; Paper I, 1995, AJ, 110, 591

ABSTRACT

I present a new method of deriving the shape of the dark matter (DM) halos of spiral galaxies. The method relies on the comparison of model predictions with high spectral and spatial resolution H I observations of the gas layer. So far, determinations of the flaring of the gas layer (i.e. the increase of the thickness with galactocentric radius) have been used to determine the mass-to-light ratio, $M/\mathcal{L}$, of the stellar disk of several edge-on galaxies. In this paper I describe a method which can be used to determine the shape of DM-halos. This technique will be applied in a forthcoming paper.

I show that the model predictions of the gas layer width are best calculated using a global approach, in which the potential arising from the *total* mass distribution of the galaxy is used in the calculation of the vertical distribution of the gas. I developed a new algorithm to calculate the force field of an arbitrary, azimuthally symmetric, density distribution. This algorithm is used to calculate the forces due to the radially truncated stellar disk as well as of the flaring gas layer.

I use a simple two-parameter family of disk-halo models which have essentially the same observed equatorial rotation curve but different vertical forces. This mass model is composed of a stellar disk with constant $M/\mathcal{L}$, and a DM-halo with a given axial ratio (see Sackett & Sparke ApJ, 1990, 361, 408). I approximate the radial force due to the gaseous disk, and iteratively determine the vertical force due to the global distribution of the gas.

In agreement with Maloney (ApJ, 414, 41, 1993) I find that beyond the Holmberg radius, the thickness of the gaseous disk is sensitive to both the flattening of the DM-halo and the self-gravity of the gas. I also show that the inferred DM-halo flattening is not sensitive to the particular choice of disk-halo decomposition.

I show that the determination of the thickness of the gas layer is not restricted to edge-on galaxies, but can be measured for moderately inclined systems as well. Thus, in combination with detailed modeling, high resolution H I imaging of nearby galaxies with extended H I envelopes will enable us to determine the shape of the DM halo of these galaxies.

2.1. INTRODUCTION

Until recently, little was known about the shape of dark halos: measurements of the equatorial rotation curve provide a one dimensional probe to the potential only. It is possible to construct both spherical and flat mass models which generate the same rotation curve. Therefore, the shape of the dark halo can not be inferred from galactic rotation curves alone.

Existing methods to estimate the shape of dark matter (DM) halos have given mixed results. It has been suggested (Dekel & Shlosman 1983; Toomre 1983; Sparke & Casertano 1988) that galactic warps, frequently observed in the outer parts of gaseous disks, result if the galactic disk is tilted with respect to the plane of a flattened dark halo. Hoffner & Sparke (1994) investigated the time evolution of such warps and concluded that only one out of the five systems they studied requires a halo as flattened as E6. Polar ring galaxies, early type spirals with material rings in a plane perpendicular to the galaxy plane, probe the dark halo potential in two perpendicular directions. Careful analysis of the stellar and gas dynamics of the polar ring system NGC 4650A, led Sackett & Sparke (1990, hereafter to referred as SS90) and Sackett *et al.* (1994, hereafter to referred as SRJF94) to conclude that the dark halo in this system could be as flattened as E6-E7 (q=0.4 to 0.3). Recently, Sackett *et al.* (1994b) reported the discovery of a rather flattened ($q \approx 0.5$) *luminous* halo around the edge-on spiral NGC 5907, which can account for the observed rotation curve provided that this luminous material has a large mass-to-light ratio (≈ 450).

Like the polar rotation curve, the thickness of the gas layer depends on the vertical force, K_z, and hence on the shape of the DM halo (Maloney 1992; Olling & van Gorkom 1992; Kundić *et al.* 1993, KHG93; Maloney 1993, M93). That the shape of the dark halo influences the vertical distribution of the gas can be easily understood. Consider a round halo, with a certain density distribution, then decrease the vertical scale-height while maintaining the total mass. Consequently, the densities as well as the exerted gravitational forces will increase, resulting in a thinner H I disk and higher rotation speeds. In order to keep the same equatorial rotation curve, one has to deform the DM halo in a very specific way (Appendix 2.5., Fig. 2) as a result of which the DM-halo densities (at large distances) will be roughly inversely proportional to the flattening q $(= c/a)$. In accordance with Maloney (1993), I find that (at large distances) the thickness of the gas layer is roughly proportional to the square root of the halo flattening, q. Several authors have employed measurements of the gas layer thickness to infer the total mass density in the plane of spiral galaxies. Van der Kruit (1981) showed that the width of a galaxy's gaseous disk should increase radially in an exponential fashion. From the thickness measurements of the gaseous disk of the edge-on galaxy NGC 891 by Sancisi & Allen (1979) van der Kruit concluded that the $M/\mathcal{L}$ ratio of the stellar disk does not change significantly with radius and that the dark halo can not be as flat as the stellar disk. Rupen (1991), using more sensitive and much higher resolution data, found that the width of the gaseous disks of NGC 891 and NGC 4565 increase exponentially, but for NGC 891 with large scatter. Our galaxy has been studied in much greater detail by Knapp (1987) and Merrifield (1992) in H I , and by Malhotra (1994) in CO. These authors find that the radial scale-length of the midplane mass density compares well with the radial scale-length of the stellar disk, implying a constancy of the mass-to-light ratio for the stellar disk.

Van der Kruit (1988) concludes that in the outer parts of stellar disks the local mass density is no longer dominated by the stellar disk so that the self-gravity of the gas will become important. Here I extend van der Kruit's pioneering work by incorporating the stellar and gaseous disks as well as the DM halo in a self consistent way: rather than making a *local* approximation to the vertical force, K_z is calculated from the *global* mass distribution of the galaxy. In Sec. 2.3.6. I show that the self-gravity of the gas can play an important role and must be included in a self-consistent manner. Furthermore I will concentrate on the flaring behaviour in the region beyond the optical disk where the vertical force is dominated by the DM halo, and is hence sensitive to its *shape*. In combination with the newly achievable large sensitivities and the high resolution of H I synthesis observations (Rupen, 1991), these improved modelling methods allow for the determination of the shape of dark halos of spiral galaxies.

In Sec. 2.2. I review some of the properties of dark halos and determine (in Sec. 2.2.1.) how, for a given rotation curve, the core radius and central density depend on the flattening of such an isothermal halo (see also Appendix 2.5.). The disk-halo conspiracy is investigated in section Sec. 2.2.2. (and Appendix 2.6.) where I derive general formulas for the three unknowns (the disk's $M/\mathcal{L}$ ratio and the halo's core radius and central density) which determine the overall mass distribution of the galaxy. In Sec. 2.3. I list the assumptions made to determine the z-distribution of the gas from the galaxian potential. I compare the local and global approaches in Sec. 2.3.2., where I also present the vertical force arising from the total mass distribution of the galaxy, $K_{\mathrm{z,tot}}$, for several radii (an analytic solution to the multi-component local approximation, as proposed by Bahcall (1984), is given in Appendix 2.7.). In Sec. 2.3.8. I calculate how the thickness of the gas layer depends on the two free parameters ($M/\mathcal{L}$ and q) and discuss how the thickness of the gas layer can be used to determine the flattening of the DM halo. In the discussion, Sec. 2.4., I will indicate which systems might be suitable for an analysis as proposed in this paper.

2.2. PROPERTIES OF DARK HALOS

From the observational fact that rotation curves of spiral galaxies are "flat", it has been concluded (e.g., Bosma 1978; Rubin *et al.* 1980; Bahcall & Casertano 1985; Begeman 1987) that there is an unseen mass component present in these galaxies. The standard assumption has been that this dark mass distribution is spherical and isothermal, characterized by the core radius, R_c, and central density, $\rho_{\mathrm{h},0}$ [Eqn. (15), with $q = c/a$=1]. The luminous matter consists of a stellar disk, for some systems a bulge, and a gaseous disk. The $M/\mathcal{L}$ ratios of the bulge and the disk are free parameters but are normally taken to be constant with radius. An upper limit to the $M/\mathcal{L}$ ratio of the disk, and hence a lower limit to the dark halo mass, can be obtained by assuming that the observed peak rotation velocity is due to the stellar disk only. A more common approach is to scale the mass-to-light ratios down in such a way as to avoid halos with hollow cores. This is commonly known as the "maximum-disk" hypothesis (van Albada & Sancisi 1986). Other means of constraining the $M/\mathcal{L}$ ratio exist: van der Kruit (1981) calculates $M/\mathcal{L}$ from the thickness of the gas layer, Efstathiou *et al.* (1982) invoke disk stability arguments, Athanassoula *et al.* (1987, hereafter referred to as ABP87) apply spiral instability criteria, and Bottema (1993) uses stellar velocity dispersion measurements. The mass-to-light ratios found by these authors range from 50 to 100 % of the maximum-disk value.

The values for the DM-halo core radius and the $M/\mathcal{L}$ ratio of the stellar disk are highly correlated: low mass-to-light ratios require small core radii, and large $M/\mathcal{L}$ ratios correspond to large values for the DM-halo core radius. In fact, many different disk-halo decompositions produce acceptable fits to a given observed rotation curve (van Albada *et al.* 1985, hereafter referred to as ABBS85; Lake & Feinswog 1989, hereafter referred to as LF89). The dark to luminous mass ratio may decrease with increasing mass (e.g., ABP87; Persic & Salucci 1988; Casertano & van Gorkom 1992; Broeils 1992, Chapter 10). This could be inferred from galactic rotation curves: dwarf galaxies have rising rotation curves (e.g., Carignan & Freeman 1988) while the rotation curves of massive galaxies fall in the outer parts (Casertano & van Gorkom 1991; Broeils 1992). For those galaxies where the rotation curves in the outer parts are flat, the luminous and dark matter "conspire": the increase of V^2_{halo} and the decline in V^2_{disk} are such that their sum remains approximately constant with galactocentric radius.

In view of the uncertainties in the disk-halo decomposition, it is clear that more and a different kind of data are needed for a unique determination of the halo flattening. In Sec. 2.3.8. I show that beyond the optical disk the thickness of the gas layer is rather sensitive to the flattening of the halo. In order to investigate this sensitivity quantitatively I construct a two parameter galaxy model, where these parameters are γ (= the fraction of the peak rotation curve due to the stellar disk, see Sec. 2.2.2. and Appendix 2.6.) and q, the flattening of the DM halo (Sec. 2.2.1. and Appendix 2.5.). The explicit dependence of the thickness of the gas layer upon these two parameters is discussed in Sec. 2.3.8..

2.2.1. Flattened Dark Halos

As mentioned above, the equatorial rotation curve (i.e. the rotation curve in the plane of the stellar disk) does not constrain the actual shape of the halo. In Appendix 2.5. I determine a family of flattened DM-halo models which have the same (to within 1.4 %) equatorial but different polar rotation curves. Thus, for this family of halo models, the radial force does not depend significantly on the shape of the DM halo, while the vertical force (and hence the thickness of the gas layer) does. In Fig. 1, I present the rotation curves for this DM-halo family graphically. The lower panel shows the rotation curves for these DM-halo models, while in the top panel I present the ratio of the flattened to the round DM-halo rotation curve. Although the residuals show systematic behavior, the amplitudes ($\leq 1.4\%$) are smaller than the routinely obtained observational errors (e.g., Begeman 1989, hereafter referred to as BE89; Broeils 1992). In conclusion: rotation curves of flattened DM halos are indistinguishable from their round equivalents. This family of flattened DM-halo models is fully specified by Eqns. (22) and (23) which relate the core radius and the central density to the flattening of the DM halo and is graphically presented in Fig. 2. For a given rotation curve, flattened DM-halo models have larger core radii and central densities than their round equivalents. Notice that both the central density and the core radius have an almost linear dependence on the halo flattening for moderately flattened DM halos.

Of course the DM-halo model I have chosen to work with might very well be different from the true DM halo mass distribution. However all mass distributions, with similar rotation curves, share the general feature that flatter distributions have larger vertical forces. Therefore, a DM halo flattening determined using the formalism outlined in this paper serves as an indicator of the true flattening of the (unknown) dark matter density distribution.

2.2.2. On the Disk-Halo Conspiracy

As we can measure the light distribution of the stellar disk only, and have no a priori knowledge of the mass-to-light ratio of stellar disks, the relative contributions of luminous and dark matter are not known (ABBS85 and LF89). However, as I will show in Appendix 2.6., the galaxy mass model described above is fully determined by *one* parameter regulating the relative importance of stars and dark matter.

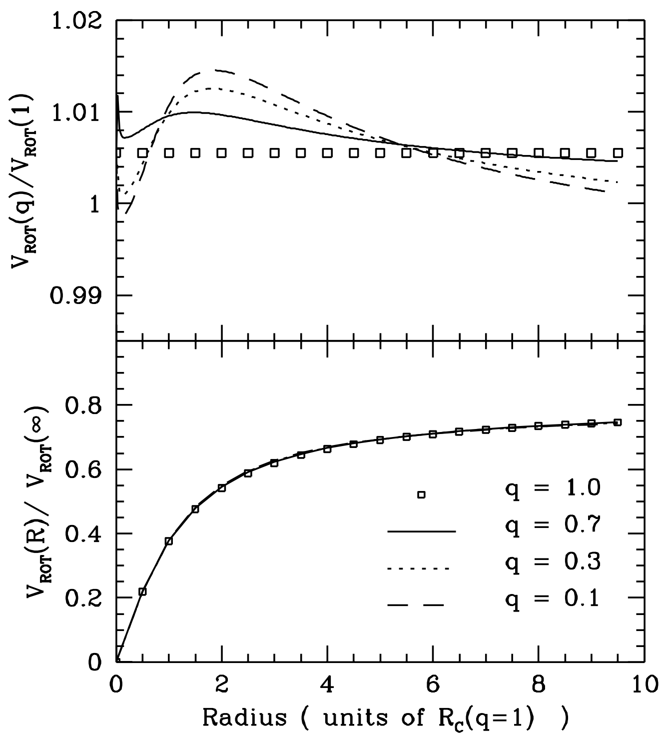


Fig. 1.— The lower panel shows the rotation curves of a family of DM-halo models with varying flattening ($q = c/a = 1.0, 0.7, 0.3, 0.1$), central density [$\rho_{\mathrm{h},0}(q)$, Eqn. (23)] and core radius [$R_{\mathrm{c}}(q)$, Eqn. (22)]. The fact that all curves fall on top of each other illustrates that the equatorial rotation curve alone is not sufficient to determine the shape of the DM halo. In the top panel I present the ratio of the rotation curve of the flattened to the round DM halo for the same q's as in the bottom panel. Although the residuals show systematic behavior, the amplitudes are smaller than the routinely obtained observational uncertainties (i.e. BE89)

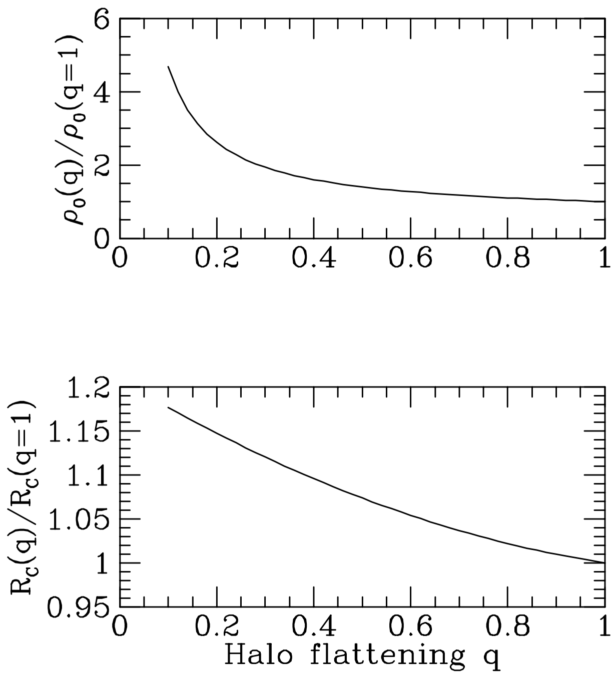


Fig. 2.— In this figure I present the q-dependence of the DM-halo central density [top panel, Eqn. (23)] and core radius [lower panel, Eqn. (22)]. With these functional forms for $\rho_{\mathrm{h},0}(q)$ and $R_{\mathrm{c}}(q)$, the resultant equatorial DM-halo rotation curve is almost independent of q (see Fig. 1).

Following Bottema (1993) I choose this parameter, γ, to be the fraction of the peak observed rotation curve which is due to the stellar disk [defined by Eq. (29)].

Different choices of γ result in quite different values for the stellar mass-to-light ratio, the halo's central density and core radius. As an example, I present the dependence of these three parameters on γ for the galaxy NGC 3198 (rotation curve and optical parameters were taken from BE89) in Fig. 3, and algebraically by Eqns. (30), (32) and (33). In Fig. 3, I have also indicated how a 5% uncertainty in the slope of the rotation curve affects the results. To obtain the core radius and central density of a flattened dark halo, one uses Eqns. (32) and (33) and multiplies these values by $\mathcal{C}(q)$ [Eqn. (22)] and $\mathcal{H}(q)$ [Eqn. (23)] respectively.

In agreement with LF89, I find that observed equatorial rotation curves do not constrain the core radius of the DM halo (or γ in our terminology) very well. This is illustrated in Fig. 4, where I present the rotation curve of a model galaxy which resembles the Sc galaxy, NGC 3198[4]. Several acceptable fits, made with different γ's, are shown.

In conclusion: the parameterization of the disk-halo conspiracy, as presented in Appendix 2.6., provides a useful way to perform the disk-halo decomposition to acceptable accuracy, and allows for a straightforward way to investigate the dependence of the thickness of the gas layer on the γ value (mass-to-light ratio) of the stellar disk.

[4]The optical parameters ($L(0)$=207.1 $L_\odot/pc^2$, $h_R = 2.3$ kpc, $z_e = 0.23$ kpc), gaseous surface density distribution and rotation curve ($V_{obs}(2.3h_R) = 146.4$, $V_{obs}(10.0h_R) = 148.0$, $V_{gas}(2.3h_R) = 13.4$ and $V_{gas}(10.0h_R) = 38.0$ km s^{-1}) were taken from BE89 who used Kent's (1987) photometry to calculate the rotation curve due to the stellar disk. The stellar disk is truncated at $6h_R$ (van der Kruit 1988).

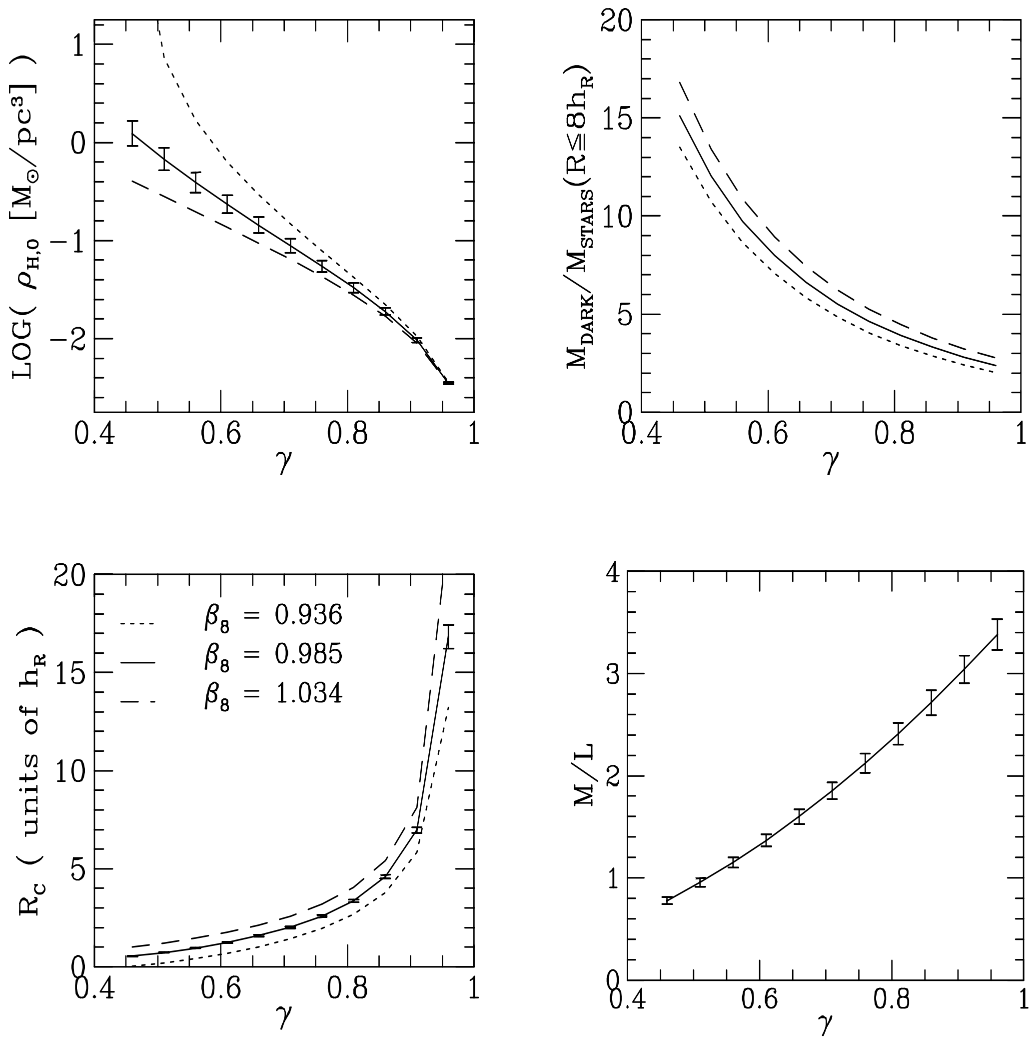


Fig. 3.— Here I show the dependence of the DM-halo parameters on the mass-to-light ratio of the stellar disk. As an example I used the parameters valid for NGC 3198 ($L(0)$=207.1 $L_\odot/pc^2$, h_R=2.3 kpc, z_e=0.23 kpc). The gaseous surface density distribution and rotation curve ($\mathcal{V}_{obs}(2.3h_R) = 145.8$) were taken from BE89, who used Kent's (1987) photometry to calculate the rotation curve due to the stellar disk (f_m=0.899 and $f_8 = 0.628$). In the lower right panel the $M/\mathcal{L}$ -ratio is presented as a function of γ [defined by Eq. (29)]. In the upper left and lower left panel I show the DM-halo central density, $\rho_{h,0}$, and its core radius, R_c respectively. The dark-to-stellar mass ratio is presented in the upper right panel. To indicate the sensitivity of the halo parameters on the slope of the rotation curve, I calculated them for three values: the appropriate β_8-value for NGC 3198 (0.985) and the ± 5% values.

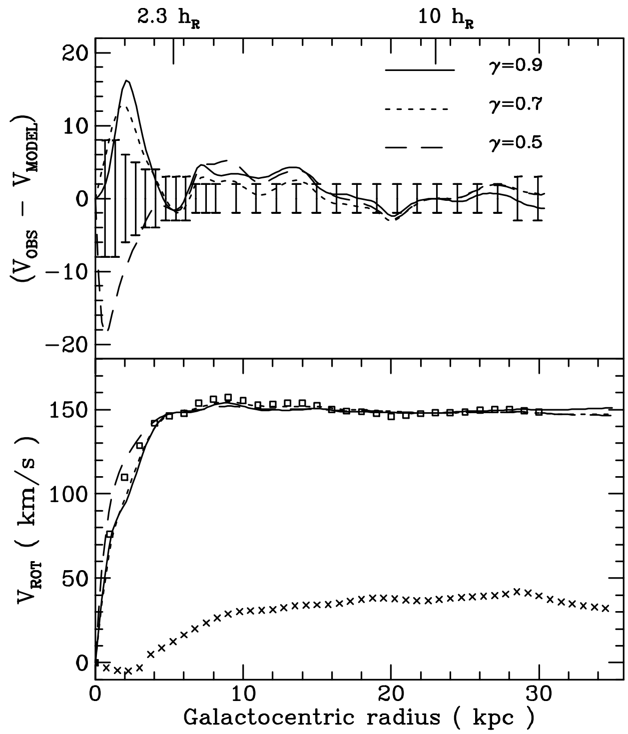


Fig. 4.— In this figure I present fits to the rotation curve of NGC3198 with different γ values. In the lower panel I present the observed rotation curve (open squared), the rotation curve due to the H I -disk (crosses) and several disk-halo combinations [full line, gas+(γ=0.9-stellar-disk)+DMhalo, $M/\mathcal{L}$ =2.96,$\rho_{\mathrm{h},0} = 11.2, R_{\mathrm{c}} = 6.26$; dotted line, gas+($\gamma$=0.7-stellar-disk)+DMhalo, $M/\mathcal{L}$ =1.79,$\rho_{\mathrm{h},0} = 92.6, R_{\mathrm{c}} = 2.00$; dashed line, gas+($\gamma$=0.5-stellar-disk)+DMhalo, $M/\mathcal{L}$ =0.92,$\rho_{\mathrm{h},0} = 684, R_{\mathrm{c}} = 0.73$], where the central densities are expressed in units of m$M_{\odot}$/pc^3 and the core radii in units of kpc. The upper panel shows the velocity difference between observations and model. The large $\gamma = 0.9$-value corresponds to the “maximum-disk” solution. A better fit is the $\gamma = 0.5$-solution (low $M/\mathcal{L}$ -disk) close to the value proposed by Bottema (1993). Furthermore, since the velocity field of the inner parts of NGC 3198 deviates from axisymmetric rotation (Corradi *et al.* , 1991), the discrepancies between observed and model curves in the inner regions seem irrelevant. The only free parameter in the fitting procedure I used (described in Appendix B) is γ [defined by Eq. (29)]. The observed rotation curve (and the errors, i.e. the difference between both sides, are indicated by the error bars), the rotation curve due to the gaseous disk, the rotation curve due to the stellar disk and the optical parameters were taken from BE89.

2.3. THE METHOD

In this section I derive the vertical distributions of the H I layer from the potential of the whole galaxy, $\Phi_{\rm galaxy}(R,z)$, and the equation of hydrostatic equilibrium. This derivation is subject to several simplifying assumptions which are listed below:

- The system is in steady state,
- and is azimuthally symmetric.
- The velocity dispersion tensor of the gas is symmetric and round, so that
- The equation of hydrostatic equilibrium is a good approximation to the vertical Jeans equation.
- The vertical velocity dispersion of the H I gas is constant (isothermal) with z-height.
- Magnetic and cosmic ray pressures are neglected

Obviously this is just an approximate description of reality. Spiral density waves violate the first two assumptions, while non-thermal pressure terms (magnetic, cosmic ray heating, ...) are not included in the equation of hydrostatic equilibrium. Keeping these assumptions in mind, I take as the starting point the vertical Jeans equation in cylindrical coordinates (as usual, R, θ, and z denote the radial, the tangential and the vertical direction respectively):

$$\frac{d\ \left(\rho(z)\sigma(R,z)^2_{\rm zz}\right)}{d\ z} \;=\; \rho(z)\ K_{\rm z}(z) - \tfrac{1}{R}\tfrac{\partial}{\partial R}\left(R\ \rho\sigma^2_{\rm Rz}\right) - \tfrac{1}{R}\tfrac{\partial}{\partial\theta}\left(\rho\sigma^2_{\theta{\rm z}}\right), \tag{1}$$

to determine the vertical distribution of the gas ($\rho(z)$). Here $K_{\rm z}(z)$ is the gravitational force (per unit mass) in the $+z$-direction and $\sigma(R,z)$ the gaseous velocity dispersion. Due to the assumption of azimuthal symmetry, the last term in Eqn. (1) vanishes. The $\sigma_{\rm Rz}$-term essentially measures the tilt of the velocity dispersion ellipsoid, for cylindrical rotation it is identically zero. In the most extreme case the velocity dispersion ellipsoid points towards the galactic center[5]. At small z/R, $\sigma_{\rm Rz}$ is approximately $(\sigma^2_{\rm RR}-\sigma^2_{\rm zz})z/R$ so that its contribution is expected to be small[6]. Thus, the non-diagonal terms of the velocity dispersion tensor (i.e. $\sigma^2_{\rm Rz}$) will vanish. Then, the vertical Jeans equation reduces to the equation of hydrostatic equilibrium:

$$\frac{d\ \left(\rho_{\rm gas}(z)\sigma(R,z)^2_{\rm z,gas}\right)}{d\ z} \;=\; \rho_{\rm gas}(z)\ K_{\rm z}(z). \tag{2}$$

---

[5] I thank the referee, Phil Maloney, for pointing this out.

[6] In the Galaxy, $\sigma_{\rm RR}$ and $\sigma_{\rm zz}$, as measured by Malhotra's (1994) and Blitz *et al.* (1984) are almost equal: 7.8 ± 3 versus 5.7 ± 1.2 km s^{-1}.

With the assumption that the gas is isothermal in the z-direction it follows that:

$$\sigma^2_{\rm z,gas}\,\frac{d\,\ln\rho_{\rm gas}(z)}{d\,z} \;=\; -\frac{d\,\Phi_{\rm galaxy}(R,z)}{d\,z}, \tag{3}$$

so that

$$\rho_{\rm gas}(R,z) \;=\; \rho_{\rm gas}(R,0)e^{-\Phi_{\rm galaxy}(R,z)/\sigma(R)^2_{\rm z,gas}} \tag{4}$$

$$\approx\; \rho_0\exp\left(-z^2/2W^2_{\rm gas}\right). \tag{5}$$

We see that the gaseous density distribution can be calculated once the gaseous velocity dispersion and the potential of the galaxy, $\Phi_{\rm galaxy}(R,z)$, are known.

Generally speaking, the vertical distribution of the gas will not be a "simple" function of z. Since the observational data does generally not allow for a more sophisticated analysis than the fitting of Gaussian functions to the measurements, I choose to fit Gaussians to the model density distributions [Eqn. (4)] as well[7]. In the remainder of this paper I will use the dispersion, $W_{\rm gas}$, of this Gaussian, as defined by Eqn. (5), as a measure of the width of the gas layer.

I incorporate three components in the present study: 1) a stellar disk with constant scale-height, 2) a flattened isothermal DM halo, and 3) a gaseous disk. The stellar density distribution for the stars must be close to the one derived from surface brightness measurements by van der Kruit & Searle (vdKS81a&b and vdKS82a&b), namely:

$$\begin{aligned}\rho_s(R,z) &= M/\mathcal{L}\;\;\rho_s(0,0)\,e^{-R/h_R}\,{\rm sech}^2(z/2z_e) \qquad && R\le R_{\rm max} \qquad [M_\odot/{\rm pc}^3] \\ &= 0.0 && R > R_{\rm max},\end{aligned} \tag{6}$$

with $R_{\rm max}=(4.7\pm0.7)h_R$, $2z_e\approx h_R/(4.7\pm1.8)$, and $M/\mathcal{L}$ the average mass per unit luminosity. Note that Barteldrees & Dettmar 1993 (BD93) find that the optical disks are truncated at significantly smaller radii: $R_{\rm max}=(3.0\pm0.2)h_R$, $2z_e\approx h_R/(4.0\pm0.3)$. In

[7] Since the *shape* of the potential changes with galactocentric radius, the gaseous density distribution will also change shape. Thus, the accuracy of the approximate density distribution [Eqn. (5)] will also change with galactocentric radius. Other indicators for the width of the gas layer could be used as well. For example the normalized second moment of the density distribution, $W_{\rm MOM2}=\sqrt{\int_{-\infty}^{+\infty}z^2\,\rho(z)\,dz\;/\;\int_{-\infty}^{+\infty}\rho(z)\,dz}$, is more sensitive to the high-z parts of the density distribution, yielding *larger* values for the "width" of the gas layer. On the other hand, the width could be calculated from the Full Width at Half Maximum ($W_{\rm FWHM}=FWHM/2.35$), a calculation which is more sensitive to the low-z part of the density distribution, so that *smaller* values for the "width" are found. Significant differences between these three width measures indicate that the true density distribution is not Gaussian. For the toy model discussed below the $W_{\rm FWHM}$ and $W_{\rm MOM2}$ width estimators can be as much as 20% larger and smaller than $W_{\rm gas}$ respectively (inside the optical disk). Beyond the optical disk, the gaseous distribution is close to Gaussian.

external galaxies it is very hard to observationally distinguish a sech-squared distribution from an exponential distribution (i.e. Wainscoat *et al.* 1989). For the Galaxy on the other hand, there is a large body of evidence suggesting that the vertical distribution can be better represented by an exponential distribution[8]: $\rho_s(z) = \rho_s(0)\, e^{-z/z_e}$ (Gilmore & Reid 1983; Pritchet 1983; van der Kruit 1986; Yoshi *et al.* 1987). Therefore, I calculate model potentials for the exponential distribution. I chose the flattened isothermal distribution as proposed by SS90 [Eqn. (15)], discussed above, to represent the DM-halo mass distribution.

The vertical force of a truncated stellar disk as well as of a flaring disk is calculated from the potential:

$$\Psi(R,z) = -2G\int_0^\infty r dr \rho(r,0)\int_{-\infty}^{\infty} dz'\rho(r,z')\int_0^\pi \frac{d\theta}{|\overline{r}-\overline{R}|} \quad , \tag{7}$$

$$K_z(R,z) = -\frac{d}{dz}\Phi(R,z) \tag{8}$$

$$= -2G\int_0^\infty r dr \rho(r,0)\int_{-\infty}^{\infty} dz' \frac{\rho(r,z')(z-z')f(R,z,r,z')}{\sqrt{(R+r)^2+(z-z')^2}\,([R-r]^2+[z-z']^2)} \quad , \tag{9}$$

where $\rho(r,z')$ is the density distribution for which to calculate the forces. At the center of the galaxy, $f(0,z,r,z')$ equals π, for all other galactocentric radii $f(R,z,r,z')$ is the complete elliptic integral of the second kind: $f = E(k) = \int_0^\pi d\phi\sqrt{1-k^2\sin^2\phi}$, with $k^2 = 4r/[(1+r)^2+(z-z')^2]$. With special care for the region around (R,z) this integral can be solved numerically and takes about 8 seconds per point on a SPARC 2 processor. I use an exponential for the vertical density distribution, $\rho(0,z')$, of the stars, while I approximate the gaseous distribution by a Gaussian. As neither the vertical scale-height nor the midplane density, $\rho(r,0)$, are generally analytic functions, I store their values in tabular form and determine the interpolated value whenever the integrating routine (QROMB or QROMO, Press *et al.* 1990) requires so. I used the double exponential stellar disk (Kuijken & Gilmore, 1989, hereafter KG89, their Eqn. [27]) as a test case and found that the two methods of calculation agree to within 1 part in 1,000[9].

2.3.1. The Global Approach

As a first step I use the radial part of the potential to determine the structural parameters of the stellar disk and DM halo by requiring that they reproduce the observed equatorial rotation curve. The radial force due to the gas is calculated (using the ROTMOD program in GIPSY, van der Hulst *et al.* 1992) assuming that it is infinitely thin. I use the

---

[8]In this paper I do not include any thick disk component. Note that these isothermal and exponential vertical distributions have the same slope at large z-heights. Thus for a given surface density and high-z slope, the exponential distribution has a twice larger value of the midplane density than the isothermal distribution (see also van der Kruit, 1988).

[9]This accuracy equals ten times the accuracy obtained in the integrating routines.

observed photometry and a constant $M/\mathcal{L}$ ratio to calculate the rotation curve due to the stars (also by using ROTMOD). I then perform the disk-halo decomposition outlined in Appendix 2.6..

I use the values for the mass-to-light ratio of the stellar disk and the core radius and central density of the DM halo, as found in the disk-halo decomposition, to calculate the vertical force due to the stellar disk [Eqn. (9)], and the DM halo [SRJF94's Eqn. (6)]. These forces are integrated numerically to yield the potential, from which the gaseous volume density distribution, $\rho_{\rm gas}(R,z)$, is calculated [using Eqn. (4), an assumed gaseous velocity dispersion, and an observed surface density distributions].

I follow an iterative procedure, where the gaseous density distribution, calculated from an approximation to the true total potential, is used to estimate the gaseous contribution to the potential, which must then be added to the contributions from the stellar disk and DM halo to yield a better approximation to the true total potential, from which a better approximation to true gaseous density distribution can be calculated, ... etc.

Thus, the vertical distribution of the gas is not fully consistent with the Poisson equation

$$4\pi G\,\rho_{\rm tot}(R,z) \;=\; -\frac{1}{R}\,\frac{\partial(R\,F_R)}{\partial R} \;+\; \frac{1}{R^2}\,\frac{\partial^2\Phi}{\partial\phi^2} \;-\; \frac{\partial K_{\rm z}}{\partial z}, \tag{10}$$

since I take the vertical distribution of the gas layer to be Gaussian, and because the radial force (F_R) due to a thick gas layer differs from the radial force arising from an infinitely thin disk. These inconsistency are expected to be small because of the small scale-height and low mass of the gaseous disk. This procedure to calculate the thickness of the gas layer is what I term the *global approach.*

In order for the global model to be self-consistent, the stellar velocity dispersion tensor ($\sigma(R,z)^2_{*;{\rm i,j}}$, with ij all the possible coordinate combinations) has to be such that for the assumed stellar density distribution and the calculated total potential, the vertical Jeans equation (1) will be satisfied. Thus, every mass model described above makes two predictions, firstly the radial variation of the width of the gaseous layer, and secondly the stellar velocity dispersion. In this paper I concentrate on the radial variation of the thickness of the gas layer.

2.3.2. The Local Approach

To date, most authors have determined the distribution of the gas above the plane from the *local* mass densities only. In this section I compare this local approach with the global approach described in the previous section. To separate variables I will postpone investigating the effects of the gaseous self-gravity till Sec. 2.3.6..

First I describe how in the local approach the vertical force is calculated from the local mass densities and gradients in the rotation curve. Assuming that, in the region where the gas is found, the rotation speed does not depend on z-height and that the vertical force does not depend on radius, the Poisson equation (10) reduces to the local Poisson equation:

$$4\,\pi\,G\,\rho(z) \;=\; -\frac{1}{R}\,\frac{d(R\,F_R)}{dR} - \frac{dK_z}{dz}. \tag{11}$$

Combining the appropriate equation of hydrostatic equilibrium (3) with Eqn. (11) I find:

$$\sigma^2_{\mathrm{z,gas}}\,\frac{d\,\ln\rho_{\mathrm{gas}}(z)}{d\,z} \;=\; -4\,\pi\,G\int_0^z dz'\left\{\rho_{\mathrm{matter}}(z') + \left(\frac{-2}{4\,\pi\,G}\right)\left(\frac{V_{\mathrm{rot}}}{R}\,\frac{dV_{\mathrm{rot}}}{dR}\right)\right\}.$$

Setting the rotation curve term to ρ_{rot}, it follows that:

$$\sigma^2_{\mathrm{z,gas}}\,\frac{d\,\ln\rho_{\mathrm{gas}}(z)}{d\,z} \;=\; K_{\mathrm{z}}(z) \;=\; -4\,\pi\,G\int_0^z dz'\left\{\rho_{\mathrm{matter}}(z') + \rho_{\mathrm{rot}}(z')\right\}. \tag{12}$$

Negative and positive rotation densities (ρ_{rot}) correspond to rising and falling rotation curves respectively. The effect of a rising rotation curve is to increase the thickness of the gaseous and stellar disks, while in a falling rotation curve a disk will be thinner relative to the case where $\rho_{\mathrm{rot}} = 0$. Bottema *et al.* (1987) find that the rotation term can contribute significantly in the rising parts of galactic rotation curves. For the case of a flat rotation curve, Eqn. (12) leads to the plane-parallel-sheet case. Eqn. (12) can be solved either iteratively or analytically (if more simplifications are introduced): the multi-component and Gaussian-equivalent methods described below.

2.3.3. The Gaussian-equivalent method

In order to arrive at the (analytic) Gaussian-equivalent method, we simplify matters further by assuming that the self-gravity of the gas is negligible and that the total mass density does not vary significantly with height above the plane. I then arrive at:

$$\sigma^2_{\mathrm{z,gas}}\,\frac{d\,\ln\rho_{\mathrm{gas}}(z)}{d\,z} \;=\; -4\,\pi\,G\,\rho_{\mathrm{tot}}(0)\,z,$$

so that the vertical distribution is Gaussian with dispersion $W_{\mathrm{gas,G}}$:

$$\rho_{\mathrm{gas}}(z) \;=\; \rho_{\mathrm{gas}}(0)\exp\left(-z^2/2W^2_{\mathrm{gas,G}}\right).$$

For which the following relation holds:

$$W_{\mathrm{gas,G}} \;=\; \frac{\sigma_{\mathrm{z,gas}}}{\sqrt{4\,\pi\,G\,\rho_{\mathrm{tot}}(0)}}\qquad. \tag{13}$$

In the Gaussian-equivalent method one then assumes that the Gaussian width ($W_{\rm gas,G}$) can be equated with the observed width W_{gas} [$\approx FWHM/2.35$, as defined by Eq. (5)], so that the inferred midplane mass density ($\rho_{\rm inf}$) is given by:

$$\rho_{\rm inf} \;=\; \frac{1}{4\pi G}\left(\frac{\sigma^2_{\rm z,gas}}{W^2_{\rm gas}}\right). \tag{14}$$

Inside the stellar disk, the Gaussian width ($W_{\rm gas,G}$) will always be smaller then the true width because, for any density distribution, the volume density decreases with z-height. The inferred midplane mass density, as calculated from the true width $W_{\rm gas}$ (which is larger than $W_{\rm gas,G}$) and Eqn. (14), will therefore underestimate the true midplane mass density. This effects is illustrated in Fig. 5, where I plot the ratio of the inferred midplane mass density ($\rho_{\rm inf}$) and the actual midplane density ($\rho_{\rm tot}$). For several mass models resembling NGC 3198, this method recovers 50 to 110 % of the true midplane density. Especially for the round halo case ($q = 1.0$) we clearly see the effects of the truncation of the stellar disk at $R = 6h_R$. The truncation of the stellar disk of NGC 3198 occurs at a relatively large distance ($6h_R$ versus $4.7h_R$ on average, vdKS81a&b and vdKS82a&b, and BD93) so that it's effect may be relatively small for NGC 3198. On the other hand, the effects of the truncation of the stellar disk are smaller if the vertical distribution of the stars is sech^2 rather than exponential (i.e. less concentrated towards the midplane). It should be noted that these results depend somewhat on the width estimator being used (see footnote 7), for example the radial variation of $\rho_{\rm inf}/\rho_{\rm tot}$ arrises partly because the vertical distribution of the gas gradually changes from a distribution which is strongly centrally peaked (in the inner two scale-lengths) to a Gaussian (beyond the optical disk). Including the rotation density will improve the Gaussian-equivalent method beyond the truncation of the optical disk (right-hand side panel). The radial variation in the density ratio beyond the optical disk results from small scale structure in the model rotation curve. A solution to Eqn. (12) is possible only if the sum of the rotation and DM densities is larger than zero see Eqn. (13) and Appendix 2.7.]. For example, this is *not* the case for a maximum-disk model of NGC 3198 in the inner two radial scale-lengths.

Although its accuracy varies systematically with radius, the Gaussian-equivalent method works reasonably well, especially beyond the stellar disk.

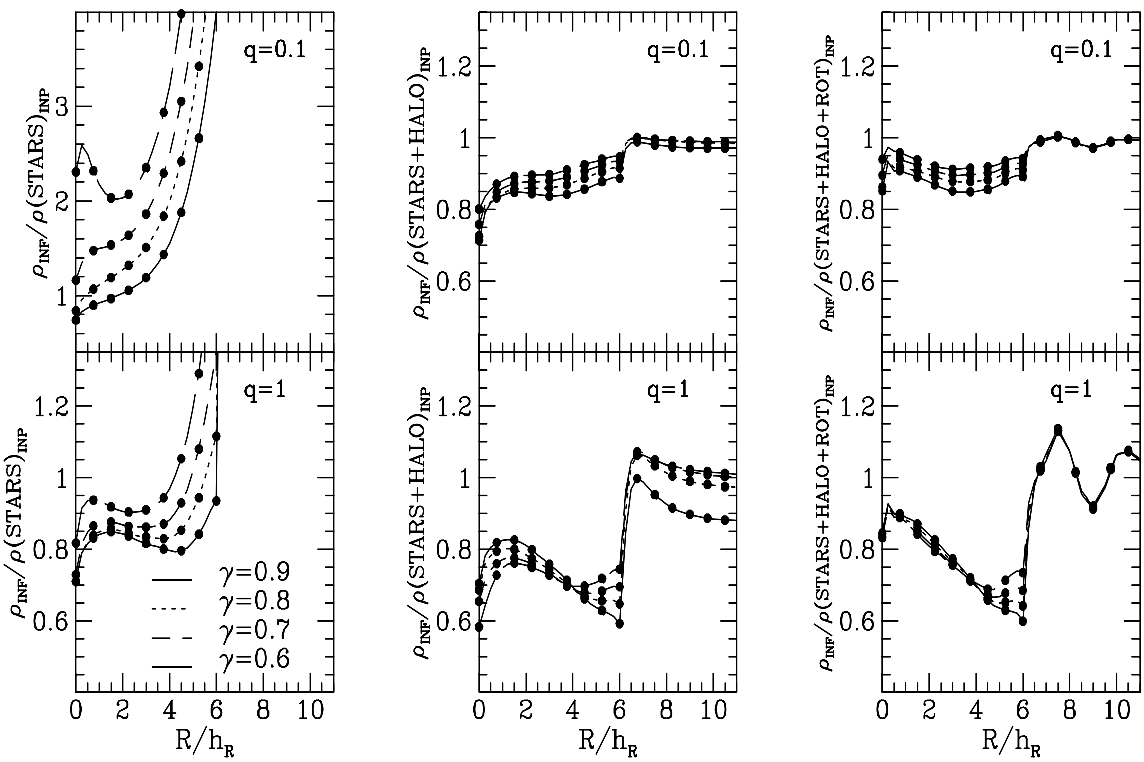


Fig. 5.— For a model galaxy (consistent with the parameters of NGC 3198 as observed by BE89) we compare the midplane mass densities inferred from the thickness of the gas layer [$\rho_{\rm INF}$, using the local approach via Eqn. (14), where $\sigma_{\rm z,gas}$=7.5 km s^{-1}] with the true stellar midplane densities ($\rho(\rm STARS)_{INP}$, left-hand set of panels), the true (stellar+DM-halo) densities ($\rho(\rm STARS + HALO)_{INP}$, middle set of panels), and the true (stellar+DM-halo+rotation) densities ($\rho(\rm STARS + HALO + ROT)_{INP}$, right-hand set of panels). The truncation of the stellar disk at $6h_{\rm R}$ is easily recognized for almost all disk-halo models. The self-gravity of the gas has not been included in these calculations. The bottom two panels were calculated for a round halo, while the top panels corresponds to a flattened (q = 0.1) DM halo. The lines correspond to models with different γ values (full line: γ = 0.9, dotted line: γ = 0.8, short dashed line : γ = 0.7 and the long dashed line: γ = 0.6). For most models, the ratio of derived-to-input densities (vertical axis) deviates significantly and systematically from unity so that the Gaussian-equivalent method can not be used to reliably determine the midplane volume densities (see also Sec. 2.3.2.)