Astronomy 422 Cosmology
Homework Assignment #1 – Due 11:00 am February 18th
80 points

Please explain your answers in complete sentences where called for. Although collaboration is encouraged, remember to answer in your own words. Where math is involved, please be sure to show all your work, and answer in no more than a reasonable number of significant digits.

1. (10 pts) You were told that homogeneity and isotropy conditions lead to a linear Hubble law; namely that $v = H_0 d$ rather than $H_0 d^2$ or $H_0 \ln(d)$, etc. It's time to convince yourself.
   a. (easier) Show that if we assume Hubble's law from our perspective, then all observers would measure the same law from their perspective (thereby demonstrating Hubble's law implies homogeneity and isotropy).
   b. (trickier) Now do the reverse. Demonstrate that isotropy and homogeneity actually *require* the form of Hubble's law for local motions (i.e., for small redshift $z \ll 1$). You might start by considering two arbitrary observers separated a distance d apart, and show that if they see the same law for an arbitrary *third* object, the only possibility is that the apparent recession speed must be proportional to the distance. For this problem, assume that both redshift and distance can be measured with arbitrary precision, and that all objects move with the Hubble flow.

2. (25 pts) The CMB is the universe's best example of a blackbody. Our observations of it since the elder days of CoBE (early 1990's) have shown that its energy distribution follows a Planck-like spectrum to within about 1 in 10^5, (more on that later in the semester). The equation given in your book (2.8) for the energy density of a blackbody in an interval of df around f (same as $d\nu$ around ν in your class notes) is:

$$\epsilon(f)df = \frac{8\pi h}{c^3}\frac{f^3 df}{e^y - 1} \qquad y = hf/kT \qquad \text{(hw1.1)}$$

   a. Rewrite this as the energy density in an interval of $d\lambda$ around λ. Note: you can't *just* algebraically substitute using $\lambda f = c$!
   b. That unitless $y = hf/kT$ is a very useful thing; rewrite hw1.1 above as $\epsilon(y)dy$ and comment on its dependence on T (think Stefan-Boltzmann law here). Using your favorite plotting program, plot $\epsilon(y) \times A$ *vs.* y where $A = c^3 h^2/(8\pi(kT)^3)$ (in other words, recreate fig 2.8). Do you understand why A is **not** $A = c^3 h^3/(8\pi(kT)^4)$ as you might expect looking at equation 2.9?
   c. Plot both $\epsilon(f)df$ *vs.* f *and* $\epsilon(\lambda)d\lambda$ *vs.* λ for T = 2.725K. If you need helpful suggestions, come by during office hours. You will need to toy with your frequency and wavelength ranges to make these look like part b.'s plot.
   d. Did you expect $\lambda_{max} = c/f_{max}$? Ha! Show (using calculus) that the two maxima are indeed, where you found them on the two plots and verify that they are **not** related so simply as $\lambda_{max} = c/f_{max}$.
   e. What are the frequencies of the receivers on WMAP (type "WMAP specs" in your favorite search engine)? Plot these on your graph of $\epsilon(f)$ *vs.* f . Are you surprised? What might the receivers be measuring *besides* the CMB and why?

3. (5) Problem 2.1 in our text. This is a rough calculation and one significant digit precision is perfectly acceptable.

4. (10) Compute the total energy density of the CMB. Compare your answer to number 3. You will have to think about the fact that mass is energy to do this part.

5. (10) (This problem brazenly stolen from Cole Miller) In practice, most surveys are **flux-limited,** meaning that bright things can be seen farther than dim things. Consider the following simple case. A catalog of sources, which all emit equally and steadily in all directions, has objects whose luminosity (energy per time) ranges from $L_{min} \rightarrow L_{max} \gg L_{min}$ That is, any individual object in the catalog has a completely constant luminosity, but different objects can have different luminosities within that range. Assume that the physics of these objects motivates us to believe that there is an equal probability of a source having a luminosity anywhere in this range. Therefore, if we had a truly representative sample, we would find an average luminosity of $\bar{L} = (L_{min} + L_{max})/2 \approx L_{max}/2$ In reality, though, we're given a real survey which is only complete (i.e., no missed objects) for fluxes $F > F_{\min}$, but sees no objects at all with fluxes lower than $F_{\min}$.

    a. What average luminosity do you infer from the objects you detect? Discuss whether this average should be larger or smaller than $L_{\max}/2$. For this part, ignore any effect of redshift.

    b. In reality, a deep survey would begin to see an effect from redshift on its completeness. Discuss what effect this would have on your estimate for $\bar{L}$

6. (10) Energy is conserved. No, really. The Hamiltonian H=T+V of a conserved system is really just the total energy in the system (eq 3.6). Let's add a cosmological constant to the Friedmann equation (eq 3.10) with malice aforethought (don't worry about the 1/3: it's for convenience later, or is related to the original GR derivation, your choice):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \qquad \text{(hw1.2)}$$

    Work backwards towards something in the form of equation 3.7 and derive the original term that must have led to and explain what kind of force it must lead to on a "particle," i.e., is it attractive? Does it grow or shrink as a function of distance from the particle, etc.

7. (10) Starting with eq hw1.2 above, derive what the "acceleration equation" (eq 3.18) must look like with such a new beast. You can check your answer easily enough by scouting ahead to eq 7.2 in the book, but you must show all your work!