Astronomy 422 Cosmology **Homework Assignment #2 – Due 11:00 am March 23rd**
80 points

Please explain your answers in complete sentences where called for. Although collaboration is encouraged, remember to answer in your own words. Where math is involved, please be sure to show all your work, and answer in no more than a reasonable number of significant digits.

1. (15) You were given a dreadful lecture by me regarding theoretical physicists' knack for setting $G, \hbar, c, k_B$ equal to 1. By the time you get around to doing this homework, I should have fixed this by giving a brilliant one (we can only hope it made more sense the second time around). By thinking about units, we can show that combinations of these constants tell us about the scales where different physics is relevant.
    a. (5) What is the "Planck length"? Start with $G^{\alpha} \hbar^{\beta} c^{\delta} k_B^{\epsilon}$ and make sure you choose α, β, δ, and ϵ so that the various units cancel and give you only meters (warning: some of these powers might be zero!). Once you know the powers, work out the actual number in meters. You can check your answers online, but I really want to see you start generally with the powers and work out what they need to be; I don't just want you looking this up. **Show all your work for full credit.**
    b. (2) What is the "Planck time"? You could use the same method, but this one's really simple if you do part a. first!
    c. (5) What is the "Planck mass"? Same method.
    d. (2) What is the "Planck energy"? Again, easy if you do part c. first.
    e. (1) What is the "Planck temperature"? Dead easy given part d.
2. (20) **Show your work!** We define ρ_{crit} as follows:
$$\rho_{crit} = \frac{3H^2}{8\pi G} \quad \text{(hw2.1)}$$
Liddle's book evaluates the constants for this critical density and finds that:
$$\rho_{crit,0} = 1.88 h^2 \times 10^{-26}\,\mathrm{kg/m^3} \quad \text{(hw2.2)}$$
where I remind you that the zero subscript is the traditional, grandfathered, and confusing convention for labeling "today's" values (if it helps, think of the zero subscript as redshift since "today" is "here and now" where $z=0$). Also, h represents our uncertainty in H_0 and has nothing to do with quantum mechanics (yet).
    a. (5) Plug in the various constants and show that you get the same number.

    But wait a gosh darn minute! Units like kilograms and meters are just not relevant to cosmology. You don't measure a lecture in nanoseconds (though I suppose it can seem to drag on!), and you don't measure a roadtrip in lightyears (though if only we could!).
    b. (3) Convert (as the book does) ρ_{crit} into solar masses per Mpc^3. This is basically "~one galaxy per cubic Mpc." See how simple things are in the right units? In cosmology, the basic mass unit is a galaxy, and the basic length is (at least) a Mpc.

    Note that we are not saying $h=1$ here, but it's darn close to one, so casting things in terms of h accents the cosmologi-ness of what we're doing. *(2. continued on next page)*

c. (2) Convert ρ_{crit} into units of Planck critical density ("WTF?!" I hear you cry...don't panic...start with your answer from 2a. and use your answers from 1a. and 1c. – this is fairly easy; just be sure to leave the h^2 alone in your answer)

d. (5) Comment on whether Planck units make sense to use for the physics of today's large scale universe. Would they make more sense if H were much larger or much smaller? In general, as we let $a \rightarrow 0$ (go far back in time) which does H do?

You'll note that apart from h^2, this ρ_{crit} number is just a constant independent of anything else about the universe. Why? To help answer this next question, consider that h is a **scale, not a constant.**

e. (5) Why does the Hubble scale show up as h^2 in ρ_{crit} and not, say as h^5 or $h^{1/2}$? Think about what a density is and discuss how our uncertainties in measuring mass by measuring luminosity and distance might play a role here.

3. (20) Let's solve for the scale factor, a, as a function of time for whenever each of four constituents is dominant (i.e., ignoring the others). Most of this is also in the book.

   a. (8) Start by rederiving the densities of our four favorite constituents as functions a. Specifically, just as I did in class, start from the first law of thermodynamics (our version = "fluid equation"), and derive (hw2.3) below by assuming the appropriate equations of state $p=f(\rho)$. Be sure to show all your work for at least one of them, and then show you understand how the others follow suit.

$$\rho_{radiation}=\frac{\rho_{radiation,0}}{a^4} \qquad \rho_{matter}=\frac{\rho_{matter,0}}{a^3} \qquad \rho_k=\frac{\rho_{k,0}}{a^2} \qquad \rho_\Lambda=\frac{\rho_{\Lambda,0}}{a^0} \qquad \text{(hw2.3)}$$

   b. (12) Now solve the FRWL equation four times assuming in turn that each constituent is all there is (i.e., is the overwhelming dominant source for the ρ term). Comment on whether you can do this for all values of curvature constant k.

4. (15) Let's begin with our old friend, the FRWL equation (hw1.2 or 3.19 in the book), and generalize it using a series of $\Omega_{i,0}$ for each constituent, explicitly putting in the scale factor a dependence. Eq. 6.7 in the book and your answers from 3 above will be helpful.

   a. (10) Show that the FRWL equation can be written as:

$$H^2=H_0^2\left(\frac{\Omega_{R,0}}{a^4}+\frac{\Omega_{m,0}}{a^3}+\frac{\Omega_{k,0}}{a^2}+\Omega_{\Lambda,0}\right) \qquad \text{(hw2.4)}$$

   b. (5) Show that if $\Omega_{R,0}+\Omega_{m,0}+\Omega_{\Lambda,0}=1$ that it *always* equals 1, i.e., even if we don't use the zero subscripts and are talking about any old redshift.

5. (10) Do 4.2 in the book.