Astronomy 422 Cosmology **Homework Assignment #4 – Due 5:00 pm April 30th**
120 points

Please explain your answers in complete sentences where called for. Although collaboration is encouraged, remember to answer in your own words. Where math is involved, please be sure to show all your work, and answer in no more than a reasonable number of significant digits.

1. (25) Basic 1st order perturbation equations for the growth of structure ignoring pressure can be "simplified" into a 2nd order differential equation for δ:

$$\ddot{\delta}+2\frac{\dot{a}}{a}\delta-4\pi G\bar{\rho}\delta=0 \qquad \text{(hw4.1)}$$

   a. (10) Assume the universe is completely matter-dominated. Show that (hw4.1) can be rewritten as a differential equation with only $\delta, \dot{\delta}, \ddot{\delta}, t$ and some constants, (i.e., get rid of $\bar{\rho}, \dot{a}$ and a by rewriting them in terms of t).

   b. (10) Now assume $\delta \propto t^n$. Solve for δ. This is a 2nd order DE, so there are two possible solutions for n.

   c. (5) Comment on whether I care about both solutions...

2. (30) Go to the WMAP page http://map.gsfc.nasa.gov/resources/camb_tool/index.html and take a look at the WMAP CMB Analyzer. On it, they've kindly placed the data points from WMAP's mission along with a theoretical curve you can play with. Dial the various parameters until you get the gratifying message, "Success!!" in the background. You've matched theory to observation! For these questions, don't forget that large angle (large scale) on the sky is small l.

   a. (10) Leaving everything else dialed "correctly," play with the redshift of reionization. Explain what you see when $z_{\text{reionization}}$ is < 11 or > 11 and why.

   b. (10) Put $z_{\text{reionization}}$ back to 11 ("but this one goes to 11!") and now leave everything else dialed "correctly" except Ω_Λ. (It's labeled more generically as dark energy.) Explain what you see when $\Omega_\Lambda < 0.74$ or > 0.74 and why.

   c. (10) Now put Ω_Λ back to 0.74 and leave everything else dialed correctly except n_S. Explain what you see in terms of large and small scale power as you change the spectral index.

3. (20) Problem 12.2 in Andrew Liddle's book.

4. (20) Wherever possible, use energy units $(c=k_B=\hbar=1)$.

   a. (10) Start by assuming that the energy density of the universe is the Planck energy density (Planck energy divided by Planck length cubed) at the Planck time. (Before that, who knows?) So, consider this to be the ultimate initial condition. Write this down as your ρ_0.

   b. (10) Ignoring inflation, What is the temperature as a function of time based on these values? Reduce it to a function of *T, t* and numbers (not unspecified constants). (Recall the relationship between energy density and temperature!) If you get around ~1 MeV at ~1 second, you're probably on the right track!

5. (25) The simplest inflaton is a scalar field with a simple potential (and for the record, it's probably wrong).

   a. (5) The energy density of such a beast is $\rho=\frac{1}{2}\dot{\phi}^2+V(\phi)$ and it can be shown that the pressure content of such a field goes like $p=\frac{1}{2}\dot{\phi}^2-V(\phi)$. Assume that the universe undergoes a Λ-like period of inflation – what general constraint does this put on what $\dot{\phi}^2$ can be compared to $V(\phi)$?

   b. (5) Using the fluid energy equation $\dot{\rho}+3H(p+\rho)=0$, plug in the expressions above and show what the fluid energy equation becomes as a function of $\phi, \dot{\phi}$ and $\ddot{\phi}$.

   c. (5) From your answer to part b., treat the $\frac{-dV}{d\phi}$ term as a force and the $3H\dot{\phi}$ term as a friction term. In particular, presume that ϕ reaches a "terminal velocity." Given the constraint from part a. above, show that we must have:
   $$\left(\frac{dV}{d\phi}\right)^2 \ll 9H^2V$$
   This is heading towards the idea of "slow roll."

   d. (5) But, you know what *H* has to be during inflation! Rewrite it in terms of $V(\phi)$ to show that
   $$\left(\frac{dV}{d\phi}\right)^2 \ll 24\pi G V^2$$, i.e., $$\left(\frac{E_{Planck}}{V}\frac{dV}{d\phi}\right)^2 \ll 1$$

   e. (5) Inflation cools the universe absurdly. *T* still follows our favorite equation $T \sim \frac{1}{a}$. Assume the inflaton (whatever $V(\phi)$ might be) starts at $t=10^{-36}$s and causes 100 e-folds, what is the temperature at the end of inflation?
   [We know that can't be right, so when inflation ends it has to "reheat" the universe back up to the T *it had before by dumping energy into regular matter and energy at the GUT scale (10^{16} GeV).]*