Zhou bi suan jing

The *Zhou bi suan jing* is a collection of ancient Chinese texts on astronomy and mathematics probably assembled during the western Han dynasty in the first century B.C.E. But its name, "The Gnomon of the Zhou Dynasty" shows that traditionally it is reputed to have been written during the Western Zhou dynasty about a thousand years earlier. It includes both determinations of the size of the heavens and calendar calculations. To parallel the Eratosthenes and Aristarchus works of ancient Greece we will just work out the Sun–Earth distance and comparative sizes.

What?

"Long ago, Rong Fang asked Chen Zi 'Master, I have recently heard something about your Way. Is it really true that your way is able to comprehend the height and size of the Sun, the [area] illuminated but its radiance, the amount of its daily motion, the figures for its greatest and least distances, the extent of human vision, the limits of the four poles, the lodges (constellations) into which the stars are ordered, and the length and breadth of heaven and Earth?'

'It is true' said Chen Zi."

Why?

One of the roles of the Chinese Emperor's was to provide order in both the human and heavenly spheres. Visible disorder in the natural world was then a sign of problems in the human world, and could be interpreted as evidence of a failure on the Emperor's part. Chinese astronomers were then to try to find order, as much as possible, in all astronomical phenomena. Until the end of imperial China all governments maintained a staff of astronomers, and in the Forbidden City there is an observatory for the Emperor's use.

How?

"Rong Fang asked 'Although I am not intelligent Master, I would like you to favor me with an explanation. Can someone like me be taught this Way?'

Chen Zi replied 'Yes. All things can be attained to you by mathematics. Your ability in mathematics is sufficient to understand such matters if you sincerely give reiterated thought to them.'"

Reference:
Astronomy and mathematics in ancient China: *The Zhou bi suan jing* by Christopher Cullen, Cambridge University Press 1996.

Goals:
1) To see the implications of two different model universes derived from similar observational data.
2) To use the same method to disprove a common misconception of today.

Background information:
The ancient Chinese used a metric system for small linear measurements; the units were *fen, cun, chi,* and *zhang.*
    10 *fen*   =   1 *cun*
    10 *cun*   =   1 *chi*
    10 *chi*   =   1 *zhang*
The *li* is a much larger linear unit of measure (20 *li* ≈ 1 km).

One important thing to keep in mind in doing the following is that for the Chinese, at this time, ***the universe was envisioned as a flat circular sky above a flat square Earth.***
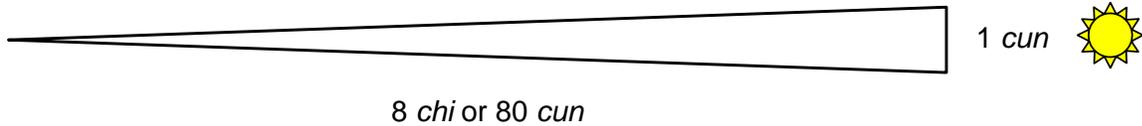
1) The standard gnomon, or *bi*, in the *Zhou bi* is 8 *chi* in length (about the height of a person). At summer solstice an 8 *chi bi* cast a shadow 1 *chi* 6 *cun* in length, at winter solstice the shadow length is 1 *zhang* 3 *chi* 5 *cun* in length. The *Zhou bi* states that "16,000 li to the south from here at summer solstice, and 135,000 *li* to the south from here at winter solstice, if one sets up a *bi* (gnomon) at noon it casts no shadow. This single [fact is the basis of] the numbers of the Way of Heaven". What is then the decrease of shadow length (*cun* per *li*) on the flat Earth? (10 points)

2) To get the distance from the Earth to the Sun, The *Zhou bi* tells us to do the following. "Wait until the base (the shadow length of an 8 *chi* gnomon) is 6 *chi*." (30) points)

    a) How far south is the no shadow point from the gnomon?

    b) What is the Earth–Sun (vertical) distance at the no shadow point?

    c) What is the Earth–Sun (slant) distance at the location of the gnomon?

3) To get the size of the sun, the *Zhou bi* then goes to say "Then take a bamboo tube of diameter 1 *cun* and of length 8 *chi*. Catch the light [down the tube] and observe it: the bore exactly covers the Sun" This measurement is done where the shadow is 6 *chi* in length. What is then the diameter of the Sun? You can use similar triangles or the small angle approximation (sin(Θ) = Θ and tan (Θ) = Θ, when Θ is in radians). (20 points)

1 *cun*

8 *chi* or 80 *cun*

4) Which is larger the Earth or the Sun (by these measurements)? (10 points)

5) Extra credit Do you notice anything peculiar about the answers to problems 1) and 2)? Hint, ask yourself why is it necessary to wait until the shadow length is 6 *chi*? (5 points)

6) So you are now taking one class about astronomy (sort of). This means that your friends will now ask you to verify (or disprove) all sorts of astronomical facts that they will come across on the Internet. One such request appeared in my work e–mail a couple of summers ago. Below is the e–mail, verbatim:

> Two moons on 27 August.
>
> Planet Mars will be the brightest in the night sky starting August
>
> It will look as large as the full moon to the naked eye. This will culminate on Aug. 27 when Mars comes within 34.65 Million Miles of earth. Be sure to watch the sky on Aug 27 12:30 am. It will look like the earth has 2 moons. The next time Mars may come this close is 2287.
>
> Share this with your friends as NO ONE ALIVE TODAY will ever see it again.

Names have been removed to hide the gullible. (But if you don't believe me I can show you the real thing.) I want you to disprove the claim in this e–mail. But, dismissing it out of hand by saying that it can't be true without showing some kind of proof is **NOT** an answer. You may assume that the distance from the Earth to Mars in the e–mail is correct; other needed data are (25 points):

Distance to the Moon: 239,000. Miles
Diameter of the Moon: 2,160. Miles
Diameter of Mars: 4,218. Miles
Diameter of Jupiter (largest planet): 85,376. Miles

You can answer all of the following questions using similar triangles and ratios as did the ancient Greeks and the Chinese.

a) What is the correct ratio of the diameter of Mars to the diameter of the Moon?

b) If the e-mail was correct, what would the ratio of the diameter of Mars to the diameter of the Moon have to be?

c) If the e-mail was correct, what would be the largest planet and what would its diameter be?