Learning Graph Theory

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Research Goal

Learn graph theory at a graduate level by reading *Graph Theory* by Reinhard Diestel and solving problems from selected chapters.

Methodology

The methodology for my research is simply how I go through the book. My original goal was to read the chapters and solve chapter exercises in sequential order: chapter 1, chapter 1 exercises, chapter 2, chapter 2 exercises, etc. After reading chapter 1 and doing some exercises from it, my mentor suggested I choose an interesting chapter and focus on it deeply as the chapters are mostly self-contained and do not rely much on results and terms in earlier sections except for chapter 1. I agree that this is a better approach, so I moved onto chapter 3 after doing a few more challenging exercises from chapter 1. After chapter 3, I started working on chapter 4. My goal after chapter 4 is 5, then 7, 9.

Results, Data, and Analysis

The results and data are simply what I've learned and any solutions to problems in the book that I wrote down. Analysis would consist of reflecting on what I've really learned about graph theory. So far, I've only done exercises from chapter 1. While solving the exercises from the end of the 1st chapter, I noticed that many problems hardly depended on the introduction at all. Several could be solved just by knowing the definitions of the terms in the question and prior knowledge. I also recognized a few exercises due to having done them before ever reading the book. There are some problems about modifying or generalizing theorems and lemmas from the chapter, and those require going back and carefully looking at the proofs, then seeing what needs to be changed.

When reading chapter 3, I learned about just how dense a graduate level textbook can get. The chapter is only 14 pages, but lots of detail is crammed into every proof and definition. When Diestel writes "clearly" or "it is easy to see that…", it may take anywhere from 30 seconds to several minutes to understand why the observation that follows is true. Similarly, when a previous theorem or lemma is cited, exactly how it is applied is often left out. It may take hours to get through a proof that only takes up a few paragraphs. Although details were also hidden inside many small observations in chapter 1, it was not nearly at the same rate as chapter 3. The textbook contains decades of results by dozens of people in graph theory, and researchers make each jump in their proofs as large as possible without making it impossible to cross on your own. Such a method of writing has been chosen as the middle ground between code written in a formal theorem prover with every application of an axiom, definition, or earlier result spelled out with parameters describing what variables or objects are being used, how the axiom, definition, or result is used, and why it can be used (which would require further steps proving it is applicable), and no proofs at all.

Future Research

In the future, I hope to read more of the book in the exact same way that I've been reading it so far. There are 12 chapters, and if I gather enough knowledge, it may be possible to even do a research project in graph theory that contributes original research. Note that what I'm doing right now is not entirely original since all results in the book has already been discovered and presumably all exercises have been solved by at least one person, namely the author.

Acknowledgements

This project would not have been possible without the help of Ivan Hu, who replaced my previous mentor after he could not continue. The author Diestel also deserves some credit for making an book that starts with the basics and goes very far.

Introduction and Context

Graphs are objects made up of vertices connected by edges. Even if you haven't heard the term before, you've definitely seen one. Whenever you draw a shape such as a hexagon or cube, you're drawing a graph where the vertices and edges are, to no surprise, the vertices and edges of that shape. For example, figure 1 shows the graph of a 4D hypercube. It has 16 vertices, 32 edges, a diameter (length of longest path between 2 vertices) of 4, girth (length of longest cycle) of 4, and chromatic number (minimum number of colors needed for vertices so that no edge has same color on both ends) of 2, and these are just some of the graph properties we study. While graph theory has many practical applications, my project will focus on theoretical graph theory. I plan to learn as much theoretical graph theory as possible by reading an advanced textbook on the subject and solving problems.

Figure 1: Hypercube Graph

T. Piesk, Wikimedia Commons

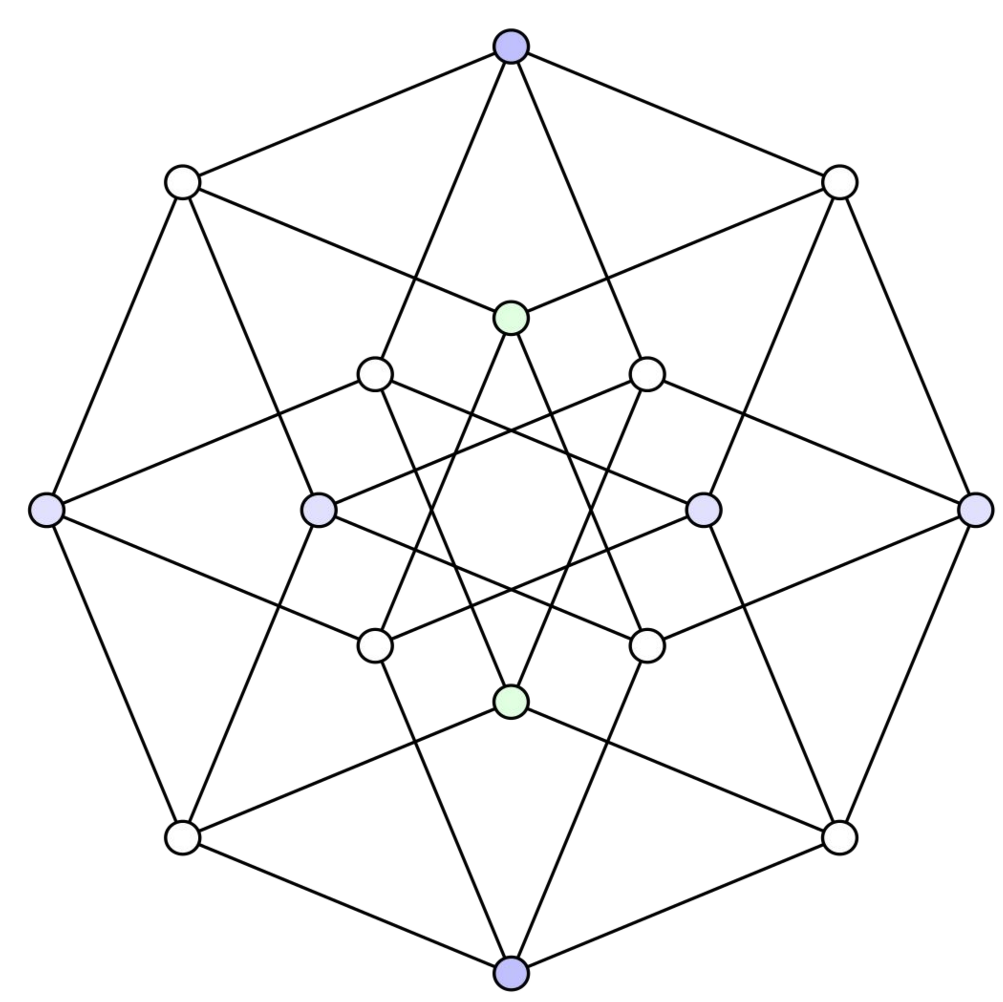
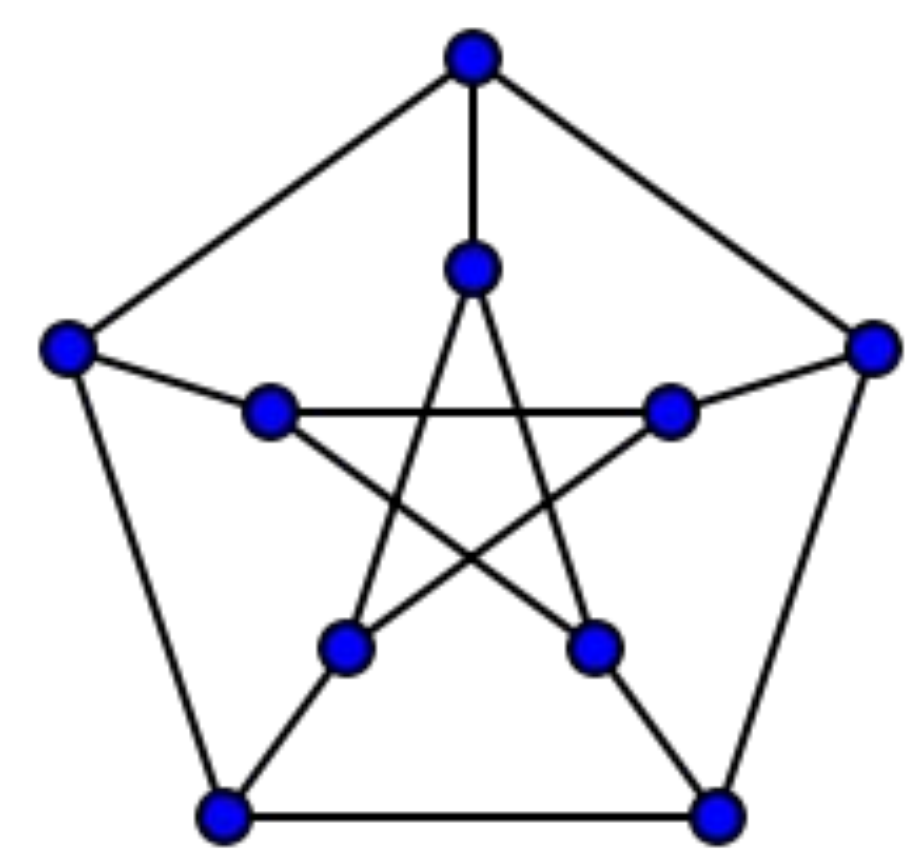


Figure 2: Petersen Graph

Leshabirukov, Wikimedia Commons



Limitations

The limitations are clear: only material that is in the book can be learned by reading the book, so some recent but accessible results such as the disproof of Hedetniemi's conjecture are not covered. However, the book offers so much that this is not really a concern.

Works Cited

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