Neutron Stars

Accreting Compact Objects-

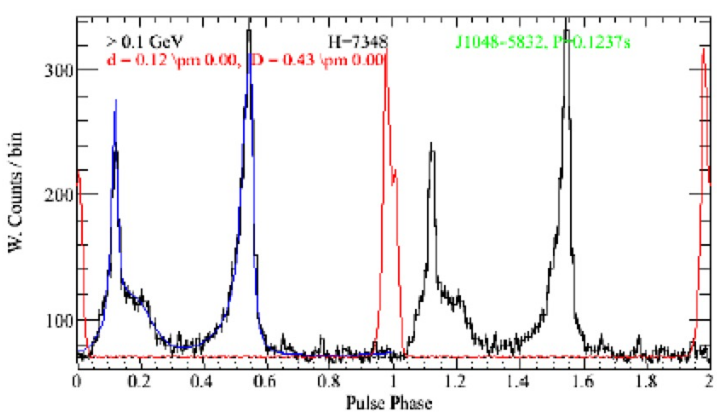
see Chapters 13 and 14 in Longair

Within the observable universe, there are ~ 30 SN every single second- producing NS and BHs galore

Neutron Stars

Accreting Compact Objects-

see Chapters 13 and 14 in Longair and Review article by Lattimer and Prakash [2004Sci...304..536L]



Radio (black) and γ-ray (red) pulse profiles from a neutron star

Origin and Basic Properties

- Neutron stars are the remnants of massive stars whose cores collapse during the supernova explosions at the end of their nuclear fusion lifetimes.
- Conservation of angular momentum and magnetic flux (?) of the progenitor star during the collapse gives the neutron star an extremely high spin rate and a high magnetic field.

Nuclear density $n = A / (4/3 \pi R_0^{\ 3}) = 3/4 \pi (1.25 \ fm)^3 = 0.122 \ (fm)^{-3}$

A is the mean mass number ; $R_0 \sim 1.2 fm$

Origin and Basic Properties

- The collapse ends when the degeneracy pressure of neutrons balances the gravitational forces of the matter (ignoring the strong force-baryonic interactions).
- The term neutron star refers to a star with a mass M on the order of 1.5 $M_\odot$, radius R of~12 km, and a central density as high as 5 to 10 times the nuclear equilibrium density

$(n_0 \simeq 0.16 * fm^{-3} = 2.7 x 10^{17} kg)$ of neutrons and protons in 'normal' nuclei.

Nuclear density $n = A / (4/3 \pi R_0^{\ 3}) = 3/4 \pi (1.25 \ fm)^3 = 0.122 \ (fm)^{-3}$

A is the mean mass number ; $R_0 \sim 1.2 fm$

Stellar Evolution and Supernovae- Review

•Stellar evolution – a series of collapses and fusions

H => He => C => Ne => O => Si

•Outer parts of star expand to form opaque and relatively cool envelope (red giant phase).

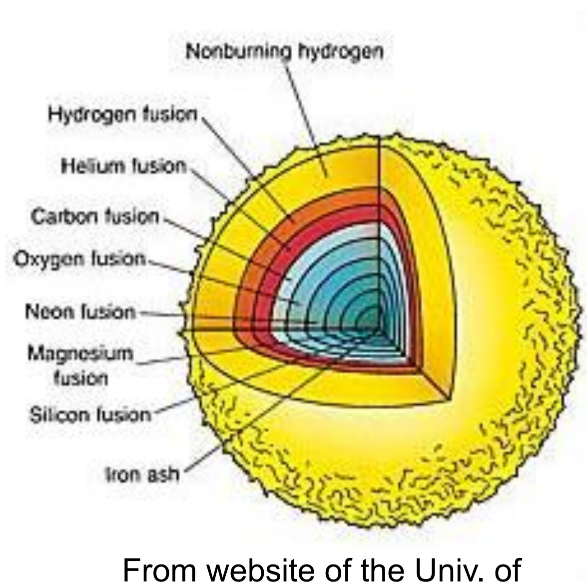
•Eventually, Si => Fe: most strongly bound of all nuclei

•Further fusion would *absorb energy* so an inert Fe core formed

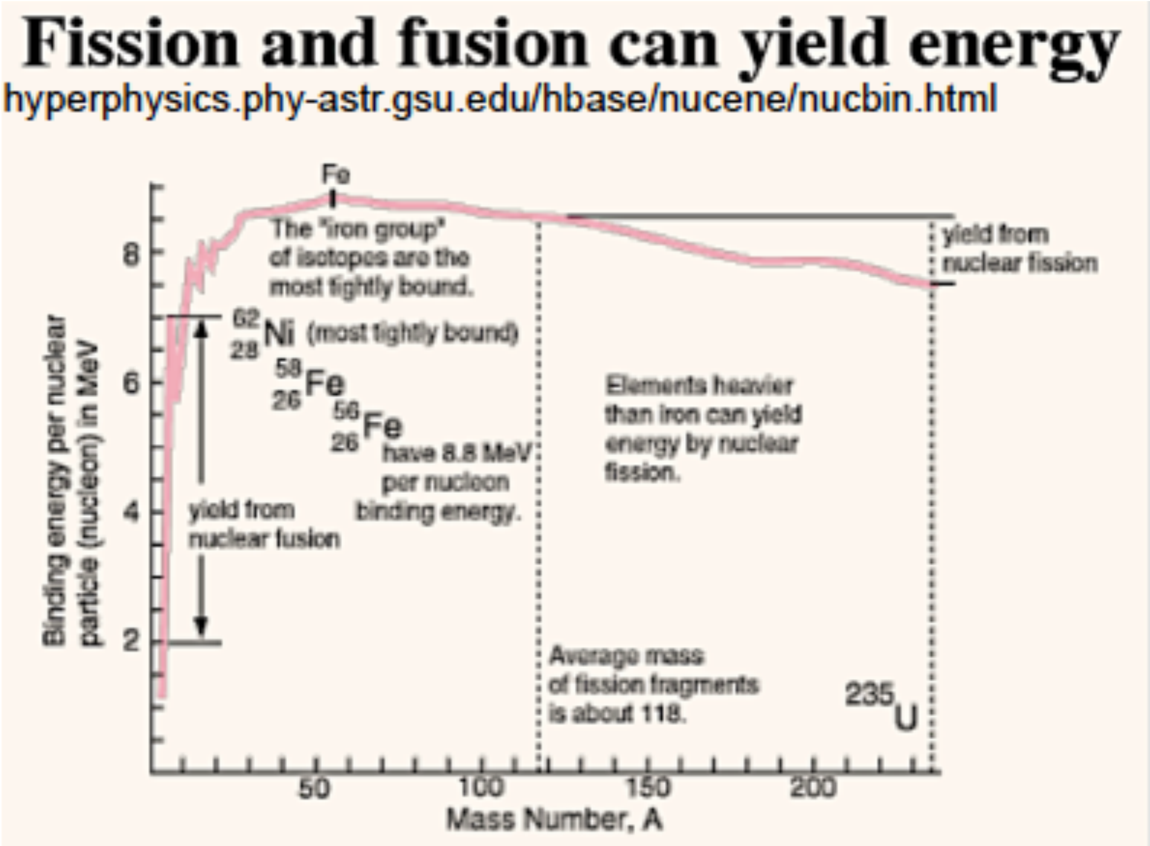
•Fuel in core exhausted hence star collapses

I.7 : Core collapse in a massive star

- End of a massive star's life ($M>8M_{sun}$)
  - Center of star has fused all of the way to iron
  - Shells of other elements surround iron core
  - Only takes ~day to build up "dead" Chandrasekhar mass iron core
  - Core is held up by electron degeneracy pressure



From website of the Univ. of Mississippi



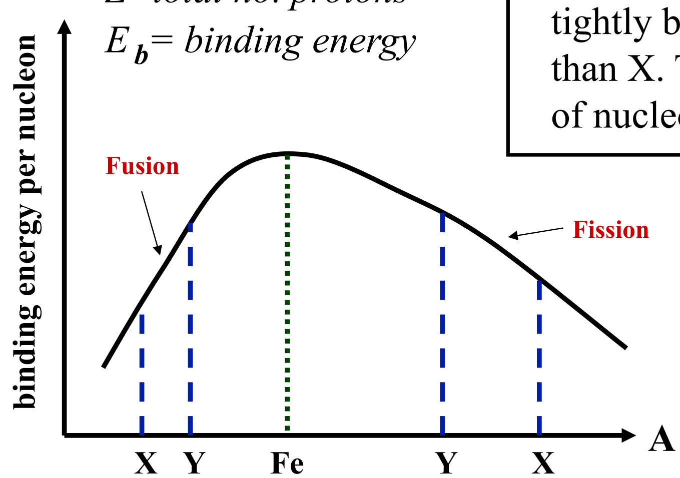
Binding energy of Nuclei - why stellar burning stops generating energy

A=total no. nucleons

Z=total no. protons

E_b= *binding energy*

Change from X to Y emits energy since Y is more tightly bound per nucleon than X. This is a function of nucleon mass



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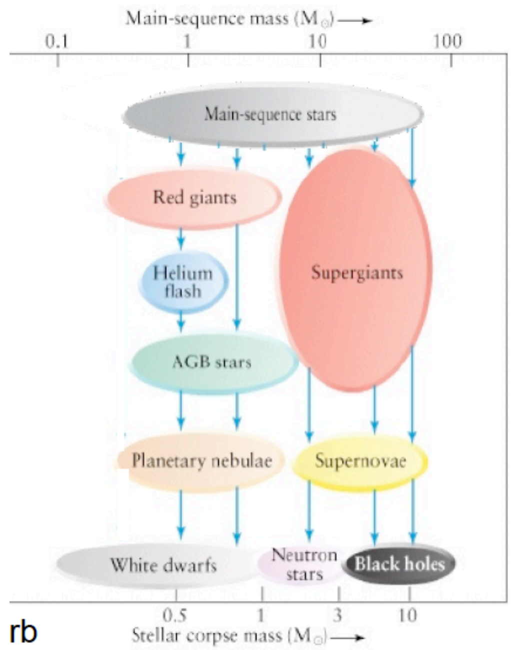
Progenitors of Compact Objects

- Main sequence stars evolve and at the end of their 'life' (period of nuclear burning) end up as compact objects
- $t_{MS}/t_{sun} \sim (M/M_{sun})^{-2.5}$
- The most massive end up as black holes
- The least massive as white dwarfs (the main sequence lifetime of stars less than 1/2 M_{sun} is greater than the Hubble time so they have never got to white dwarfs)



rb

Samar Safi-Harb

Review from SN

- Stars with a defined mass range evolve to produce cores that can collapse to form Neutron Stars
- Following nuclear fuel exhaustion, core collapses gravitationally; this final collapse supplies the supernova energy
- Collapse to nuclear density, in ≈ few seconds, is followed by a rebound in which the outer parts of the star are blown away
- The visible/X-ray supernova results due to radiation
  - i. From this exploded material
  - ii. Later from shock-heated interstellar material
- Core may
  - i. Disintegrate
  - ii. Collapse to a Neutron star
  - iii. Collapse to a Black Hole

  according to its mass which in turn depends on the mass of the original evolved star- not a linear relationship

From L. Cominsky

Creation of Neutron Stars

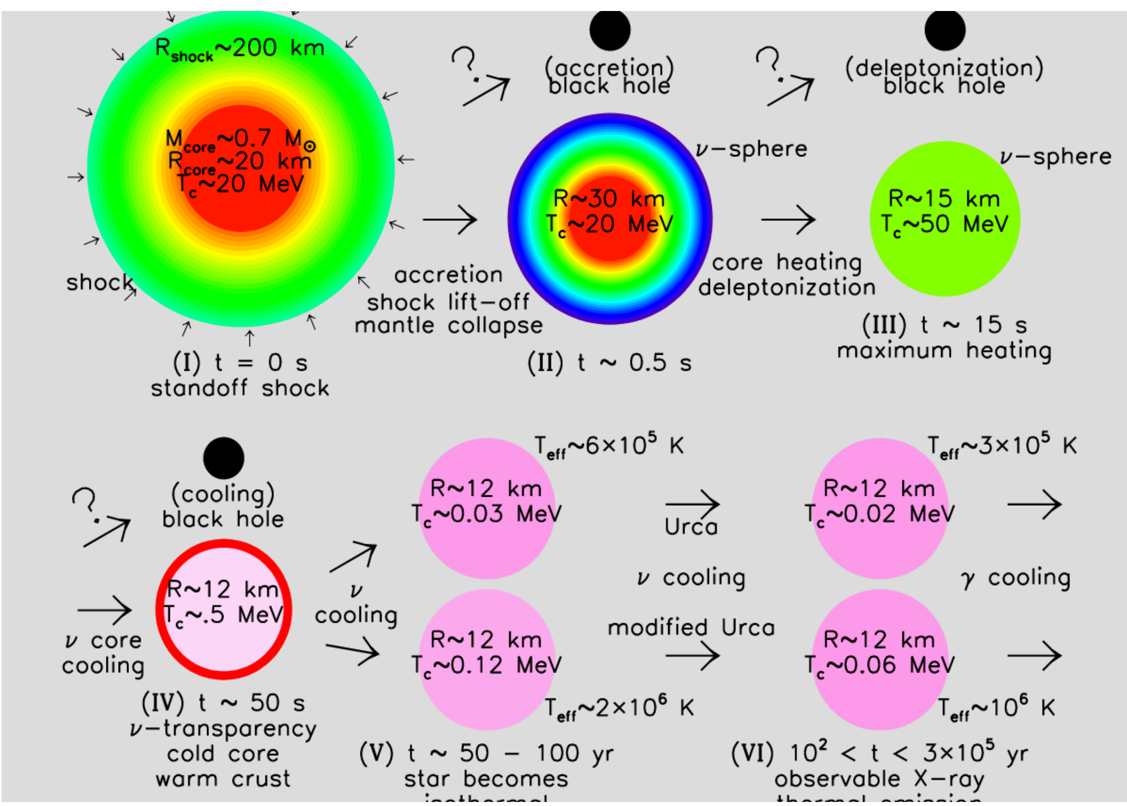
II/Ib/Ic Core-Collapse of Massive Progenitor at the end of its evolution

Collapse to nuclear density, in $\approx$ few seconds, is followed by a rebound in which the outer parts of the star are blown away

- Massive stellar progenitor core forms neutron star or black hole
- Explosive nucleosynthesis products near core (Si and Fe) plus hydrostatically formed outer layers (O, Ne) are expelled
- *Most of the explosion energy is carried away by neutrinos*
- Uncertain explosion mechanism details involve neutrinos, probably large-scale shock instabilities, rotation, possibly magnetic fields
- Star must be at least $8M_{\odot}$; core at least $1.4\ M_{\odot}$.

(U. Hwang 2007)

NS- Formation Lattimer and Prakash 2004



The Previous Slide in Words....pg 3,4 in L&P

- Neutron stars are created in the aftermath of the gravitational collapse of the core of a massive star ($>8M_{\odot}$)

- Newly-born neutron stars or proto-neutron stars are rich in leptons, mostly e^- and ν_e

- The gravitational binding energy released is ~$3GM/5R^2 \approx 3\times10^{53}$ erg ~ 10% of Mc^2. The kinetic energy of the expanding remnant is on the order of ~10^{51} erg to and the energy radiated in photons is ~10^{49} erg

- The proto-neutron star left behind rapidly shrinks because of pressure losses from neutrino emission- which forces electrons and protons to combine, making the matter neutron-rich

- Once the dead core exceeds $1.4M_{sun}$, electron degeneracy (later) pressure cannot support it.
- Core starts to collapse
  – $\rho\approx10^9$ kg/m^3 - Density of core when collapse begins (onset of relativistic effects in electron motions)
  – $\rho\approx10^{10}$ kg/m^3 - Fermi energy exceeds neutron-proton mass difference…

  Inverse beta decay becomes energetically preferable to normal beta decay

$$p + e^- \rightarrow n + \nu$$

- Nuclei become very neutron rich… neutronization

And then...

– $\rho \approx 10^{14}$ kg/m^3 - Individual nuclei are so neutron rich that they start to fall apart

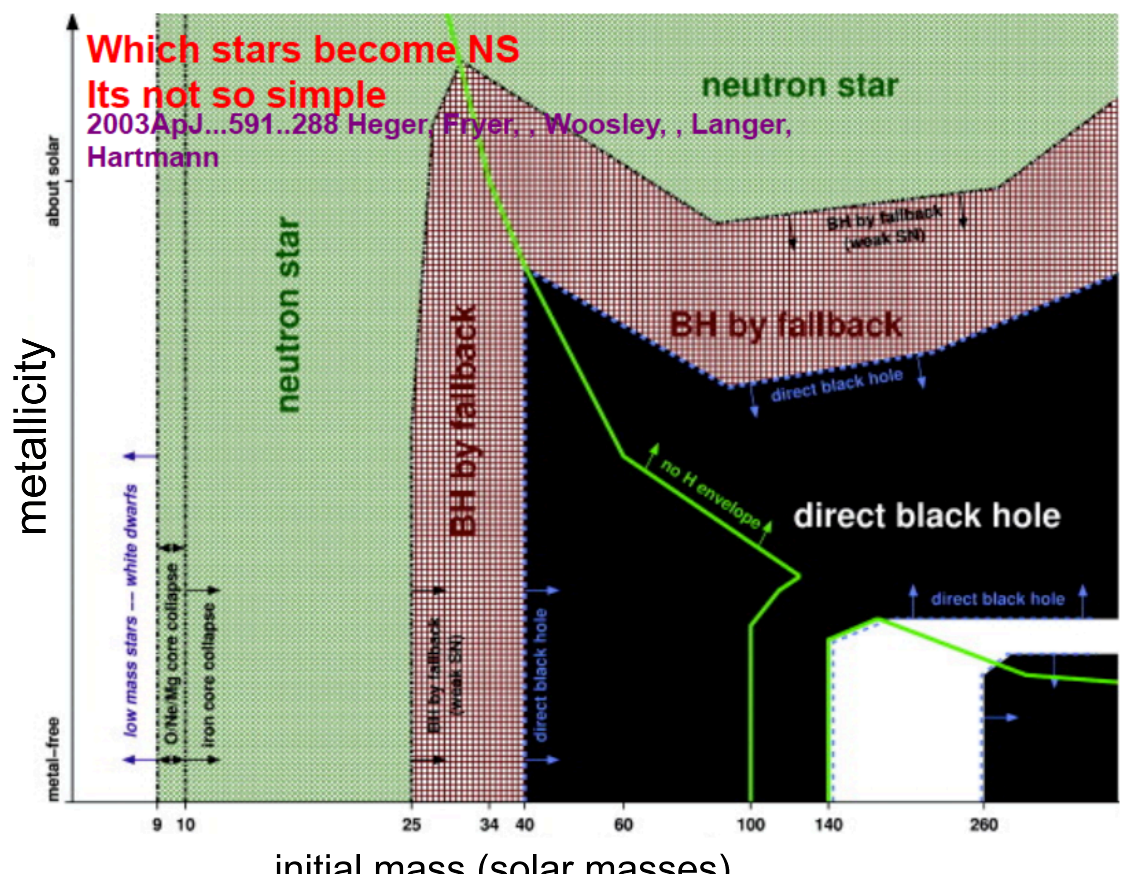
- Remaining nuclei surrounded by sea of free neutrons

  This is called the neutron drip phase

– $\rho \approx 10^{16}$ kg/m^3 - Neutron degeneracy pressure starts to become important

– **$\rho \approx 10^{18}$ kg/m^3 - Neutron degeneracy finally halts the collapse provided that $M < 3M_{sun}$ or maybe not and a BH forms**

– End up with a neutron star... typical mass of $1.4M_{sun}$ with a radius of 10km- theoretical mass radius relation is not well understood due to the effects of QCD



UpDated Plot of NS Progenitor Mass

- Raithel, et al 2018 arXiv:1712.00021
- NS masses in purple, black hole in gray
- (progenitor star mass in orange, core in green)

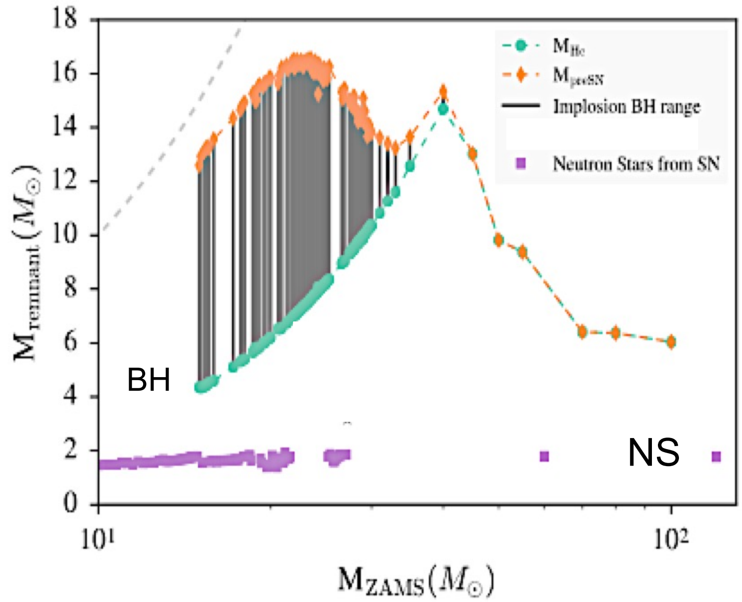


FIG. 1.— Baryonic remnant masses as a function of the progenitor ZAMS mass, for the central engine W18. Neutron star remnant masses from successful explosions are shown in purple. The range

Neutron Stars- see **The Physics of Neutron Stars**
J.M. Lattimer, M. Prakash Science 2004

- Predicted theoretically by Volkoff and Oppenheimer (1939)
  - First 'discovered' by Hewish et al 1967 (Noble prize) as the counterparts to radio pulsars

- short period from radio pulsars (<1s) can only be obtained from a compact object via either rotation or oscillations;
  - the period derivatives are small and for radio pulsars periods always increase (slow down)

- **All characteristic timescales scale as $\rho^{-1/2}$ (ρ is density)**

  $\omega=1/\mathrm{sqrt}(GM/r^3) =1/\mathrm{sqrt}(G\rho)$

Neutron Stars- see **The Physics of Neutron Stars**
J.M. Lattimer, M. Prakash Science 2004

Shortest periods ~1.5ms- light travel time arguments give a size (ct~ 500km)

White dwarfs have $\rho \sim 10^7 – 10^8$ gmcm^{-3}

maximum rotation periods $P = 2\pi/\Omega \sim 1–10$ s

- **To get periods of ~1ms (radio pulsars) need $\rho \sim 10^{14}$ gmcm^{-3}** the maximum spin rate is governed by the density profile of neutron stars

- The observed spin rate of the object can be used to determine a minimum average density…

- The minimum period (P) of a star is that for which the surface layers are "in orbit"…

$$\frac{v_{\rm rot}^2}{R} < \frac{GM}{R^2}$$

$$\frac{4\pi^2 R}{P^2} < \frac{GM}{R^2}$$

$$\bar{\rho} \equiv \frac{3M}{4\pi R^3} > \frac{3\pi}{GP^2}$$

Putting in Numbers

- $P=1s \Rightarrow \rho>1\times10^{11}$ kg/m^3
- $P=10^{-1}s \Rightarrow \rho>1\times10^{13}$ kg/m^3
- $P=10^{-2}s \Rightarrow \rho>1\times10^{15}$ kg/m^3

Maximum Spin

- An absolute upper limit to the neutron star spin frequency is the mass-shedding limit:
  - *the velocity of the stellar surface equals that of an orbiting particle suspended just above the surface.*
  - For a rigid Newtonian sphere this frequency is the Keplerian rate
  - $\nu_K= (2\pi)^{-1}\sqrt{GM/R^3}= 1833\ (M/M_\odot)^{1/2}(10\ \mathrm{km}/R)^{3/2}$Hz.
  - remember ρ~M/R^3
- However, both deformation and **GR** effects are important
- The highest observed spin rate, 641Hz from pulsar PSR B1937+21 implies a radius limit of 15.5 km for 1.4 $M_\odot$

Lattimer and Prakash 2004

Observational Intro to Neutron Stars

- Neutron stars are a very diverse population, in their observational properties.
- Radiate over a broad band but most of their energy is at X-ray and gamma-ray wavelengths

Their electromagnetic emission *can* be powered by

- rotation (spin down)
- accretion
- residual heat
- magnetic fields
- nuclear reactions
- **But all are a subset of the same sort of object**

A. Harding 2013

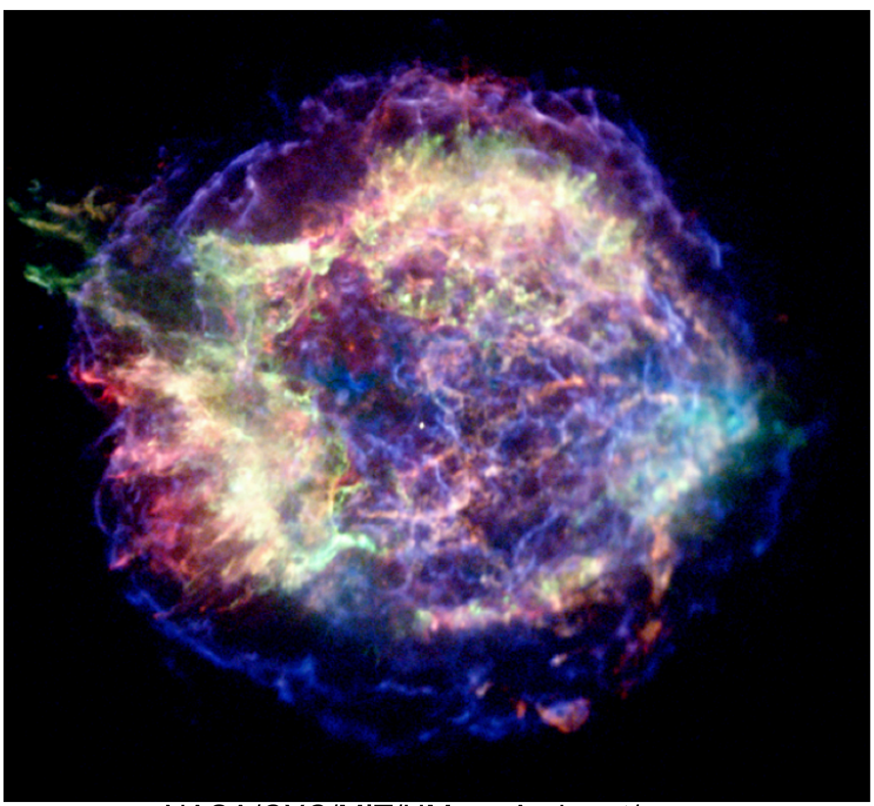
Observational Intro to Neutron Stars

- They show an amazing variety of pulsating and bursting behaviors.
- The 'zoo' of names is base on observational properties classified according to the primary power source for their emission and spin evolution.
- **But all are a subset of the same sort of object**

A. Harding 2013

"Classes' of Neutron Stars

- Rotation-powered pulsars (RPP) derive their energy primarily from the rotation of the NS
- Magnetars from magnetic field energy
- Isolated NSs (INS) from the latent heat of the NS matter from the SN
- "X-ray" pulsars from accretion
- Bursters from nuclear energy



NASA/CXC/MIT/UMass Amherst/
M.D.Stage et al.

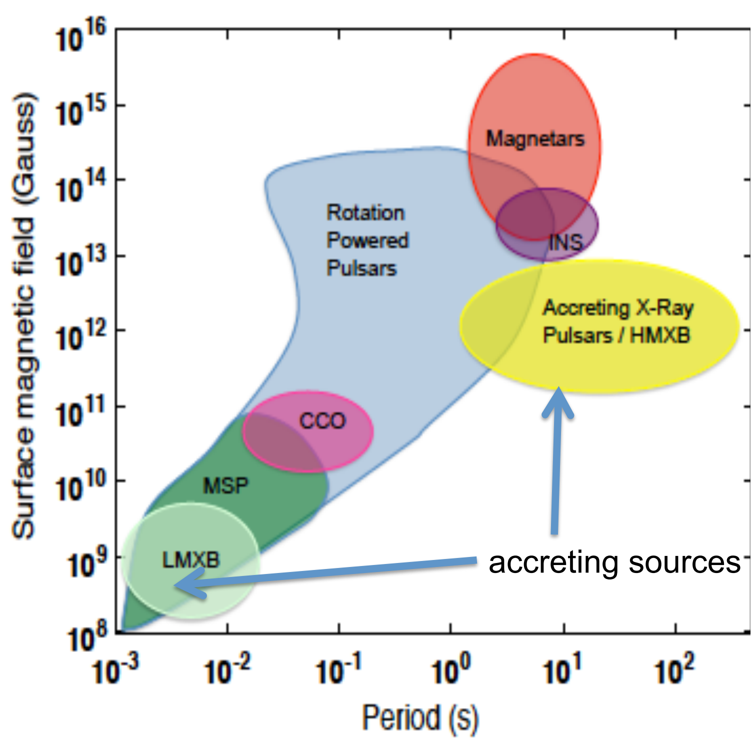
"Classes' of Neutron Stars

- Accretion-powered NSs from the energy released by matter accreting onto the NS from a binary companion.
  - A subclass of accreting NSs are X-ray bursters whose bursts are powered by thermo-nuclear explosions or a instability in the magnetosphere (will not cover in class) pg 481-483 in Longair
- Central Compact Objects (CCO), are soft X-ray point sources inside supernova remnants and are dim at all other wavelengths- physics is not understood.

Period and Magnetic Field

CCO- central compact objects
MSP= millisecond pulsar
LMXBs- low mass x-ray binaries

A. Harding 2013



Population

- The most 'common' observational population are non-accreting pulsars (>3000 known, https://www.atnf.csiro.au/people/pulsar/psrcat/)
- Periods (P) from 0.001-100 secs
- 22 orders of magnitude range in dP/dt
- dipole magnetic

$B_s \sim 10^{19}(P/dP/dt)^{1/2}$

P is in seconds, B in gauss

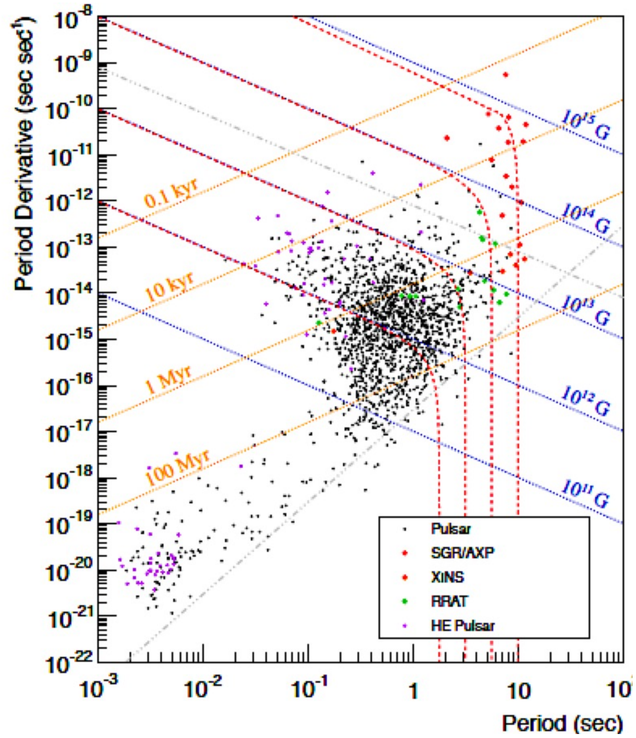
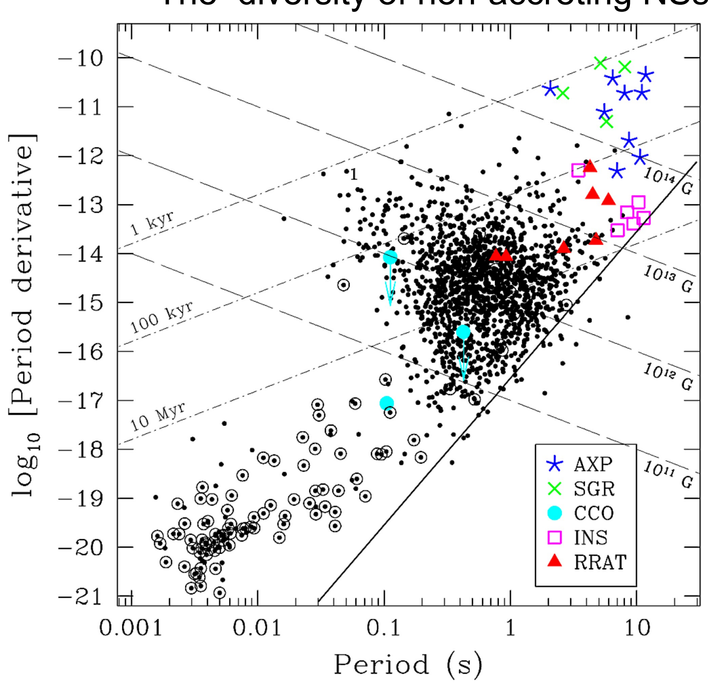


Figure 1: The P-$\dot{P}$ diagram of pulsars shown with lines of constant dipole magnetic field, B, and spin-down age ($\tau_c = \frac{P}{2\dot{P}}$). The black dots show the majority of radio-discovered pulsars believed to be rotation-powered, and the red circles show the X-ray or gamma-ray discovered magnetars addressed in this white paper (see §1.1). Source: ATNF pulsar catalog

- 'Millisecond pulsars' are rotation- powered, but have different evolutionary histories, involving long-lived binary systems and a 'recycling accretion episode which spun-up the neutron star and quenched its magnetic field
- **We will not discuss**
  - X ray-Dim Isolated NSs (XDINSs), Central Compact Object (CCOs) Rotating Radio Transients (RRATs),AXPS and Magnetars...

Longair 13.5.3-13.5.5

The diversity of non-accreting NSs



Open circles are in binaries

Degenerate Compact Objects

- The determination of the internal structures of white dwarfs and neutron stars depends upon detailed knowledge of the equation of state of the degenerate electron and neutron gases

The Chandrasekhar mass

- Relativistic degeneracy pressure
- The maximum mass of a white dwarf
- The fate of a white dwarf pushed over the limit…

Electron degeneracy pressure and white dwarfs

- Quantum mechanics of cold particles in a box
- Electron degeneracy pressure
- Structure of white dwarfs

Electron degeneracy pressure and white dwarfs

According to the Heisenberg uncertainty principle the distance between particles at which quantum effects become dominant is

$\delta x \sim h/\delta p$ (p is the momentum)

At the objects density increases δx gets smaller and thus δp gets larger and for a fixed mass the velocity increases, increasing the pressure.

When the pressure due to this "Heisenberg motion" exceeds that of the pressure from the thermal motions of the electrons, the electrons are referred to as degenerate-

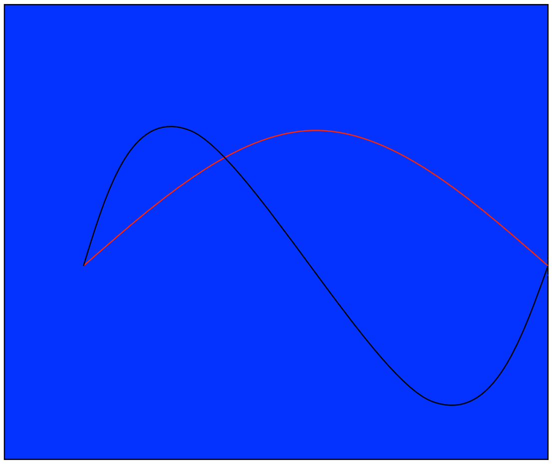
this is calculated from Fermi gas theory

$P=(2/3)E/V$ (V is the volume, E is the total energy)

$P=[(3\pi^2)^{2/3}h^2/(5m)]\rho$ $m=m_e$ for white dwarfs, m_n for NS and ρ is the density

Cold particle in a box

- Consider three electrons in a box. Suppose they are as cold as you can make them… they are in their quantum ground state.



Length L

$$\lambda_1 = 2L$$
$$\lambda_2 = 2L$$
$$\lambda_3 = L$$

$$mv = h/\lambda$$

Use Heisenberg uncertainty principle with L taking place of δX
p=mv,δxδp=h

- For a given particle, its kinetic energy is

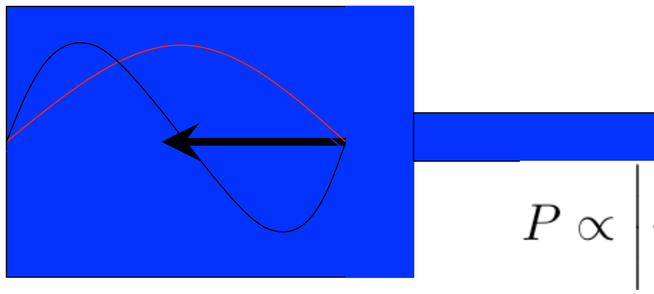
$$E = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{h^2}{2m\lambda^2}$$

- So total kinetic energy over the 3 particles is

$$E = \frac{h^2}{2m}\left(\sum_{i=1}^{3}\frac{1}{\lambda_i^2}\right)$$

$$E = \frac{h^2}{2m}\left(\frac{1}{4L^2} + \frac{1}{4L^2} + \frac{1}{L^2}\right) = \frac{3h^2}{4mL^2}$$

- What happens if we squeeze the box?



$$P \propto \left|\frac{dE}{dL}\right| = \frac{3h^2}{2mL^3}$$

- Particle energies go up as L is decreased- the allowed values of δx are smaller and thus δp is larger. We must be doing work on the box. We are doing work against <u>degeneracy pressure</u>.

Degneracy and All That- Longair pg 395 sec **13.2.1**

- Thus when things are squeezed together and δx gets smaller the momentum, p, increases, particles move faster and thus have more pressure
- In the box- with a number density,n, of particles are hitting the wall;the number of particles hitting the wall per unit time and area is 1/2nv
- the momentum per unit time and unit area (**P**ressure) transferred to the wall is 2nvp; **P**~nvp=(n/m)p^2 (m is mass of particle)

Degeneracy- continued

- The average distance between particles is the cube root of the number density and if the momentum is calculated from the Uncertainty principle $p \sim h/(2\pi\delta x) \sim hn^{1/3}$
- and thus $\mathbf{P} = h^2 n^{5/3}/m$- if we define matter density as

  $\rho = [n/m]$ then
- $\mathbf{P} \sim \rho^{5/3}$ **independent of temperature**
- Dimensional analysis gives the central pressure as
  - $\mathbf{P} \sim GM^2/r^4$
- If we equate these we get $r \sim M^{-1/3}$ e.q **a degenerate star gets smaller as it gets more massive**

- So, an approximate expression for radius of white dwarf is:

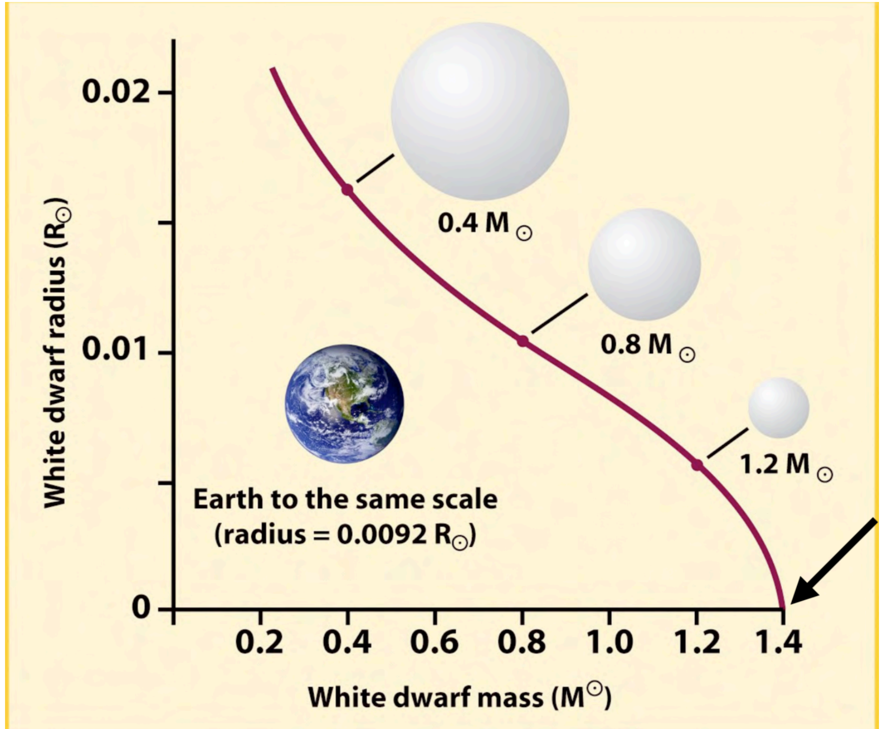
$$R \sim \frac{K}{GM^{1/3}}$$

$$R \sim 1.2 \times 10^4 \left(\frac{M}{M_\odot}\right)^{-1/3} \mu^{-5/3}\,\mathrm{km}$$

- Exact calculation gives

$$R \sim 1.13 \times 10^4 \left(\frac{M}{M_\odot}\right)^{-1/3} \left(\frac{\mu}{2}\right)^{-5/3}\,\mathrm{km}$$

White Dwarf Mass Size Relation



As mass increases size decreases

$M \propto R^{-3}$

- Detailed calculation shows that (provided it is not too dense or relativistic) the electron degeneracy pressure,**P**, of a cold "gas" is

$$P = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5 m_e \mu^{5/3} m_p^{5/3}} \rho^{5/3} \quad \mu \equiv \frac{\rho}{m_p n_e}$$

- The maximum electron momentum is

$$mv = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

Relativistic effects on degeneracy pressure

- Consider degenerate cold gas.
- Maximum momentum of particle is

$$p = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

- Assume non-relativistic physics… maximum velocity given by

$$v = \frac{1}{m_e}\left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

- So, particles have to be moving at relativistic speeds (~c) when density satisfies:

$$n_e > \frac{8\pi}{3}\left(\frac{m_e c}{h}\right)^3 \qquad \rho > 2 \times 10^{12}\,\mathrm{kg\,m^{-3}}$$

- For relativistic degeneracy, the mass is determined…

- Complete calculation gives

$$M \sim \left(\frac{K'}{G}\right)^{3/2}$$

- This is called the Chandrasekhar mass… it is the maximum possible mass of a White Dwarf.

$$M \approx 1.457\left(\frac{2}{\mu}\right)^2\,\mathrm{M}_\odot$$

Degeneracy- continued

- At higher densities the material gets 'relativistic' e.g. the velocities from the uncertainty relation get close to the speed of light- this changes things and $P \sim \rho^{4/3}$;
- this is important because when we use **P**$\sim GM^2/r^4$ we find that the pressure does not depend on radius and just get an expression that depends on mass- this is the Chandrasekar mass. (see 13.2.2 in Longair)

- Particles are moving at relativistic speeds when density satisfies:

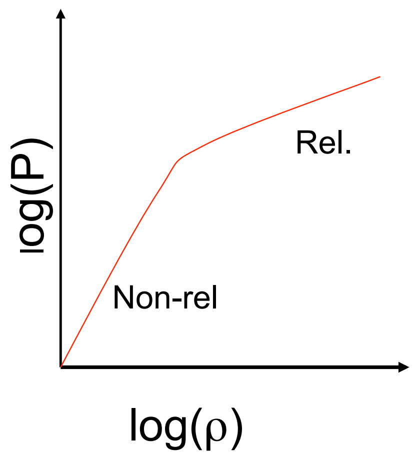
$$n_e > \frac{8\pi}{3}\left(\frac{m_e c}{h}\right)^3 \quad \rho > 2 \times 10^{12}\,\mathrm{kg\,m^{-3}}$$

This comes from setting $\delta p \approx m_e c$ Longair 13.9

$\rho \sim m_p/(\delta x)^3 \sim m_p(m_e c/h)^3 \sim 3 \times 10^{10}$ kg m^{-3} .

- Onset of relativistic effects makes degeneracy pressure less effective than it would otherwise be

$$P = \frac{3^{1/3}\pi^{2/3}\hbar c}{4m_p^{4/3}\mu^{4/3}}\rho^{4/3}$$

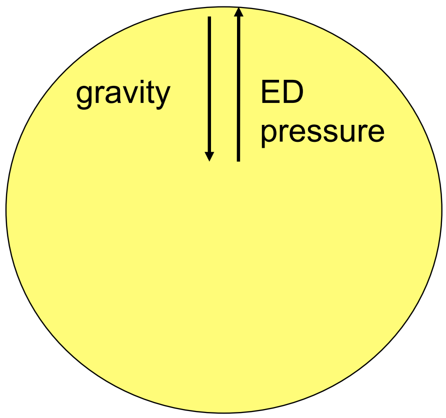


Degneracy and All That- Longair pg 395 sec **13.2.1**

- In *white dwarfs*, internal pressure support is provided by electron degeneracy pressure and their masses are roughly the mass of the Sun or less
- the density at which degeneracy occurs in the non-relativistic limit is proportional to $T^{3/2}$
- This is a quantum effect

white dwarfs…

- Pressure-Density relation



Pressure gradient = weight

$$\frac{dP}{dR} = -\frac{GM}{R^2}\rho$$

$$\frac{P_c}{R} \sim \frac{GM}{R^2}\rho_c$$

with

$$P_c = K'\rho_c^{4/3}$$

$$\rho_c \sim M/R^3$$

Independent of temperature

Degenerate Compact Objects- See Longair pg 394-398

- pressure is independent of the temperature for degenerate stars, use the first two equations
- $dp/dr = -GM\rho/r^2$; $dM/dr = 4\pi\rho r^2$.
- In eqs 13.16-13.24 the eqs for a white dwarf are derived
- $M = 5.836/\mu^2_e M_\odot$. $\mu_e=2$ for white dwarf and thus the **Chandrasekar mass for a white dwarf $M_{Ch}=1.46M_\odot$** (eq. 13.24)
- For NS general relativity is important..

White Dwarfs…

$$R \sim \frac{K}{GM^{1/3}}$$

$$P_c = K\rho_c^{5/3} \qquad P = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m_e\mu^{5/3}m_p^{5/3}}\rho^{5/3}$$

Mass of particle producing degeneracy pressure- important

Number of nucleons per degenerate particle

Neutron Stars

- By analogy, <u>neutron stars</u> have (to a crude approximation)…

$$R_n \sim \frac{K_n}{GM^{1/3}}$$

– I.e., degenerate particles have mass m_n, and μ=1

$$P_n = K_n\rho^{5/3} \qquad P_n = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m_n^{8/3}}\rho^{5/3}$$

- So, we can try to estimate radius of neutron star given what we know about white dwarfs
  - We know that $$\frac{R_n}{R_{wd}} \sim \frac{m_e}{m_n} 2^{5/3}$$
  - So we expect $$R_{wd} \sim 10^4\,\mathrm{km}$$

$$R_n \sim 16\,\mathrm{km}$$

- For relativistic degeneracy, the mass is determined…

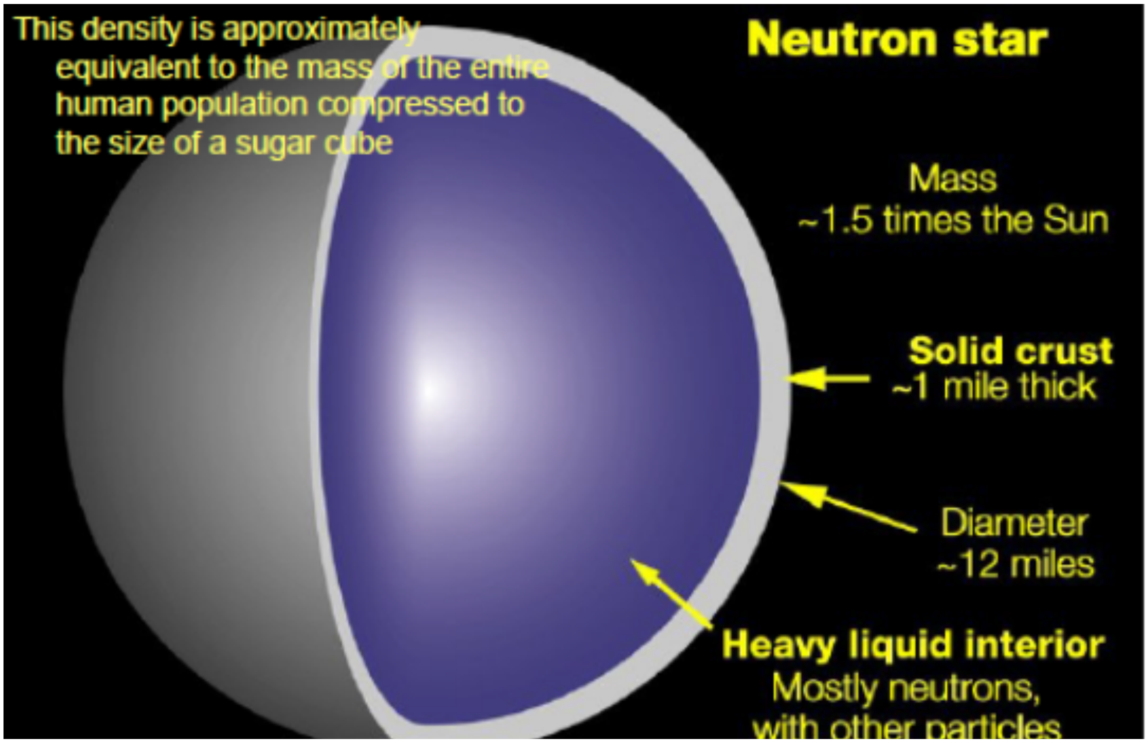
- Complete calculation (pg 398-399 in Longair) gives

$$M \sim \left(\frac{K'}{G}\right)^{3/2}$$

- This is called the Chandrasekhar mass… it is the maximum possible mass of a White Dwarf.

$$M \approx 1.457 \left(\frac{2}{\mu}\right)^2 \mathrm{M}_\odot$$

Inside Neutron Stars



Inside a Neutron Star