

Summary Last Time

- The luminosity that results from accretion is roughly
 - $L \sim \epsilon c^2 \dot{M}$ ($dM/dt = \dot{M}$) Where $\epsilon = GM/Rc^2$ (the depth of the potential)
 - $\epsilon \sim 3 \times 10^{-4}$ for a white dwarf and 0.1 for a neutron star
- If the gas flow is spherically symmetric and steady state the luminosity should not exceed the **Eddington limit** (outward force from Compton scattering balances gravity) if the main cross section is the Thompson cross section

$$L_{\text{Edd}} \sim 1.3 \times 10^{38} M_{\odot} \text{ ergs/sec}$$

Summary from Last Lecture

- Two natural temperatures
 - Free fall $kT = 3/16 \epsilon m_p c^2 = 2 \times 10^5 \epsilon \text{ keV}$; ϵ is efficiency of conversion of KE into heat
 - Black body temperature: minimum temperature for the object to radiate the observed luminosity
 - $T_{\text{BB}} \sim (L/A\sigma)^{1/4}$; A is the area and σ is the Stefan-Boltzman constant
 - about 0.2 keV for a white dwarf and 2 keV for a neutron star

Last Time

- Time scales:

$\tau_{\text{dyn}} \sim (r^3/GM)^{1/2}$ This is about 0.1 ms for matter at $r = 10$ km, and 2 ms at $r = 100$ km.

The typical orbital period of circulating matter,

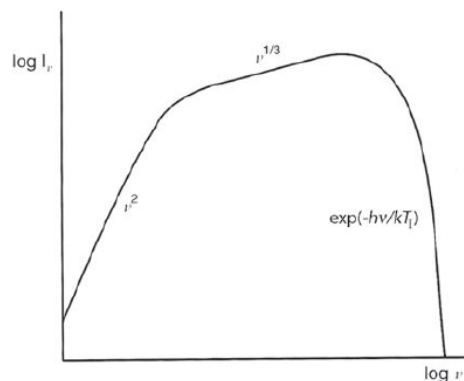
$$P_{\text{orb}} = 2\pi\tau_{\text{dyn}} \sim 1 \text{ ms:}$$

- Characteristic velocity is $\sim (GM/R)^{1/2} \sim 0.5c$.
- The two main accretion mechanisms are
 - Roche lobe over flow, which most often occurs in low-mass binaries (LMXB, low B field, accretion disk and boundary layer dominated)
 - and stellar wind capture, which is common for high-mass binaries with super-giant companions (high B fields, pulsars)

Total Spectrum

- If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$ (Longair 14.54)
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$
- At lower frequencies the spectrum has a Rayleigh-Jeans ν^2 shape and at higher energies has an exponential cutoff corresponding to the maximum temperature ($e^{-h\nu/kT_{\text{inner}}}$)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

If the disk 'cuts off' at some radius r_{inner} then the temperature profile is $T(r) = 3GM\dot{m}/8\pi\sigma r^3 [1 - (r_{\text{inner}}/r)^{1/2}]^{1/4}$ eq in 14.7.1.



the emission spectrum of an optically thick accretion disc. The exponential cut-off at high energies occurs at frequency $\nu = kT_1/h$, where T_1 is the temperature of the innermost layers of the thin accretion disc. At lower frequencies the spectrum tends towards a Rayleigh-Jeans spectrum $I \propto \nu^2$.

Basics of Accretion

- Because of angular momentum considerations an **accretion disk**, **almost** always forms
- Matter is thought to form a physically thin (but optically thick* disk) which has Keplerian rotation
- Matter falls in by losing angular momentum via viscosity

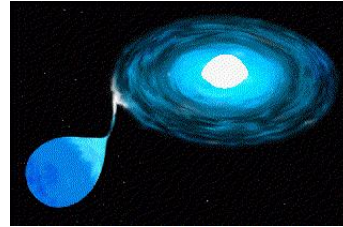
the angular velocity is $\Omega_k = \sqrt{GM/r^3}$

The binding energy of a parcel of the disk is $E = GM_{\text{disk}} M_x / 2R = 1/2 L_{\text{acc}}$

The other half of L_{acc} is released very close to the star surface (the boundary layer) as matter in the disk tries to co-rotate with the NS (what happens for a black hole??)

If the star spins more slowly than the innermost part of the accretion disk (angular speed ω_k), the BL must release a large amount of energy as the accreting matter comes to rest at the stellar surface. Some of this is used to spin up the star, but there remains an amount

$GM_x/2R(1 - \omega_k/\Omega_k)^2$ which is radiated



* Optically thick means that a photon emitted inside the disk always interacts with matter at least once before 'escaping'

Accreting Neutron Stars-Mass transfer Fuels Accretion, Creating X-rays

- Two types- based on mass of companions
 - Low mass x-ray binaries-NS star tend to have *low* magnetic field ($B \sim 10^{8-10}$ G) and accrete via Roche lobe overflow
 - High mass-NS tends to have *high* magnetic field ($B \sim 10^{11-13}$ G), accrete from stellar winds
- A bit about observations
 - LMXBs tend to be rather luminous and not show pulsations- spectra are 'quasi-thermal' due to radiation from accretion disk and surface of NS
 - HMXBs are often 'pulsars', spectra are non-thermal dominated by effects of energy generation and transfer in a high B field. Can measure B field via detection of cyclotron emission/absorption features.

$$\hbar\omega_c = \hbar eB/m_e c = 11.6 B_{12} \text{ keV,}$$

Neutron stars cyclotron absorption

- Direct evidence for strong magnetic fields

$$E_{\text{cyc}} = n / (1+z) h e B / [m_e c] \approx n(1+z) 11.6 \text{ [keV]} \times B_{12}$$

z is the gravitational redshift due to the NS mass and n is the number of Landau levels involved: e.g., $n=1$ is the case of a scattering from the ground level to the first excited Landau level

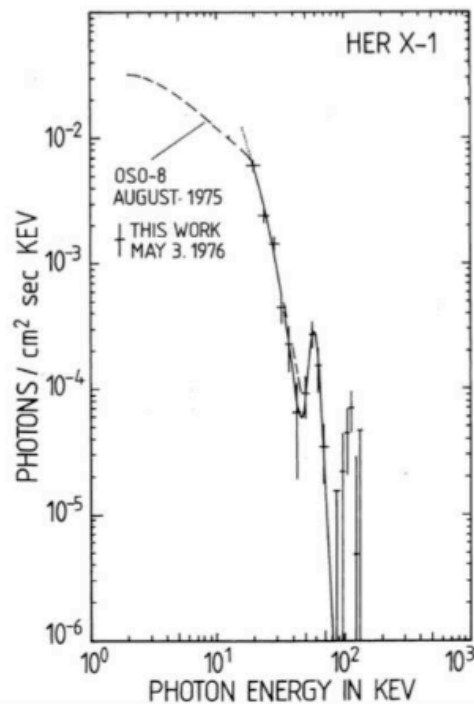
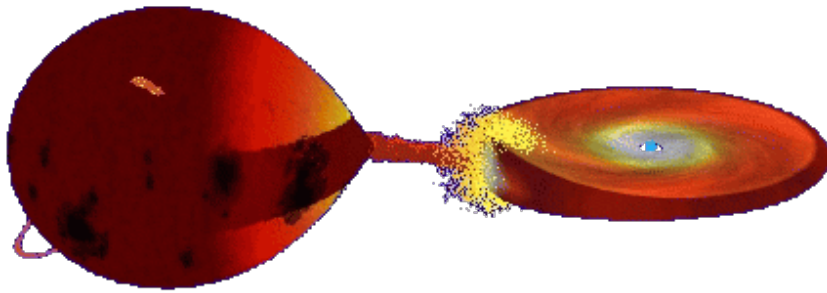


Fig. 1. X-ray spectrum of Her X-1 as obtained in a balloon observation in 1976, constituting the first detection of a cyclotron line (from Trümper et al. 1978).

For More Information on NS

<http://www.astro.umd.edu/~miller/nstar.html>

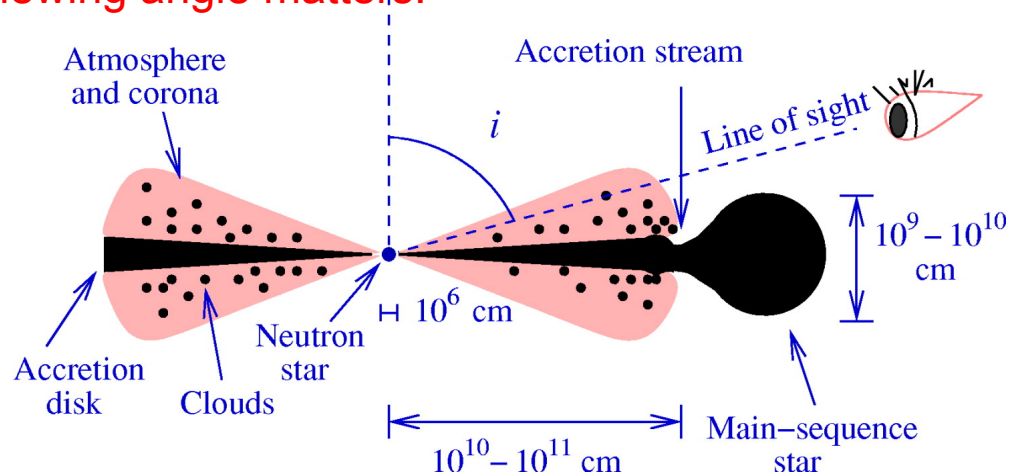
Accretion from a Low Mass Companion



- http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif

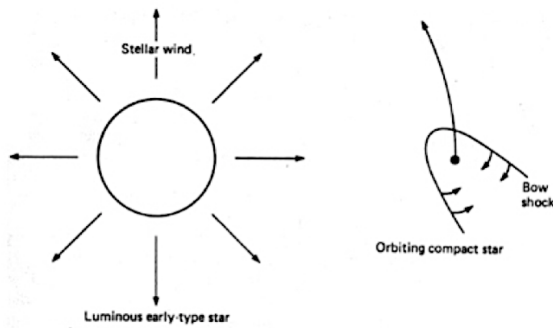
Geometry of heated accretion disk + corona in LMXB

Viewing angle matters!



Jimenez-Garate et al. 2002

Two Modes of Accretion- Longair 14.5.2



Accretion from a stellar wind

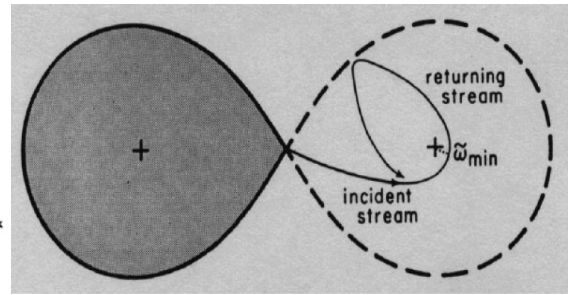
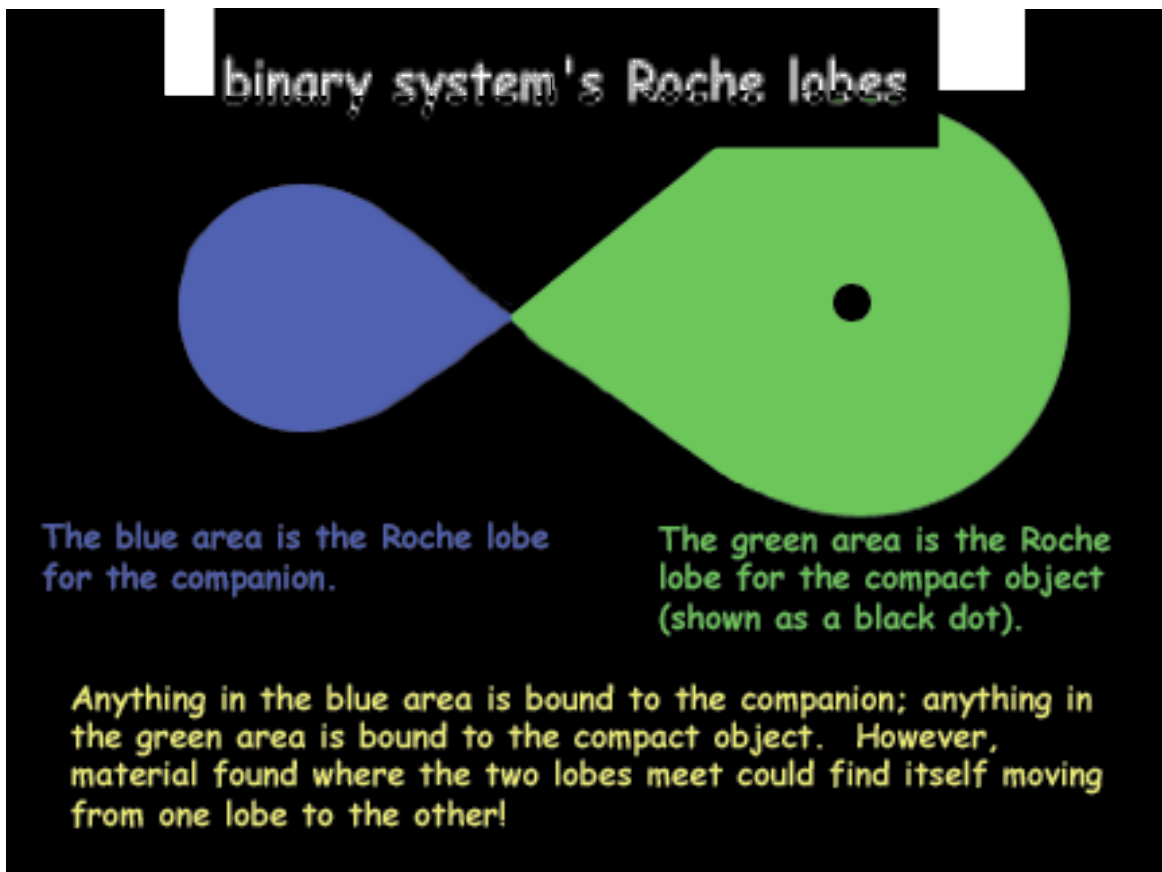


Figure 6: Roche lobe geometry, showing the incident mass stream through the inner Lagrangian point and the intersection of the returning stream with the newly arriving material. The low mass companion on the left hand side completely fills its Roche lobe, while the position of the compact object is denoted by a plus sign (+) inside the right hand lobe.²⁴

Accretion from Roche Lobe Overflow

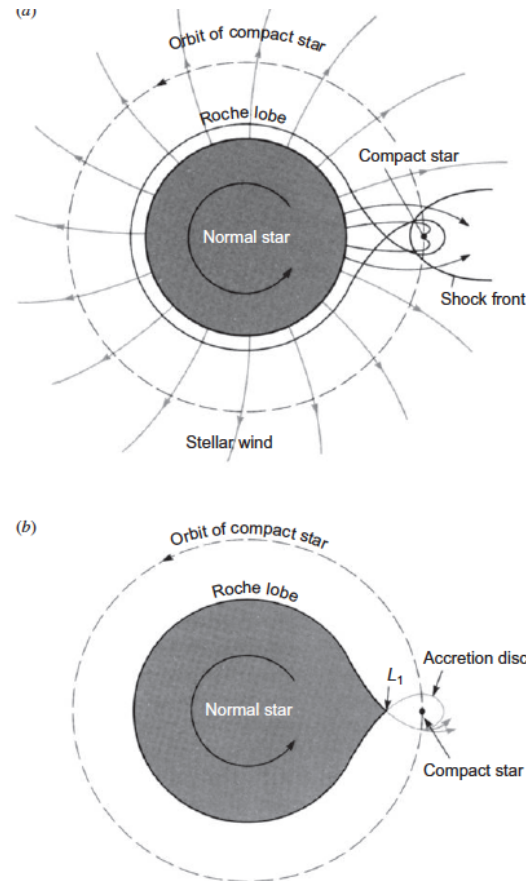
Cominsky (2002)



Two Modes of Accretion – Longair

Fig 14.13

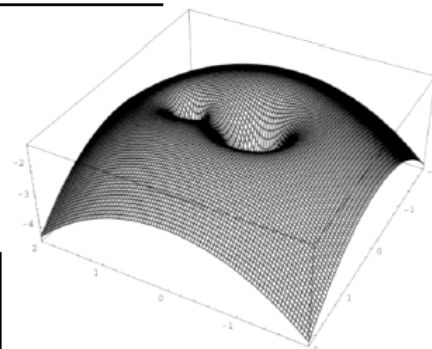
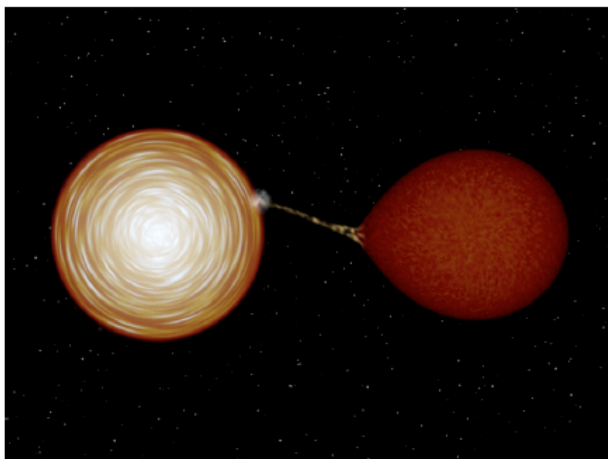
- Capture from a stellar wind: Luminous O and B stars emit stellar winds, with mass loss rates as high as $10^{-5} M_{\odot} \text{yr}^{-1}$ (Top figure the compact companion is embedded in an outflowing stellar wind)
- *Roche lobe overflow*. The equipotential surfaces of a close binary star system are distorted in the rotating frame of reference when the stars fill a substantial fraction of the Roche lobe (bottom figure)



Roche Lobe Overflow Systems

Sample

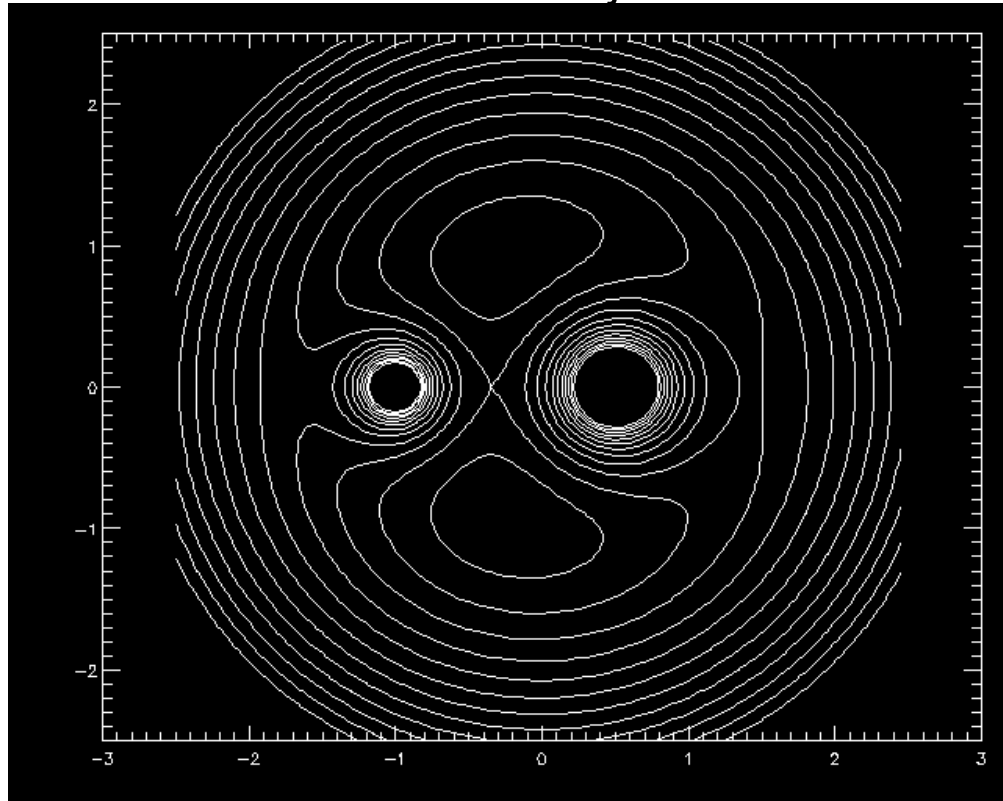
- Almost all LMXBs and IMXBs
- Small fraction of HMXBs



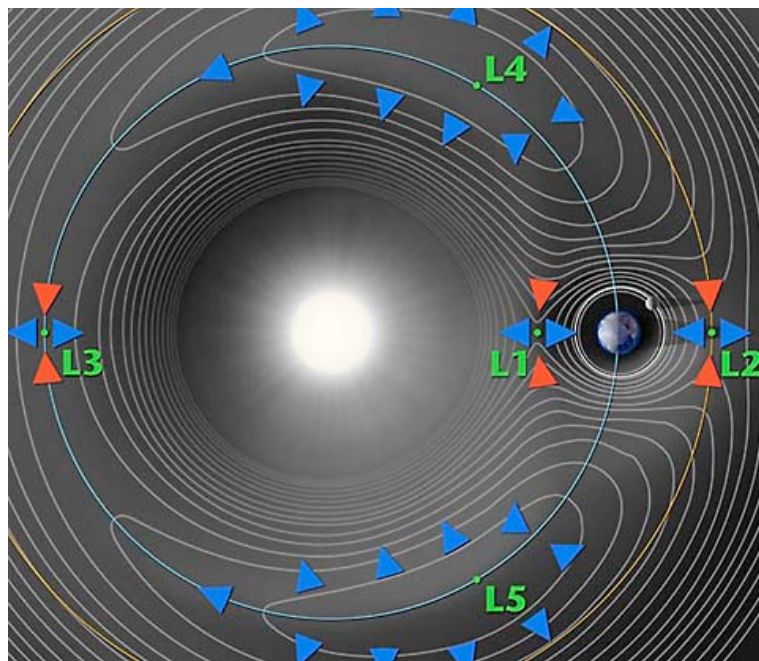
From Frank et al., 2002, *Accretion Power in Astrophysics*

The precise shape of the Roche lobe depends on the mass ratio $q = M_1 / M_2$

Potential For 2 Objects



- Gravitational and centrifugal (angular momentum) terms
- ω is the angular momentum
- $\omega = [GM/a^3]^{1/2}$
- where a is the separation and $\mathcal{M} = [M_2/M_1 + M_2]$
- L1 is an unstable Lagrangian pt



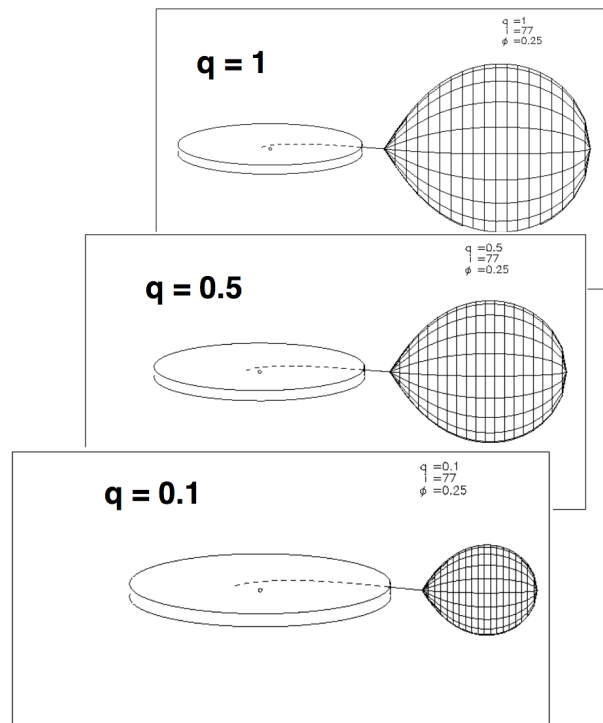
$$\Phi_R = -\frac{GM}{|\mathbf{r} - \mathbf{r}_1|} - \frac{GM}{|\mathbf{r} - \mathbf{r}_2|} - \frac{1}{2}(\omega \times \mathbf{r})^2$$

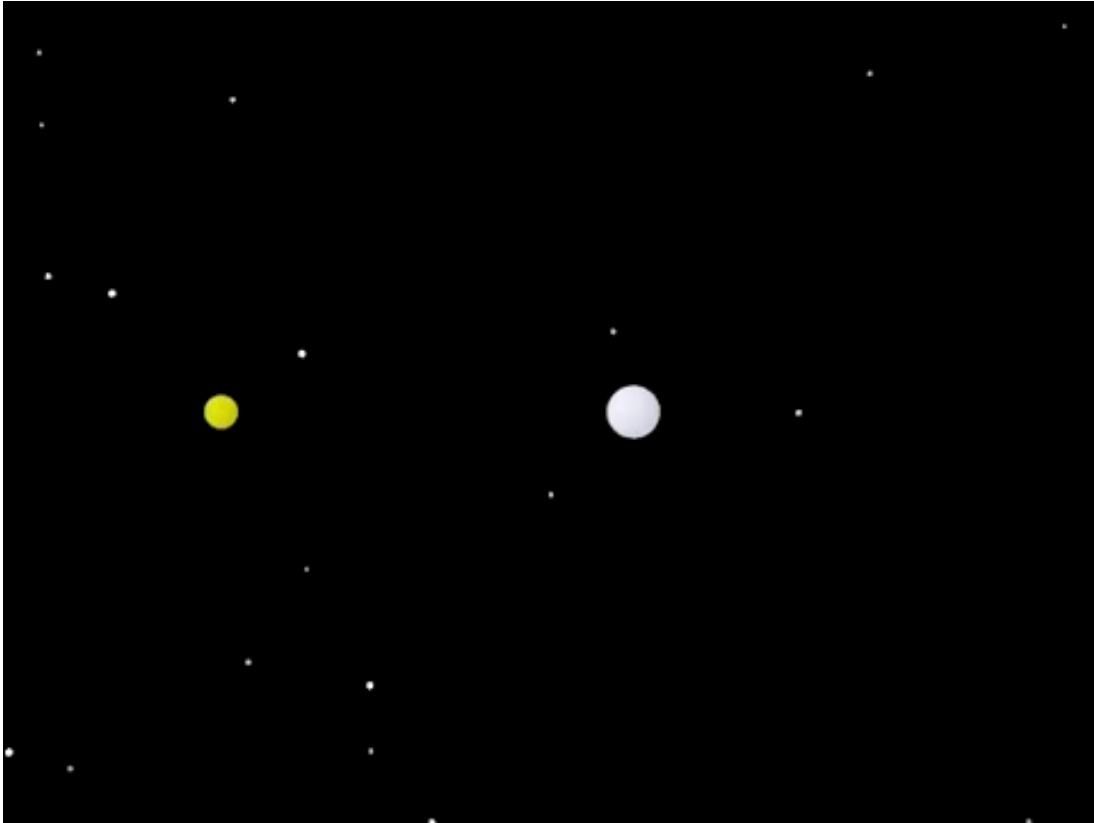
Lagrangean Points

- when $\nabla\Phi=0$
- L1 - Inner Lagrange Point– in between two stars–matter can flow freely from one star to other–mass exchange
- L2 - on opposite side of secondary–matter can most easily leave system
- L3 - on opposite side of primary
- L4, L5 - in lobes perpendicular to line joining binary–form equilateral triangles with centers of two stars
- Roche-lobes:: surfaces which just touch at L1

Effect of Mass Ratio q

<http://star-www.st-and.ac.uk/~kdh1/bsad/bs9.pdf>





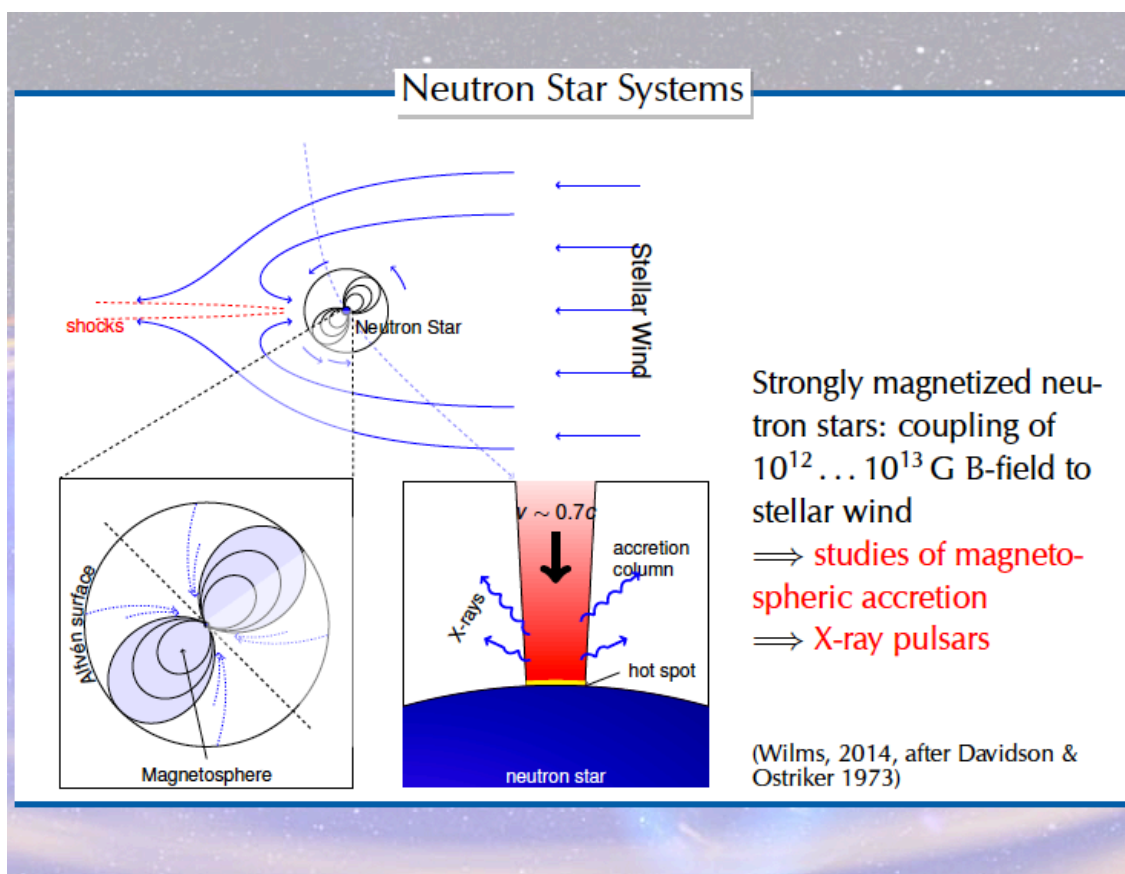
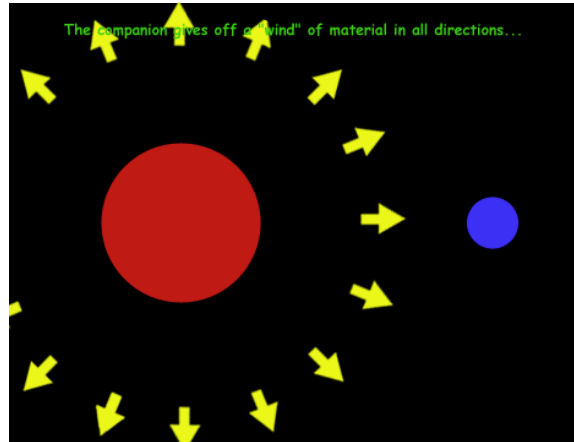
HMXBs T. Kallman

- Among the first discovered extra-solar sources (eg. Vela X-1, Cyg X-1, Cyg X-3, Her X-1)
- Often contain pulsar
- Often eclipsing
- Pulse timing + stellar radial velocity + eclipses = mass, orbital separation, inclination determination
- Accretion can occur from wind from primary, or from Roche-lobe overflow
- Two different subtypes:
 - Be binaries
 - Supergiant binaries
- Statistics: ~50 known in galaxy
- Young population, lifetime $\sim 10^5$ yrs: mass transfer is unstable- wide range in luminosities

High Mass X-ray Binary

The high mass companion sheds mass through a wind.

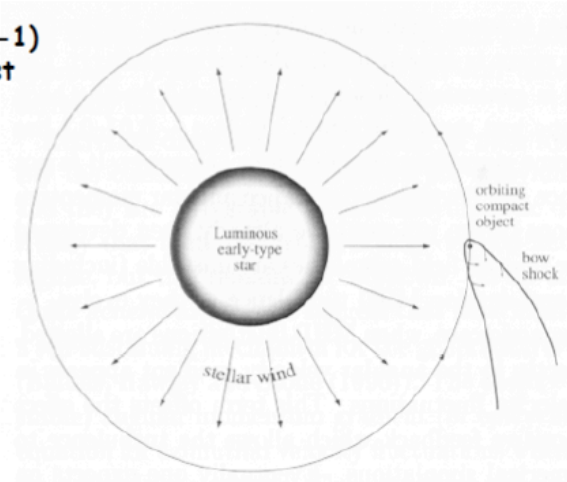
- This wind flows isotropically from the companion, so a portion runs into the compact object.
- This material releases its potential energy as X-rays.



Wind Fed Systems

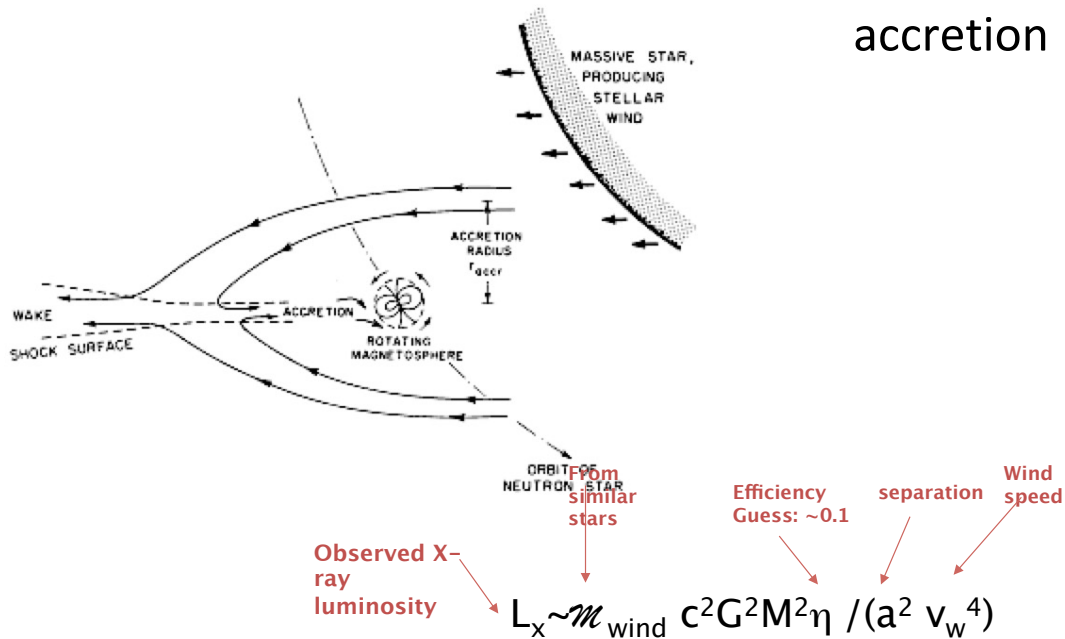
Sample

- Some HMXBs with supergiant companions $\sim 1/3$ of the systems
- Both persistent (e.g. Vela X-1) and transient (supergiant fast X-ray transients)
- Symbiotic X-ray binaries (e.g. GX 1+4)



From Frank et al., 2002, Accretion Power in Astrophysics

HMXB- wind accretion



Theory of wind accretion:
(Davidson and Ostriker 1973):

$$\rho v_w^2 / 2 = GM\rho / r$$

Accretion From a Wind Longair 14.6.4

- The process is called Bondi accretion
- Consider a star of mass m , traveling through a gas of density ρ at relative velocity v_{rel} .
- Material inside a cylinder of radius $r_{\text{acc}} = 2GM/v_{\text{rel}}^2$ can lose enough energy to fall onto the star at an accretion rate of $\dot{S} = \pi r_{\text{acc}}^2 v_{\text{rel}} \rho f$ (where f is a fudge factor due to things not properly modeled due to radiation pressure effects and gas dynamics)

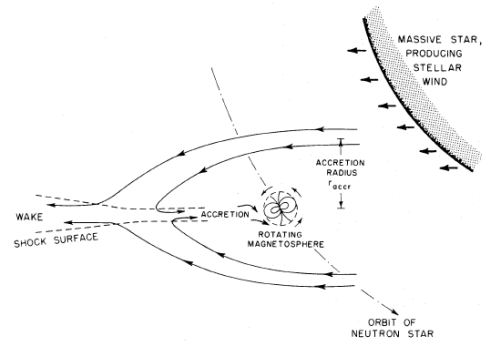


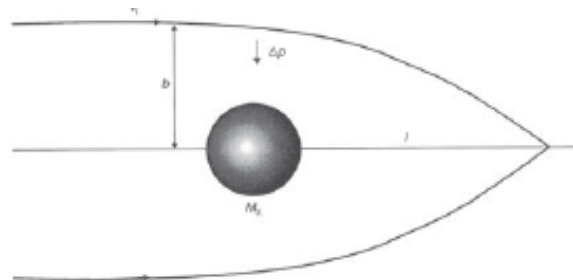
FIG. 3.—Streamlines of stellar-wind material, relative to an accreting neutron star. Dimensions are not to scale.

giant stars stellar wind speeds are
~700km/sec giving

$$r_{\text{acc}} \sim 5 \times 10^{10} \text{ cm}$$

Accretion From a wind – Following Longair

- the impulse which a particle receives on passing a stationary mass is given by the inward force at the distance of closest approach b (see 5.2)



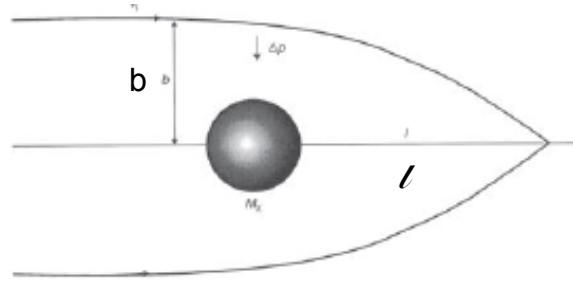
process of accretion by a star of mass M_x in a stellar wind of velocity v_t .

The gravitational force of attraction per unit mass at distance b is GM_x/b^2 and the duration of this force is $2b/v_t$. The momentum impulse inwards is

$$\delta p = 2GM_x/bv_t$$

Accretion From a wind – Following Longair

At distance l downstream, the particles with collision parameter b collide on the axis of the flow.



: process of accretion by a star of mass M_x in a stellar wind of velocity v_w .

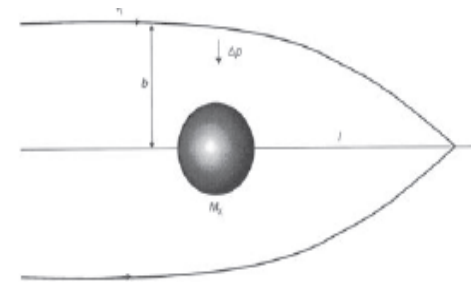
The perpendicular component of the velocity goes to zero and the condition that the matter be captured by the star is: gravitational potential energy of matter at l be greater than its KE outwards $\sim 1/2 v_t^2$

Giving a capture radius

$$R_c = 2GM_x / [v_x^2 + v_w^2]$$

Accretion From a wind – Following Longair

- $L_x \approx [\eta \dot{m}_p / 4] (2GM_x / R_p)^2 v_w^{-4}$
- \dot{m}_p the mass loss rate from the donor star
- accretion rate is $\sim (\dot{m}_p / 4) (R_c / R_p)^2$
- R_p is the distance of the compact object from the donor star
- R_c is the critical (capture radius)
- **Wind velocity $v_w \gg$ orbital velocity v_x**



: process of accretion by a star of mass M_x in a stellar wind of velocity v_w .

So X-ray luminosity is directly proportional to the mass-loss rate of the donor star, \dot{m}_p , and is very sensitive to the wind velocity

If the Magnetic Field is Strong (As In NS Pulsars)

- If magnetic pressure dominates over thermal pressure the magnetic field channels the accretion flow and matter flows along field lines that connect to the magnetic polar regions:
- As a result, almost all of the accretion energy is released in a “hot spot” near the two magnetic poles.
- If the magnetic axis is not aligned with the rotational axis, then as the star rotates we see more or less of the hot spot, and hence see pulsations in the X-rays.

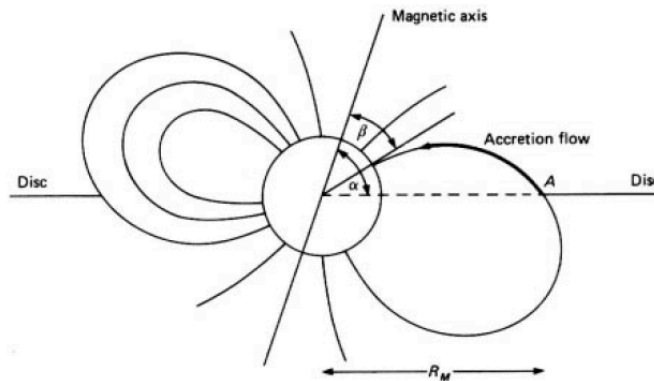


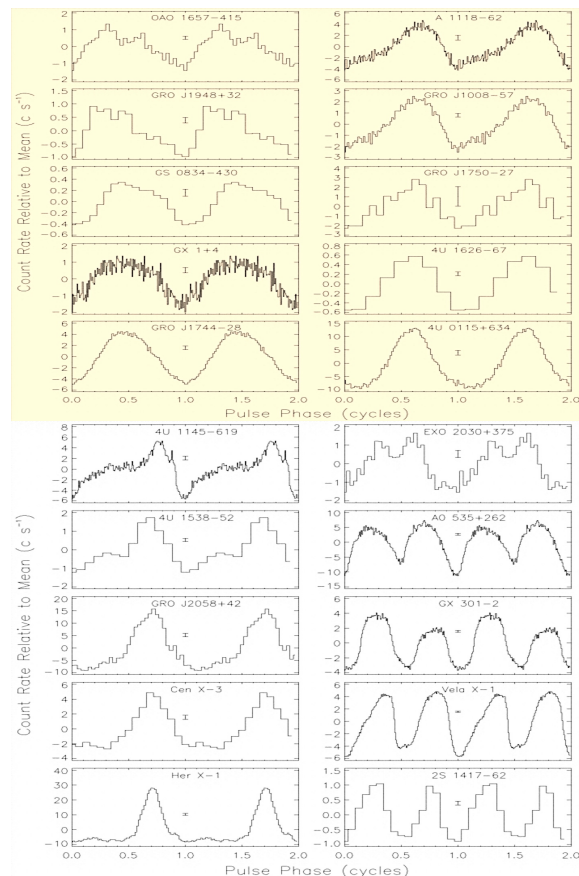
Figure 8: Accretion in a strong ($\sim 10^{12}$ Gauss) magnetic field. Note that the accretion disk is held off the neutron star surface by the centrifugal barrier formed by the rotating magnetosphere.²³

Longair 14.5.3

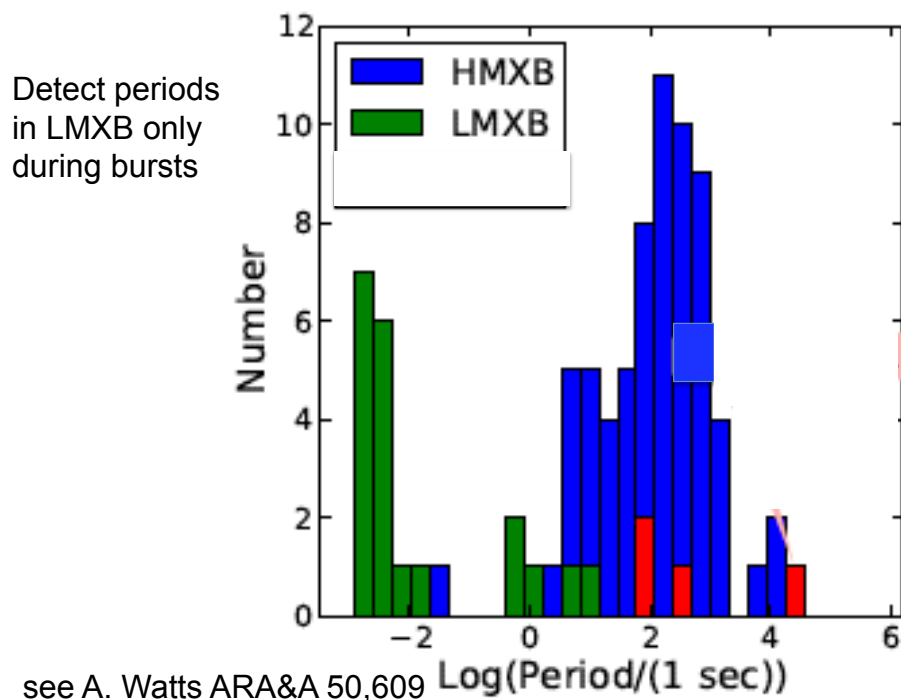
Cominsky (2002)

X-ray Pulsars

- Accrete matter through wind or via disk from a high mass companion. Because of a large magnetic field strength (typically 10^{12} G) the material is channeled onto small spots at the magnetic poles.
- the relativistically moving plasma is decelerated in a radiative shock near the surface and settles subsonically
 - **plasma radiates in the X-ray band**
- Pulsations are observed if the magnetic field is inclined relative to the rotation axis.
- Studies of the pulse profiles of individual pulsars allow one to constrain the emission pattern of the hotspots (or accretion columns) at the NS surface as well as the geometry of the magnetic field

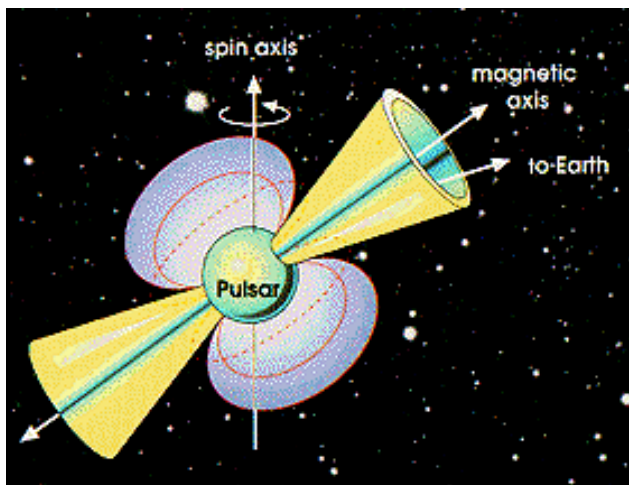


Wide range in Periods

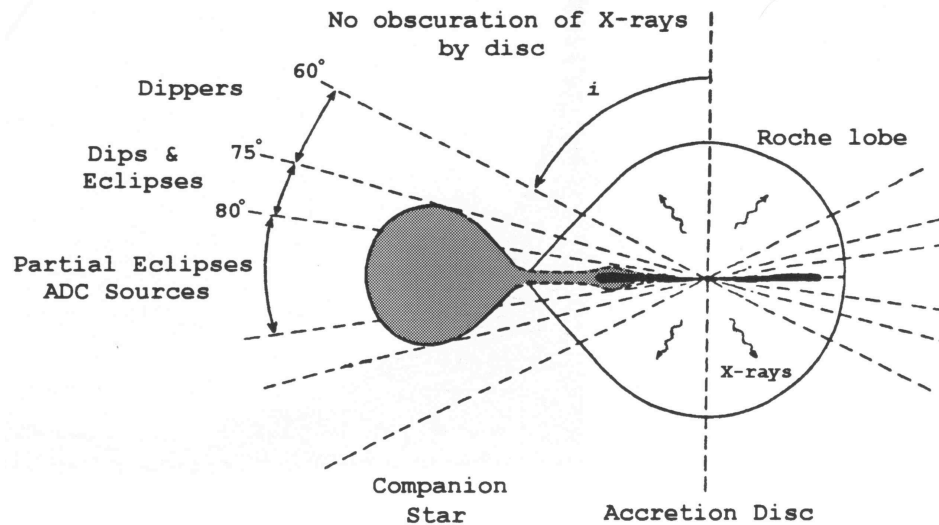


Thermonuclear Burst Oscillations

Rotating magnetic field model



Effects of Geometry on Observed Properties can be Huge (P.Charles)



Accreting Magnetic Neutron Stars Longair 14.5.3

- Effect of magnetic field
 - flow of ionized gas is channeled by the field
 - Photon production in a strong field is different (cyclotron radiation)
- When/where does the magnetic field dominate the accretion flow? (following C. Miller)

The magnetic energy density is $B^2/8\pi$, and the kinetic energy density of the accreting matter is $1/2\rho v^2$, where ρ is the density and v is the typical velocity.

For a dipolar field, $B = \mu/r^3$, (μ is the magnetic moment) and the matter radial free fall velocity is

$$v = v_{ff} = \sqrt{2GM/r}.$$

For a NS $\mu = 10^{30} \text{ G cm}^3$ corresponds to $B_s = 10^{12} \text{ G}$

Accreting Magnetic Neutron Stars

By continuity, $\rho v_{\text{ff}} = dM/dt / (4\pi r^2)$ (gas flow) ($dM/dt = \dot{M}$)

Magnetic energy density $= B^2 / 8\pi$

Notice the radial dependences

magnetic energy density goes as r^{-6}

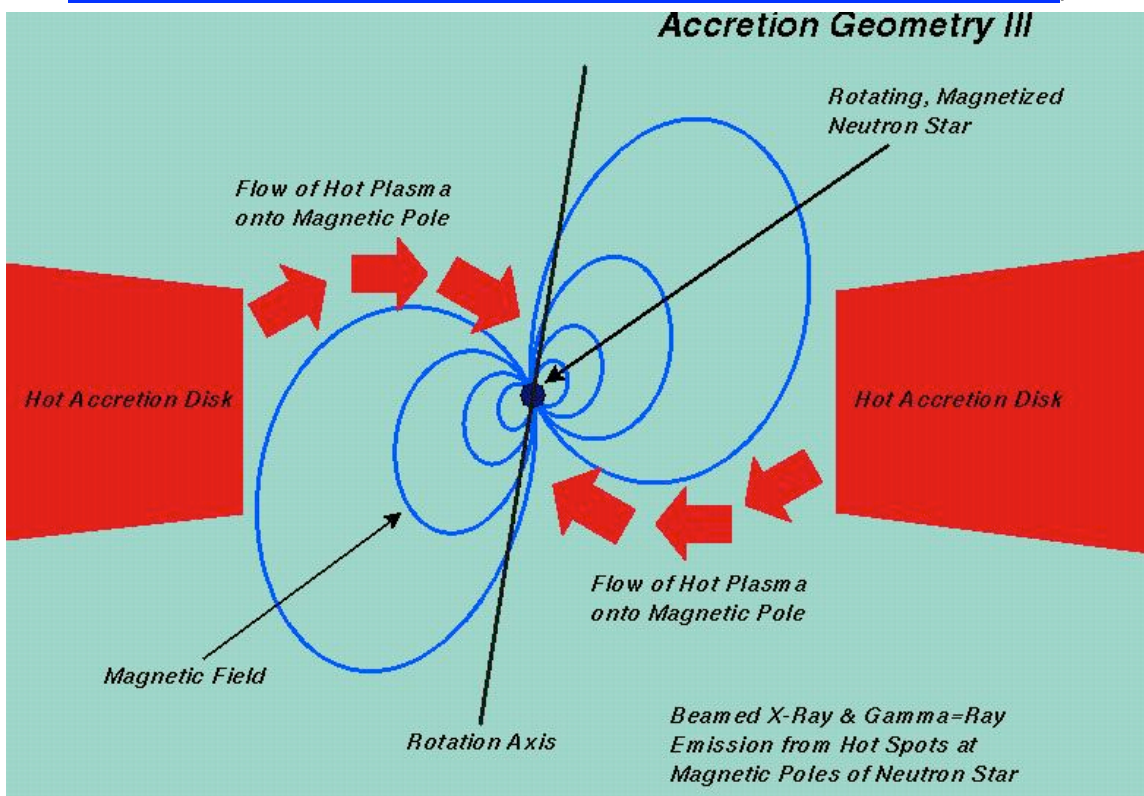
material energy density goes as $r^{-5/2}$.

Close to the star, magnetic stresses will dominate if the field is strong enough;

A magnetic moment of $\mu_{30} = 10^{30} \text{ G cm}^3$ which gives a surface field of $\sim 10^{12} \text{ G}$ is typical of neutron stars in high-mass X-ray binaries.

radius of a neutron star is $R \approx 10^6 \text{ cm}$, the accretion flow onto a strongly magnetized neutron star is dominated by the magnetic field.

www.astro.msfc.nasa.gov/xray/openhouse/ns/,



Where Does the Magnetic Field Start to Dominate?

the Alfvén radius is the radius at which the pressure due to the pulsar's magnetic field equals the ram pressure of infalling material.

$$\text{So : } \rho v_{\text{ff}}^2 = \mathcal{M} / (4\pi r^2)$$

$$\text{The free fall velocity } v_{\text{ff}} = (GM_x / 2r)^{1/2}$$

The Kinetic energy

$$E_{\text{kinetic}} = 1/2 \rho v_{\text{ff}}^2 = \mathcal{M} \sqrt{GM_x} r^{-5/2} / 8\pi \sqrt{2}$$

$$\text{The magnetic energy is } E_{\text{mag}} = B^2 / 8\pi = \mu^2 / 4\pi r^6$$

μ is the magnetic moment

Notice the dependencies

The magnetic energy density goes as r^{-6} ,

the material energy density goes as $r^{-5/2}$

Alfven radius

- The magnetic stresses thus increase much more steeply with decreasing radius than the material stresses do.
- Therefore one expects that far from the star material stresses must dominate. Close to the star, magnetic stresses will dominate if the B field is strong enough
- so there is some radius where the two balance approximately- the capture radius.
- In accretion by a weakly magnetized NS the matter can be very close to the stellar surface before it is captured by the magnetic Field
 - thus the material can end up over a large surface area making pulsations unlikely

Alfven radius

Since High-mass X-ray binaries. have strong B fields , the field is able to capture and channel matter at large radii

The capture radius should be $\sim 10^8$ cm, compared with 10^6 cm for the radius of the NS.

Therefore matter flows along the field lines that connect to the magnetic poles:

the equation of a dipole field is $r = \sin^2\theta = \text{const}$, so

$$\theta = R_{\text{star}}/r_A \sim 10^{-1} - 10^{-2}$$

Therefore, most of the accreting matter falls on a small region which has a 10^{-3} of the NS surface area

Where Does the Magnetic Field Start to Dominate?

the Alfvén radius is the radius at which the pressure due to the pulsar's magnetic field equals the ram pressure of infalling material.

$$\rho v^2 = \frac{B_s^2}{2\mu_0} \left(\frac{R_*}{r_M} \right)^6 .$$

- $\rho v^2 = (B^2/8\pi)(R_*/r_A)^6$

since mass accretion rate is $\dot{M} = 4\pi r^2 \rho v$

one finds that $r_A \sim (B^4 R_*^{12} / M_* \dot{M}^2)^{1/7}$ where M_* is the mass of the star and R_* is the radius of the star (Longair 14.6)

Where Does the Magnetic Field Start to Dominate?

the Alfvén radius is the radius at which the pressure due to the pulsar's magnetic field equals the ram pressure of infalling material.

- Notice the dependence $r_A \sim B^{4/7} \mathcal{M}^{-2/7}$

Or putting in typical numbers

$r_A \sim 3.2 \times 10^8 \mathcal{M}_{17}^2 \mu_{30}^{4/7} M^{1/7}_{\odot} \text{ cm}$: **since a NS has a typical radius of 10^6 cm the magnetic field controls the flow at relatively large radii ($10^8 \gg 10^6$)**

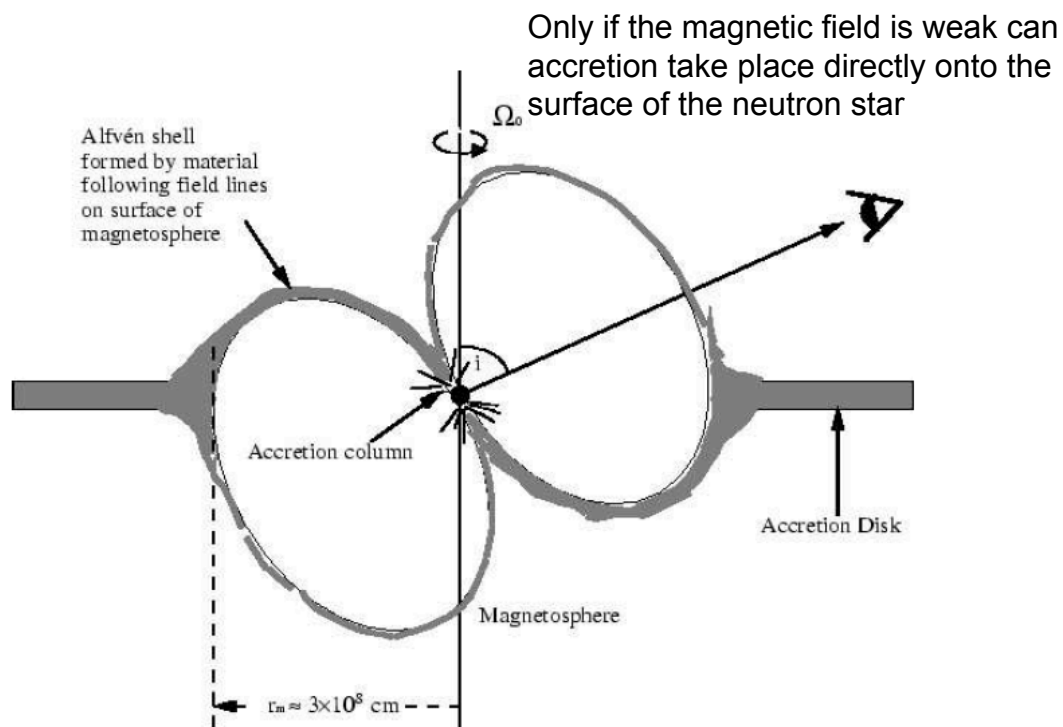
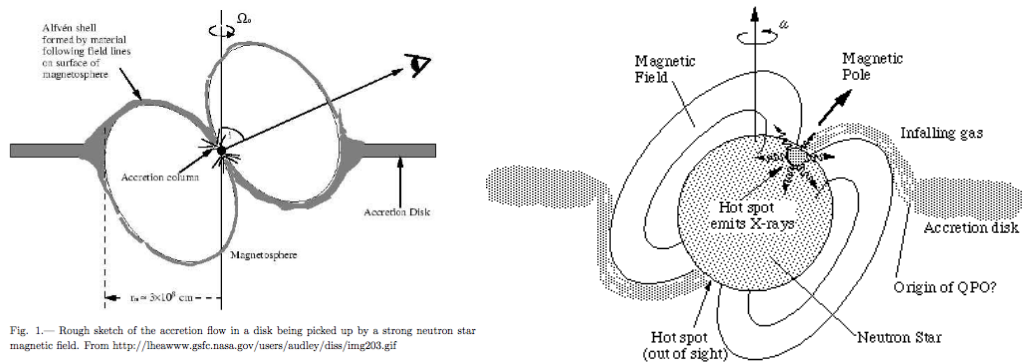


Fig. 1.— Rough sketch of the accretion flow in a disk being picked up by a strong neutron star magnetic field. From <http://lheawww.gsfc.nasa.gov/users/audley/diss/img203.gif>

- Putting in typical numbers the radius where magnetic and material stresses are equal is the Alfvén radius

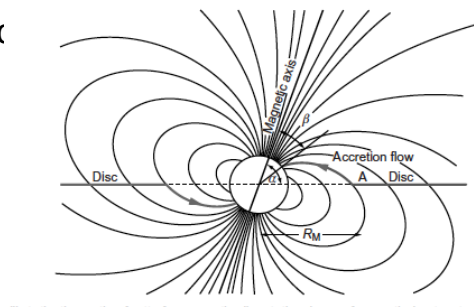
$$r_A = \left(\frac{\mu^4}{2GM\dot{M}^2} \right)^{1/7} = 3.2 \times 10^8 \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} \left(\frac{M}{M_\odot} \right)^{-1/7} \text{ cm} .$$

\dot{M}_{17} is the accretion rate in units of 10^{17} gm/sec- why do we scale it this way??



So How Does Matter Get In??

- For luminous X-ray sources, the immediate vicinity of the neutron star is magnetically dominated
 - Matter can, however, be accreted onto the surface of the neutron star, if the matter flows along the magnetic field lines onto the poles of the rotating neutron star
 - releasing the binding energy of the infalling matter as radiation
- in an *accretion column* associated with the infall of matter onto strongly magnetic



High-mass X-ray binaries.

- since these NS have strong B fields , the field is able to capture and channel matter far from the surface
- The capture radius is $\sim 10^8 \text{cm}$, compared with 10^6cm for the radius of the NS.
- Therefore matter flows along the field lines that connect to the magnetic poles:
- the equation of a dipole field is $r = \sin^2 \theta = \text{const}$, so
- $\theta = R_{\text{star}}/r_A \sim 10^{-1} - 10^{-2}$
- Therefore ,most of the accreting matter falls on a small region which has a 10^{-3} of the NS surface area

Violation of Eddington Limit ??

- The accretion rate of, ~ 0.1 the Eddington limited accretion rate falls onto a surface area only 10^{-3} of the star !
- **So the local flux generated \gg Eddington limit**
- For such accretion to persist, the radiation cannot escape back up the accretion funnel (remember the incoming material is interacting with the radiation for the Eddington limit to be defined).
-

Violation of Eddington Limit ??

- Instead the radiation has to come out where there is little or no accreting material (out the sides).
- The Eddington flux is a limit only for spherically symmetric systems, and in this case we have a system that is very aspherical
- the radiation pattern can be a “fan beam” (radiation escaping out the sides), so that we might get two peaks per cycle from the funnel (one from one side, one from the other) as opposed to the one peak we would expect if this were just a thermally glowing hot spot.

Origin of Field ?

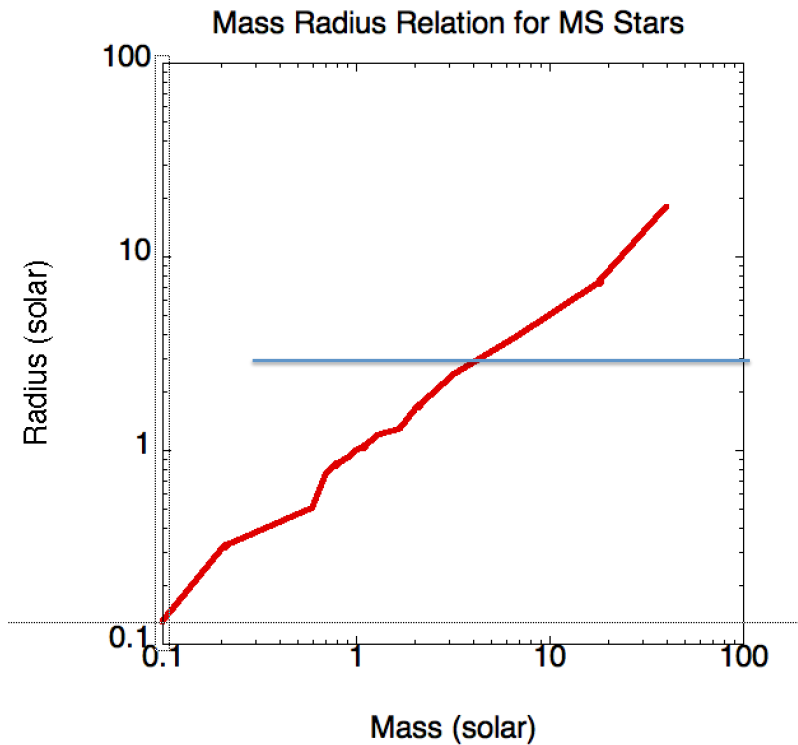
If the field is due to the 'original' star

- The fields in Main Sequence (MS) stars are $\sim 1\text{G}$.
- For a MS progenitor of mass, $10M_{\odot}$, and radius $4 \times 10^{11}\text{cm}$ (the sun has a radius of $7 \times 10^{10}\text{cm}$), the star would contain a magnetic flux of $\sim 5 \times 10^{23}\text{ Gcm}^2$ ($\pi r^2 B$)
- If flux is conserved during the collapse then a neutron star with the same flux would have surface field strength of $5 \times 10^{11}\text{G}$, sufficient for a pulsar
- However no one really knows if flux is conserved in the formation of the NS during the Supernova explosion and collapse and there are good reasons to believe that this is not true (see Spruit 2007 arxiv 0711.3650.pdf)

- **How can a neutron star have a magnetic field if it is composed of neutrons?**

A neutron star is not *entirely* composed of neutrons. It also contains some number of protons and electrons (probably about 10% each of the number of neutrons). It is those particles, which are electrically charged, that can produce currents and therefore sustain a magnetic field. (C. Miller)

Mass Radius Relation for Main Sequence Stars



Eclipsing Pulsing Neutron Stars

- A breakthrough in the understanding of these objects was the discovery of eclipses and pulse timing.

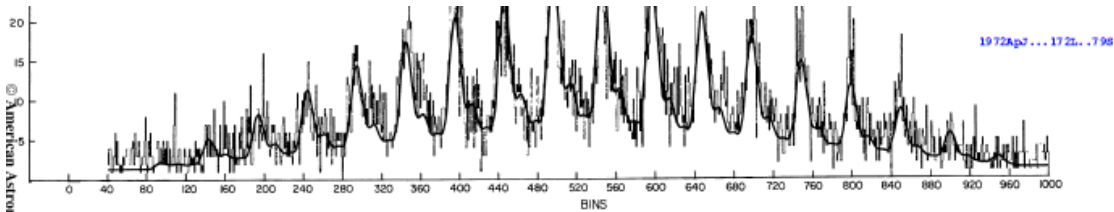
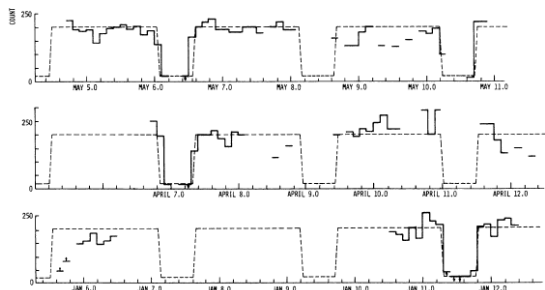


FIG. 3.—The counts accumulated in 0.096 bins from Cen X-3 during a 100-s pass on 1971 May 7 are plotted as a function of bin number. The functional fit obtained by minimizing χ^2 is also shown.

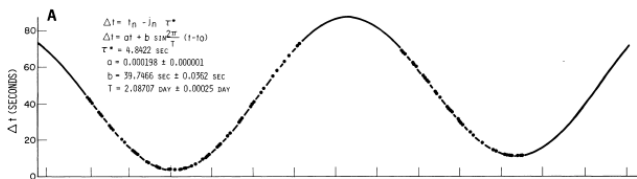


FIG. 4.—(a) The difference Δt between the time of occurrence of a pulse and the time predicted for a constant period is plotted as a function of time. A best-fit function and the values of the parameters are given. (b) The dependence of the pulse period P on time as derived from the best-fit phase function above is shown and the values of the parameters given. (c) The intensity observed and the light curve predictions are shown for the same set of data. Note the coincidence of the null points of the period function with the centers of the high- and low-intensity states.

Doppler curve of pulsar- difference between the predicted and observed pulse arrival time vs time