Downwards to Black Holes!- Longair 13.11

- a neutron star has a maximum mass
- If this mass is exceeded on has a complete gravitational collapse to a black hole

- Basic anatomy of a black hole
- Observational discovery of black holes

Black Holes

- What do you mean 'black holes' ?
- We know of objects whose mass (derived from observations of the lines from the companion objects and Newton's (Einstein) laws which are larger than possible for a NS or white dwarf.
- They have other unusual properties (related to their x ray spectrum and timing behavior)
- **Big differences- no surface, no (?) magnetic field, higher mass strong GR effects**.

Table 4.3. *Candidate black hole binaries*[a]

Source	RA(2000)	DEC(2000)	r_x[b]	BH trait[c]	Grade[d]	Referen
1354–645 (BW Cir)	13 58 09.74	-64 44 05.2		LH,HS	A	1,2,
1524–617 (KY TrA)	15 28 16.7	-61 52 58		LH,HS	A	5,
4U 1630–47	16 34 01.61	-47 23 34.8		LH,HS	A	8,9,10,11
XTE J1650–500	16 50 01.0	-49 57 45		LH,HS,VH	A	12,13,14,15
SAX J1711.6–3808	17 11 37.1	-38 07 06		LH,HS	B	17
GRS 1716–249[e]	17 19 36.93	-25 01 03.4		LH	B	19,20
XTE J1720–318	17 19 59.06	-31 44 59.7		LH:,HS	C	22,23
KS 1730–312	17 33 37.6	-31 13 12	30″	LH,HS	C	25
GRS 1737–31	17 40 09	-31 02.4	30″	LH	B	27,28
GRS 1739–278	17 42 40.03	-27 44 52.7		LH,HS,VH	A	30,31,32,33
1E 1740.7–2942	17 43 54.88	-29 44 42.5		LH,HS,J	A	35,36,37,38
H 1743–322	17 46 15.61	-32 14 00.6		HS,VH	A	40,41,42,80,81
A 1742–289	17 45 37.3	-29 01 05		HS:	C	43,44,45
SLX 1746–331	17 49 50.6	-33 11 55	35″	HS:	C	47,48
XTE J1748–288	17 48 05.06	-28 28 25.8		LH,HS,VH,J	A	50,51,52,53
XTE J1755–324	17 55 28.6	-32 28 39	1′	LH,HS	B	55,56,57
1755–338 (V4134 Sgr)	17 58 40.0	-33 48 27		HS	B	59,42,60,61
GRS 1758–258	18 01 12.67	-25 44 26.7		LH,HS,J	A	63,38,64,65
EXO 1846–031	18 49 16.9	-03 03 53	11″[f]	HS	C	
XTE J1908+094	19 08 53.08	+09 23 04.9		LH,HS	B	68,69,70
1957+115 (V1408 Aql)	19 59 24.0	+11 42 30		HS	C	72,42,73,74
XTE J2012+381	20 12 37.70	+38 11 01.2		LH,HS	B	76,77,78

How Can We Observe Black Holes

- If a black hole is a 'place' where radiation cannot escape to infinity how can they be observed ?
- Dynamical effects on 'nearby' material
- "Shining" black holes- a black hole can be a place where accretion occurs and as we have seen the process of accretion around a compact object can produce huge amounts of energy and radiation- making the black hole 'visible'

General properties of emission from black hole systems

- Emission usually variable on wide variety of timescales
  - Galactic black hole binaries : millisecond and up
  - AGN : minutes and up
  - Most rapid variability approaches light-crossing timescale limit of physical size of object (τ~R/c)
- Significant emission over **very** broad spectral range (radio to hard X-ray or gamma-rays)-NS and WDs tend to have 'thermal' like spectra (relatively narrow in wavelength)
- Lack of a signature of a surface - not a pulsar, no boundary layer emission (no x-ray bursts), no 'after glow' from cooling

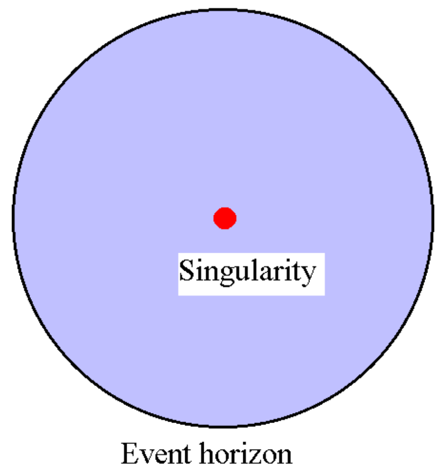
Basic anatomy of a black hole

- Complete gravitational collapse inevitably leads to a black hole (Hawking)
- Space-time singularity
  - Where the mass-energy resides
  - Place where GR breaks down and laws of quantum gravity must be applied

Event horizon

Point of no return for light or matter

Events inside horizon can have no causal effect on universe outside of the horizon



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- Event horizon
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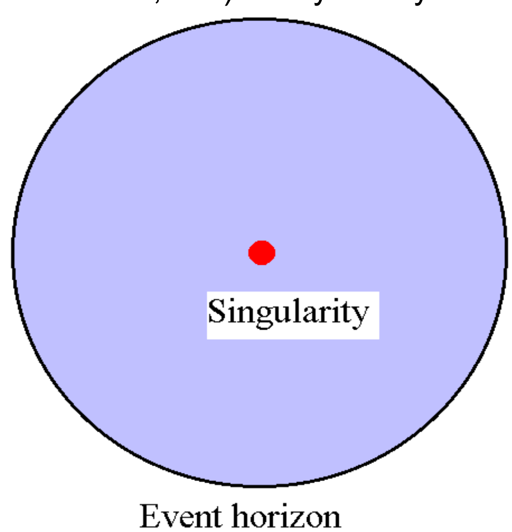
  - Events inside horizon can have no causal effect on universe outside of the horizon
  - Analogous to the point of no return in a waterfall

*black holes have no hair

3 parameters <u>mass, angular momentum, and electric charge</u> completely characterize black holes

Everything else (quadrupole terms, magnetic moments, weak forces, etc.) decays away*.



So what is the actual size ?

$R_G \sim 1.5\ (M / M_\odot)$ km

So how close are neutron stars to being black holes ?

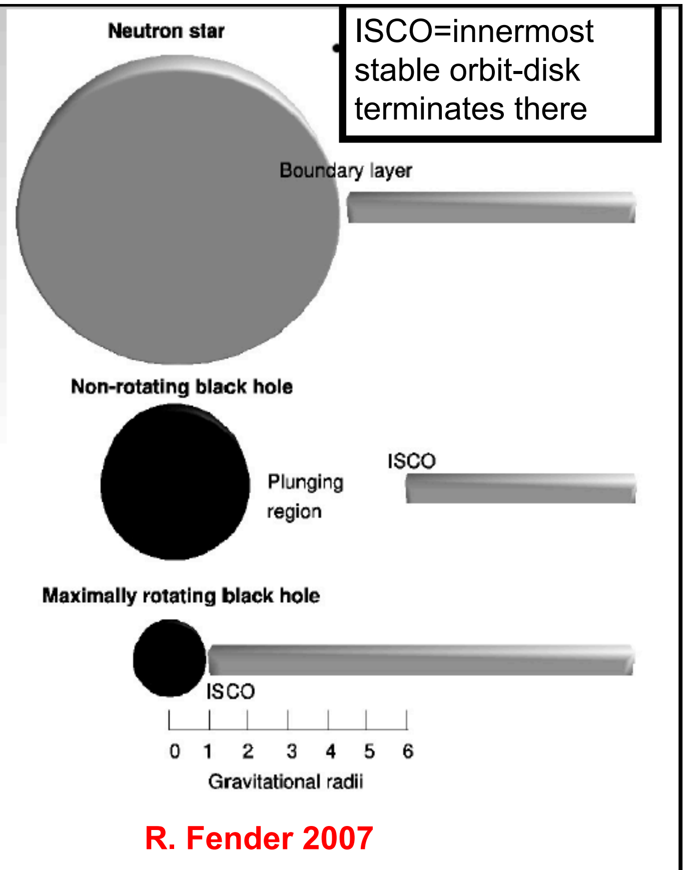
Neutron stars are only about a factor 2—3 larger than their event horizons

What about spin ?

A non-rotating ("Schwarzschild") black hole has its event horizon at 2 R_G and its ISCO at 6 R_G

A maximally rotating ("Maximal Kerr") black hole has both its event horizon and ISCO at R_G

→ Spinning black holes are more compact → potentially more radiatively efficient



Schwarzschild Radius-AKA the Event Horizon for a Non-Spinning Black Holes -Longair 19.1

- $R_s = 2GM/c^2$; $3(M/M_\odot)$km

- The Schwarzschild radius is the radius of 'no return' for a non-rotating black hole- ***it is not the singularity. Its apparent singularity is a due to a choice of coordinates (Longair pg 433) .***

- Events inside that horizon cannot be seen by any external observer
- inside the event horizon the radius becomes a timelike coordinate, and the time becomes a spacelike coordinate. Specifically, that means that once inside R_S, you must go to smaller radii, just as now you must go forward in time
- once you're inside the event horizon **one cannot avoid the singularity at r = 0**
- These are the 'simplest' macroscopic objects-the only instance of an exact description of a macroscopic object..., almost by definition,the most perfect macroscopic objects there are in the universe
- **Timescales are VERY sort $T \geq R_s/c = 2GM/c^3 \approx 10^{-5}\ (M/M_\odot)$ sec**

More features of Schwarzschild black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)
- Observer who enters event horizon would only "feel" “strange” gravitational effects if the black hole mass is small, so that R_s is comparable to observer’s size
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
- Thus black hole would produce “shadow” on sky

'The' Singularity

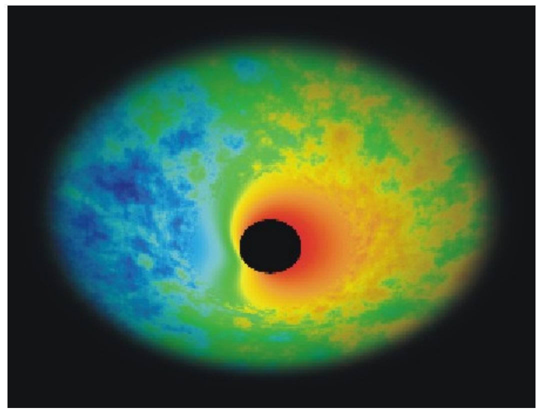
- At $r = 0$, there is a real *physical singularity* and, according to classical general relativity, the infalling matter collapses to a singular point.
- However the Schwarzschild singularity is unobservable because no information can arrive to the external observer from within the Schwarzschild radius R_s.
- see discussion on pg 433

For the Mathematically Inclined see sec 13.11.1

- The Schwarzschild solution for the metric around a point mass in spherical coordinates is
- $ds^2=-(1-2GM/c^2r)c^2dt^2+(1-2GM/c^2r)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)$ eq 13.52

  s is the length element
- Notice singularity at $r=2GM/c^2$ (can be gotten rid of in a coordinate transformation)

- A static observer measures proper time $c^2d\tau^2=-ds^2=-(1-2GM/c^2r)c^2dt^2$
- $d\tau/dt=\text{sqrt}(1-2GM/c^2r))=1+z_{grav}$



Example metrics

- Normal (flat) 3-d space in cartesian coordinates

- Flat spacetime of Special Relativity in cartesian coordinates

$$ds^2 = dx^2 + dy^2 + dz^2$$

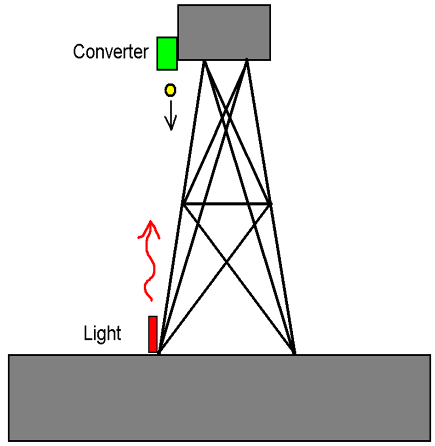
- Non-spinning, uncharged black hole (Schwarzschild metric) in "Schwarzschild coordinates"

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = -c^2\left(1-\frac{2GM}{c^2r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2GM}{c^2r}\right)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Gravitational redshift- Longair 19.21

- Thought experiment:
  - Send photon upwards in a gravitational field
  - Convert that energy into mass and drop the mass
  - Convert mass back into photon
- Conservation of energy $\Rightarrow$ photon must lose energy as it climbs in the gravitational field
- Another way of thinking about this - the escape speed from the object has to be less than the speed of light (assuming, incorrectluy, that light could slow down and fall).
- In Newton mechanics the escape speed is $v^2 = 2GM/r$, so $v^2 = c^2$ at $r = 2GM/c^2$
- Redshift of light $Z=(\lambda_0-\lambda_e)/\lambda_e$; λ_0 = wavelength as measured by the observer, λ_e as emitted



Gravitational redshifts near a black hole

- Gravitational redshift is really a form of relativistic time dilation
- As observed from infinity, time near a (non-spinning, non-charged) black hole runs slow by a fac

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - 2GM/c^2}}$$

- The event horizon is the "infinite redshift" surface where (as observed from infinity) time appears to stop!
- But… a free falling observing would fall through the event horizon without noticing anything unusual.
- The wavelength of light is redshifted ($Z=(\lambda_0-\lambda_{e)}/\lambda_e$; λ_0 = wavelength as measured by the observer, λ_e as emitted at radius r
-

Longair 13.58 in frequency units
$\nu_\infty=\nu_{em}$ sqrt(1-2GM/rc^2)

$$z = \frac{1}{\sqrt{1 - \left(\frac{2GM}{rc^2}\right)}} - 1$$

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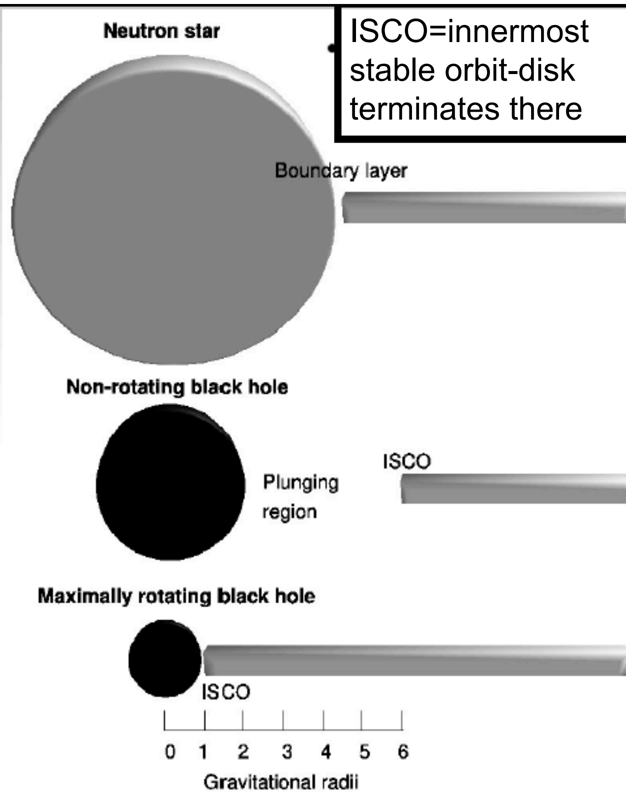
ISCO=innermost stable orbit-disk terminates there

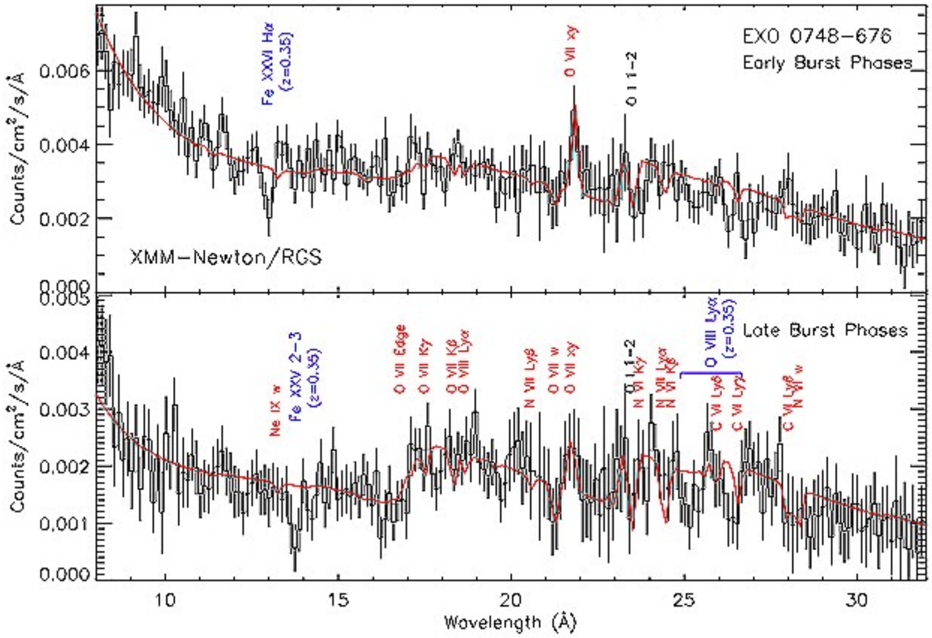


R. Fender 2007

Question for class- what is the redshift from the surface of a NS?

- M ~M_{sun} ; R=10km (set by nuclear physics)

Emission of line radiation from highly ionized atoms of Fe And O from near the surface of a ***NS***



Redshifted absorption lines from a neutron star surface
Cottam, Paerels & Mendez (2002)

-β<9 (DeDeo & Psaltis 2003)

What is the Event Horizon

- The event horizon is the "infinite redshift" surface where (as observed from infinity)
  - the radius at which the gravitational redshift (z) is infinite
  - $z=\Delta\lambda//\lambda$; $1+z=1/\mathrm{sqrt}(1-R_s/R)$; $R_s=2GM/c^2$
- But… a free falling observing would fall through the event horizon without noticing anything unusual.

- As observed from infinity, time near a (non-spinning, non-charged) black hole run slow by a factor of

$$t_0 = t_f\sqrt{1-\frac{2GM}{rc^2}} = t_f\sqrt{1-\frac{r_s}{r}}$$

•Also time appears to stop !

A Bit More- Longair 13.11.2, 19.6.2

- For a non-rotating BH the last stable orbit is at $3R_s$; this is due to the form of the potential which is NOT 1/r
- Particles on the last stable orbit about a Schwarzschild black hole move at half the speed of light.

For those interested the potential can be expressed as
$U(r)_{eff} = (1-r_S/r)(mc^2+\ell^2/mr^2)$ where ℓ is the angular momentum (K.Griest)

For a Schwarzschild black hole, the maximum energy which can be released corresponds to the binding energy of the matter on the last stable circular orbit at $r = 3\,R_s = 6GM/c^2$.

A fraction $[1 - (8/9)^{1/2}] = 0.0572$ of the rest mass energy can thus be released

What are the possible energy sources?

- Accretion?
  - Release of gravitational potential energy as matter falls into black hole
  - YES! Thought to be primary power source of all systems just discussed

- Rotational energy of black hole? Spinning (Kerr) BGs see 13.11.2
  - Tapping the rotational energy of a spinning black hole $1/2I\Omega^2$ can be very large
  - May be important in some settings… but can only be tapped if accretion occurring!

Rotating black holes- remember the extra special nature of accelerated frames

- Roy Kerr (1963)
  - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has "**static surface**" inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: "**frame dragging**".
  - **Ergosphere** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Spinning BH- Longair sec 13.11.2

- A black hole with angular momentum $\boldsymbol{J}$ has a metric

$ds^2=(1-2GMr/\rho c^2)dr^2-\{(1/c^2)[4GMra\sin^2\theta/\rho c]\,dr d\phi\}+(\rho/\Delta)dr^2+\rho d\theta^2+(r^2+a^2+2GMra^2\sin^2\theta/\rho c^2)\sin^2\theta\, d\phi^2]$-- Longair eq 13.63

r,θ,ϕ– usual polar coordinates

where $a=(\boldsymbol{J}/Mc)$ is the angular momentum per unit mass (dimensions of distance) and $\Delta=r^2-(2GMr/c^2)+a^2$; $\rho=r^2+a^2\cos^2\theta$

Just like Schwarschild metric it becomes singular but at a radius where $\Delta=r^2-(2GMr/c^2)+a^2=0$; the larger root is

$r_+=GM/c^2+[(GM/c^2)^2-(\boldsymbol{J}/Mc)^2]^{1/2}$

for J>0 this is smaller than the Schwarschild radius

there is a maximum angular momentum ***$J=GM^2/c$;*** *for this value of J the horizon is at* ***$r_+=GM/c$; 1/2 of the Schwarzschild radius***