

## New Result !

### A noninteracting low-mass black hole–giant star binary system Science

01 Nov 2019:

Vol. 366, Issue 6465, pp. 637-640 Todd A. Thompson et al

- Almost all black holes are discovered by their x-ray emission.
- Thompson et al. observed light from a giant star that is Doppler shifted, indicating an orbit around a binary companion.
- The companion object must weigh more than 2.6 solar masses, but it emits no light, including x-rays.
- This indicates the presence of a black hole that is not currently consuming any material.
- There may be a population of similarly hidden black holes that have been missed by x-ray observations.

The optical spectra exhibit only a single set of absorption lines ,i.e., a single-lined spectroscopic binary.

- The system has a nearly circular orbit with orbital period  
 $P_{\text{orb}} \approx 83.2 \pm 0.06$  days,
- radial velocity semi-amplitude  $K \approx 44.6 \pm 0.1$  km s<sup>-1</sup>, and eccentricity  $e \approx 0.0048 \pm 0.0026$ .
- The mass function

$$f(M) \equiv \frac{M_{\text{CO}}^3 \sin^3 i_{\text{orb}}}{(M_{\text{giant}} + M_{\text{CO}})^2}$$
$$= \frac{K^3 P_{\text{orb}}}{2\pi G} (1 - e^2)^{3/2} \approx 0.766 \pm 0.006 M_{\odot}$$

Thus J05215658 likely consists of a  
 $M \approx 3.2 \pm 1.0 M_{\odot}$  giant star **and a noninteracting low-mass black hole  
companion with  $M_{\text{CO}} \approx 3.3 (+2.8, -0.7) M_{\odot}$ .**

# Compact Object Masses Determined via Dynamics

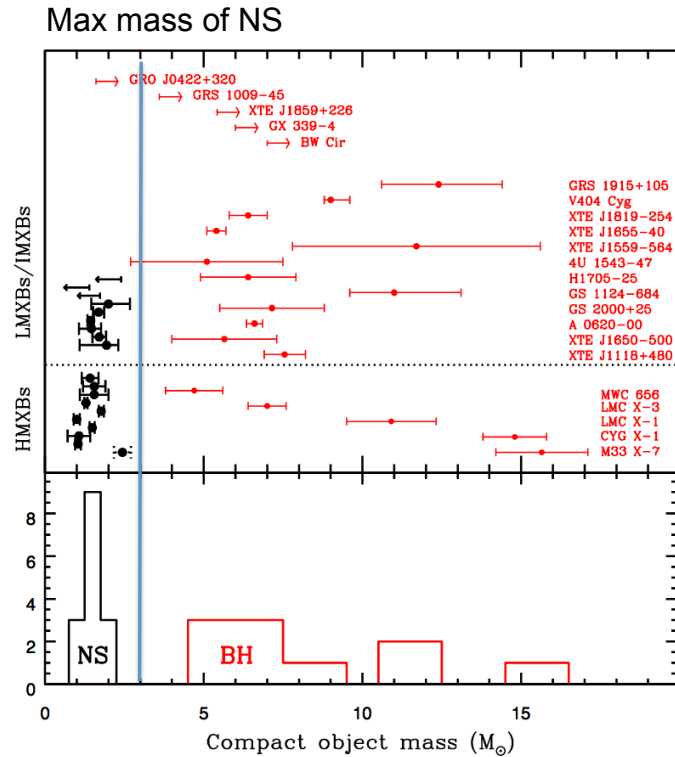
NS in black

BHs in red

is there a gap  
between  $\sim 2-5 M_{\odot}$

the most massive black  
holes ( $\sim 15 M_{\odot}$ ) are  
found in HMXBs.

J. Casares, P.G. Jonker, and  
G. Israelian 1701.07450.pdf



## Homework Common Errors

- In problem 4 very few students mentioned that one needs to observe the lines of highly ionized elements to determine the abundance

Also many students said this technique could fail since the x-ray emission is relatively short lived – while this is not totally wrong, the 'life time' of the SNR in the x-rays is not much shorter than that in radio or optical and thus is not a 'best answer' Two better answers are 1) if the remnant is a plerion (synchrotron dominated remnant) and thus had either weak or absent lines or 2) if the swept up material dominates the x-ray emission reducing the impact of the ejecta on the x-ray emission lines

Another answer (not as good, but correct) is that the remnant may show strong spatial variations in emission lines (e.g. Cas-A) and thus one may draw a wrong conclusion if looking at just one spot.

5) This problem should have been written more clearly- I should have said 'observational evidence'

The 'lifetime' of CR is, of course, energy dependent; ignoring this the 'confinement time' of CR in the MW (1407.5223.pdf) is  $\sim \tau_{\text{esc}} = 15.0 \pm 1.6$  Myr (compared to the 5 Myr gave in the problem). The paper argues for a timescale of  $\sim 5$  Myr

## Recent Reviews

- Nature Astronomy Jan 08 2019 Cosmic celebrities with gravitas  
Black holes have the distinct honor of being the most popular and potentially the least well-understood objects in the Universe. This issue's Insight explores how far black hole research has come since its inception, though it still has a long way to go.
- Observing black holes spin: Christopher S. Reynolds [arXiv:1903.11704](https://arxiv.org/abs/1903.11704)
- Observational constraints on the feeding of supermassive black holes  
Thaisa Storchi-Bergmann [arXiv:1904.03338](https://arxiv.org/abs/1904.03338)
- The many definitions of a black hole: Erik Curiel [arXiv:1808.01507](https://arxiv.org/abs/1808.01507)  
Reviews for Science Special Issue on black holes
- Stellar Mass Black Holes and Ultraluminous X-Ray Sources  
Rob Fender & Tomaso Belloni [arXiv:1208.1138](https://arxiv.org/abs/1208.1138)
- The Formation and Evolution of Massive Black Holes M. Volonteri  
*Science* 03 Aug 2012:  
Vol. 337, Issue 6094, pp. 544-547

## Gravitational Waves

- See **Classical Black Holes: The Nonlinear Dynamics of Curved Spacetime** Kip S. Thorne *Science* 03 Aug 2012:  
Vol. 337, Issue 6094, pp. 536-538
- and Black holes, gravitational waves and fundamental physics: a roadmap [1806.05195.pdf](https://arxiv.org/abs/1806.05195)

black holes are objects made wholly and solely from curved spacetime, objects whose curvature is generated by the energy stored in the curvature. General relativity, predicts that black holes can spin and oscillate.

The gravitational waveforms from a merging black-hole binary in principle carry detailed information about the initial black holes (their masses, spins, and orbit), the final merged black hole (its mass and spin), and the binary orbital plane orientation, the eccentricity, the distance to the source and the sky location

## Newtonian Concept

In 1783 at the Royal Society of London in, John Michell noted that,

- if a star were sufficiently massive, the escape velocity from its surface would exceed the speed of light and so light would not escape from it (Michell, 1784).
- In classical terms, the escape velocity from the surface of a star of mass  $M$  and radius  $r$  is  $v = (2GM/r)^{1/2}$ .
- Setting  $v$  equal to the speed of light  $c$ , the radius of such a star would be  $r_g = 2GM/c^2$ .

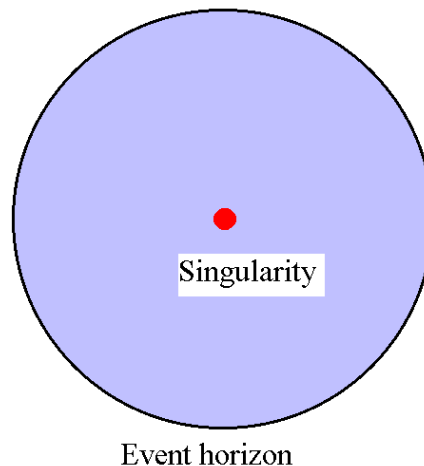
exactly the same as the Schwarzschild radius of a black hole of mass  $M$  in general relativity,

although the coordinate  $r$  has a different meaning from the Newtonian definition of distance.

This spherical surface plays the role of the surface of the black hole.

## Basic anatomy of a black hole

- Complete gravitational collapse inevitably leads to a black hole (Hawking)
  - Event horizon  
Point of no return for light or matter  
Events inside horizon can have no causal effect on universe outside of the horizon
- Space-time singularity
  - Where the mass-energy resides
  - Place where GR breaks down and the (unknown) laws of quantum gravity must be applied



## Properties of Black Holes

- 1. Once one falls into the sphere of influence of a black hole, there is no escape.
- 2. The region of space time black holes encompass only ever increases.
- 3. It is impossible, by any physical process, to reduce the effect of black holes to zero
- 4. All black holes are essentially the same on the outside, no matter the fine details of their inner constitution.
- 5. One cannot make a black hole not be a black hole.
- 6. Once a black hole gets hold of information, it is forever lost

## A Tiny History (Erik Curiel)

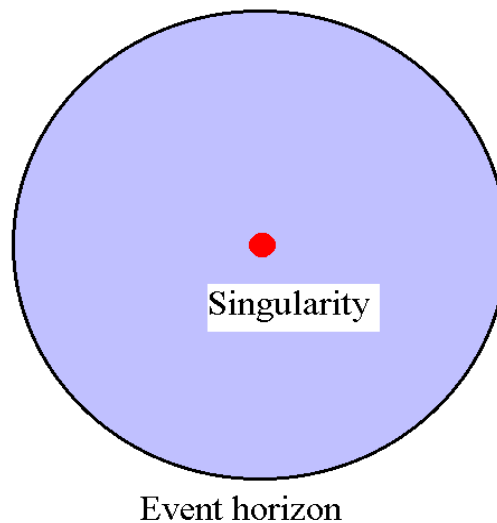
- Hawking showed that when quantum effects are taken into account, black holes should emit thermalized radiation like an ordinary blackbody. This appears to point to a deep connection among our three most fundamental theories, general relativity, quantum field theory, and thermodynamics.
- ...the community achieved something like unanimous agreement on the existence and relevance of black holes only in the early 2000s, with the demonstration that SgrA\*, the center of the Milky Way, holds a supermassive black hole,

## A Few Definitions (E. Cürel)

Field	Core Concepts
astrophysics	<ul style="list-style-type: none"> <li>compact object</li> <li>region of no escape</li> <li>engine for enormous power output</li> </ul>
classical relativity	<ul style="list-style-type: none"> <li>causal boundary of the past of future null infinity (event horizon)</li> <li>apparent horizon</li> <li>quasi-local horizon</li> </ul>
mathematical relativity	<ul style="list-style-type: none"> <li>apparent horizon</li> <li>singularity</li> </ul>
semi-classical gravity	<ul style="list-style-type: none"> <li>same as classical relativity</li> <li>thermodynamical system of maximal entropy</li> </ul>
quantum gravity	<ul style="list-style-type: none"> <li>particular excitation of quantum field</li> <li>ensemble or mixed state of maximal entropy</li> <li>no good definition to be had</li> </ul>
analogue gravity	<ul style="list-style-type: none"> <li>region of no escape for finite time, or for low energy modes</li> </ul>

## Basic anatomy of a black hole

- Complete gravitational collapse inevitably leads to a black hole (Hawking)
  - Space-time singularity
    - Where the mass-energy resides
    - Place where GR breaks down and laws of quantum gravity must be applied*
  - Event horizon
    - Point of no return for light or matter
    - Events inside horizon can have no causal effect on universe outside of the horizon
    - Analogous to the point of no return in a waterfall
- 3 parameters mass, angular momentum, and electric charge completely characterize black holes. Everything else (quadrupole terms, magnetic moments, weak forces, etc.) decays away\*.



\*black holes have no hair

So what is the actual size ?

$$R_G \sim 1.5 (M / M_\odot) \text{ km}$$

So how close are neutron stars to being black holes ?

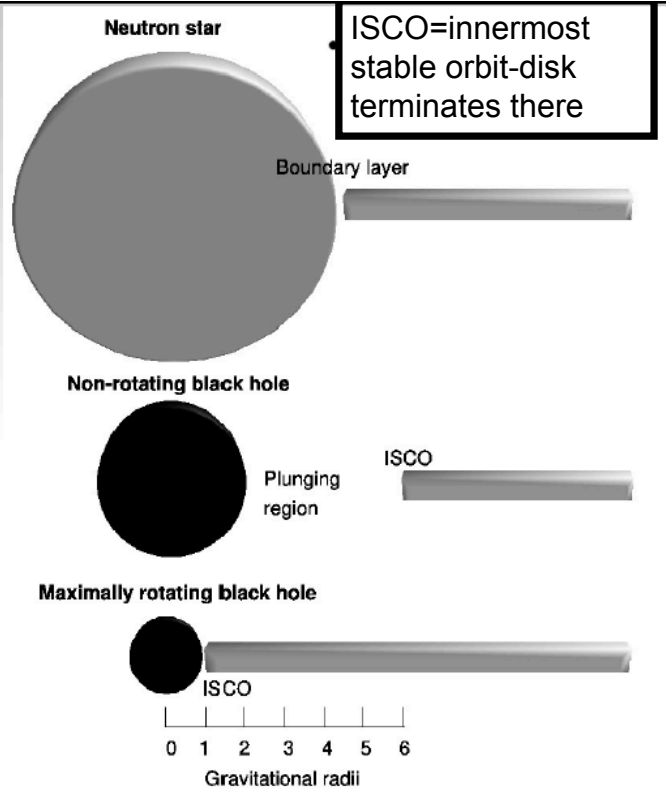
Neutron stars are only about a factor 2—3 larger than their event horizons

What about spin ?

A non-rotating (“Schwarzschild”) black hole has its event horizon at  $2 R_G$  and its ISCO at  $6 R_G$

A maximally rotating (“Maximal Kerr”) black hole has both its event horizon and ISCO at  $R_G$

→ Spinning black holes are more compact → potentially more radiatively efficient



**R. Fender 2007**

## Schwarzschild Radius-AKA the Event Horizon for a Non-Spinning Black Holes -Longair 19.1

- $R_s = 2GM/c^2; 3(M/M_\odot)\text{km}$
- The Schwarzschild radius is the radius of 'no return' for a non-rotating black hole- ***it is not the singularity. Its apparent singularity is a due to a choice of coordinates (Longair pg 433) .***

## Schwarzschild Radius-AKA the Event Horizon for a Non-Spinning Black Holes -Longair 19.1

- Events inside that horizon cannot be seen by any external observer
- inside the event horizon the radius becomes a timelike coordinate, and the time becomes a spacelike coordinate. Specifically, that means that once inside  $R_s$ , you must go to smaller radii, just as now you must go forward in time
- once you're inside the event horizon **one cannot avoid the singularity at  $r = 0$**
- These are the 'simplest' macroscopic objects-the only instance of an exact description of a macroscopic object..., almost by definition, the most perfect macroscopic objects there are in the universe
- **Timescales are VERY short  $T \sim R_s/c = 2GM/c^3 \approx 10^{-5} (M/M_\odot) \text{ sec}$**

### More features of Schwarzschild black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)
- Observer who enters event horizon would only "feel" "strange" gravitational effects if the black hole mass is small, so that  $R_s$  is comparable to observer's size
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Stable, circular orbits are not possible inside  $3R_s$  : inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance  $1.5R_s$  would be bent around into a circle
- Thus black hole would produce "shadow" on sky



## 'The' Singularity

- At  $r = 0$ , there is a real *physical singularity* and, according to classical general relativity, the infalling matter collapses to a singular point.
- However the Schwarzschild singularity is unobservable because no information can arrive to the external observer from within the Schwarzschild radius  $R_s$ .
- see discussion on pg 433 of Longair

For the Mathematically Inclined see sec 13.11.1

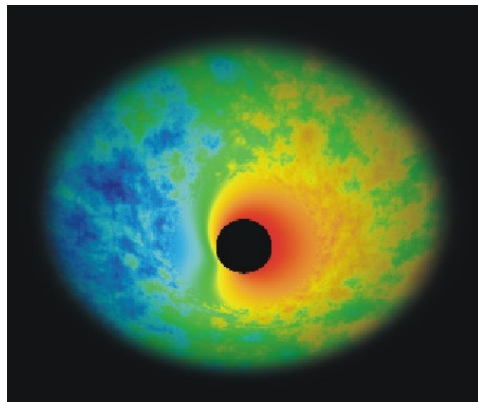
- The Schwarzschild solution for the metric around a point mass in spherical coordinates is
- $ds^2 = -(1 - 2GM/c^2r)c^2dt^2 + (1 - 2GM/c^2r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$  eq 13.52

$s$  is the length element

– Notice singularity at  $r = 2GM/c^2$  (can be gotten rid of in a coordinate transformation)

- A static observer measures proper time

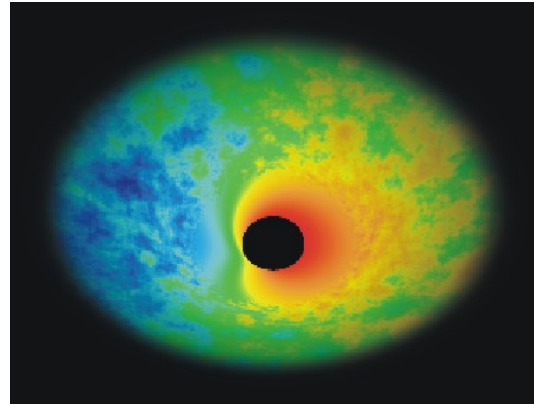
$$c^2d\tau^2 = -ds^2 = -(1 - 2GM/c^2r)c^2dt^2$$



## For the Mathematically Inclined see sec 13.11.1

time is slowed down in a  
gravitational field

- $\delta\tau = \sqrt{1 - 2GM/c^2r}$   $\delta t = 1 + z_{\text{grav}}$   
- (the time interval  $\delta\tau$  corresponds to the period of the waves emitted at the point  $r$ , the observed period of the wave at infinity is  $\delta t$ )



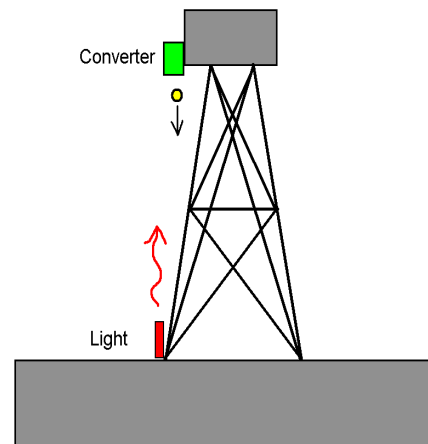
The frequency of radiation emitted from radius  $r$  as seen by a distant observer is  $\nu_{\infty}$  where  $\nu_0$  is the emitted frequency

$\nu_{\infty} = \nu_0 \sqrt{1 - 2GM/c^2r}$  - if  $r = 2GM/c^2$  is zero - e.g. **redshifting of radiation in a gravitational field.**

Picture is the frequency of light emitting by an accretion disk- **red is redshift**, **blue is blue shift** ( Quiz-how do we get blue shifts??)

## Gravitational redshift- Longair 19.21

- Thought experiment:
  - Send photon upwards in a gravitational field
  - Convert that energy into mass and drop the mass
  - Convert mass back into photon
- Conservation of energy  $\Rightarrow$  photon must lose energy as it climbs in the gravitational field
- Another way of thinking about this - the escape speed from the object has to be less than the speed of light (assuming, incorrectly, that light could slow down and fall).
- In Newton mechanics the escape speed is  $v^2 = 2GM/r$ , so  $v^2 = c^2$  at  $r = 2GM/c^2$
- Redshift of light  $Z = (\lambda_0 - \lambda_e) / \lambda_e$ ;  $\lambda_0$  = wavelength as measured by the observer,  $\lambda_e$  as emitted



## Gravitational redshifts near a black hole

- Gravitational redshift is really a form of relativistic time dilation
- As observed from infinity, time near a (non-spinning, non-charged) black hole runs slow by a factor

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - 2GM/c^2}}$$

- The event horizon is the “infinite redshift” surface where (as observed from infinity) time appears to stop!
- But... a free falling observer would fall through the event horizon without noticing anything unusual.
- The wavelength of light is redshifted ( $z = (\lambda_0 - \lambda_e) / \lambda_e$ ;  $\lambda_0$  = wavelength as measured by the observer,  $\lambda_e$  as emitted at radius  $r$ )
- 

Longair 13.58 in frequency units  
 $v_\infty = v_{em} \sqrt{1 - 2GM/rc^2}$

$$z = \frac{1}{\sqrt{1 - \left(\frac{2GM}{rc^2}\right)}} - 1$$

## What are the possible energy sources?

- Accretion?
  - Release of gravitational potential energy as matter falls into black hole
  - **YES!** Thought to be primary power source of all systems just discussed
- Rotational energy of black hole? Spinning (Kerr) BHs see Longair 13.11.2
  - Tapping the rotational energy of a spinning black hole  $1/2I\Omega^2$  can be very large
  - May be important in some settings... but can only be tapped if accretion occurring!

## Example metrics

- Normal (flat) 3-d space in cartesian coordinates
- Flat spacetime of Special Relativity in cartesian coordinates

$$ds^2 = dx^2 + dy^2 + dz^2$$

- Non-spinning, uncharged black hole (Schwarzschild metric) in “Schwarzschild coordinates”

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

## What is the Event Horizon

- The event horizon is the “infinite redshift” surface where (as observed from infinity)
  - the radius at which the gravitational redshift (z) is infinite
  - $z = \Delta\lambda/\lambda$ ;  $1+z = 1/\sqrt{1-R_s/R}$ ;  $R_s = 2GM/c^2$
- But... a free falling observing would fall through the event horizon without noticing anything unusual.
- As observed from infinity, time near a (non-spinning, non-charged) black hole run slow by a factor of

$$t_0 = t_f \sqrt{1 - \frac{2GM}{rc^2}} = t_f \sqrt{1 - \frac{r_s}{r}}$$

•Also time appears to stop !

So what is the actual size ?

$$R_g \sim 1.5 (M / M_\odot) \text{ km}$$

So how close are neutron stars to being black holes ?

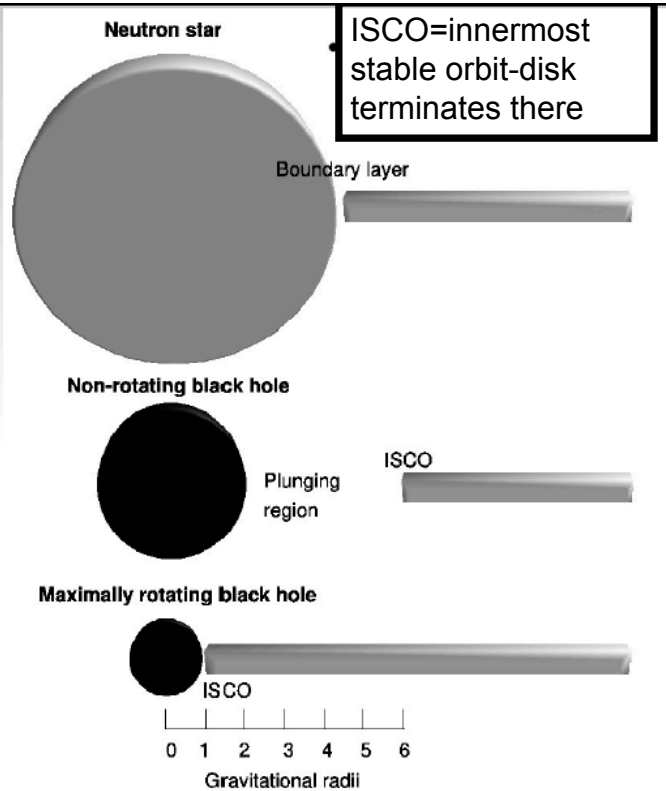
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**R. Fender 2007**

## Why is it a Black Hole-Longair pg 433

If the specific angular momentum of a particle is  $l \leq 2r_g c$ , **it inevitably falls** into  $r = 0$ , where  $r_g$  is the Schwarzschild radius,  $r_g = 2GM/c^2$ .

There is a last stable circular orbit about a mass with radius  $r = 3r_g$ .

There do not exist circular orbits with radii less than this value, particles rapidly spiral into  $r = 0$ .

This is why the black hole is called a ‘hole’ – matter inevitably collapses in to  $r = 0$  if it comes too close to the point mass.

## A Bit More- Longair 13.11.2, 19.6.2

- For a non-rotating BH the last stable orbit is at  $3R_s$ ; this is due to the form of the potential which is NOT  $1/r$
- Particles on the last stable orbit about a Schwarzschild black hole move at half the speed of light.

For those interested the potential can be expressed as

$U(r)_{\text{eff}} = (1 - r_s/r)(mc^2 + \ell^2/mr^2)$  where  $\ell$  is the angular momentum (K.Griest)

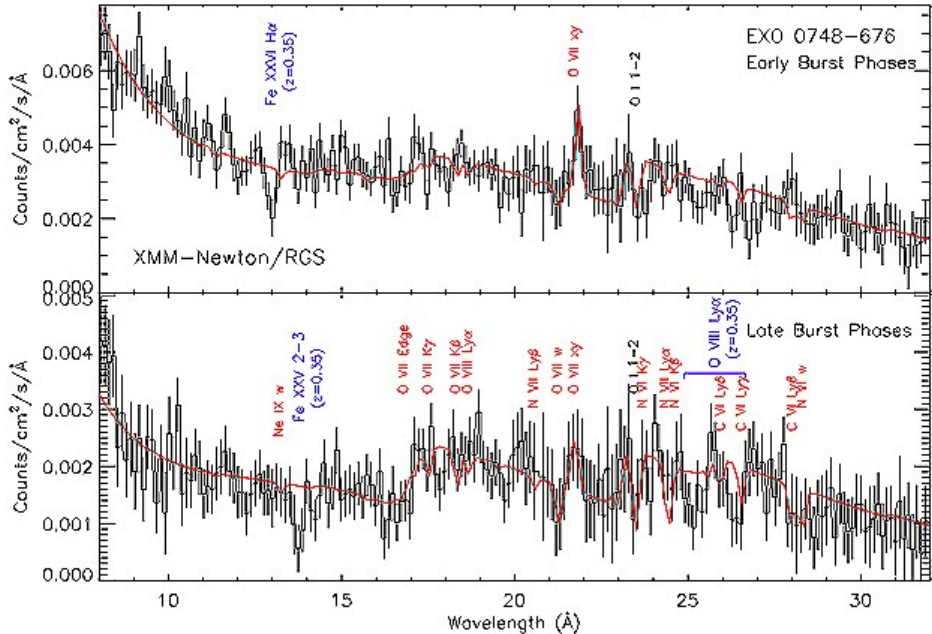
For a Schwarzschild black hole, the maximum energy which can be released corresponds to the binding energy of the matter on the last stable circular orbit at  $r = 3 R_s = 6GM/c^2$ .

A fraction  $[1 - (8/9)^{1/2}] = 0.0572$  of the rest mass energy can thus be released

Question for class- what is the redshift from the surface of a NS?

- $M \sim M_{\text{sun}}$  ;  $R=10\text{km}$  (set by nuclear physics)

Emission of line radiation from highly ionized atoms of Fe  
And O from near the surface of a **NS**



**Redshifted absorption lines from a neutron star surface**  
Cottam, Paerels & Mendez (2002)

**Rotating black holes-** remember the extra special nature of accelerated frames

- **Roy Kerr (1963)**
  - Discovered solution to Einstein’s equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- **Features of the Kerr solution**
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has “**static surface**” inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: “**frame dragging**”.
  - **Ergosphere** – region where space-time dragging is so intense that it is impossible to resist rotation of black hole.

## Spinning BH- Longair sec 13.11.2

- A black hole with angular momentum  $J$  has a metric  

$$ds^2 = (1 - 2GM/r/c^2)dr^2 - \{(1/c^2)[4GMra \sin^2 \theta / \rho c] dr d\phi\} + (\rho/\Delta)dr^2 + \rho d\theta^2 + (r^2 + a^2 + 2GMra^2 \sin^2 \theta / \rho c^2) \sin^2 \theta d\phi^2$$
 -- Longair eq 13.63

$r, \theta, \phi$  – usual polar coordinates

where  $a = (J/Mc)$  is the angular momentum per unit mass (dimensions of distance) and

$$\Delta = r^2 - (2GM/c^2)r + a^2 \quad ; \quad \rho = r^2 + a^2 \cos^2 \theta$$

- If the black hole is non-rotating,  $J = a = 0$  and the Kerr metric reduces to the Schwarzschild metric

## Spinning BH- Longair sec 13.11.2

Just like Schwarzschild metric it becomes singular but at a radius where

$$\Delta = r^2 - (2GM/c^2)r + a^2 = 0;$$

the larger root is

$$r_+ = GM/c^2 + [(GM/c^2)^2 - (J/Mc)^2]^{1/2} \quad \text{Longair 13.65}$$

for  $J > 0$  this is smaller than the Schwarzschild radius

there is a maximum angular momentum  $J = GM^2/c$ ; for this value of  $J$  the horizon is at

$r_+ = GM/c^2$ ; 1/2 of the Schwarzschild radius

- If the black hole is not rotating ( $a = J/M = 0$ ), the Kerr line element reduces to the Schwarzschild line element