What is the Event Horizon

- The event horizon is the "infinite redshift" surface where (as observed from infinity)
  - the radius at which the gravitational redshift (z) is infinite
  - $z=\Delta\lambda/\lambda$; $1+z=1/\text{sqrt}(1-R_s/R)$; $R_s=2GM/c^2$
- But… a free falling observing would fall through the event horizon without noticing anything unusual.

- As observed from infinity, time near a (non-spinning, non-charged) black hole run slow by a factor of

$$t_0 = t_f\sqrt{1-\frac{2GM}{rc^2}} = t_f\sqrt{1-\frac{r_s}{r}}$$

•time appears to stop ar $r=r_s$!

Example metrics

- Normal (flat) 3-d space in cartesian coordinates

$$ds^2 = dx^2 + dy^2 + dz^2$$

- Flat spacetime of Special Relativity in cartesian coordinates

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

- Non-spinning, uncharged black hole (Schwarzschild metric) in "Schwarzschild coordinates" general relativity- notice new terms!

$$\boxed{ds^2 = -c^2\left(1-\frac{2GM}{c^2r}\right)dt^2 + \frac{dr^2}{(1-\frac{2GM}{c^2r})} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)}$$

So what is the actual size ?

$R_G \sim 1.5\ (M / M_\odot)$ km

So how close are neutron stars to being black holes ?

Neutron stars are only about a factor 2—3 larger than their event horizons

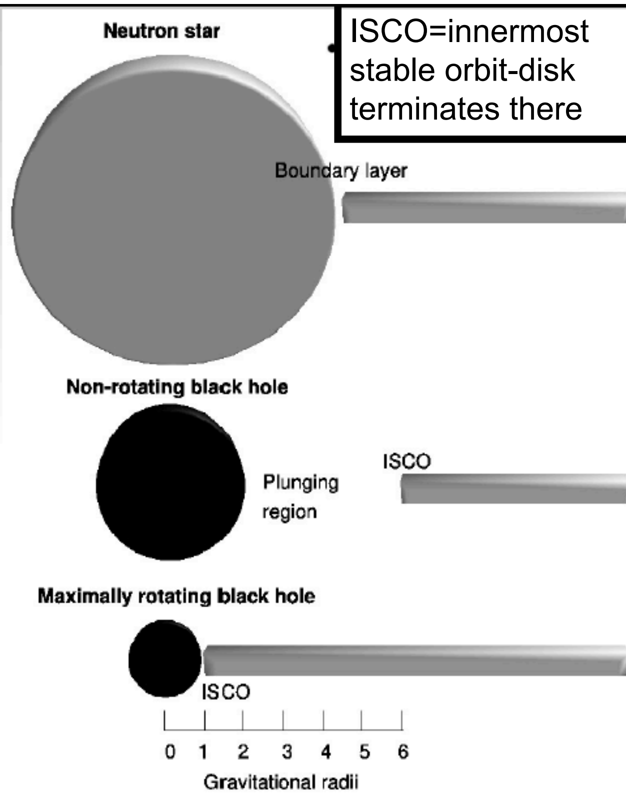
What about spin ?

A non-rotating ("Schwarzschild") black hole has its event horizon at 2 R_G and its ISCO at 6 R_G

A maximally rotating ("Maximal Kerr") black hole has both its event horizon and ISCO at R_G

→ Spinning black holes are more compact → potentially more radiatively efficient

ISCO=innermost stable orbit-disk terminates there



R. Fender 2007

Why is it a Black Hole-Longair pg 433

If the specific angular momentum of a particle is $l \leq 2r_g c$, it inevitably falls into $r = 0$, where r_g is the Schwarzschild radius, $r_g = 2GM/c^2$.

There is a last stable circular orbit about a mass with radius $r = 3r_g$.
There do not exist circular orbits with radii less than this value, particles rapidly spiral into $r = 0$.

This is why the black hole is called a 'hole' – matter inevitably collapses in to $r = 0$ if it comes too close to the point mass.

A Bit More- Longair 13.11.2, 19.6.2

- For a non-rotating BH the last stable orbit is at $3R_s$; this is due to the form of the potential which is NOT 1/r
- Particles on the last stable orbit about a Schwarzschild black hole move at half the speed of light.

For those interested the potential can be expressed as
$U(r)_{eff} = (1-r_S/r)(mc^2+\ell^2/mr^2)$ where ℓ is the angular momentum (K.Griest)

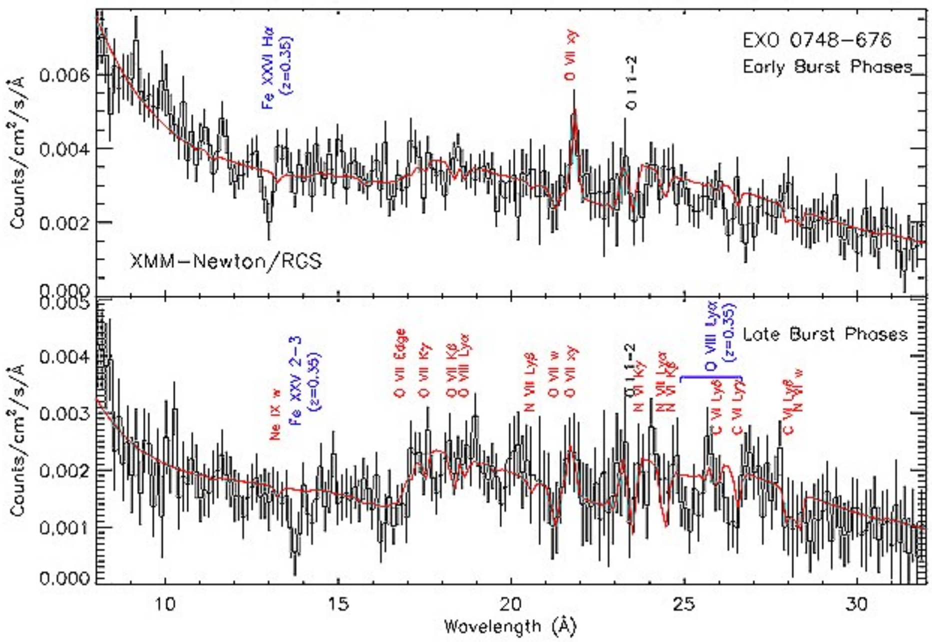
For a Schwarzschild black hole, the maximum energy which can be released corresponds to the binding energy of the matter on the last stable circular orbit at $r = 3\ R_s = 6GM/c^2$.

A fraction $[1 - (8/9)^{1/2}] = 0.0572$ of the rest mass energy can thus be released

Question for class- what is the redshift from the surface of a NS?

- $M \sim M_{sun}$; R=10km (set by nuclear physics)

Emission of line radiation from highly ionized atoms of Fe And O from near the surface of a ***NS***



Redshifted absorption lines from a neutron star surface
Cottam, Paerels & Mendez (2002)

-β<9 (DeDeo & Psaltis 2003)

Rotating black holes- remember the extra special nature of accelerated frames

- Roy Kerr (1963)
  - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
  - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
  - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
  - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
  - Also has "**static surface**" inside of which nothing can remain motionless with respect to distant fixed coordinates
  - Space-time near rotating black hole is dragged around in the direction of rotation: "**frame dragging**".
  - **Ergosphere** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Spinning BH- Longair sec 13.11.2

- A black hole with angular momentum ***J*** has a metric

$ds^2=(1-2GMr/\rho c^2)dr^2-\{(1/c^2)[4GMra \sin^2\theta/\rho c]\, dr d\phi\}+(\rho/\Delta)dr^2+$
$\rho d\theta^2+(r^2+a^2+2GMra^2 \sin^2\theta/\rho\, c^2) \sin^2\theta\, d\phi^2]$-- Longair eq 13.63

r,θ,ϕ– usual polar coordinates

where $a=(J/Mc)$ is the angular momentum per unit mass (dimensions of distance) and

$\Delta=r^2-(2GMr/c^2)+a^2$; $\rho=r^2+a^2\cos^2\theta$

- If the black hole is non-rotating, J =a = 0 and the Kerr metric reduces to the Schwarzschild metric

Spinning BH- Longair sec 13.11.2

Just like Schwarschild metric it becomes singular but at a radius where

$\Delta=r^2-(2GMr/c^2)+a^2=0;$

the larger root is

$r_+=GM/c^2 +[(GM/c^2)^2-(a)^2]^{1/2}$ Longair 13.65

for $a>0$ this is smaller than the Schwarschild radius

there is a maximum angular momentum ***a;*** *for this value of a the horizon is at*

r_+=GM/c ; 1/2 of the Schwarzschild radius

- If the black hole is not rotating (a=J/M=0), the Kerr line element reduces to the Schwarzschild line element

Importance of Kerr Radius

- in the case of a rapidly rotating black hole, matter can move in stable circular orbits with a smaller radii compared with the Schwarzschild case.
- **Consequently more of the rest-mass energy of the infalling matter can be extracted as compared with the non-rotating case.**

- For a Schwarzschild black hole, the maximum energy which can be
- released corresponds to the binding energy of the matter on the last stable circular orbit at $r = 3\ r_g = 6GM/c^2$.
  - Giving [1 − (8/9)1/2] = **0.0572** of the rest mass energy can be released in this process
- For a Kerr the amount depends on the spin, for a maximal spin

the maximum binding energy is a fraction (1− 1/√ 3) =**0.423** of the rest-mass energy

Spin Energy

- All the spin energy of a Black Hole resides outside the horizon and can, in principle, be extracted
- For a spinning BH energy can be extracted by the 'Penrose' process
- For a maximally spinning BH the most energy that can be extracted is $0.29Mc^2$
  - It is not clear if this process is important in actual sources

- In a Kerr black hole particles are dragged in angle ϕ

as they fall radially inward, even though no forces act on them

- The event horizon is permeable, mass and angular momentum can fall through it and onto the singularity inside. The event horizon is a special region of spacetime but not an object.
- The accreted mass adds mass and angular momentum To use the Physics 101 example of a spinning ice skater, if you were throw a spinning ice skater dumbells that were moving faster than their hands were moving, they would spin up from the additional specific angular momentum. I think the issue is that no information can be communicated from inside the event horizon to outside, so there must be a physical process at the event horizon that communicates the angular momentum to the rest of the universe. The way I think about is that the angular momentum and mass are embedded in the gravitational field, and it's the enclosed mass inside an observer's orbit that observer feels as that belonging to the "central source". Like Gauss's law. An observer in fact would never see any mass completely penetrate the event horizon due to time dilation effects; it would appear to an outside observer that the mass is stuck on the event horizon and piles on as more accretes (if your telescope was sensitive enough).

Black Hole Spin Movie

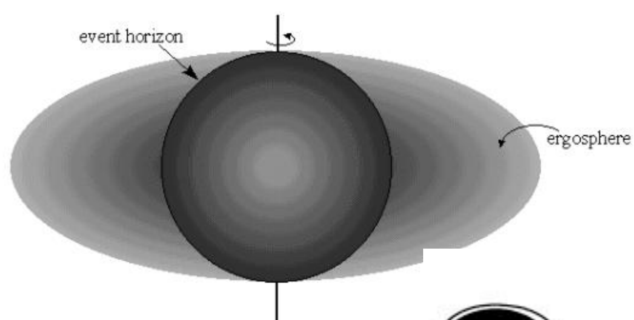
- https://kottke.org/18/04/how-to-harvest-nearly-infinite-energy-from-a-spinning-black-hole

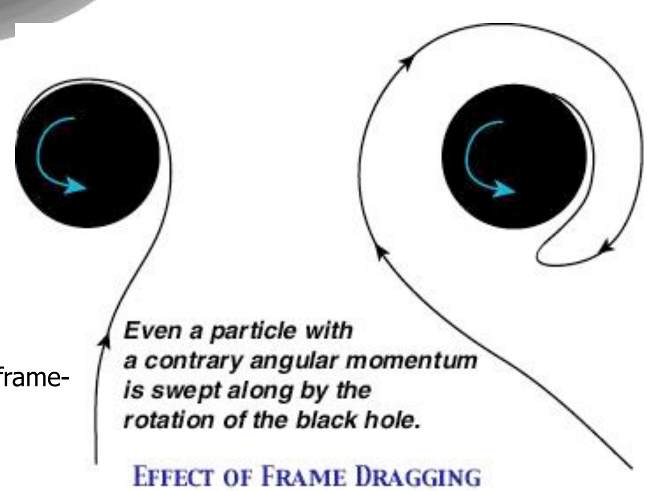
Schwarzschild and Kerr Metric

- for a <u>Schwarzschild</u> BH the innermost stable radius is $3r_G=6GM/c^2$ - there are no **stable** circular orbits at smaller radii
  - the binding energy from this orbit is 0.0572 of the rest mass energy
- For a Kerr the innermost stable radius is at $r_+=GM/c^2$ The spinning black hole drags the the inertial frame-
- The smaller critical radius allows more energy to be released by infalling matter
  - For a Kerr BH 0.423 of the energy can be released.

- There is another 'fiducial' radius in the Kerr solution, that radius within which all light cones point in the direction of rotation, the 'static' radius, r_{static}.
- Between r_{static} and r_+ is a region called the 'ergosphere' within which particles must rotate with the black hole and from energy might be extracted (Penrose process).

Frame dragging by rotating black hole





tested in 2004

http://www.universetoday.com/9984/frame-dragging-confirmed/

Graphics: University of Winnipeg, Physics Dept.

The innermost stable circular orbit (ISCO)

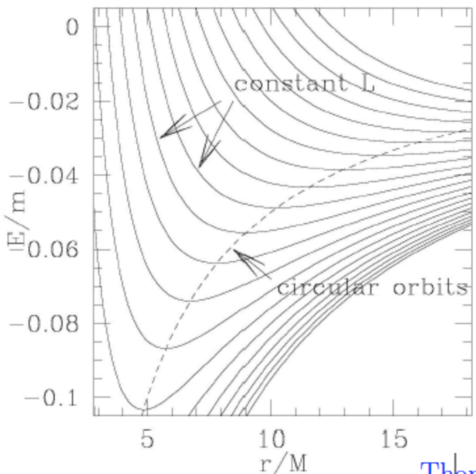
circular orbit extremizes binding energy E of test mass m at const. angular momentum L

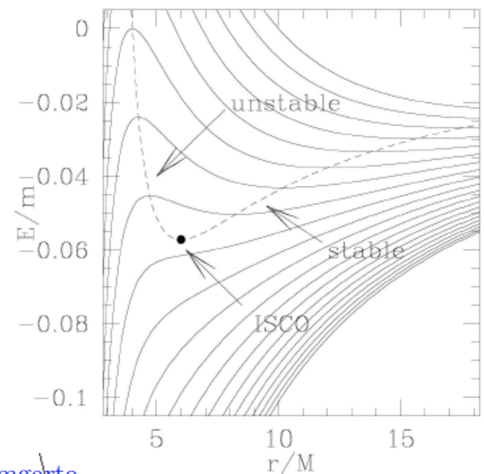
Newtonian point mass

$$\frac{E}{m} = -\frac{M}{r} + \frac{L^2}{2r^2}$$

Schwarzschild black hole

$$\frac{E}{m} = \left(\left(1-\frac{2M}{r}\right)\left(1+\frac{L^2}{r^2}\right)\right)^{1/2} - 1$$



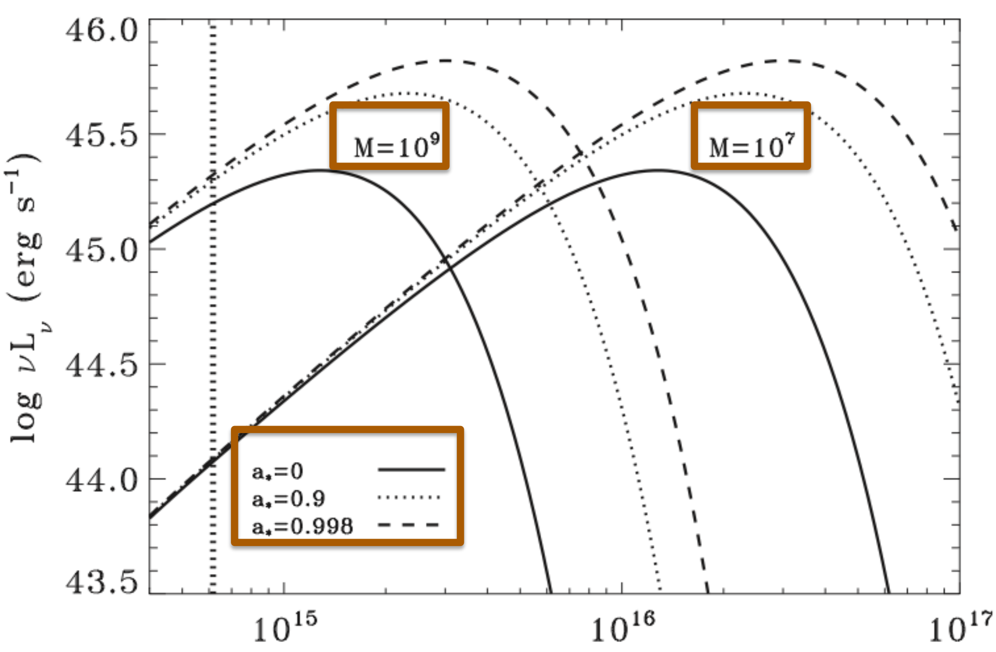


Thomas Baumgarte

Bowdoin College

Effect of BH Mass and Spin on Emitted Spectrum of "Standard" Accretion disk Spectrum

all the curves assume a rate of $1M_{\odot}$/yr (L_{Edd} for $M=4\times10^7M$ BH with 10% efficiency)



Effect of BH Mass and Spin on Emitted Spectrum of "Standard" Accretion disk Spectrum

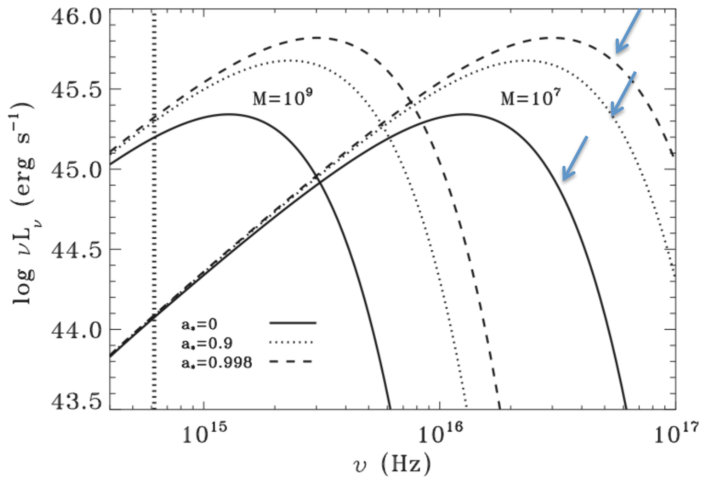
all the curves assume a rate of $1 M_{\odot}$/yr (L_{Edd} for $M=4\times10^{7}$M BH with 10% efficiency)

High spin **increases**

- the total energy emitted per unit mass accreted (integral under the curve)
- the maximum temperature of the disk

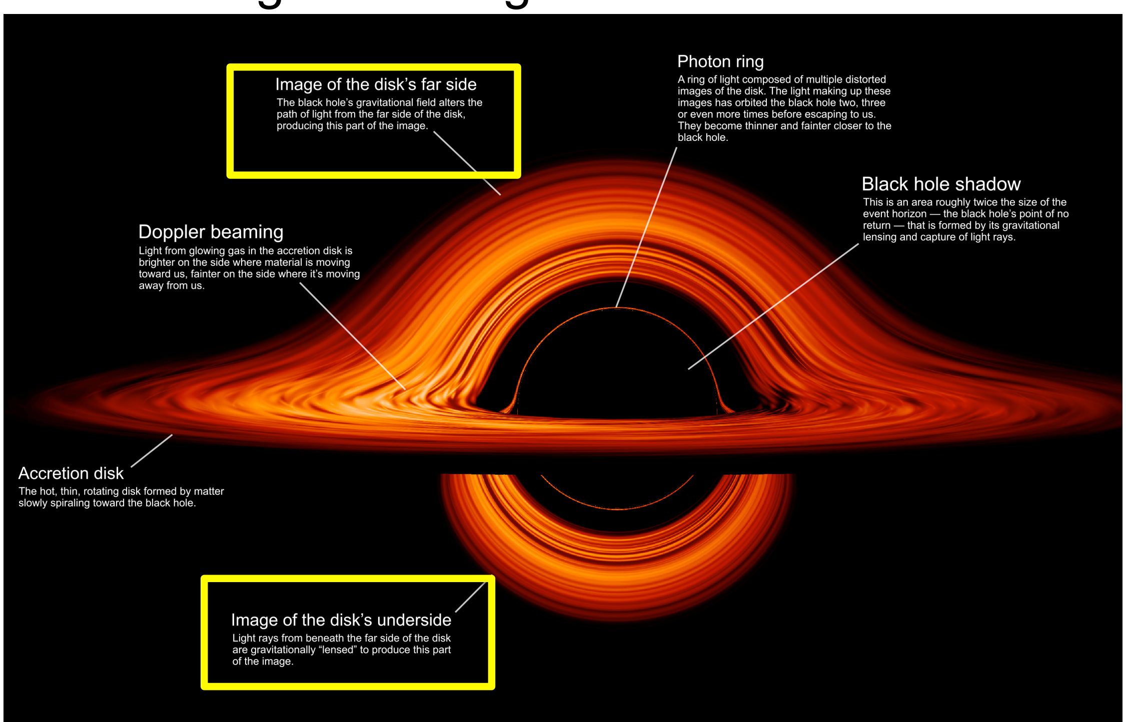
- Both these effects are related to the change in the innermost stable orbit

Temperature increases



Also (later) the GR effects are increased- e.g. redshift and light bending

Light Bending +Some Other GR Effects



https://www.nasa.gov/sites/default/files/thumbnails/image/bh_labeled.jpg

What *can* come **out** of black hole?

…more than you might think!

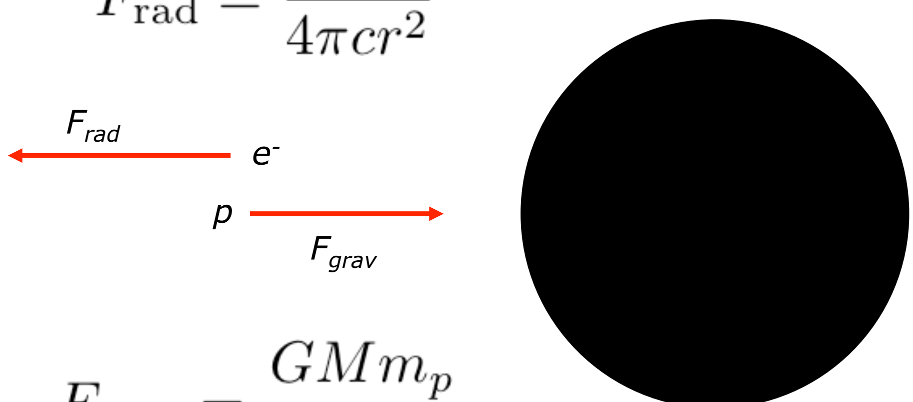
- Magnetic fields threading ergosphere can attach to and drag surrounding matter, reducing the black hole's spin and energy
- "Hawking Radiation"(pg 438): black hole slowly evaporates due to quantum mechanics effects
  - Particle/antiparticle pair is created near BH
  - One particle falls into horizon; the other escapes
  - *Energy to create particles comes from gravity outside horizon*

$$t_{evap} = 10^{10}\,yrs \times \left(\frac{M}{10^{12}\,kg}\right)^3$$

  - Solar-mass black hole would take 10^{65} years to evaporate (Jupiter 10^{55} years)!
  - Mini-black holes that *could* evaporate are not known to exist now, but possibly existed in early Universe

How luminous can an accreting black hole be?- this is the same Eddington limit as we discussed for neutron stars

$$F_{\rm rad} = \frac{L\sigma_T}{4\pi c r^2}$$



$$F_{\rm grav} = \frac{GMm_p}{r^2}$$

- The accreting matter is pushed away if

$$F_{\rm rad} > F_{\rm grav}$$

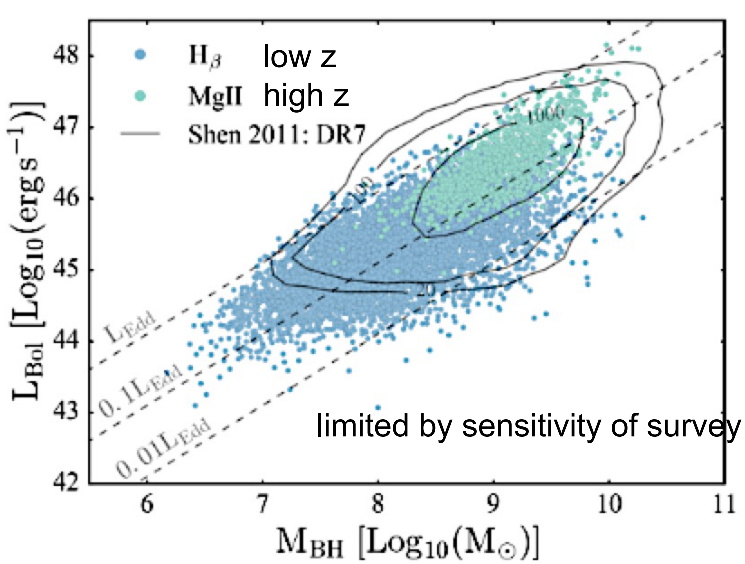
- This is the Eddington limit (L_{Edd}). Acts as effective upper limit to the luminosity of accretion powered object. Numerically:

$$L > \frac{4\pi G m_p c}{\sigma_T} M$$

$$L_{\rm Edd} \approx 1.3 \times 10^{31} \left(\frac{M}{M_\odot} \right) \, \rm W$$

Black Hole Masses and Eddington Limit

- Use of single epoch spectral masses (later) gives a very large sample.
- Confirms the 'existence' of the Eddington limit (!) Coffey et al.2019

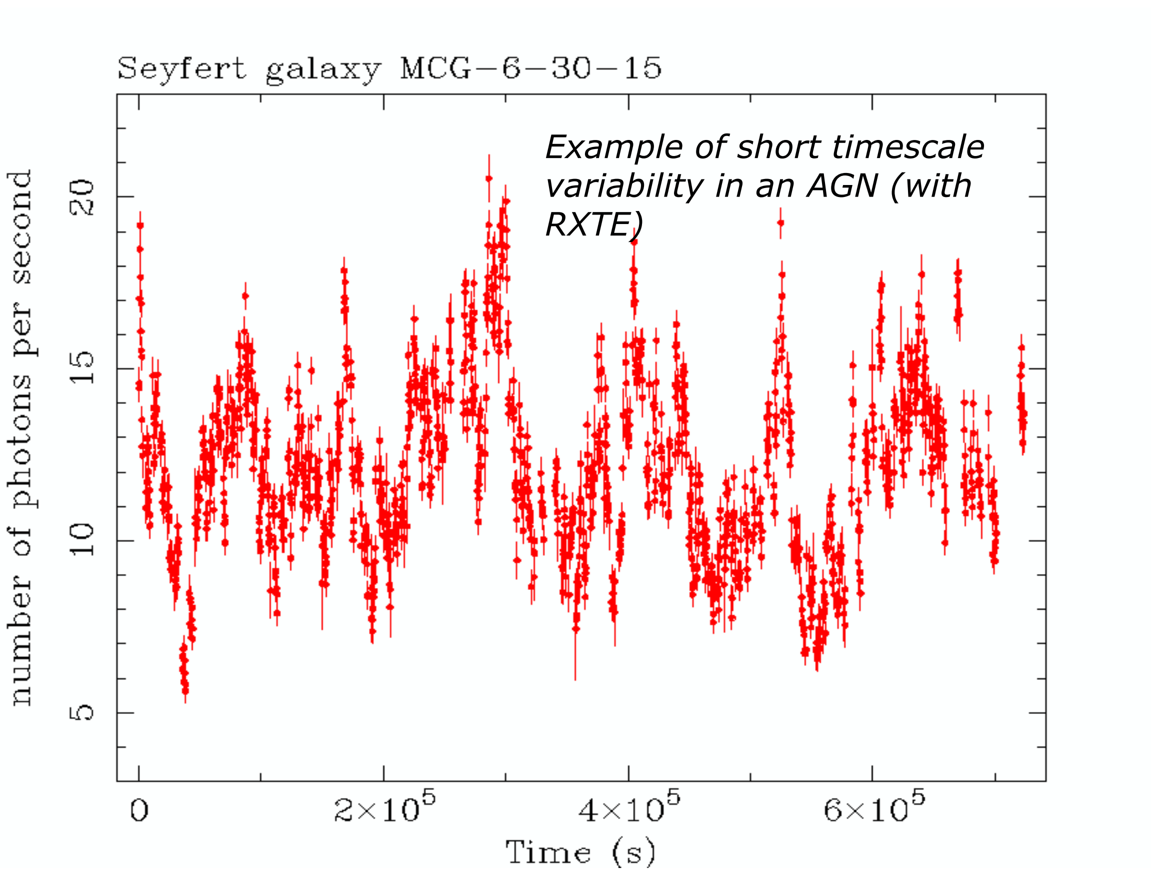


General properties of emission from black hole systems

- Emission usually variable on wide variety of timescales
  - Galactic black hole binaries : millisecond and up
  - AGN : minutes and up
  - Most rapid variability approaches light-crossing timescale limit R~cτ

- Significant emission over **very** broad spectral range (radio to hard X-ray or gamma-rays)-NS and WDs tend to have 'thermal' like spectra (relatively narrow in wavelength)

- Lack of a signature of a surface - not a pulsar, no boundary layer emission (no x-ray bursts), no 'after glow' from cooling



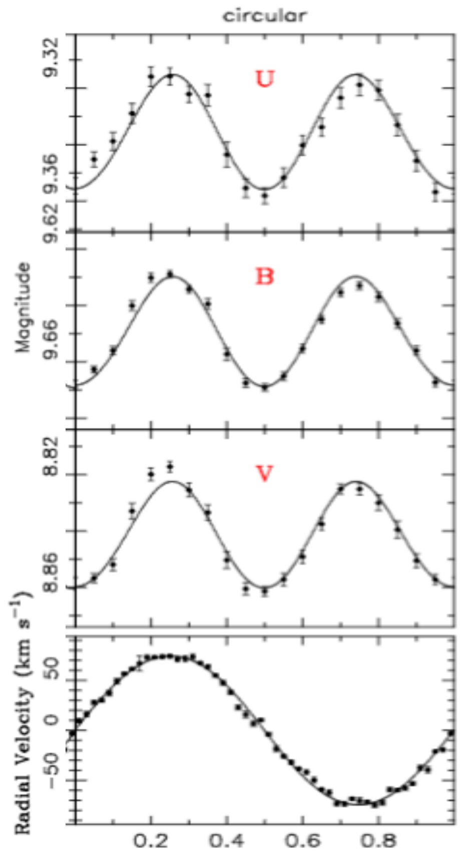
Evidence for black holes

- Galactic black hole candidates – the same sort of dynamical evidence we have for neutron stars! ~20 known
- Black hole mass from orbit of companion star- Cyg X-1 first galactic black hole discovered
  - Period 5.6 days
  - K = V sin i = 75km/s
  - Analysis of orbit shows that

$$f = \frac{K^3 P}{2\pi G} = \frac{M_1^3 (\sin i)^3}{(M_1 + M_2)^2}$$

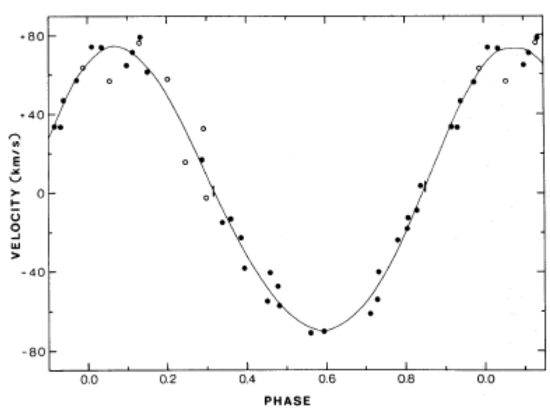
  - “Mass function” f can be measured…
  - M_{BH}>f

Cyg X-1
Orosz 2011



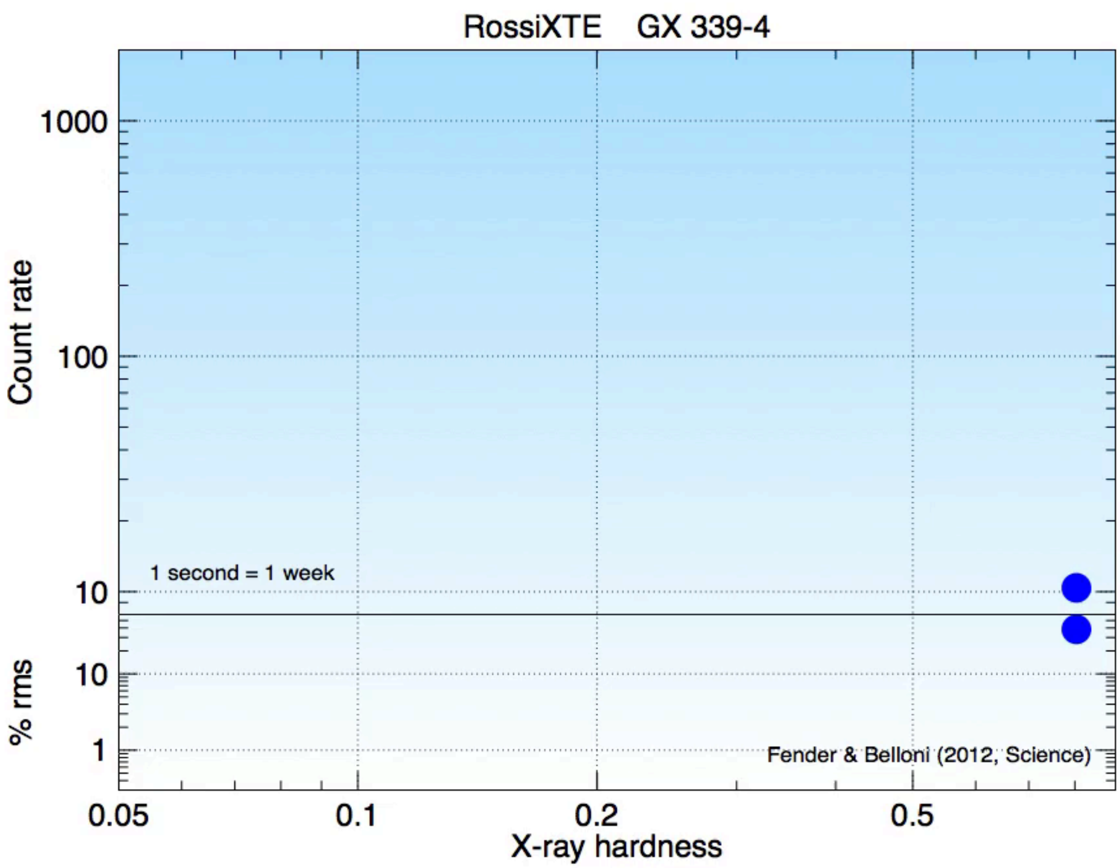
Discovery of black holes

- First evidence for an object which ‘must’ be a black hole came from discovery of the X-ray source Cygnus X-1
  - Binary star system… black hole in orbit around a massive O-star; period =5.6 days - not eclipsing
  - Mass of x-ray emitting object 7-13 $M_{\odot}$- too high for a NS. Object emits lots of x-rays, little optical light.
  - X-rays produced due to accretion of stellar wind from O-star
  - 2kpc away



Velocity curve of the stellar companion
It is a massive O star

$f(M) = P_{orb}v^2/2\pi G = M_1 \sin^3 i/(1 + q)^2$.
$q=M_2/M_{1;}$ v^2 is the maximum measured velocity
the value of the mass function is the absolute minimum mass of the compact star



RossiXTE GX 339-4
1000
100
10
10
1
Count rate
% rms
1 second = 1 week
Fender & Belloni (2012, Science)
0.05
0.1
0.2
0.5
X-ray hardness

Stellar Mass BHs Do Crazy Things

S1: An animation of the evolution of a black hole X-ray binary outburst in the hardness-intensity diagram (top panel) as well as the evolution of integrated X-ray variability as a function of the sameX-ray hardness (lower panel)
. Hard X-ray states (steady radio jet, no accretion disc wind) are on theright and are indicated by blue circles, and soft X-ray states (no jet
, strong accretion disc wind) are to the left and indicated by red circles. Intermediate X-ray states are indicated by green and yellow circles and correspond primarily to significant changes in the X-ray variability properties of the system. The point of the largest radio flare, probably corresponding to the strongest relativistic ejection event, is indicated by the yellow star and occurs in an intermediate state during the hard to soft state transition. The movie is scaled to real time with one second = one week (1:604800) and smoothly interpolated between points

Evidence for black holes- Longair 19.3.2

- For Supermassive Black Holes

Dynamics of 'Test particles'

Orbits of gas disks around mass compact objects at the centers of other galaxies- best case is NGC4258 (water maser orbits)

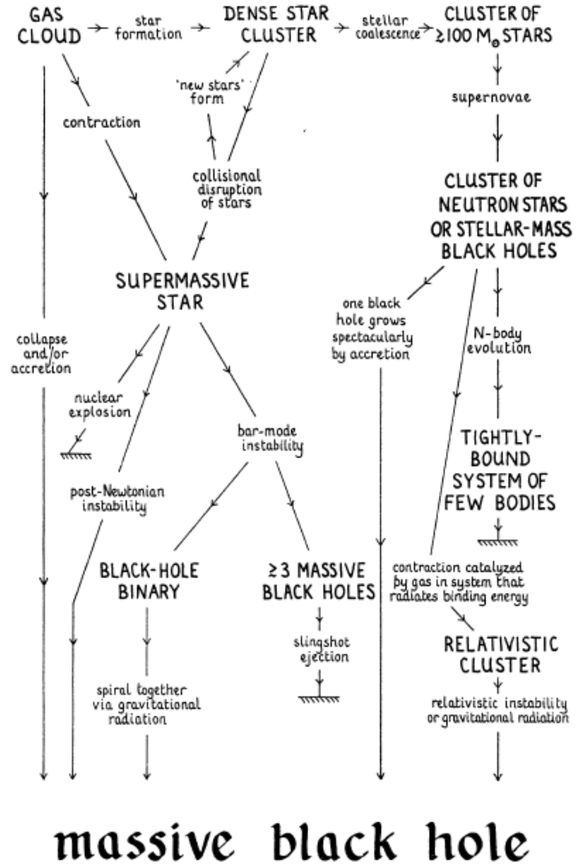
Stellar orbits around a compact mass at the center of our own Galaxy (most solid case for any black hole)

Of course what these data give is the mass inside a given radius. If the mass density is higher than (?) it must be a black hole

- Emission from the region of 'strong gravity'

Extreme gravitational redshifting of emission lines in the X-ray spectrum of some accreting black holes

In a dense region all roads lead
to a black Hole
(Ress 1984 ARAA)



Some Scales (Rees 1984)

A central mass M has a gravitational radius

$$r_g = \frac{GM}{c^2} = 1.5 \times 10^{13}\, M_8 \text{ cm}, \qquad 1.$$

where M_8 is the mass in units of $10^8\, M_\odot$. The characteristic minimum time scale for variability is

$$r_g/c \simeq 500\, M_8 \text{ s}. \qquad 2.$$

A characteristic luminosity is the "Eddington limit," at which radiation pressure on free electrons balances gravity:

$$L_E = \frac{4\pi GMm_p c}{\sigma_T} \simeq 1.3 \times 10^{46}\, M_8 \text{ erg s}^{-1}. \qquad 3.$$

Related to this is another time scale

$$t_E = \frac{\sigma_T c}{4\pi G m_p} \simeq 4 \times 10^8 \text{ yr}. \qquad 4.$$

The time scale to grow a black hole if it
Were accreting at the Eddington luminosity

The characteristic black body temperature if the Eddington luminosity is emitted at r_g

$$T_E \simeq 5 \times 10^5\, M_8^{-1/4}.$$

On to AGN!