Remnant Evolution

Free Expansion
Ejecta expand without deceleration $r \sim t$ (see movie Rudnick et al., 1996, BAAS, 188.7403.) - Core collapse SN have initial velocities of $\sim 5000\text{km/sec}$ and several $M_\odot$ of ejecta, SN Ia $\sim 10,000 \text{km/sec}$, $\sim 1 \text{M}_\odot$

Adiabatic (Sedov-Taylor, or “atomic bomb”)
Ejecta are decelerated by a roughly equal mass of ISM - $r \sim t^{2/5}$
Energy is conserved-(Cooling timescales are much longer than dynamical timescales, so this phase is essentially adiabatic e.g. net heat transfer is zero).

Evolution of density, pressure is self-similar
Temperature increases inward, pressure decreases to zero

Radiative
Dissipation of remnant energy into ISM
Remnant forms a thin, very dense shell which cools rapidly
Interior may remain hot- typically occurs when shock velocities drop to around 200 km/sec
Summary of SNR Expansion Phases

I. \( m_0 \gg M_{\text{ISM}} \)

II. \( m_0 < M_{\text{ISM}} \) - shock heated gas adiabatic due to high temperature

III. \( m_0 << M_{\text{ISM}} \) - gas cools radiatively at constant momentum
**Shock Expansion**

- At time \( t=0 \), mass \( m_0 \) of gas is ejected with velocity \( v_0 \) and total energy \( E_0 \).

- This interacts with surrounding interstellar material with density \( \rho \) and low temperature.

\[
\frac{dE}{dt} \quad \text{rad} \quad 0
\]

Note \( E_0 \approx 10^{41-45} \text{J} \)

![Shock front, ahead of ‘heated’ material. Shell velocity much higher than sound speed in ISM, so shock front of radius \( R \) forms.]

**Supernova Remnants**

Development of SNR is characterized in phases – values are averages for “end of phase”:

<table>
<thead>
<tr>
<th>Phase</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass swept up (( M_\odot ))</td>
<td>0.2</td>
<td>180</td>
<td>3600</td>
</tr>
<tr>
<td>Velocity (km/s)</td>
<td>3000</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>Radius (pc)</td>
<td>0.9</td>
<td>11</td>
<td>30</td>
</tr>
<tr>
<td>Time (yrs)</td>
<td>90</td>
<td>22,000</td>
<td>100,000</td>
</tr>
</tbody>
</table>

Phase IV represents disappearance of remnant.
Shell of swept-up material in front of shock does not represent a significant increase in mass of the system.

**ISM mass** within sphere radius $R$ is still \textbf{small}.

\[
m_0 \gg \frac{4\pi}{3} \rho_0 R^3(t)
\]

(1)

Since momentum is conserved:

\[
m_0 v_0 = (m_0 + \frac{4\pi}{3} \rho_0 R^3(t)) v(t)
\]

(2)

Applying condition (1) to expression (2) shows that the velocity of the shock front remains constant, thus:

\[v(t) \sim v_0\]

\[R(t) \sim v_0 t\]
Supernova Remnant Cartoon

Forward shock moves supersonically into interstellar/circumstellar medium
Reverse shock propagates into ejecta, starting from outside

Shocks compress and heat gas

Mass, momentum, energy conservation give relations
(for $\gamma=5/3$)

\[
\rho = 4\rho_0
\]
\[
V = \frac{3}{4} \frac{v_{\text{shock}}}{1000 \text{ km/s}}
\]
\[
T=1.1 \frac{m_{\text{H}}}{m} \left( \frac{v}{1000 \text{ km/s}} \right)^2 \text{ keV}
\]

X-rays are the characteristic emission

These relations change if significant energy is diverted to accelerating cosmic rays

The shock is “collisionless” because its size scale is much smaller than the mean-free-path for collisions (heating at the shock occurs by plasma processes) coupled through the structure of turbulence in shocks and acceleration

Collisions do mediate ionizations and excitations in the shocked gas
• Forward shock into the ISM- is a 'contact discontinuity' - outside of this the ISM does not yet 'know' about the SN blast wave

• Reverse shock- information about the interaction with the ISM travels backwards into the SN ejecta
• Shell like remnants
• Shell velocity much higher than sound speed in ISM, so shock front of radius R forms.

Supernova Remnants

See Longair 16.7

• Explosion blast wave sweeps up CSM/ISM in forward shock
  - spectrum shows abundances consistent with solar or with progenitor wind

• As mass is swept up, forward shock decelerates and ejecta catches up; reverse shock heats ejecta
  - spectrum is enriched w/ heavy elements from hydrostatic and explosive nuclear burning
The Shock Longair 11.3

- A key ingredient in SNR dynamics is the strong (high Mach number) shock which is “collisionless” (see Longair sec 11.2.1 and eg 11.17)
- the effect of the shock is carried out through electric and magnetic fields generated collectively by the plasma rather than through discrete particle–particle collisions
- the shock system is given by the synonymous terms “adiabatic” and “non-radiative” to indicate that no significant energy leaves the system in this phase
- a “radiative” shock describes the case where significant, cooling takes place through emission of photons
- For standard 'collionless' shocks

\[ kT = \frac{3}{16} \mu_m V_s^2 \sim 1.2(V_s/1000 \text{km/sec})^2 \text{keV} \]

Phase II - adiabatic expansion-
Adapted from L. Culhane

Radiative losses are unimportant in this phase - no exchange of heat with surroundings.

Large amount of ISM swept-up:

\[ m_0 \ll \frac{4\pi}{3} \rho_0 R^3(t) \] (3)
Thus conservation of momentum becomes:

\[ m_0v_0 = \frac{4\pi}{3} \rho_0 R^3(t)v(t) \]  

since \( m_0 \) is small

\[ = \frac{4\pi}{3} \rho_0 R^3(t) \frac{dR(t)}{dt} \tag{4} \]

Integrating:

\[ m_0v_0t = \frac{\pi}{3} \rho_0 R^4(t) \tag{5} \]

\[ R(t) = 4v(t)t \]

\[ v(t) = \frac{R(t)}{4t} \]

\[ e.g. \ E = \frac{1}{2}nv^2 \text{volume} = \frac{1}{2} \text{cm}^{-3} (\text{cm/sec})^2 \text{cm}^3 \]

Sedov Solution

- When the swept-up mass becomes greater than the ejected mass, the dynamics are described by the adiabatic blast-wave similarity solution of Taylor and Sedov.
- During this phase, the overall dynamics are determined by the total mass of expanding gas, which is mostly swept up interstellar gas, and the energy released in the initial explosion.
- Dimensional analysis: only variables are \( n \) (the particle density); \( E \) is the energy of the explosion, time (t) and radius (r).
- Dimension of \( E/ n \) are length\(^5\) time\(^{-2}\) so the dimensionless variable is \((E/ n)t^2r^{-5}\); therefore we can write \( r(t) \sim (E/ n)^{1/5}t^{2/5} \)
• Taking a full calculation for the adiabatic shock wave into account for a gas with $\gamma = 5/3$:
\[
R(t) = 1.17 \left( \frac{E_0}{\rho_0} \right)^{\frac{1}{5}} t^{\frac{2}{5}} \quad \text{and} \quad v(t) = 0.4 \frac{R(t)}{t}
\]

• **Temperature behind the shock, $T \propto v^2$, remains high – little cooling**
\[
T \approx \frac{3}{16} \frac{m}{k} v^2
\]

• **Typical feature of phase II – integrated energy lost since outburst is still small:**
\[
\int \left( \frac{dE}{dt} \right)_{RAD} dt \ll E_0
\]

---

**Sedov Solution**

• Kinetic energy of expansion (KE) is transferred into internal energy - total energy remains roughly the same (e.g. radiative losses are small)

• The temperature of the gas is related to the internal energy
\[
T \sim 10^6 k E_{51}^{1/2} n^{-2/5} (t/2 \times 10^4 \text{yr})^{-6/5}
\]

• $n$ is the particle density in cm$^{-3}$; $E_{51}$ is the energy of the explosion in units of 10$^{51}$ ergs

• for typical explosion energies and life times the gas emits primarily in the x-ray band

• measuring the size ($r$), velocity ($v$) and temperature $T$ allows an estimate of the age

• $t_{\text{Sedov}} \sim 3 \times 10^4 T_6^{-5/6} E_{51}^{1/3} n^{-1/3} \text{yr}$

• at $T \sim 10^6-10^7$ k the x-ray spectrum is line dominated

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• If the density (gm/cm\(^3\)) in the ISM/circumstellar gas is \(\rho_{\text{ism}}\) then the radius of the shock when it has swept up an equal mass to the eject \(M_{\text{ejecta}}\) is
\[
r_1=2pc(M_{\text{ejecta}}/M_\odot)^{1/3} (\rho_{\text{ism}}/10^{-24}\text{ gm/cm}^3)^{1/3}
\]
• to get an estimate of the time this occurs assume that the shock has not slowed down and the total input energy remains the same (radiation losses are small) and travels at a velocity
\[
v_{\text{ejecta}}=(v_{\text{ejecta}}/10^4\text{ km/sec}) \text{ to get}
\]
\[
t_1=r_1/(v_{\text{ejecta}}/10^4\text{ km/sec}) =200 \text{ yr} (E_{51})^{-1/2} (M_{\text{ejecta}}/M_\odot)^{5/6} (\rho_{\text{ism}}/10^{-24}\text{ gm/cm}^3)^{1/3}
\]
• To transform variables total energy \(E=1/2M_{\text{ejecta}}v^2\sim r^3\rho_{\text{ism}}v^2\) to get
\[
r\sim(E/\rho_{\text{ism}})^{1/5} t^{2/5}
\]
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**Sedov-Taylor Solution- Again**

• nice discussion in Draine sec 39.1.2
• assume a spherical shock of radius \(R\) and it has a a power law dependence on energy of the explosion, \(E\), time since explosion, \(t\), and density of the medium into which it is running \(\rho\).
• \(R=AE^\alpha \rho^{\beta} t^\eta\)
• with dimensional analysis (e.g. the powers to which mass length and time appear
• get mass \(\alpha+\beta=0\), length \(1-2\alpha-3\beta=0\), -2\(\alpha+\eta=0\) time
one solves this to get
\(\alpha=1/5, \beta=-1/5, \eta=2/5\)

or \(R=AE^{1/5} \rho^{-1/5} t^{2/5}\)

putting in the physics and numbers
\(R=1.54\times10^{19}\text{ cm } E_{51}^{1/5} n^{-1/5} t_3^{2/5}\)

(we have switched units, \(n\) is particle density, \(t_3\) is in units of \(10^3\) years, \(E_{51}\) is in units of \(10^{51}\) ergs.
\(v_s=1950\text{ km/s } E_{51}^{1/5} n^{-1/5} t_3^{-3/5}\)

\(T=5.25\times10^7 k E_{5}^{2/5} n^{-2/5} t_3^{-6/5}\)
Sedov-Taylor Solution

- \( R \sim (E/\rho)^{2/5}t^{2/5} \)
- \( v \sim (2/5)(E/\rho)^{2/5}t^{-3/5} \)

- Just behind the shock wave
  
  \( \rho_1 = \rho_0 (\gamma + 1/\gamma - 1) ; \gamma \) is the adiabatic index
  
  \( v_1 = (4/5)(1/\gamma + 1) (E/\rho_0)^{2/5}t^{-3/5} \)
  
  Pressure \( P_1 = (8/25)(\rho_0/\gamma + 1)(E/\rho_0)^{2/5}t^{-6/5} \)

Limit of Strong Shocks - Longair pg 315-318

- Ratio of temperatures behind and in front of the shock is related to the Mach speed, \( \mathcal{M} \), of the shock (shock speed/sound speed in gas)
  
  \( T_2/T_1 = (2\gamma(\gamma - 1)\mathcal{M}^2)/(\gamma - 1)^2 = (5/16)\mathcal{M}^2 \) if the adiabatic index \( \gamma = 5/3 \) (ideal gas)
  
  Longair eq 11.74

In the strong shock limit, the temperature and pressure can become arbitrarily large, but the density ratio attains a maximum value of \((\gamma + 1)/(\gamma - 1) = 4\)
Sedov-Taylor phase

This solution is the limit when the swept-up mass exceeds the SN ejecta mass - the SNR evolution retains only vestiges of the initial ejecta mass and its distribution.

The key word here is **SELF SIMILAR** *(solutions can be scaled from solutions elsewhere)*

$$f(r, t) \rightarrow f\left(\frac{r}{r_{\text{ref}}}\right) \cdot f\left(r_{\text{ref}}\right)$$

(SKIPPING THE EQUATIONS)

$$R_s = 12.4 \, \text{pc} \ (\text{KE} \frac{51}{n_1})^{1/5} \ t_4^{2/5}$$

$$t = 390 \, \text{yr} \ R_s \ T_{\text{meas}}^{-1/2}$$

$R_s$ is the shock radius, $T$ is the temperature

In the Sedov-Taylor model one expects thermal emission coming from a thin shell behind the blast wave. As the shock expends the pressure drops between the shock wave and the material ejected.

The 4 Phases in the Life of a SNR

- 4 limits
  1) blast wave, \( v=\text{const} \)
  2) Sedov: \( E=\text{const} \)
  3) Snow plough \( \text{momentum}=\text{constant} \)
  4) no longer expands, merges with ISM

*see fig 4.9 in R+B*
**Summary**

- **Free Expansion Phase**
  The ejecta expands freely into the interstellar medium. The expanding envelope compresses the ISM, creates a shock wave because of its high velocity, and sweeps up the ISM. During this initial phase, the mass of gas swept up is \(<<\) mass of the ejecta and the expansion of the envelope is not affected by the outer interstellar gas and it keeps its initial speed and energy.

- **Adiabatic Expansion Phase**
  When mass of gas swept up > mass of ejecta the kinetic energy of the original exploded envelope is transferred to the swept up gas, and the swept up gas is heated up by the shock wave roughly independent of the physics of the explosion. The radiative losses from the swept up gas are low (energy is conserved) - adiabatic expansion phase.

  The evolution during this phase is determined only by the energy of explosion \(E_0\), the density of interstellar gas, and the elapsed time from the explosion \(t\). A self similar solution relating the density, pressure, and temperature of the gas, and the distribution of the expansion velocity exists (Sedov-Taylor)
Next 2 Phases

• Constant Temperature Expanding Phase
The expansion velocity decreases with time and, radiative cooling behind the shock front becomes important. When the radiative cooling time of the gas becomes shorter than the expansion time, the evolution deviates from the self similar one. In this phase, the SNR evolves, conserving momentum at a more or less constant temperature and the radius of the shell expands in proportion to the $1/4$ power of the elapsed time since the explosion.

Phase III - Rapid Cooling

• SNR cooled, $\Rightarrow$ no high pressure to drive it forward.
• Shock front is coasting
  \[
  \frac{4}{3} \pi R^3 \rho_0 v = \text{constant}
  \]
  
• Most material swept-up into dense, cool shell.
• Residual hot gas in interior emits weak X-rays.
Radiative/Snow plough phase

$T$ drops as a steep function of radius

$===>$ at some point, $T$ is below $T_{\text{recomb}} \sim 1 \text{ keV}$ - the cooling function increases steeply and the gas recombines rapidly; $v_{\text{shock}} \sim 150 \text{ km/sec}$

Age of SNR when this happens depends on models for cooling functions, explosion energy and density.

roughly $t_{\text{cool}} \sim n k T/n^2 \Lambda(T) \sim 4 \times 10^4 \text{ yr} T_6^{3/2}/n$

($\Lambda(T)$ is the cooling function)

phase starts when $t_{\text{cool}} < t_{\text{Sedov}}$: $T_6 < E_6^{1/7} n^{2/7}$

Between 17,000 and 25,000 years (assuming standard $E_o$ and $n_1$) -

Then: THE END... SNR merges with surrounding medium

End of Snowplough Phase- Draine sec 39.1.4

- The strong shock gradually slows (radiative losses and accumulation of 'snowplowed' material)
- Shock compression declines until $v_{\text{shock}} \sim c_s$ (sound speed); no more shock
- Using this criteria the 'fade away' time
- $t_{\text{fade}} \sim ((R_{\text{rad}}/t_{\text{rad}})/c_s)^{7/5} t_{\text{rad}}$
- $t_{\text{fade}} \sim 1.9 \times 10^6 \text{ yrs} \ E_5^{0.32} n^{-0.37}(c_s/10\text{km/sec})^{-7/5}; c_s=0.3\text{km/sec(T/10k)}^{1/2}$
- $R_{\text{fade}} \sim 0.06\text{kpc} \ E_5^{0.32} n^{-0.37}(c_s/10\text{km/sec})^{2/5}$
Plasma takes time to come into equilibrium

- particle ("Coulomb") collisions in the post-shock plasma will bring the temperature of all species, including the free electrons, to an equilibrium value:
  \[ kT = \frac{3}{16} \mu m_p v_s^2 \]
- However it takes time for the system to come into equilibrium and for a long time it is in non-equilibrium ionization (NEI)
  \[ \tau \sim n_e t \sim 3 \times 10^{12} \text{cm}^{-3} \text{s} \]
  if the plasma has been shocked recently or is of low density it will not be in equilibrium

- Timescale to reach equilibrium depends on ion and temperature-solution of coupled differential equations.
- Relevant parameter is \( n_e t \) (density x time)
Time-Dependent Ionization

Oxygen heated to 0.3 keV  
(Hughes & Helfand 1985)

Ionization is effected by electron-ion collisions, which are relatively rare in the ~1 cm\(^{-3}\) densities of SNRs. Ionization is time-dependent.

Ionization timescale = \(n_e t\)  
electron density x time since impulsively heated by shock

Ionization equilibrium attained at \(n_e t \sim 10^4\) cm\(^{-3}\) yr

Ionizing gas can have many more H- and He-like ions, which then enhances the X-ray line emission.

Inferred element abundances will be too high if ionization equilibrium is inappropriately assumed for an ionizing gas.
Instabilities

irregular shock boundaries
mixing between ejecta layers
mixing between ejecta and ISM

What it really looks like
SNR are Thought to Be the Source of Galactic cosmic rays

- SNR need to put ~ 5-20% of their energy into cosmic rays in order to explain the cosmic-ray energy density in the Galaxy (~2 eV/cm$^3$ or $3 \times 10^{38}$ erg/s/kpc$^2$), the supernova rate (1-2/100yrs), the energy density in SN (1.5x10$^{41}$ ergs/sec~2x10$^{39}$ erg/s/kpc$^2$)

- particles are scattered across the shock fronts of a SNR, gaining energy at each crossing (Fermi acceleration)

- Particles can travel the Larmor radius

$R_L \sim E_{17}/B_{10 \mu G} Z \text{kpc}$

many young SNRs are actively accelerating electrons up to 10-100 TeV, based on modeling their synchrotron radiation

See Longair 17.3

Fermi acceleration-1949:

- charged particles being reflected by the moving interstellar magnetic field and either gaining or losing energy, depending on whether the "magnetic mirror" is approaching or receding. energy gain per shock crossing is proportional to velocity of shock/speed of light - spectrum is a power law

DeCourchelle 2007

Nice analogy- ping pong ball bouncing between descending paddle and table
Fermi Acceleration

2nd Order energy gained during the motion of a charged particle in the presence of randomly moving "magnetic mirrors". So, if the magnetic mirror is moving towards the particle, the particle will end up with increased energy upon reflection.

- energy gained by particle depends on the mirror velocity squared. - also produces a power law spectrum
- the average increase in energy is only second-order in \(V/c\). This result leads to an exponential increase in the energy of the particle since the same fractional increase occurs per collision.

Longair 17.15
Test of Fermi Acceleration Hypothesis

- Shock waves have moving magnetic inhomogeneities - Consider a charged particle traveling through the shock wave (from upstream to downstream). If it encounters a moving change in the magnetic field, it can reflect it back through the shock (downstream to upstream) at increased velocity. If a similar process occurs upstream, the particle will again gain energy. These multiple reflections greatly increase its energy. The resulting energy spectrum of many particles undergoing this process turns out to be a power law.
How Does the Fermi $\gamma$-ray Signal 'Prove' CRs are Accelerated?

$\gamma$-rays can originate in SNR in 3 separate ways

- Inverse Compton scattering of relativistic particles
- Non-thermal bremsstrahlung
- Decay of neutral pions into 2 $\gamma$-rays

- the first 2 have broad band ~power law shapes
- pion decay has a characteristic energy $E_\gamma=67.5$ MeV - need to convolve with energy distribution of CR protons
- The $\pi_0$ meson has a mass of 135.0 MeV/c$^2$. The main $\pi_0$ decay mode, is into two photons: $\pi_0 \rightarrow 2 \gamma$.

Fit of Fermi $\gamma$-ray data to Pion model

- One of the fitted parameters is the proton spectrum need to product the g-ray spectrum via pion decay.
- This indicates that the proton spectrum is not a pure power law but has a break (change in slope) at high energies
• an incoming proton with 135 MeV of kinetic energy cannot create a neutral pion in a collision with a stationary proton because the incoming proton also has momentum, and the collision conserves momentum, so some of the particles after the collision must have momentum and hence kinetic energy.

• Assume initially two protons are moving towards each other with equal and opposite velocities, thus there is no total momentum. in this frame the least possible K.E. must be just enough to create the \( \pi_0 \) with all the final state particles (p,p,\( \pi_0 \)) at rest. Thus if the relativistic mass of the incoming protons in the center of mass frame is \( m \), the total energy \( E=2m_p c^2+m_{\pi_0} c^2 \) and using total energy \( =m_p \sqrt{1-v^2/c^2} \)

• rest mass energy of proton is 931 MeV gives \( v/c=0.36c \); use relativistic velocity addition to get total velocity or a needed 280MeV of additional energy-- threshold for \( \pi_0 \) production
Sn1006

- The first SN where synchrotron radiation from a 'thermal' remnant was detected - direct evidence for very high energy particles.

Direct evidence is the energy of the photons emitted (~TeV) + the needed particle energies to produce synchrotron x-rays

\[ \nu_{\text{synch}} \approx 16\text{keV} (B \cdot E_{\text{TeV}}^{-2}) \text{ Hz} \]

loss time of the particles \[ t_{\text{synch}} \approx 400s \cdot B^{-2} \cdot E_{\text{TeV}}^{-1} \]

for field of 100\,\mu G one gets

\[ E \approx 100\,\text{TeV}, \quad t_{\text{synch}} \approx 15\text{ years} \quad \text{-- so need continual reacceleration} \]
SN1006

Difference Image

![SN1006 Difference Image](image)

![Graphs](graphs)

- **Azimuth Angle (degree)**
  - **Power-Median (arcsec)**
  - **Normalized Counts/sec/eV/m**