Neutron Stars

Accreting Compact Objects-
see Chapters 13 and 14 in Longair

Degenerate Compact Objects

- The determination of the internal structures of white dwarfs and neutron stars depends upon detailed knowledge of the equation of state of the degenerate electron and neutron gases

Degneracy and All That- Longair pg 395 sec **13.2.1**

- In *white dwarfs*, internal pressure support is provided by electron degeneracy pressure and their masses are roughly the mass of the Sun or less
- the density at which degeneracy occurs in the non-relativistic limit is proportional to $T^{3/2}$
- This is a quantum effect: Heisenberg uncertainty says that $\delta p \delta x > h/2\pi$
- Thus when things are squeezed together and δx gets smaller the momentum, p, increases, particles move faster and thus have more pressure
- Consider a box- with a number density,n, of particles are hitting the wall;the number of particles hitting the wall per unit time and area is 1/2nv
- the momentum per unit time and unit area (**P**ressure) transferred to the wall is 2nvp; **P**$\sim nvp=(n/m)p^2$ (m is mass of particle)

Degeneracy- continued

- The average distance between particles is the cube root of the number density and if the momentum is calculated from the Uncertainty principle $p \sim h/(2\pi\delta x) \sim hn^{1/3}$
- and thus **P**$=h^2n^{5/3}/m$- if we define matter density as $\rho=[n/m]$ then
- **P**$\sim \rho^{5/3}$ **independent of temperature**
- Dimensional analysis gives the central pressure as **P**$\sim GM^2/r^4$
- If we equate these we get $r \sim M^{-1/3}$ e.q **a degenerate star gets smaller as it gets more massive**

- At higher densities the material gets 'relativistic' e.g. the velocities from the uncertainty relation get close to the speed of light- this changes things and $P \sim \rho^{4/3;}$
- this is important because when we use **P**$\sim GM^2/r^4$ we find that the pressure does not depend on radius and just get an expression that depends on mass- this is the Chandrasekar mass. (see 13.2.2 in Longair)

- Particles are moving at relativistic speeds when density satisfies:

$$n_e > \frac{8\pi}{3}\left(\frac{m_e c}{h}\right)^3$$

$$\rho > 2 \times 10^{12}\,\mathrm{kg\,m^{-3}}$$

White Dwarfs…
Courtesy of C. Reynolds

- Where…

$$R \sim \frac{K}{GM^{1/3}}$$

$$P_c = K\rho_c^{5/3} \qquad\qquad P = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m_e\mu^{5/3}m_p^{5/3}}\rho^{5/3}$$

Mass of particle producing degeneracy pressure

Number of nucleons per degenerate particle

- So, an approximate expression for radius of white dwarf is:

$$R \sim \frac{K}{GM^{1/3}}$$

$$R \sim 1.2 \times 10^4 \left(\frac{M}{M_\odot}\right)^{-1/3} \mu^{-5/3}\,\mathrm{km}$$

- Exact calculation gives

$$R \sim 1.13 \times 10^4 \left(\frac{M}{M_\odot}\right)^{-1/3} \left(\frac{\mu}{2}\right)^{-5/3} \mathrm{km}$$

Maximum Mass of a Compact Object- Longair 13.2.2

- The Chandrasekar limit (maximum mass of a white dwarf) corresponds to the situation in which it costs less energy for a electron to fuse with a proton to form a neutron then to climb higher in the Fermi sea.
  - Above this limit the compact object becomes all neutrons (a neutron star)

An alternative way of looking at this is to calculate the equation of state (EOS) of degenerate matter and use hydrostatic equilibrium.

– P_e=(1/20)(3/π)2/3(h^2/me) ($\rho/\mu_e m_p)^{5/3}$ ρ is the total mass density and $\mu_e m_p$ is the mass per electron (composition of the material) - or more simply

– $P_e \sim \rho^{5/3}$ - non relativistic

- for relativistic matter $P_e=(1/8)(3/\pi)^{1/3}ch(\rho/\mu_e m_p)^{4/3}$ - notice the appearance of the speed of light

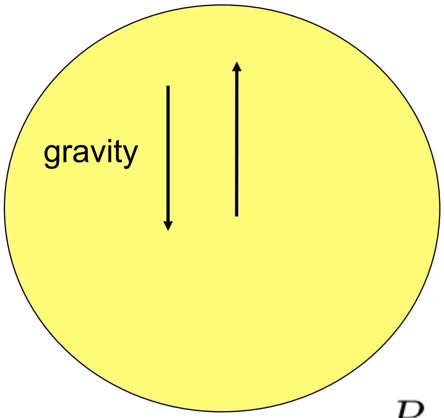
– $P_e \sim \rho^{4/3}$

- – in hydrostatic equilibrium (remember $dP(r)/dr=GM(r)\rho(r)/r^2$; $P \sim GM^2/R^4$
- Setting the 2 pressures equal produces the Chandrasekhar limit at which a

white dwarf collapses to a neutron star M~1.46M (but depends on its composition μ_e, eg an iron core??))

Chandrasekar Limit eqs 13.15-13.25

- Electron degeneracy is responsible for balancing gravity in White Dwarfs



Pressure gradient = weight

$$\frac{dP}{dR} = -\frac{GM}{R^2}\rho$$

$$\frac{P_c}{R} \sim \frac{GM}{R^2}\rho_c$$

$$P_c = K\rho_c^{5/3} \qquad \rho_c \sim M/R^3$$

- For relativistic degeneracy, the mass is determined…

- Complete calculation (pg 398-399 in Longair) gives

$$M \sim \left(\frac{K'}{G}\right)^{3/2}$$

- This is called the Chandrasekhar mass… it is the maximum possible mass of a White Dwarf.

$$M \approx 1.457 \left(\frac{2}{\mu}\right)^2 \mathrm{M}_\odot$$

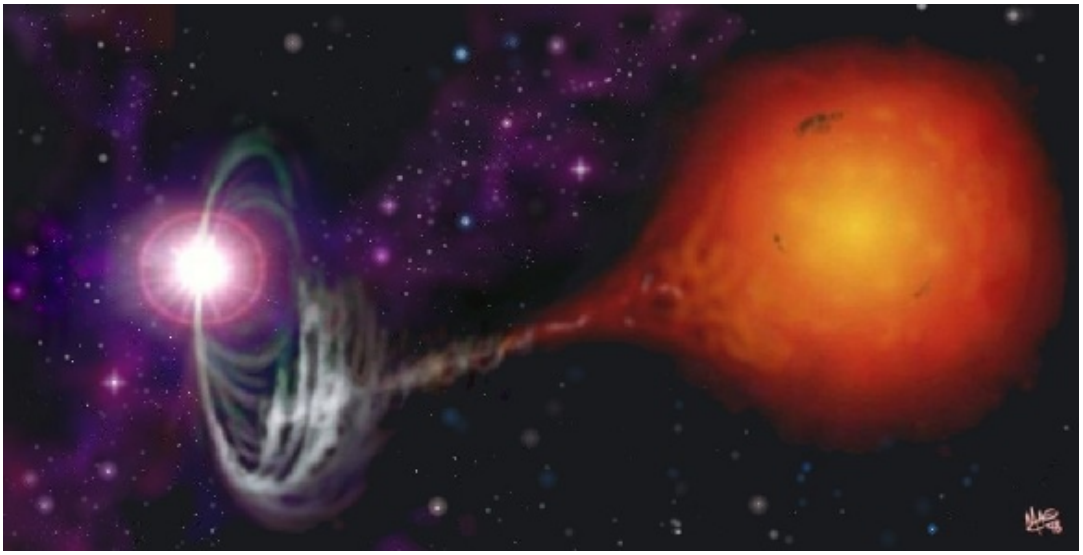
Pushing a white dwarf over the edge…

- Some white dwarfs are in a binary system and "accrete" matter from a companion star

M.A.Garlick 1998

C. Reynolds

… so a white dwarf may grow to the Chandrasekhar mass!

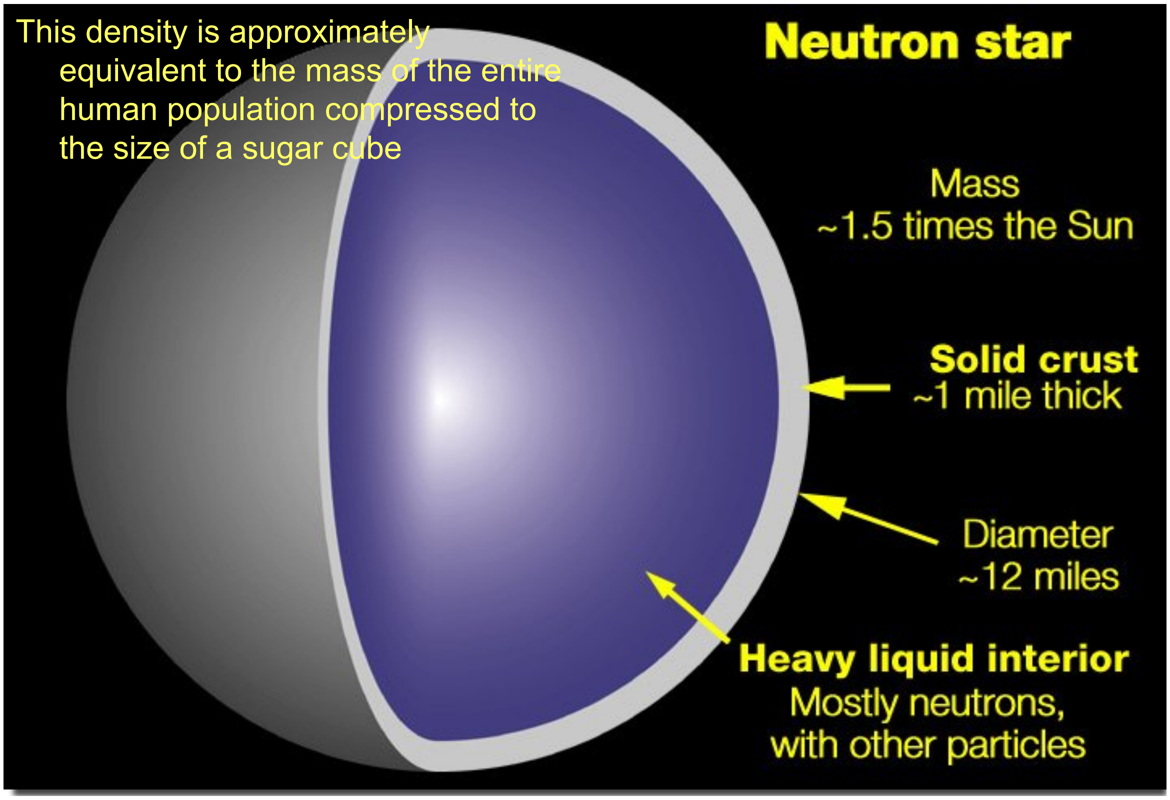


M.A.Garlick 1998

- Accretion…
  - Mass approaches Chandrasekhar mass
  - Central density and temperature increases
- Then, suddenly…
  - He or C/O in white dwarf core starts to undergo nuclear fusion
  - Energy release leads to rapid rise in temperature
  - Rise in temperature makes fusion occur faster
  - Get a thermonuclear runaway… whole star is incinerated to iron/nickel in few seconds
  - Energy release blows the star apart
  - TYPE-1A Supernova

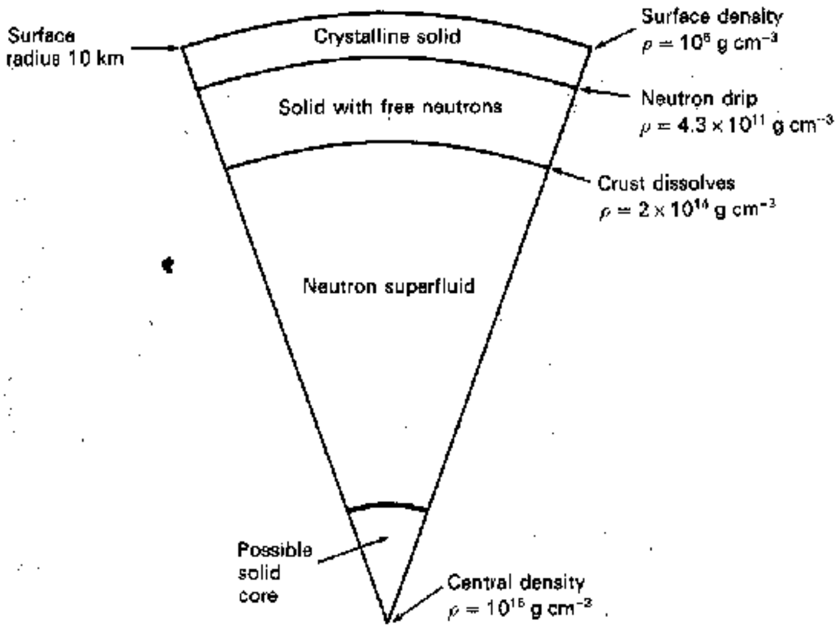
Alternatively 2 white dwarfs merge and go over the Chandrasekar limit.

Inside Neutron Stars



This density is approximately equivalent to the mass of the entire human population compressed to the size of a sugar cube

Inside a Neutron Star



Neutron Stars

- Predicted theoretically by Volkoff and Oppenheimer (1939)
  - First 'discovered' by Hewish et al 1967 (Noble prize) as the counterparts to radio pulsars
- short period from radio pulsars (<1s) can only be obtained from a compact object via either rotation or oscillations; also the period derivatives are small and for radio pulsar periods always increase (slow down)
  - All characteristic timescales scale as $\rho^{-1/2}$ (ρ is density)

$$\omega=1/\mathrm{sqrt}(GM/r^3) =1/\mathrm{sqrt}(G\rho)$$

Shortest periods ~1.5ms- light travel time arguments give a size (ct~ 500km)

White dwarfs with $\rho \sim 10^7 – 10^8$ gmcm^{-3} maximum rotation periods

$P = 2\pi/\Omega \sim 1–10$ s

- To get periods of ~1ms (radio pulsars) need $\rho \sim 10^{14}$ gmcm^{-3}
- What are the sources of energy?
  - Spin down
  - accretion

Creation of Neutron Stars

II/Ib/Ic Core-Collapse of Massive Progenitor at the end of its evolution

- Massive stellar progenitor core forms neutron star or black hole
- Explosive nucleosynthesis products near core (Si and Fe) plus hydrostatically formed outer layers (O, Ne) are expelled
- *Most of the explosion energy is carried away by neutrinos*
- Uncertain explosion mechanism details involve neutrinos, probably large-scale shock instabilities, rotation, possibly magnetic fields
- Star must be at least $8M_{\odot}$; core at least $1.4\ M_{\odot}$.

(U. Hwang 2007)

Creation of Neutron Stars

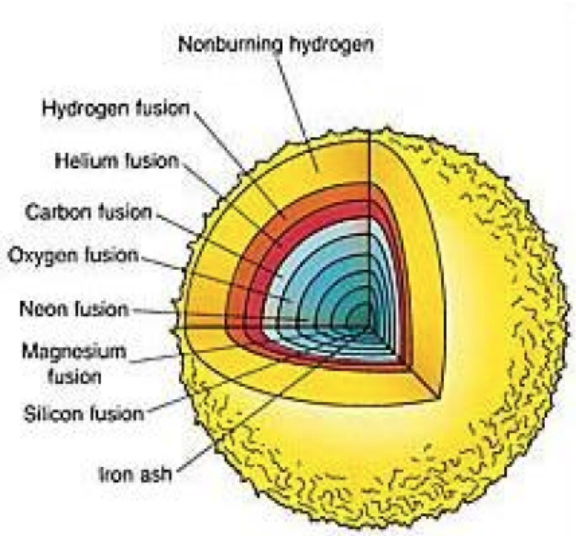
- Stellar core collapses under the force of its own gravity. At the very high pressures involved in this collapse, it is energetically favorable to combine protons and electrons to form neutrons plus neutrinos. The neutrinos escape after scattering a bit and helping the supernova happen, and the neutrons settle down to become a neutron star, with neutron degeneracy managing to oppose gravity.
- Energy set free by the collapse expels most of star's mass.

- Dense remnant, a neutron star, remains- due to the large reduction in radius and conservation of angular momentum the NS is born spinning very rapidly
- Observed spins ~1.4ms-30sec
- Very high surface gravity $7x10^{12}m/sec^2$ -10^{11}x that of the earth

(U. Hwang 2007)

I.7 : Core collapse in a massive star

- End of a massive star's life ($M>8M_{sun}$)
  - Center of star has fused all of the way to iron
  - Shells of other elements surround iron core
  - Only takes ~day to build up "dead" Chandrasekhar mass iron core
  - Core is held up by electron degeneracy pressure



From website of the Univ. of Mississippi

- Once dead core exceeds 1.4Msun, electron degeneracy pressure cannot support it.
- Core starts to collapse…oops
  - $\rho \approx 10^9$ kg/m^3 - Density of core when collapse begins (onset of relativistic effects in electron motions)
  - $\rho \approx 10^{10}$ kg/m^3 - Fermi energy exceeds neutron-proton mass difference…

    Inverse beta decay becomes energetically preferable to normal beta decay $p + e^- \rightarrow n + \nu$
- Nuclei become very neutron rich… neutronization

– $\rho \approx 10^{14}$ kg/m^3 - Individual nuclei are so neutron rich that they start to fall apart

- Remaining nuclei surrounded by sea of free neutrons

  This is called the neutron drip phase

– $\rho \approx 10^{16}$ kg/m^3 - Neutron degeneracy pressure starts to become important

– $\rho \approx 10^{18}$ kg/m^3 - Neutron degeneracy finally halts the collapse provided that $M<3M_{sun}$

– End up with a neutron star… typical mass of $1.4M_{sun}$ with a radius of 10km- theoretical mass radius relation is not well understood due to the effects of QCD

Courtesy of C. Reynolds

- So, we can try to estimate radius of neutron star given what we know about white dwarfs

  – We know that

$$\boxed{\frac{R_n}{R_{wd}} \sim \frac{m_e}{m_n} 2^{5/3}}$$

  – So we expect

$$R_{wd} \sim 10^4 \,\mathrm{km}$$

$$R_n \sim 16 \,\mathrm{km}$$

Radius of NS

- Use the 'known' density of nuclear matter

($\rho_{Neutron}$~$1.2x10^{14}$g/cm^3) and

the Chandrasekar mass

gives a radius

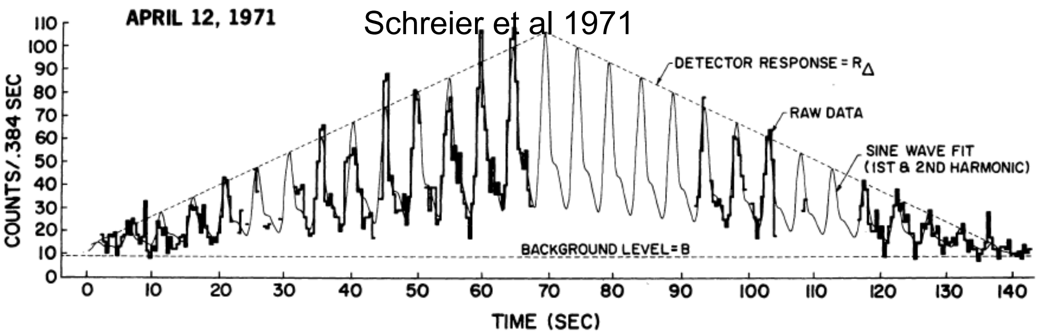
- R_{NS}~$(3M_{Chandra}/4\pi\rho_{Neutron})^{1/3}$ ~10km

consistency between the observed spin periods,

and neutron stars

Cen X-3
Schreier et al 1971

1971ApJ...167L..67G



- By analogy to white dwarfs, neutron stars have (to a crude approximation)..

$$R_n \sim \frac{K_n}{GM^{1/3}}$$

- Where…

$$P_n = K_n \rho^{5/3} \qquad P_n = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m_n^{8/3}}\rho^{5/3}$$

  - I.e., degenerate particles have mass m_n, and μ=1

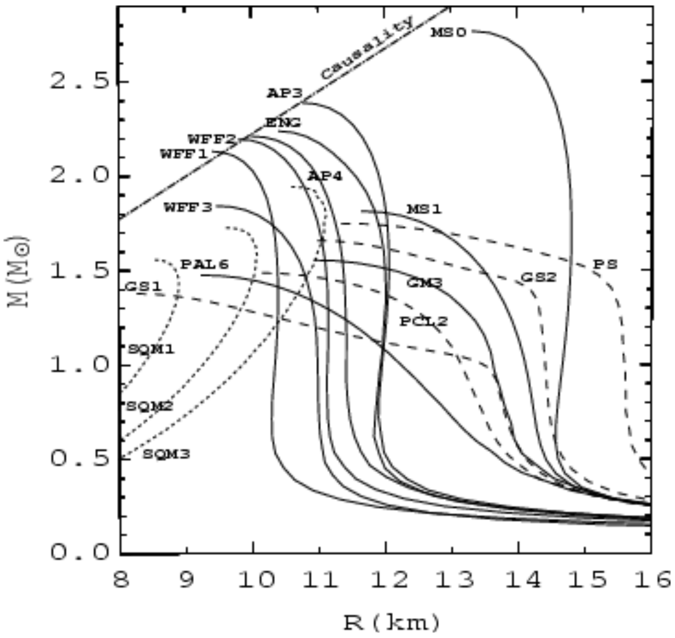
EOS of Neutron Star- Size/Mass Relation

- Rather Complex
  - – Have to use General Relativistic form of hydrostatic equilibrium equation

– Neutrons don’t behave like an ideal degenerate gas…

- strong force interactions are crucial

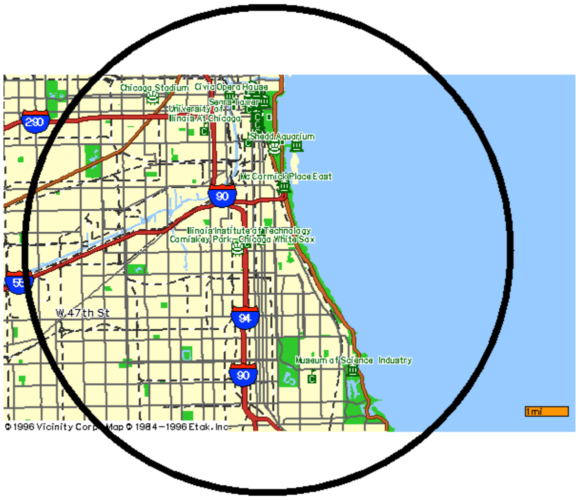
– There remain uncertainties about the “equation of state” of neutron stars



- one teaspoon of a neutron star has a mass of ~5 x 10^{12} kilograms.
- http://videos.howstuffworks.com/nasa/13498-chandra-neutron-stars-video.htm

C. Miller

Neutron star vs. Chicago

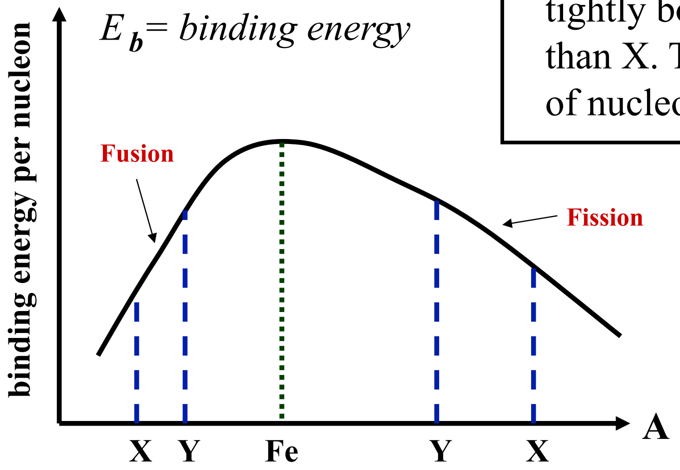


Mass=1.4 M_{sun}, Radius=10 km
Spin rate up to 38,000 rpm
Density~10^{14} g/cc, Magnetic field~10^{12} Gauss

Binding energy of Nuclei - why stellar burning stops generating energy

A=total no. nucleons
Z=total no. protons
E_b= binding energy

Change from X to Y emits energy since Y is more tightly bound per nucleon than X. This is a function of nucleon mass



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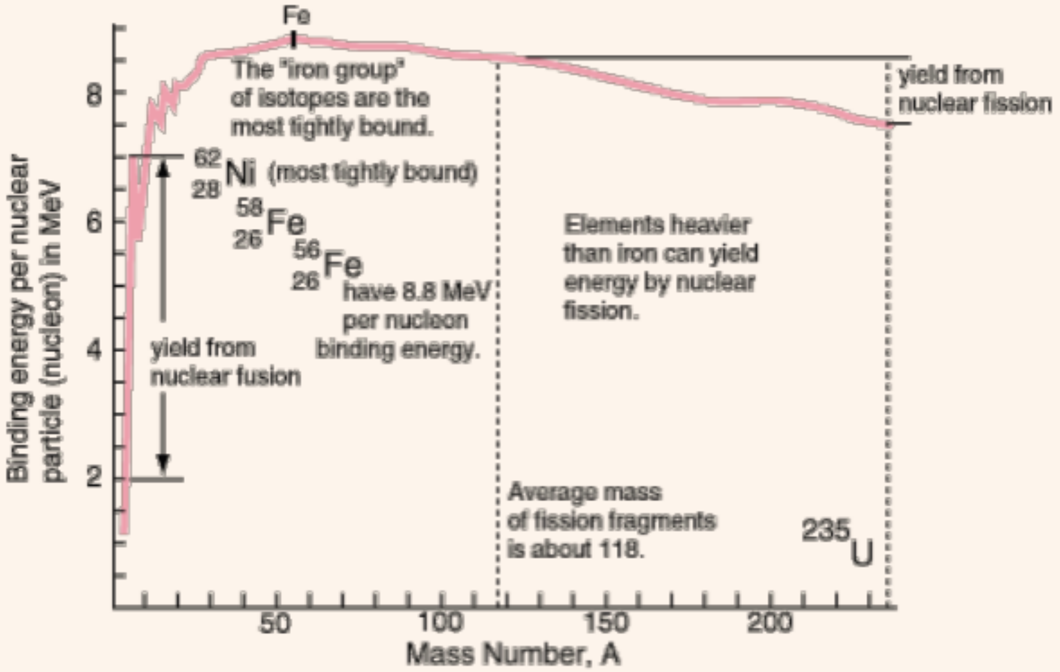
Fission and fusion can yield energy

hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin.html



Stellar Evolution and Supernovae

- •Stellar evolution – a series of collapses and fusions

H => He => C => Ne => O => Si

- •Outer parts of star expand to form opaque and relatively cool envelope (red giant phase).
- •Eventually, Si => Fe: most strongly bound of all nuclei
- •Further fusion would *absorb energy* so an inert Fe core formed
- •Fuel in core exhausted hence star collapses

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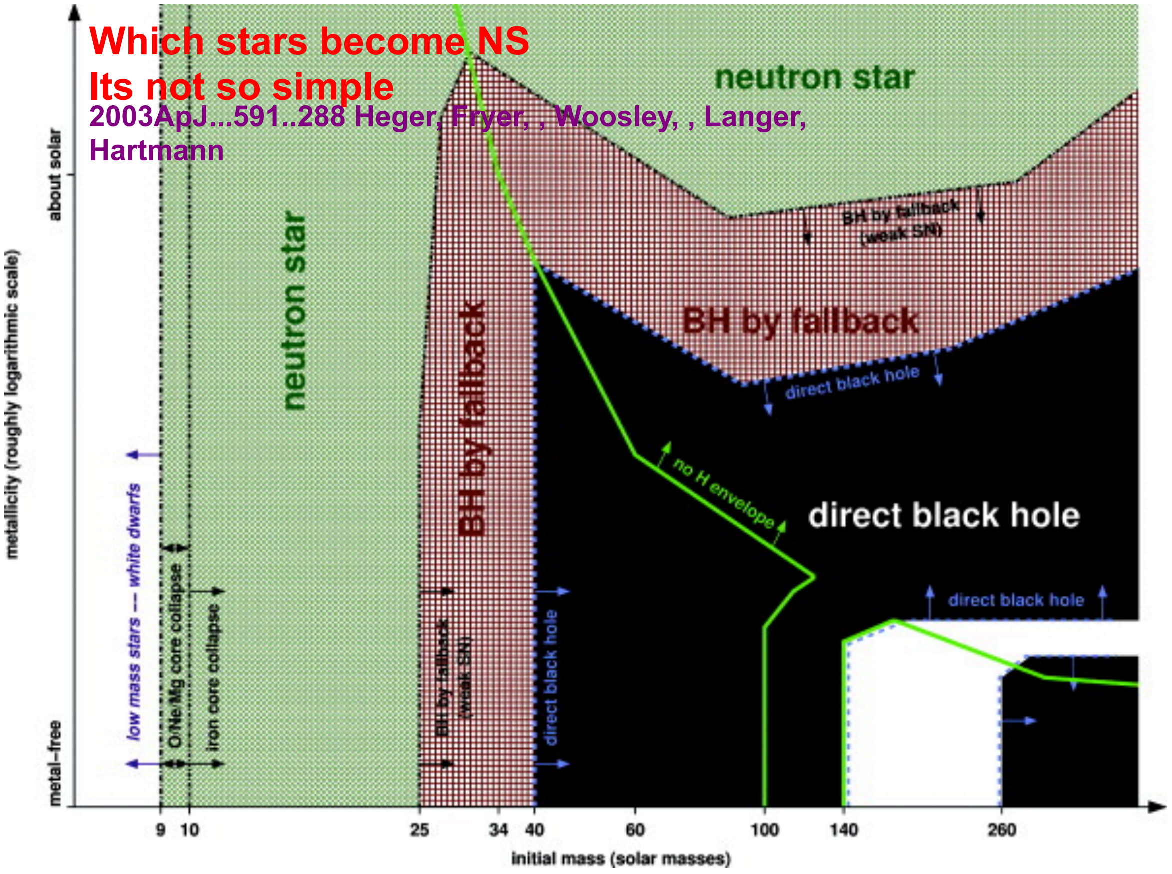
- Stars with a defined mass range evolve to produce cores that can collapse to form Neutron Stars
- Following nuclear fuel exhaustion, core collapses gravitationally; this final collapse supplies the supernova energy
- Collapse to nuclear density, in ≈ few seconds, is followed by a rebound in which the outer parts of the star are blown away
- The visible/X-ray supernova results due to radiation
  i. From this exploded material
  ii. Later from shock-heated interstellar material
- Core may
  i. Disintegrate
  ii. Collapse to a Neutron star
  iii. Collapse to a Black Hole

  according to its mass which in turn depends on the mass of the original evolved star

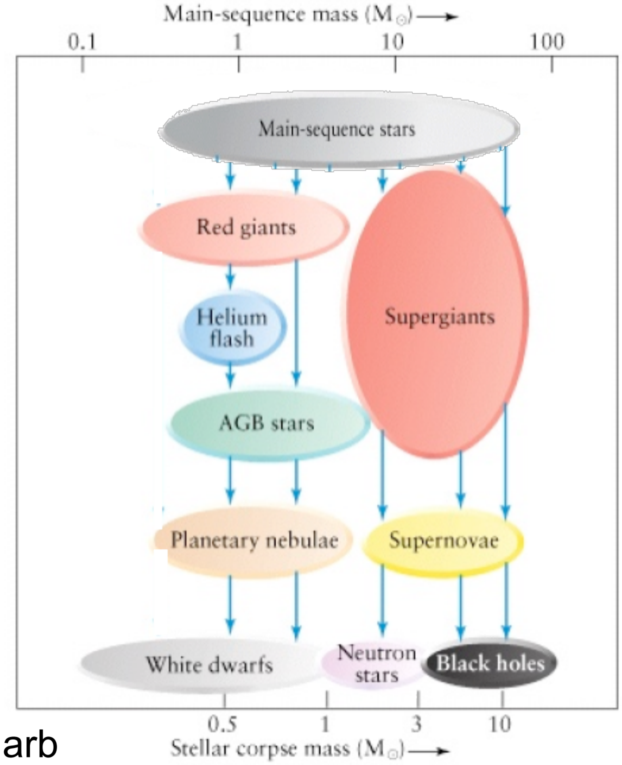
From L. Cominsky



Which stars become NS
Its not so simple
2003ApJ...591..288 Heger, Fryer, , Woosley, , Langer, Hartmann

Progenitors of Compact Objects

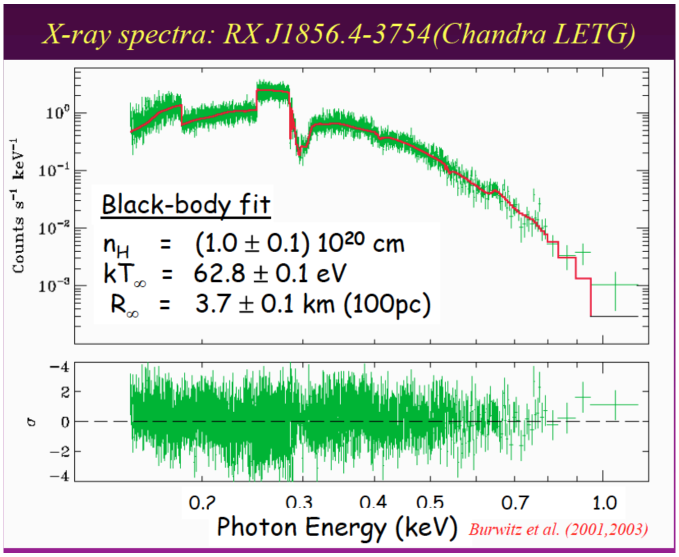
- Main sequence stars evolve and at the end of their 'life' (period of nuclear burning) end up as compact objects
- $t_{MS}/t_{sun} \sim (M/M_{sun})^{-2.5}$
- The most massive end up as black holes
- The least massive as white dwarfs (the main sequence lifetime of stars less than 1/2 M_{sun} is greater than the Hubble time so they have never got to white dwarfs)



Samar Safi-Harb

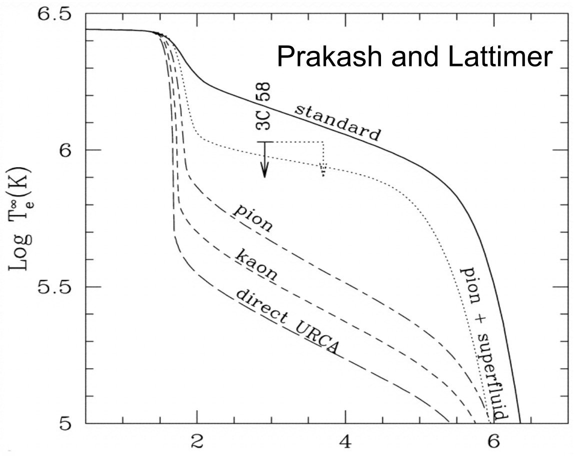
Isolated Neutron Stars- Non Accreting

- These objects are cooling from the initial high temperature of the supernova explosion
- Recent results show that they have an almost pure black body spectrum-



Burwitz et al 2001

Neutron Star Continuum Spectroscopy and Cooling

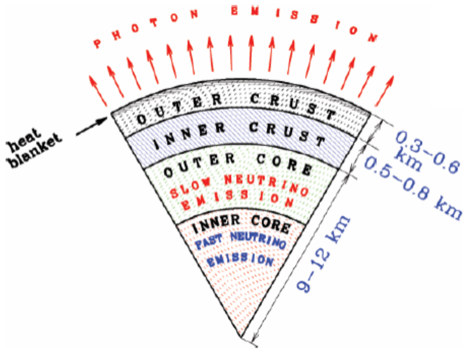


Main cooling regulators:

1. **EOS**
2. **Neutrino emission**
3. **Superfluidity**
4. **Magnetic fields**
5. **Light elements on the surface**

- After Neutron star is created in a supernova, if it is isolated it cools
- The rate at which it cools depends on the conductivity and heat capacity which depends on what it is made of and physics we do not truly understand.

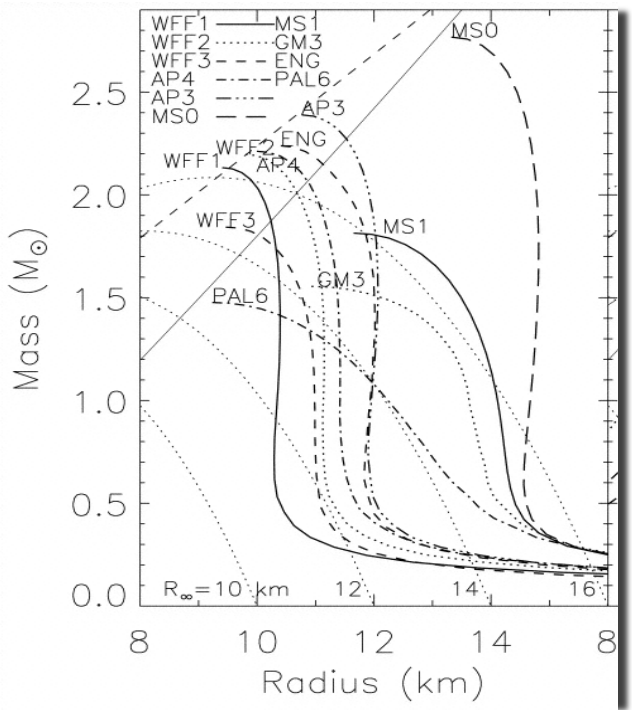
(L. Cominsky)



D.G. Yakovlev, O.Y. Gnedin*, M.E. Gusakov, A.D. Kaminker
K.P. Levenfish, A.Y. Potekhin

Fundamental Physics: The Neutron Star Equation of State (EOS)

$$dP/dr = -\rho\, G\, M(r) / r^2$$



- High mass limit sets highest possible density achievable in neutron stars (thus, in nature, “the MOST dense”).
- Radius is prop. to $P^{1/4}$ at nuclear saturation density. Directly related to symmetry energy of nuclear interaction
- Other issues: have to use general relativistic eq for hydrostatic equil
- Effect of strong interaction makes neutrons not an ideal gas
- Do not understand the eq of state (relation between pressure and density)
- Maximum mass measurements, limits softening of EOS from hyperons, quarks, other “exotica”.

Interesting Physics- Will Not Discuss Further

- The physics of how neutron stars cool depend critically on their exact composition

THEORY versus OSERVA- TIONS