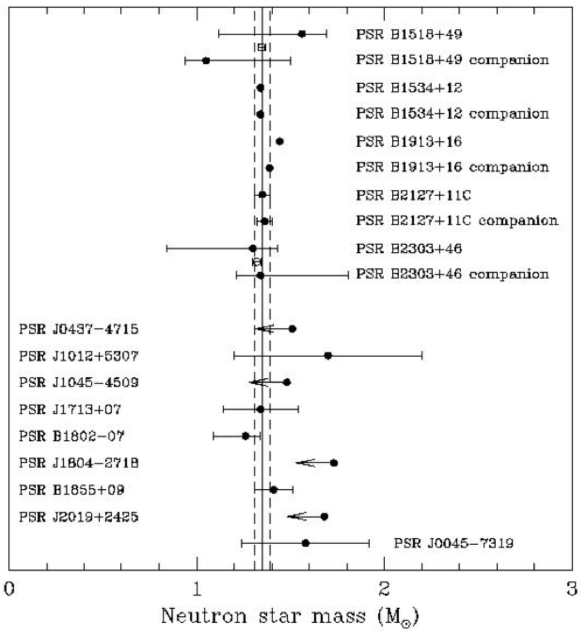
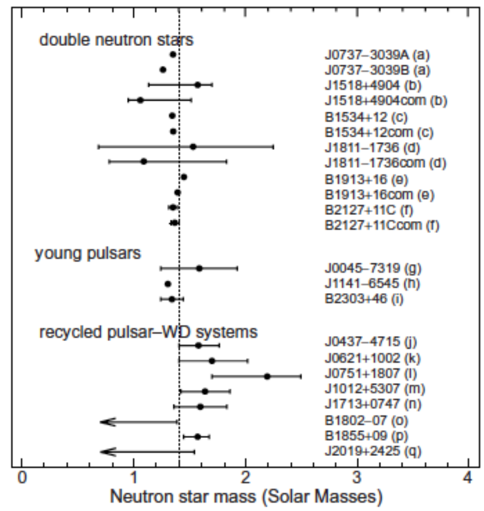
Masses of Neutron Stars – Longair pg 417

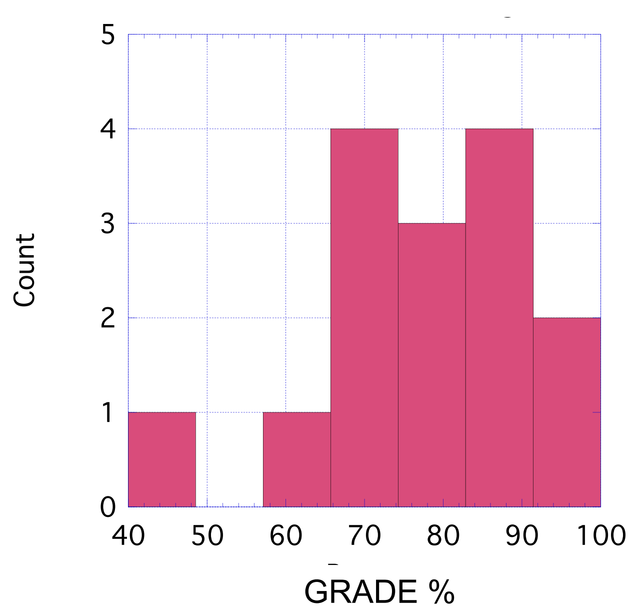
- Observed masses of non-accreting NS in binaries strongly clusters around 1.4 $M_\odot$ the Chandrasekar mass





Midterm

- Mean=77
- A>85
- 70<B<84
- 60<C<69
- The original scheme was
  - 90%+ A
  - 80-90% B
  - 70-80% C
  - 60-70% D
  - <60% F



Basics of Accretion – Longair 14.2

- If accretion takes place at a rate dM/dt= $\mathcal{M}$ then the potential energy gained by the material is
- $E=G\mathcal{M}M_x/R$ (where M_x is the mass of the accreting object) - if this energy is released as radiation it also is the luminosity L_{acc}
- Alternatively (Longair 443-444) one can calculate the free-fall velocity from infinity an $\frac{1}{2}m_p v_{ff}^2 = \frac{GMm_p}{r}$. $L=1/2 m_p v_{ff}^2$

We can write this as $L = \xi \dot{m} c^2$ and show that ε depends on r.

Normalizing the observed luminosity to a typical value of 1.3×10^{37} erg/sec gives accretion rates of

- $L_{acc}=1.3 \times 10^{37} \mathcal{M}_{17} m_x R_6$

- $\mathcal{M}_{17}$ is $\mathcal{M}$ in units of 10^{17}gm/sec= 1.5×10^{-9} M_{sun}/yr
- R_6 Is the radius in units of 10^6cm
- m_x is the mass in solar units of the accretor

Frank, King & Raine, "Accretion Power in Astrophysics",

Basics of Accretion – Longair 14.2

- For a white dwarf star with $M = M$ and $R \approx 5 \times 10^6$ m, $\xi \approx 3\ 10^{-4}$.
- Neutron star with mass $M = M$ and $R = 10$ km, $\xi \sim 0.15$.
- For nuclear energy generation the conversion of hydrogen into helium has $\xi \approx 7 \times 10^{-3}$.

Frank, King & Raine, "Accretion Power in Astrophysics",

Basics of Accretion Longair 14.2.2

Is there a limit on accretion?

If the accreting material is exposed to the radiation it is producing it receives a force due to radiation pressure

The minimum radiation pressure is
(Flux/c)xσ (σ is the relevant cross section)
assume that the infalling matter is fully ionised and that the radiation pressure force is provided by Thomson scattering of the radiation by the electrons in the plasma.
the smallest cross-section
Or
$L\sigma_T/4\pi r^2 m_p c$ (σ_T is the Thompson cross section ($6.6 x10^{-25}$ cm^2) m_p is the mass of the proton)
The gravitational force on the proton is
GM_x/R^2

Equating the two gives the Eddington limit

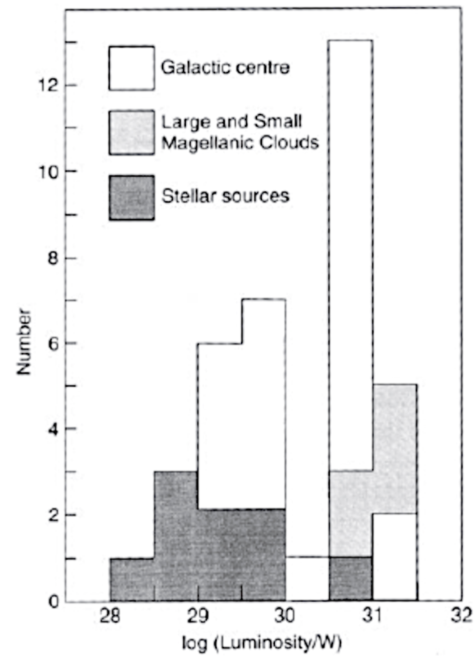
$L_{Edd}=4\pi M_x G m_p c/\sigma_T = 1.3x10^{38} M_{sun}$ erg/sec

Eddington Limit- More Detail Longair pg 446

- $f_{grav} \approx GMm_p/r^2$ force due to gravity acting on the protons
  The radiation pressure acts -upon the electron-
- Each photon gives up a momentum $p = h\nu/c$ to the electron in each collision
- force acting on the electron is the momentum communicated to it per second by the incident flux density of photons N_{ph}.
- Thus, $f_{rad} = \sigma N_{ph}\, p$ *(p is momentum,* σ is the relevant cross section , the smallest is the Thompson cross section σ_T= $6.6x10^{-29}$ m^2*)*
- *As we go away from the source of photons the flux of photons is*
  $N_{ph}/4\pi r^2$; $N_{ph}=L/h\langle\nu\rangle$; *L* is the luminosity of the source.
- so the outward force on the electron is $f = \sigma_T L/4\pi c r^2$.
- Equate this to gravity (e.g. radiation pressure and gravity balance)
  Gives **$L_E = 4\pi GMm_p c/\sigma_T$**
- maximum luminosity a spherically symmetric source of mass *M* can emit in a steady state. The limiting luminosity is independent of the radius *r* and depends only upon the mass *M* of the emitting region

Simplistic Check

- If a NS is accreting at the Eddington limit and radiating via a black body what is its temperature?
- $4\pi r^2_{NS} a T^4 = L_{edd}$
- So put in 10km for r_{ns} and 1.3×10^{31} W for L_{edd} for 1 solar mass and get
  - ($a=5.67\times10^{-8}$ Wm^{-2}K^{-4})
- $T\sim2\times10^7$K ; 'natural' for NS to radiate in the x-ray band.



Accretion -Basic idea

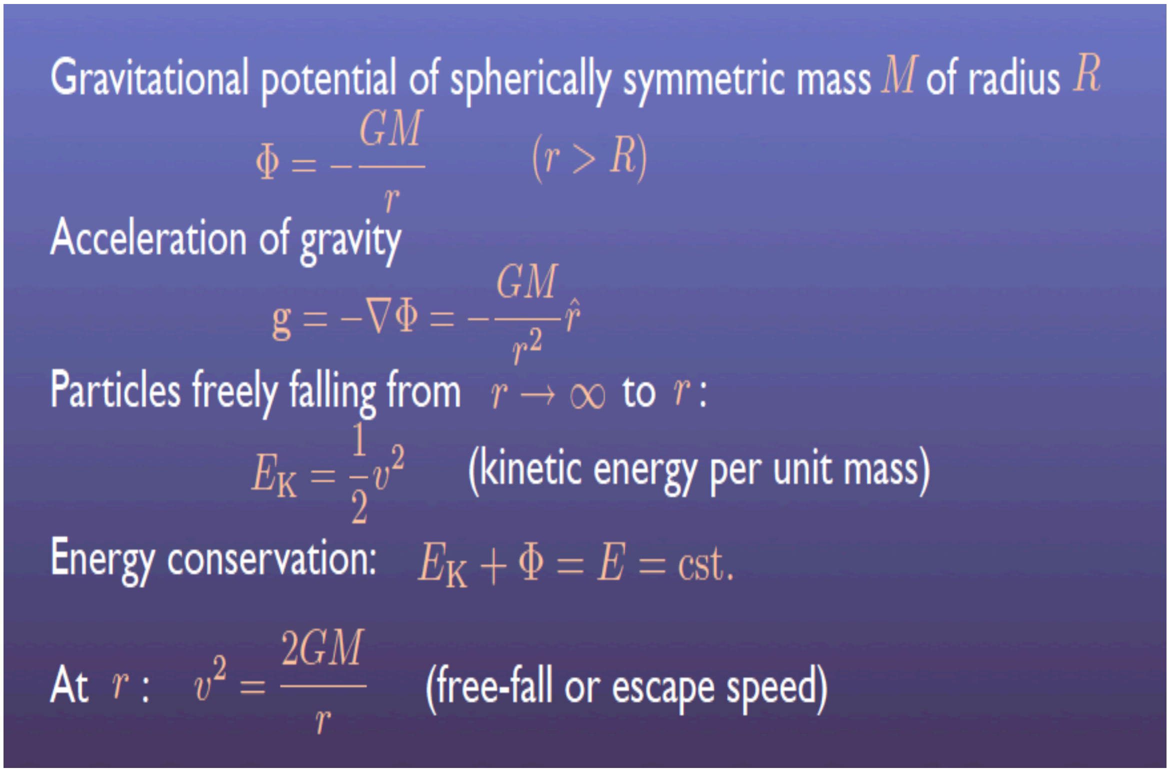
- Viscosity/friction moves angular momentum outward
  - allowing matter to spiral inward
  - Accreting onto the compact object at center

gravitational potential energy is converted by *friction* to heat Some fraction is radiated as light – physical origin of friction is complex.

Very efficient process Energy ~$GM/R=1.7\times10^{16}$ $(R/10\text{km})^{-1}$ J/kg ~$1/2mc^2$

Nuclear burning releases ~7×10^{14}J/kg (0.4% of mc^2)

- $L = 1/2\dot{m} c^2 (r_g/R)$ (14.3)
- This expression for the luminosity can be written $L = \xi \dot{m} c^2$, where ξ is the *efficiency of conversion* of the rest-mass energy of the accreted matter into heat.
- the efficiency is roughly $\xi = (r_g/2R)$ and so depends upon how compact the star is. For a white dwarf star with $M = M_\odot$ and $R \approx 5\times10^6$ m, $\xi \approx 3\times10^{-4}$.
- For a neutron star with mass $M = M_\odot$ and $R = 10$ km, $\xi \sim 0.15$.
- In the case of nuclear energy generation, the greatest release of nuclear binding energy occurs in the conversion of hydrogen into helium for which $\xi \approx 7\times10^{-3}$.
- Thus, accretion onto neutron stars is an order of magnitude more efficient as an energy source than nuclear energy generation.

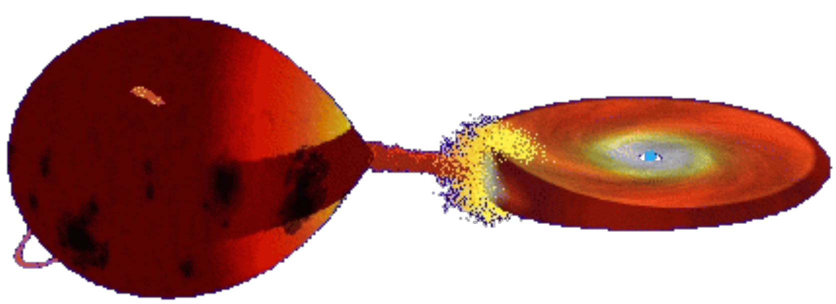


Viral temperature T_{viral}=GM/kr; for a NS M~1.4M_{sun}, R~10 km

T~10^{12}k

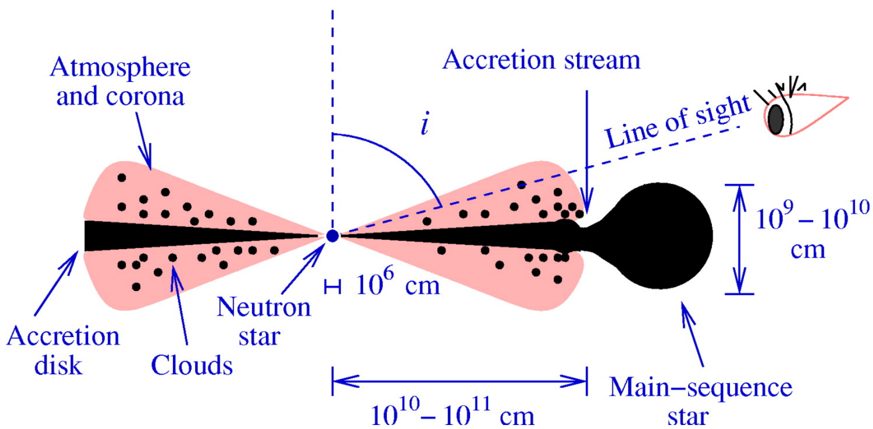
(H. Spruit)

Accretion from a Dwarf Companion

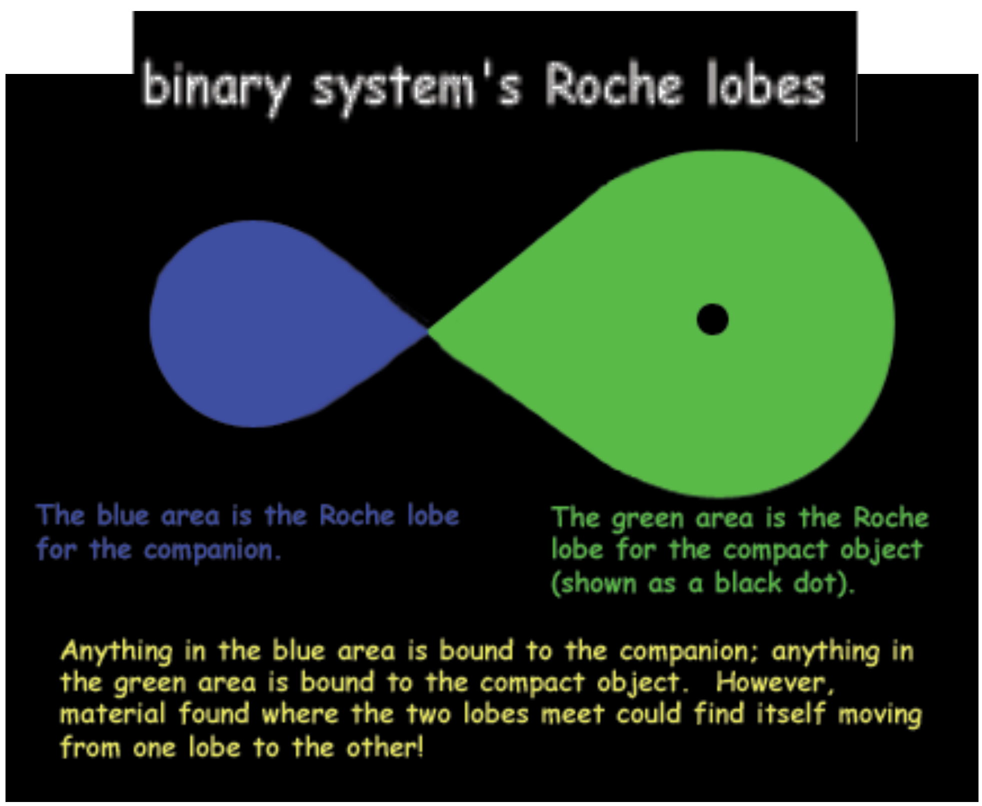


- http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif

Geometry of heated accretion disk + coronal in LMXB



Jimenez-Garate et al. 2002



Pulsars

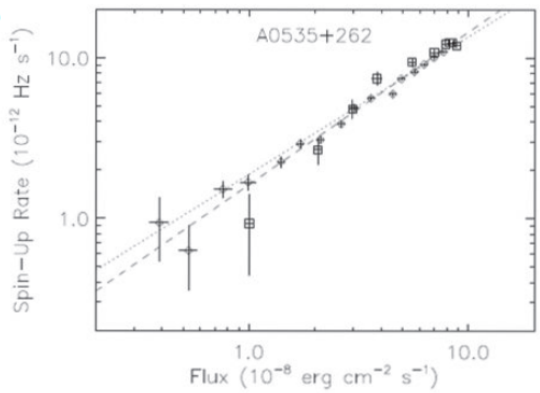
- The rate of change of the pulse period can
  - measure the orbital period of the source
  - The accreted angular momentum (e.g. the amount of material accreted)

 $(dP/dt)/P \sim (L/10^{37})^{6/7}$ (Ghosh and Lamb 1978) Longair 14.62,14.63

$\log_{10}(dP/dt)/P = -4.4 + \log_{10} P L_{37}^{6/7}$

L_{37} is the luminosity in units of 10^{37}ergs/sec

(a)



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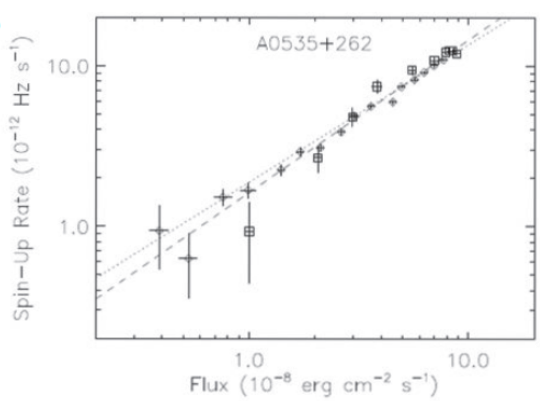


How much energy is released by accretion onto a compact object?

- Consider matter in an accretion disk assume that…
  - The matter orbits in circular paths (will always be approximately true)
  - Centripetal acceleration is mainly due to gravity of central object (i.e., radial pressure forces are negligible… will be true if the disk is thin)

- Energy is..

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

$$E = \frac{1}{2}v^2 - \frac{GM}{r} = -\frac{GM}{2r}$$

total luminosity liberated by accreting a flow of matter is - Longair 14.48

$$L = \left[0 - \left(-\frac{GM}{2r_{\rm in}}\right)\right]\dot{M} = \frac{GM\dot{M}}{2r_{\rm in}}$$

Initial energy (at infinity) — *Final energy* — *Mass flow rate*

- Total luminosity of disk depends on inner radius of dissipative
- part of accretion disk

- The matter falling in from infinity passes through a series of bound
- Keplerian orbits for which the kinetic energy is equal to half the gravitational potential energy.
- The matter dissipates half its potential energy in falling from infinity to radius *r* and this is the source of the luminosity of the disc.
- When the matter reaches the boundary layer, it has only liberated half its gravitational potential energy. If the matter is then brought to rest on the surface of the star, the rest of the gravitational potential energy can be dissipated.
- Thus, the boundary layer can be just as important a source of luminosity as the disc itself.- of course black holes do not have a surface so the energy 'disappears'

Thin accretion disks

Accretion disks form due to angular-momentum of incoming gas

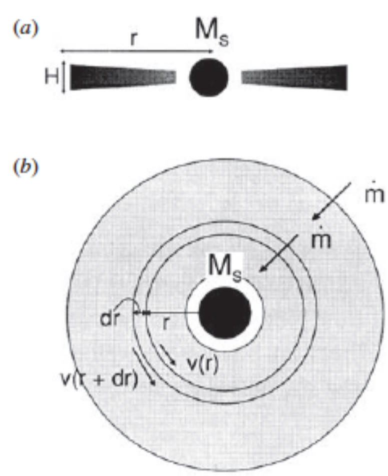
Once in circular orbit, specific angular momentum (i.e., per unit mass) is

$$J = vr = \sqrt{GMr}$$

So, gas must shed its angular momentum for it to actually accrete…

Releases gravitational potential energy in the process!

Matter goes in, angular momentum goes out!



A Simple Disk- see Longair 14.48

C. Done IAC winter school

- The underlying physics of a Shakura-Sunyaev accretion disc (a very simple derivation just conserving energy -rather than the proper derivation which conserves energy and angular momentum).
- A mass accretion rate $\mathcal{M}$ spiraling inwards from R to R-dR liberates potential energy at a rate dE/dt = L_{pot} = (GM$\mathcal{M}$/R^2)xdR.
- The virial theorem says that only half of this can be radiated, so

  dL_{rad} = GM$\mathcal{M}$ dR/(2R^2).
- If this thermalises to a blackbody then dL = (dA)xkT^4 where k is the Stephan-Boltzman constant and area of the annulus dA = 2 x 2πRxdR (where the factor 2 comes from the fact that there is a top and bottom face of the ring).
- Then the luminosity from the annulus dL_{rad} = GM$\mathcal{M}$ dR/(2R^2)= 4πdRkdRT^4 or
  - $kT^4(R)$ = (GM$\mathcal{M}$/8πR^3)
- This is only out by a factor $3(1-(Rin/R)^{1/2})$ which comes from a full analysis including angular momentum
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

The First Physical Disk Model- Longair 14.3.3

- The first physical model of a disk was developed by Shakura and Sunyaev in 1973
- They made a set of reasonable assumptions which have proved to be reasonable.
- The disk is optically thick
- The local emission should consist of a sum of quasi–blackbody spectra

Temperature Structure of Accretion disk Longair14.3.5

- Energy released by an element of mass in going from r+dr to r

Gravitational potential energy is

E_p =-GMm/2r so energy released is

E_g =-GMmdr/r^2.

the luminosity of this annulus, for an accretion rate $\mathcal{M}$, is

dL ~ GM$\mathcal{M}$ dr/r^2.

assuming the annulus radiates its energy as a blackbody

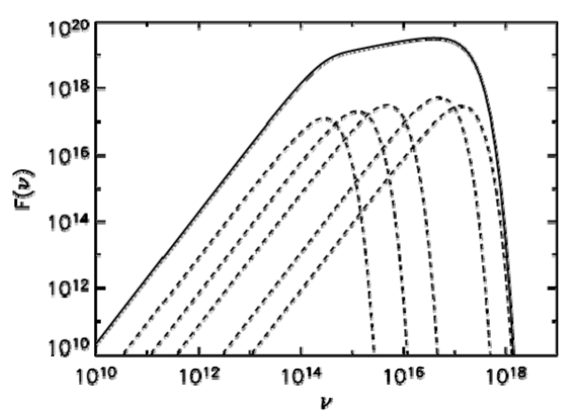
For a

- blackbody, L = σAT^4. The area of the annulus is $2\pi r dr$, and since
- L=M$\mathcal{M}$ dr/r^2 we have
- T^4 ~M$\mathcal{M}r^{-3}$, or
- **T ~$(M\mathcal{M}/r^3)^{1/4;}$ e.g. T~$r^{-3/4}$; T~$\mathcal{M}^{1/4}$**
- .

Total Spectrum- see Longair eqs 14.54-14.57

- If each annulus radiates like a black body and the temperature scales as T~$r^{-3/4}$
- The emissivity scales over a wide range of energies as $I(\nu)\sim\nu^{1/3}$
- At lower frequencies the spectrum has a Raleigh-Jeans ν^2 shape and at higher energies has a exponential cutoff corresponding to the maximum temperature ($e^{-h\nu/kT_{inner}}$)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

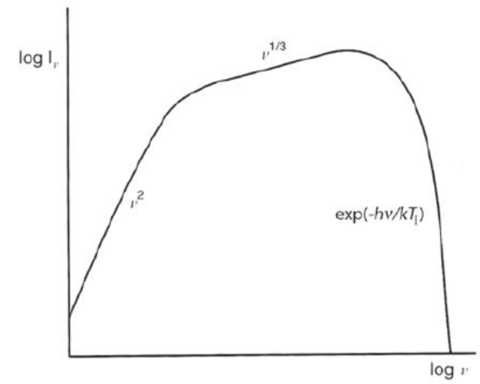
Standard Disk Spectrum



Total Spectrum

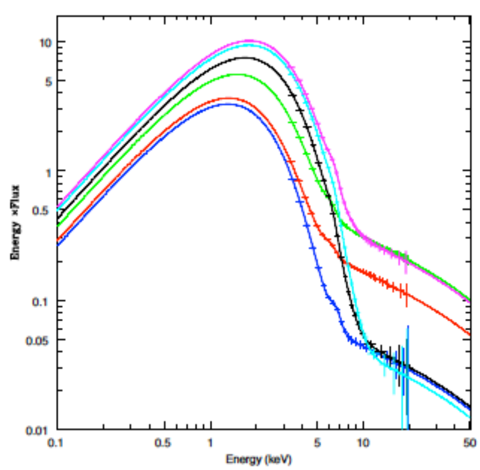
- If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$ (Longair 14.54)
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$
- At lower frequencies the spectrum has a Raleigh-Jeans ν^2 shape and at higher energies has a exponential cutoff corresponding to the maximum temperature ($e^{-h\nu/kT_{inner}}$)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

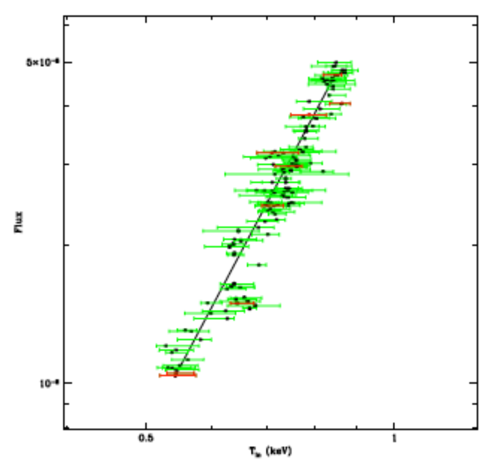
If the disk 'cuts off' at some radius r_{inner} then the temperature profile is $T(r) = 3GM\dot{m}/8\pi\sigma r^3[1 - (r_{inner}/r)^{1/2}]^{1/4}$ *eq in 14.7.1.*



the emission spectrum of an optically thick accretion disc.
The exponential cut-off at high energies occurs at frequency $\nu = kT_l/h$, where T_l is the temperature of the innermost layers of the thin accretion disc. At lower frequencies the spectrum tends towards a Rayleigh–Jeans spectrum $I \propto \nu^2$.

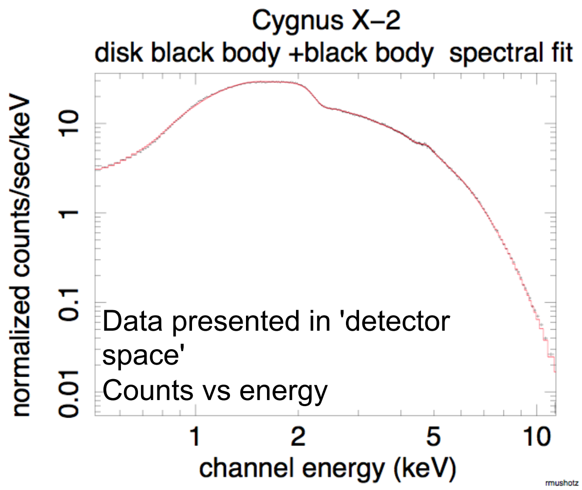
Do They Really Look Like That

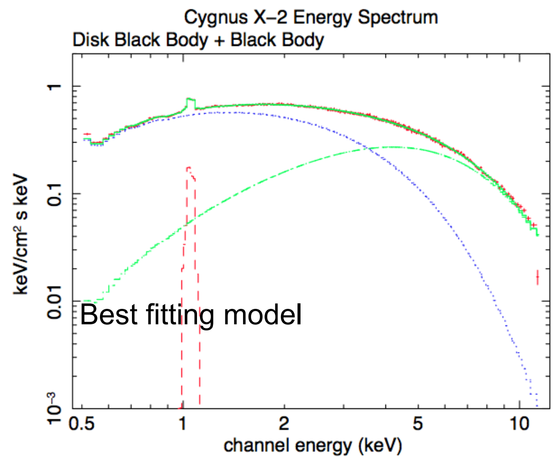




- X-ray spectrum of accreting Neutron star at various intensity levels
- Right panel is $T(r_{in})$ vs flux - follows the T^4 law

Fit to Real Data

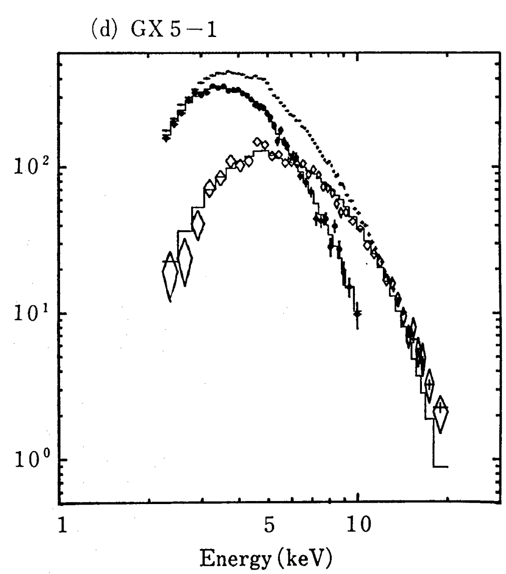




The data is of very high signal to noise
Simple spectral form fits well over a factor of 20 in energy
Emitted energy peaks over broad range from 2-6 kev

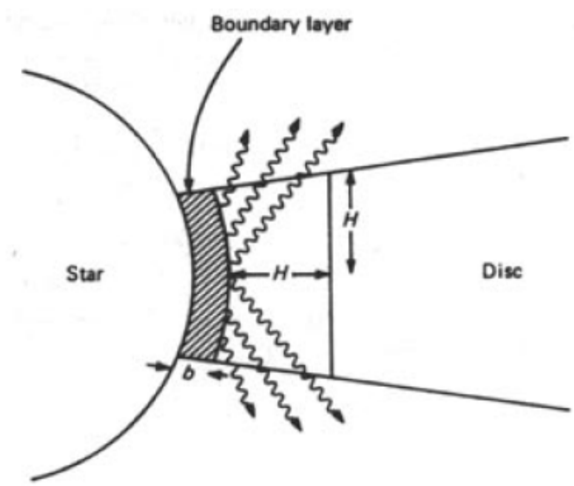
Actual Neutron Star X-ray Spectra

- Low Mass x-ray binaries (NS with a 'weak' magnetic field) have a 2 component spectrum
  - The low energy component is well described by a multi-color disk black body spectrum
  - And the hotter temperuture black body is related to the boundary layer
    - However the observed temperatures disagree with simple theory due to three effects
      - General relativity
      - The 'non-black body' nature of the radiation
      - Reprocessing of the radiation of the central regions by the outer regions and then re-emission



Where is the Accretion Occurring

- In a weak field neutron star (LMXB) has the accretion disk and the place where the material hits the star surface (boundary layer) produce the radiation



How is the Potential Energy Released

- Suppose that there is some kind of "viscosity" in the disk
  - Different annuli of the disk rub against each other and exchange angular momentum
  - Results in most of the matter moving inwards and eventually accreting
  - Angular momentum carried outwards by a small amount of material

- Process producing this "viscosity" might also be dissipative… could turn gravitational potential energy into heat (and eventually radiation)
- Physics of the 'viscosity' is very complex- it turns out that it is due to magnetic fields and an instability magnetorotational instability (MRI) , by which weak magnetic fields are amplified by differential rotation, gives the required viscosity

General Considerations

- The luminosity that results from accretion is roughly
  - $L \sim \varepsilon c^2 \mathscr{M}$ ($dM/dt = \mathscr{M}$) Where $\varepsilon = GM/Rc^2$ (the depth of the potential)
    - $\varepsilon \sim 3 \times 10^{-4}$ for a white dwarf and 0.1 for a neutron star

- If the gas flow is spherically symmetric and steady state the luminosity should not exceed the Eddington limit (outward force from Compton scattering balances gravity)
- The Compton optical depth in a spherical accretor is $\tau = (2/\varepsilon)^{1/2} L/L_{edd}$

- Two natural temperatures
  - Free fall $kT = 3/16\ \varepsilon m_h c^2 = 2 10^5 \varepsilon$ kev
  - Black body temperature: minimum temperature for the object to radiate the observed luminosity
    - $T_{BB} \sim (L/A\sigma)^{1/4}$;A is the area and σ is the Stefan-Boltzman constant

    - about 0.2 keV for a white dwarf and 2 keV for a neutron star

General Considerations

- Time scales:

$\tau_{dyn} = (r^3/G\ M)^{1/2}$ This is about 0.1 ms for matter at r = 10 km, and 2 ms at r = 100 km.

The typical orbital period of circulating matter,

$P_{orb} = 2\pi\tau_{dyn}$ ~1 ms:

- Characteristic velocity is $\sim (GM/R)^{1/2} \sim 0.5c$.

- The two main accretion mechanisms are
  - Roche lobe over flow, which most often occurs in low-mass binaries (LMXB,low B field, accretion disk and boundary layer dominated)
  - and stellar wind capture, which is common for high-mass binaries with super-giant companions (high B fields, pulsars)

Basics of Accretion

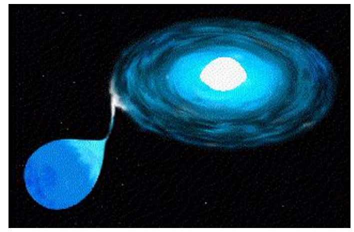
- Because of angular momentum considerations an accretion disk, **almost** always forms
- Matter is thought to form a physically thin (but optically thick* disk) which has Keplerian rotation
- Matter falls into by losing angular mometum via viscosity

the angular velocity is Ω_k=sqrt(GM/r^3)

The binding energy of a parcel of the disk is $E=GM_{disk}M_x/2R= 1/2\ L_{acc}$

The other half of L_{acc} is released very close to the star surface (the boundary layer) as matter in the disk tries to co-rotate with the NS (what happens for a black hole??)

If the star spins more slowly than the innermost part of the accretion disk (angular speed $\omega_{k)}$, the BL must release a large amount of energy as the accreting matter comes to rest at the stellar surface. Some of this is used to spin up the star, but there remains an amount $G\mathcal{M}M_x/2R(1- \omega_{k/}\Omega_k{}^{)2}$ which is radiated



* Optically thick means that a photon emitted inside the disk always interacts with matter at least once before 'escaping'

Total Spectrum

- If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$ (Longair 14.54)
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$
- At lower frequencies the spectrum has a Raleigh-Jeans ν^2 shape and at higher energies has a exponential cutoff corresponding to the maximum temperature ($e^{-h\nu/kTinner}$)
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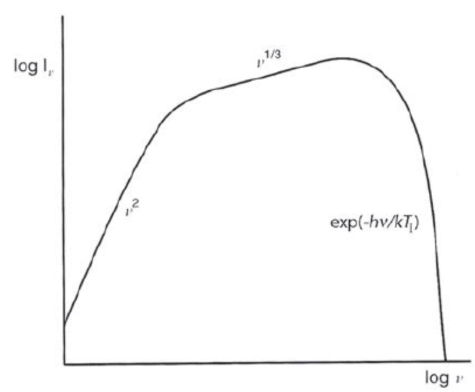

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Effects of Geometry on Observed Properties can be Huge (P.Charles)