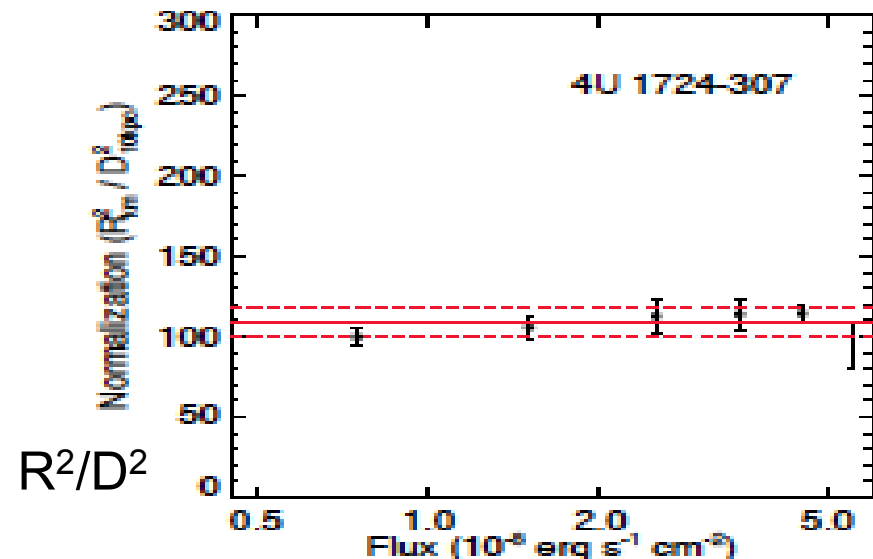
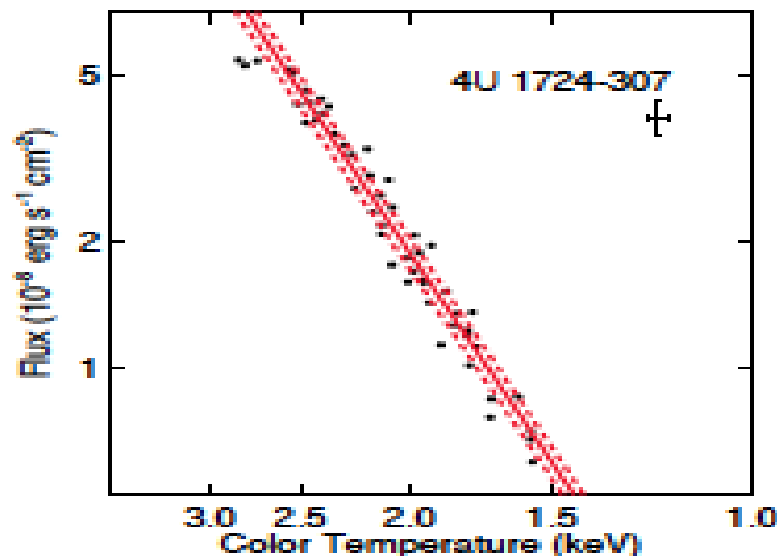


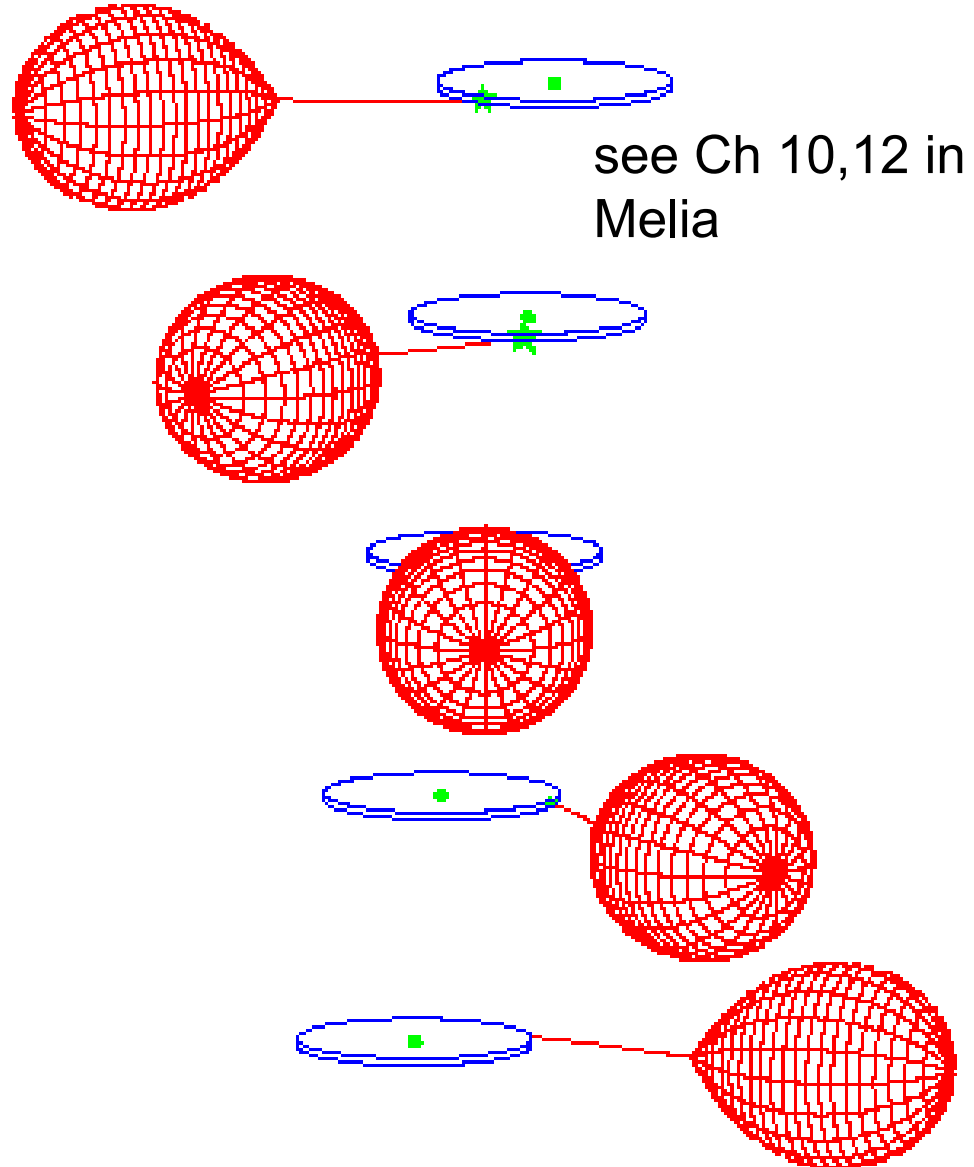
Recent Results on NS Eq of State

- Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. I. Apparent Radii T Guver, D., D. Psaltis & F Ozel 2012
- The masses and radii of low-magnetic field neutron stars can be measured by combining different observable quantities obtained from their X-ray spectra the apparent radius of each neutron star can be inferred from the X-ray flux and spectral temperature
- Need accurate absolute distances to make further progress



Black Holes

- What do you mean 'black holes' ?
- We know of objects whose mass (derived from observations of the lines from the companion objects and Newton's (Einstein) laws) which **are larger than possible for a NS or white dwarf.**
- They have other unusual properties (related to their x-ray spectrum and timing behavior)
- **Big differences- no surface, no (?) magnetic field, higher mass, stronger GR effects.**



How Can We Observe Black Holes

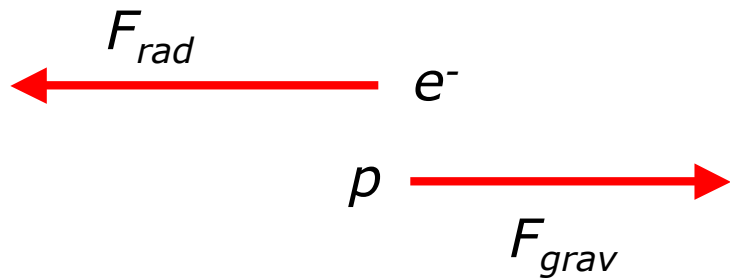
- If a black hole is a 'place' where radiation cannot escape to infinity how can they be observed ?
- Dynamical effects on 'nearby' material
- “Shining” black holes- a black hole can be a place where accretion occurs and as we have seen the process of accretion around a compact object can produce huge amounts of energy and radiation- making the black hole 'visible'

What are the possible energy sources?

- Accretion?
 - Release of gravitational potential energy as matter falls into black hole
 - YES! Thought to be primary power source of all systems just discussed
- Rotational energy of black hole
 - Tapping the rotational energy of a spinning black hole $\frac{1}{2}I\Omega^2$ can be very large
 - May be important in some settings... but can only be tapped if accretion occurring! (Blandford + Znajek 1977 ('Electromagnetic extraction of energy from Kerr black holes'))

How luminous can an accreting black hole be?-
this is the same Eddington limit as we discussed
for neutron stars

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi cr^2}$$



$$F_{\text{grav}} = \frac{GMm_p}{r^2}$$

- The accreting matter is pushed away if

$$F_{\text{rad}} > F_{\text{grav}}$$

- This is the Eddington limit (L_{Edd}). Acts effective upper limit to the luminosity of accretion powered object. Numerically:

$$L > \frac{4\pi G m_p c}{\sigma_T} M$$

$$L_{\text{Edd}} \approx 1.3 \times 10^{31} \left(\frac{M}{M_{\odot}} \right) \text{ W}$$

General properties of emission from black hole systems

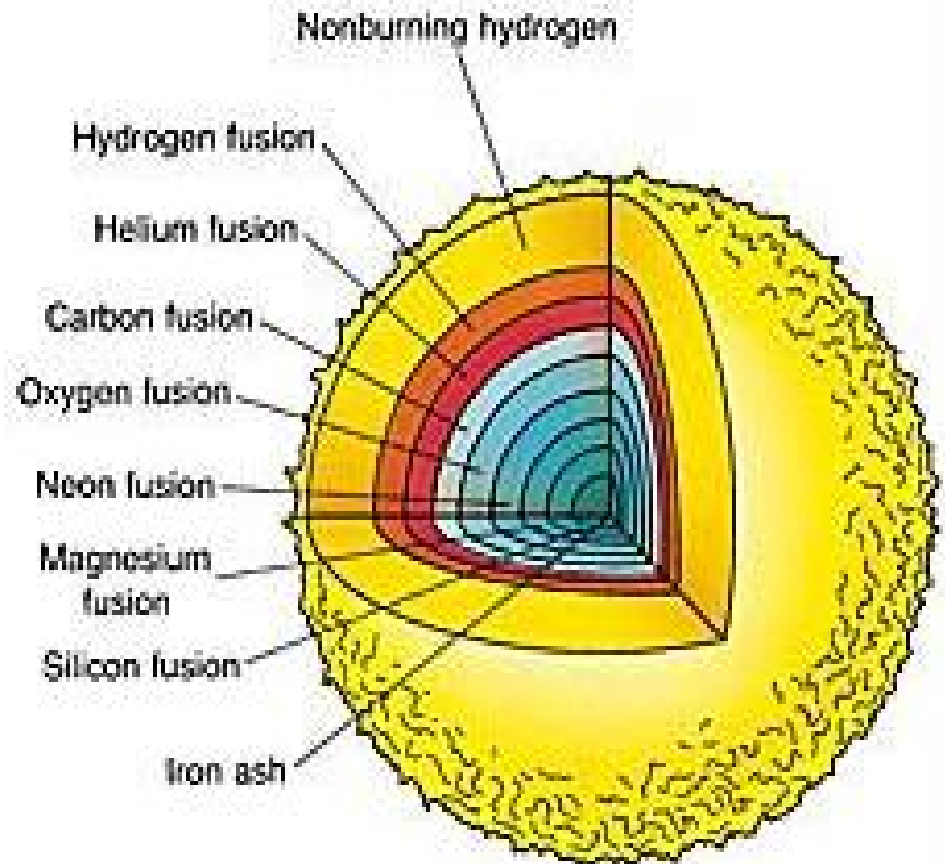
- Emission usually variable on wide variety of timescales
 - Galactic black hole binaries : millisecond and up
 - AGN : minutes and up
 - Most rapid variability approaches light-crossing timescale limit of physical size of object ($\tau \sim R/c$)
- Significant emission over **very** broad spectral range (radio to hard X-ray or gamma-rays)-NS and WDs tend to have 'thermal' like spectra (relatively narrow in wavelength)
- Lack of a signature of a surface - not a pulsar, no boundary layer emission (no x-ray bursts), no 'after glow' from cooling

Downwards to Black Holes!

- a neutron star has a maximum mass
- If this mass is exceeded on has a complete gravitational collapse to a black hole
- Basic anatomy of a black hole
- Observational discovery of black holes

Beyond neutron stars...

- Suppose collapsing core has mass that exceeds maximum mass for a neutron star...this can happen in several ways
 - Maybe a more massive iron core forms before it cools to the point that degeneracy pressure kicks in...
 - ... or initial core collapse of $1.4M$ core is followed by more infall from stellar envelope?
- What then occurs if the gravitational attraction exceeds the degeneracy pressure?
- We know of no physics that can prevent a total gravitational collapse of the core

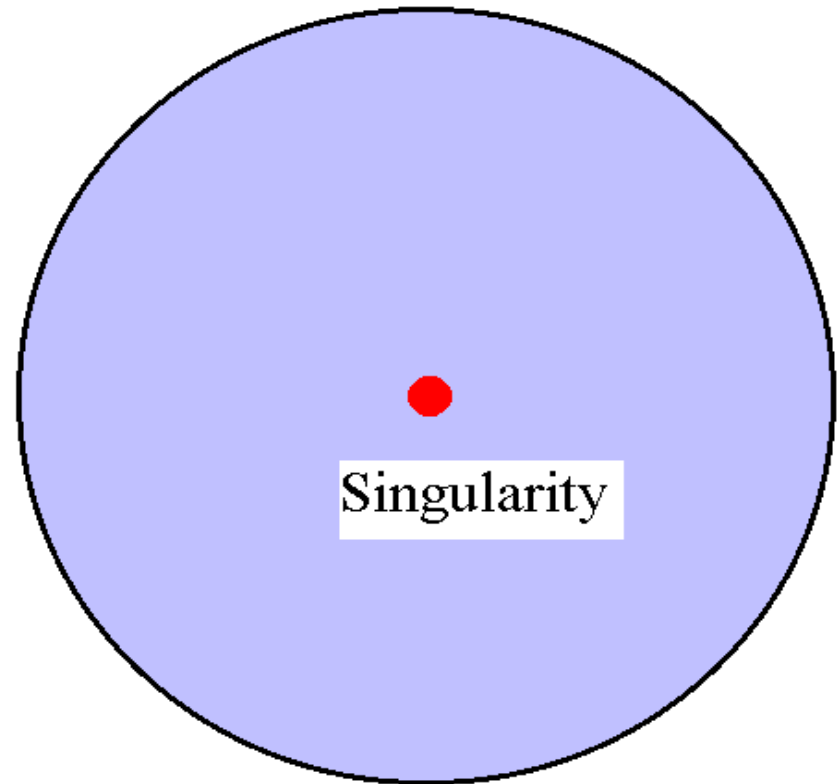


Basic anatomy of a black hole

- Complete gravitational collapse inevitably leads to a black hole (Hawking)
- Space-time singularity
 - Where the mass-energy resides
 - *Place where GR breaks down and laws of quantum gravity must be applied*
- Event horizon
 - Point of no return for light or matter
 - Events inside horizon can have no causal effect on universe outside of the horizon
 - Analogous to the point of no return in a waterfall

*black holes have no hair

3 parameters mass, angular momentum, and electric charge completely characterize black holes. Everything else (quadrupole terms, magnetic moments, weak forces, etc.) decays away*.



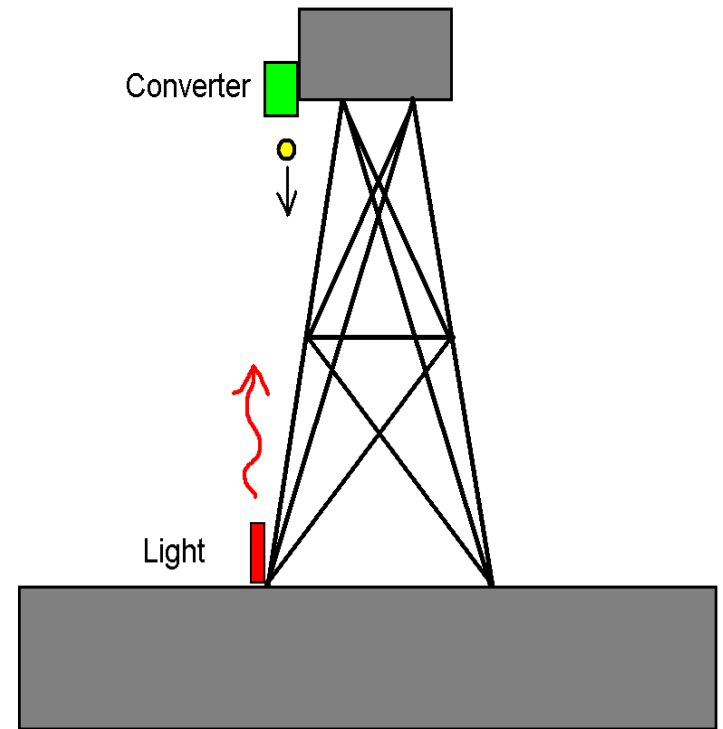
Event horizon

Schwarzschild Radius-AKA the Event Horizon for a Non-Spinning Black Holes

- $R_s = 2GM/c^2$
- The Schwarzschild radius is the radius of 'no return' for a non-rotating black hole- it is not the singularity.
- Events inside that horizon cannot be seen by any external observer
- inside the event horizon the radius becomes a timelike coordinate, and the time becomes a spacelike coordinate. Specifically, that means that once inside R_s , you must go to smaller radii, just as now you must go forward in time
- once you're inside the event horizon one cannot avoid the singularity at $r = 0$

Gravitational redshift

- Thought experiment:
 - Send photon upwards in a gravitational field
 - Convert that energy into mass and drop the mass
 - Convert mass back into photon
- Conservation of energy \Rightarrow photon must lose energy as it climbs in the gravitational field
- Another way of thinking about this - the escape speed from the object has to be less than the speed of light (assuming, incorrectly, that light could slow down and fall).
- In Newton mechanics the escape speed is $v^2 = 2GM/r$, so $v^2 = c^2$ at $r = 2GM/c^2$
- Redshift of light $Z = (\lambda_0 - \lambda_e) / \lambda_e$; λ_0 = wavelength as measured by the observer, λ_e as emitted



Gravitational redshifts near a black hole

- Gravitational redshift is really a form of relativistic time dilation
- As observed from infinity, time near a (non-spinning, non-charged) black hole runs slow by a factor of

$$\frac{\Delta t'}{\Delta t} = \frac{1}{\sqrt{1 - 2GM/c^2}}$$

- The event horizon is the “infinite redshift” surface where (as observed from infinity) time appears to stop!
- But... a free falling observing would fall through the event horizon without noticing anything unusual.
- The wavelength of light is redshifted ($Z=(\lambda_0-\lambda_e)/\lambda_e$; λ_0 = wavelength as measured by the observer, λ_e as emitted) by
- $z=(1/\text{sqrt}(1-R_s/r)) -1$

$$z = \frac{1}{\sqrt{1 - \left(\frac{2GM}{rc^2}\right)}} - 1$$

Question for class- what is the redshift from the surface of a NS?

- $M \sim M_{\text{sun}}$; $R=10\text{km}$ (set by nuclear physics)

So what is the actual size ?

$$R_G \sim 1.5 (M / M_\odot) \text{ km}$$

So how close are neutron stars to being black holes ?

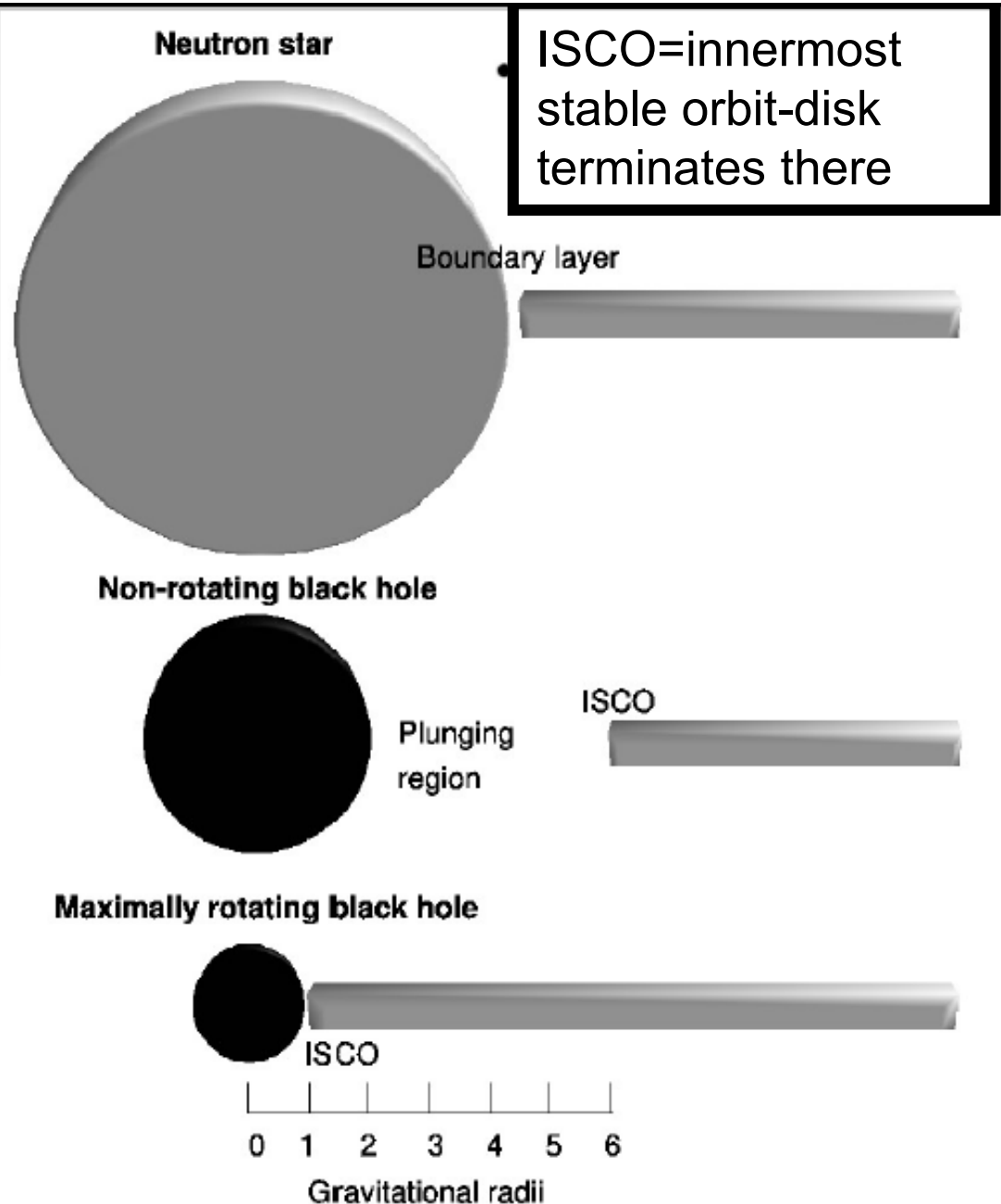
Neutron stars are only about a factor 2—3 larger than their event horizons

What about spin ?

A non-rotating (“Schwarzschild”) black hole has its event horizon at $2 R_G$ and its ISCO at $6 R_G$

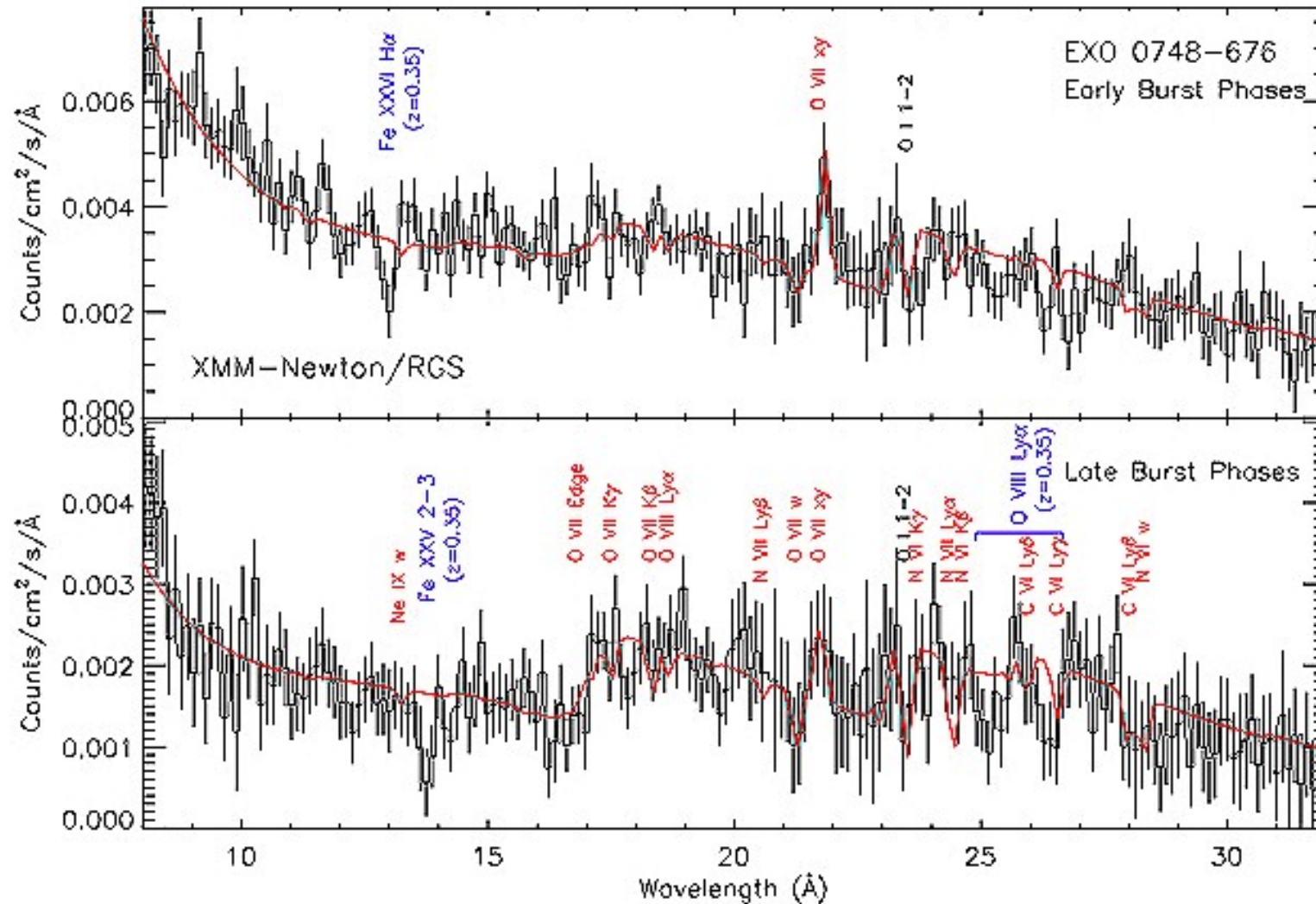
A maximally rotating (“Maximal Kerr”) black hole has both its event horizon and ISCO at R_G

→ Spinning black holes are more compact → potentially more radiatively efficient



R. Fender 2007

*Emission of line radiation from highly ionized atoms of Fe
And O from near the surface of a NS*



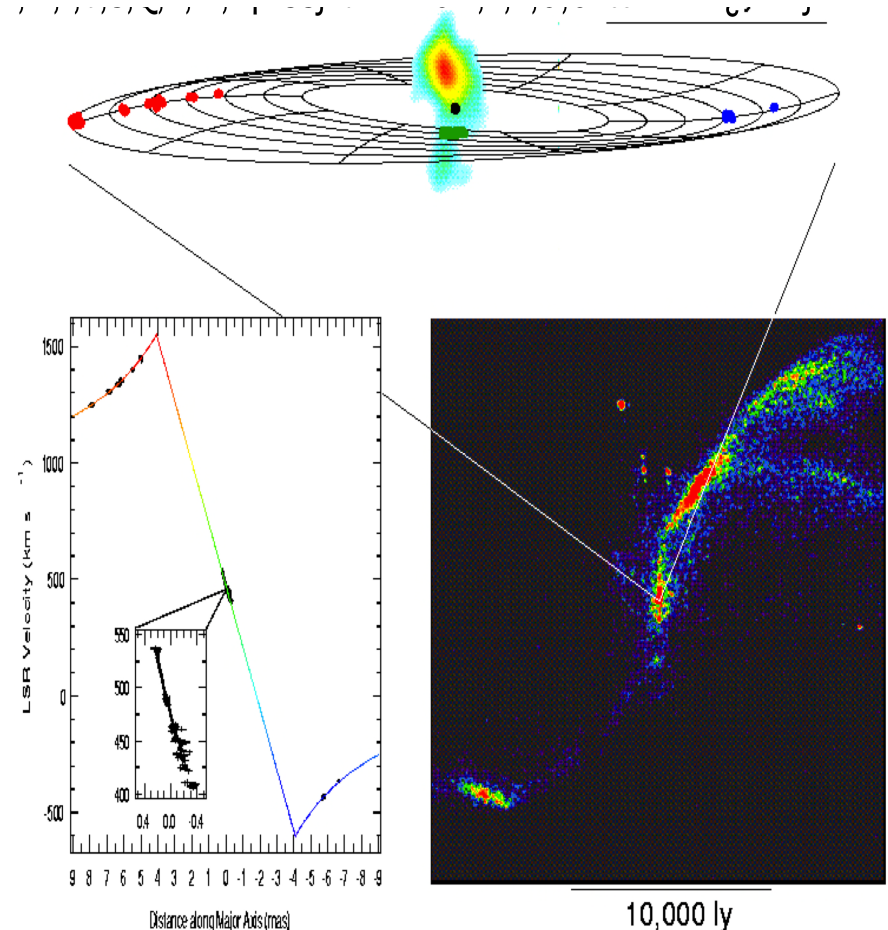
Redshifted absorption lines from a neutron star surface

Cottam, Paerels & Mendez (2002)

$-\beta < 9$ (DeDeo & Psaltis 2003)

Examples of Astrophysical Black Holes

- We know that black holes come in 2 size scales
 - 5-20 M_{sun} ; the result of stellar evolution
 - 10^6 - $10^9 M_{\text{sun}}$ super massive black holes that reside in the centers of **most** massive galaxies
 - They may also come in another size scale; intermediate mass black holes with $50 < M_{\text{sun}} < 10^3$
- Detailed stellar evolution calculations indicate that for a star with roughly solar metallicity the maximum mass of the remnant black hole is $\sim 20 M_{\text{sun}}$

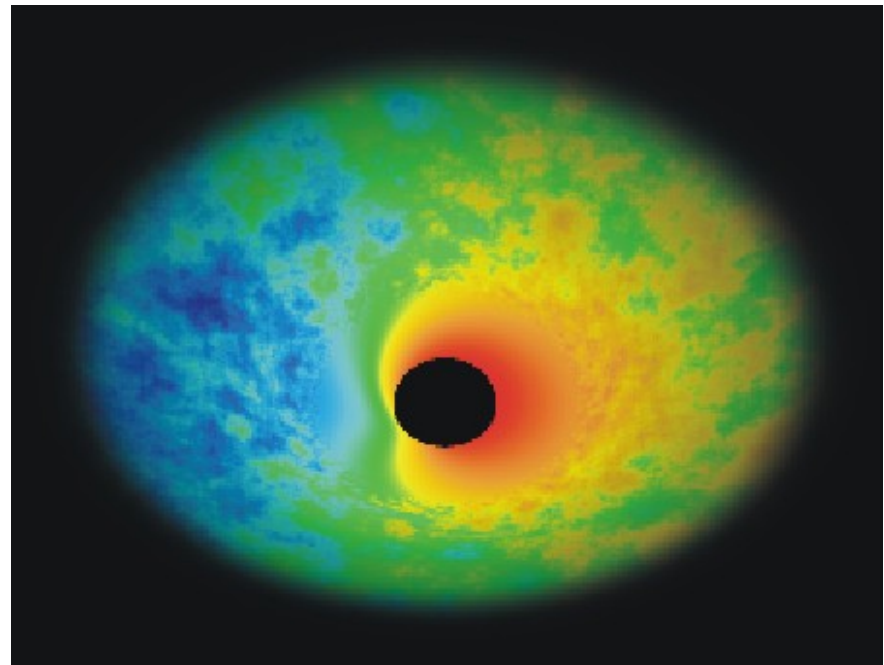


Miyoshi et al

Black Hole Metric

- The Schwarzschild solution for the metric around a point mass is
- $ds^2 = -(1 - 2GM/c^2r)c^2dt^2 + (1 - 2GM/c^2r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$
- Notice singularity at $r = 2GM/c^2$ (can be gotten rid of in a coordinate transformation)
- A static observer measures proper time $c^2d\tau^2 = -ds^2 = (1 - 2GM/c^2r)c^2dt^2$
- $d\tau/dt = \sqrt{1 - 2GM/c^2r} = 1 + z_{\text{grav}}$
- A spinning black hole is described by the Kerr metric

$$R_{Sch} = \frac{2GM}{c^2} \approx 3 \left(\frac{M}{M_{\odot}} \right) \text{ km}$$



More features of Schwarzschild black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)
- Observer who enters event horizon would only "feel" "strange" gravitational effects if the black hole mass is small, so that R_s is comparable to observer's size
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
- Thus black hole would produce "shadow" on sky

Evidence for black holes

- Galactic black hole candidates (dynamical measurements- last class using velocity of companion and period)

For Supermassive Black Holes

Dynamics of 'Test particles'

Orbits of gas disks around mass compact objects at the centers of other galaxies- best case is NGC4258 (water maser orbits)

Stellar orbits around a compact mass at the center of our own Galaxy (most solid case for any black hole)

Of course what these data give is the mass inside a given radius.
If the mass density is higher than (?) it must be a black hole

- Emission from the region of 'strong gravity'

Extreme gravitational redshifting of emission lines in the X-ray spectrum of some accreting black holes

Rotating black holes- remember the extra special nature of accelerated frames

- Roy Kerr (1963)
 - Discovered solution to Einstein's equations corresponding to a *rotating* black hole
 - Kerr solution describes all black holes found in nature
- Features of the Kerr solution
 - Black Hole completely characterized by its mass and spin rate (no other features [except charge]; **no-hair theorem**)
 - Has space-time **singularity** and **event horizon** (like Schwarzschild solution)
 - Also has “**static surface**” inside of which nothing can remain motionless with respect to distant fixed coordinates
 - Space-time near rotating black hole is dragged around in the direction of rotation: “**frame dragging**”.
 - **Ergosphere** – region where space-time dragging is so intense that its impossible to resist rotation of black hole.

Spinning BH- Longair sec 13.11.2

- A black hole with angular momentum \mathbf{J} has a metric

$$ds^2 = (1 - 2GM/r/c^2)dr^2 - \{(1/c^2)[4GMra \sin^2 \theta / \rho c] dr d\phi\} + (\rho/\Delta)dr^2 + \rho d\theta^2 + (r^2 + a^2 + 2GMra^2 \sin^2 \theta / \rho c^2) \sin^2 \theta d\phi^2] -- \text{Longair eq 13.63}$$

r, θ, ϕ – usual polar coordinates

where $a = (\mathbf{J} / Mc)$ is the angular momentum per unit mass (dimensions of distance) and

$$\Delta = r^2 - (2GM/r/c^2) + a^2 \quad ; \quad \rho = r^2 + a^2 \cos^2 \theta$$

Just like Schwarzschild metric it becomes singular but at a radius where

$\Delta = r^2 - (2GM/r/c^2) + a^2 = 0$; the larger root is

$$r_+ = GM/c^2 + [(GM/c^2)^2 - (J/Mc)^2]^{1/2}$$

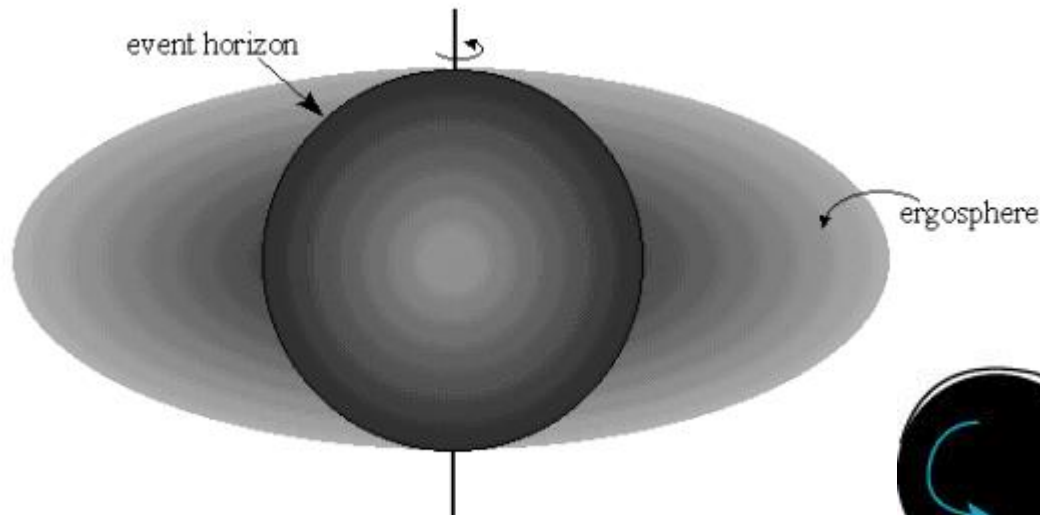
for $J > 0$ this is smaller than the Schwarzschild radius

there is a maximum angular momentum $\mathbf{J} = GM^2/c$; for this value of J the horizon is at $r_+ = GM/c$; 1/2 of the Schwarzschild radius

Schwarzschild and Kerr Metric

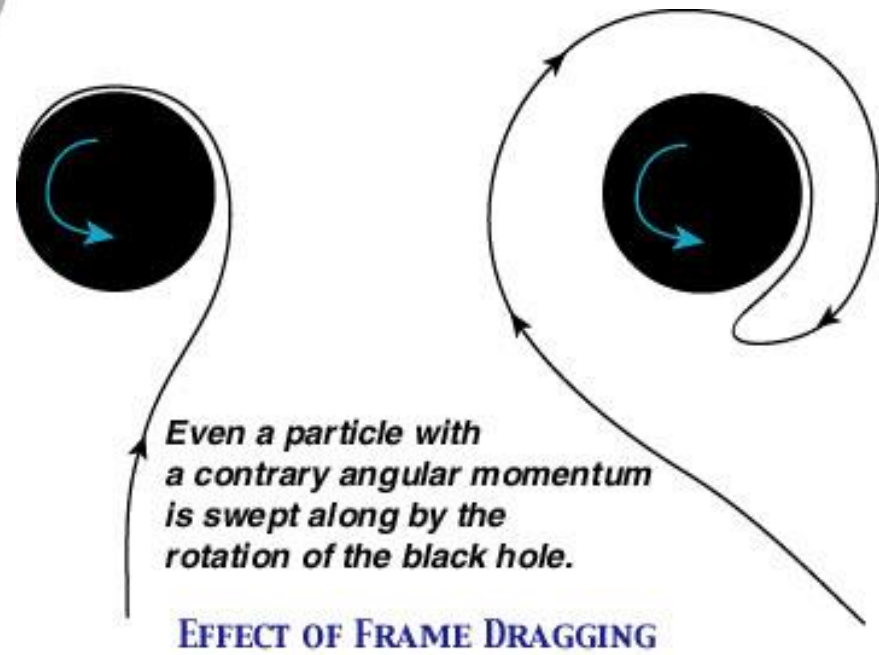
- for a Schwarzschild BH the innermost **stable** radius is $3r_G = 6GM/c^2$ - there are no stable circular orbits at smaller radii
 - the binding energy from this orbit is **0.0572 of the rest mass energy**
- For a Kerr the innermost stable radius is at $r_+ = GM/c^2$ The spinning black hole drags the the inertial frame-
- The smaller critical radius allows more energy to be released by infalling matter
 - For a Kerr BH **0.423 of the energy can be released.**
- There is another 'fiducial' radius in the Kerr solution, that radius within which all light cones point in the direction of rotation, the 'static' radius, r_{static} .
- Between r_{static} and r_+ is a region called the 'ergosphere' within which particles must rotate with the black hole and from energy might be extracted (Penrose process).

Frame dragging by rotating black hole



tested in 2004

<http://www.universetoday.com/9984/frame-dragging-confirmed/>



3/16/13

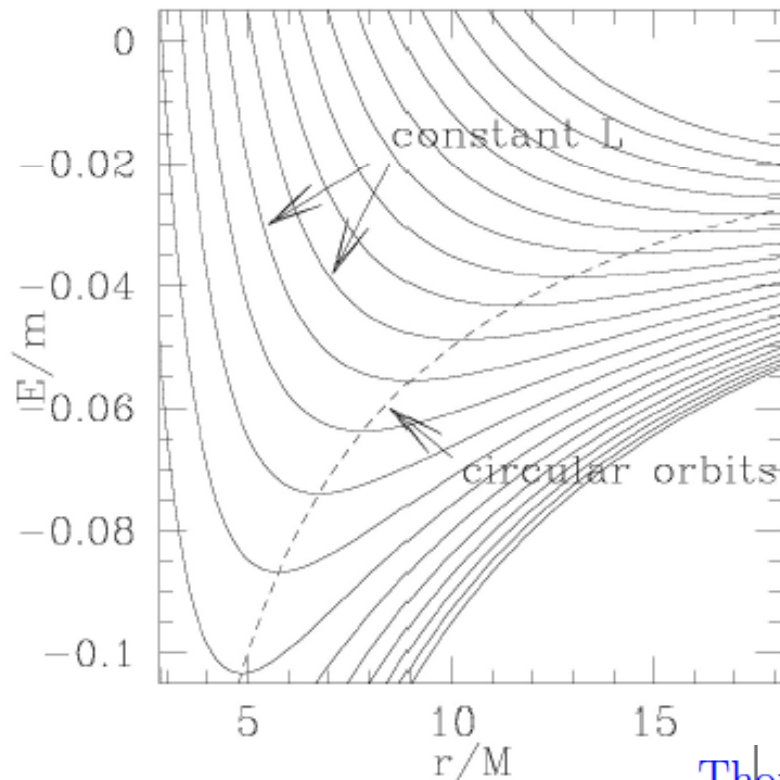
Graphics: University of Winnipeg, Physics Dept.

The innermost stable circular orbit (ISCO)

circular orbit extremizes binding energy E of test mass m at const. angular momentum L

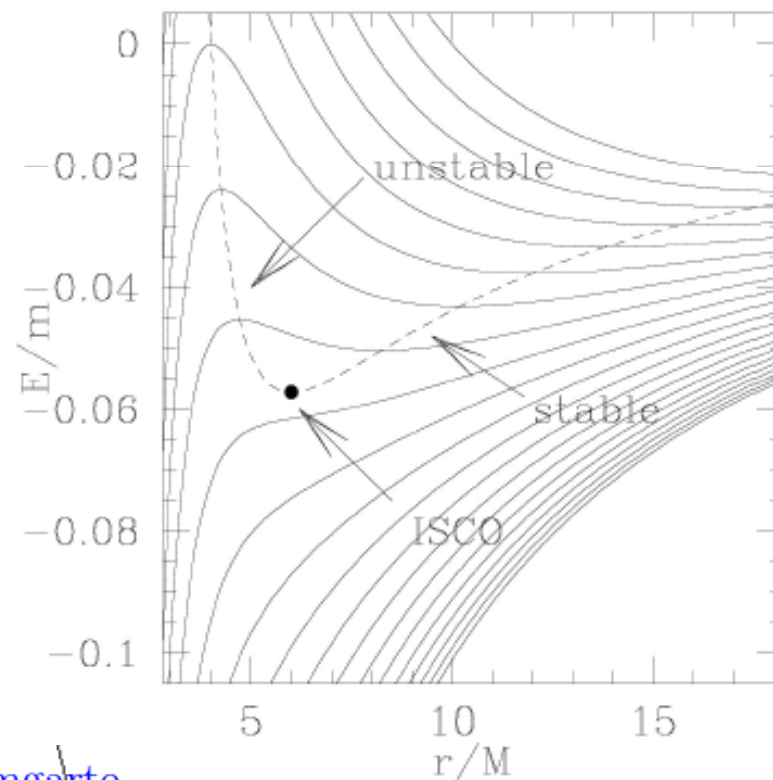
Newtonian point mass

$$\frac{E}{m} = -\frac{M}{r} + \frac{L^2}{2r^2}$$



Schwarzschild black hole

$$\frac{E}{m} = \left(\left(1 - \frac{2M}{r} \right) \left(1 + \frac{L^2}{r^2} \right) \right)^{1/2} - 1$$

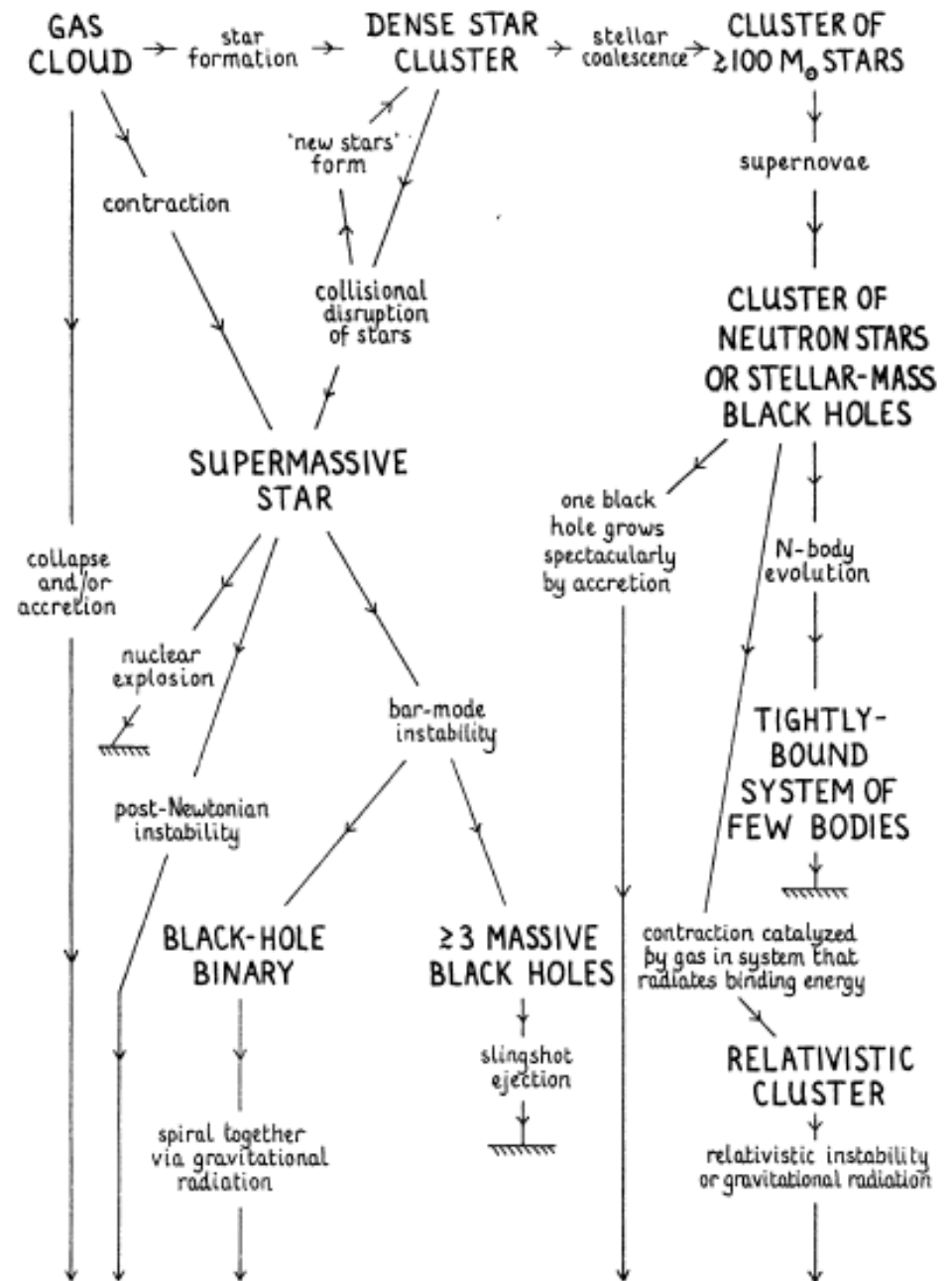


Thomas Baumgarte

Bowdoin College

Formation of SuperMassive Black Holes

In a dense region all roads lead to a black Hole (Rees 1984 ARAA)



massive black hole

Some Scales (Rees 1984)

A central mass M has a gravitational radius

$$r_g = \frac{GM}{c^2} = 1.5 \times 10^{13} M_8 \text{ cm}, \quad 1.$$

where M_8 is the mass in units of $10^8 M_\odot$. The characteristic minimum time scale for variability is

$$r_g/c \simeq 500 M_8 \text{ s}. \quad 2.$$

A characteristic luminosity is the “Eddington limit,” at which radiation pressure on free electrons balances gravity:

$$L_E = \frac{4\pi GMm_p c}{\sigma_T} \simeq 1.3 \times 10^{46} M_8 \text{ erg s}^{-1}. \quad 3.$$

Related to this is another time scale

$$t_E = \frac{\sigma_T c}{4\pi G m_p} \simeq 4 \times 10^8 \text{ yr.} \quad 4.$$

The time scale to grow a black hole if it were accreting at the Eddington luminosity

The characteristic black body temperature if the Eddington luminosity is emitted at r_g

$$T_E \simeq 5 \times 10^5 M_8^{-1/4}.$$

More massive BHs are cooler

What can come **out** of black hole?

- Magnetic fields threading ergosphere can attach to and drag surrounding matter, reducing the black hole's spin and energy
- “Hawking Radiation”: black hole slowly evaporates due to quantum mechanics effects
 - Particle/antiparticle pair is created near BH
 - One particle falls into horizon; the other escapes
 - *Energy to create particles comes from gravity outside horizon*

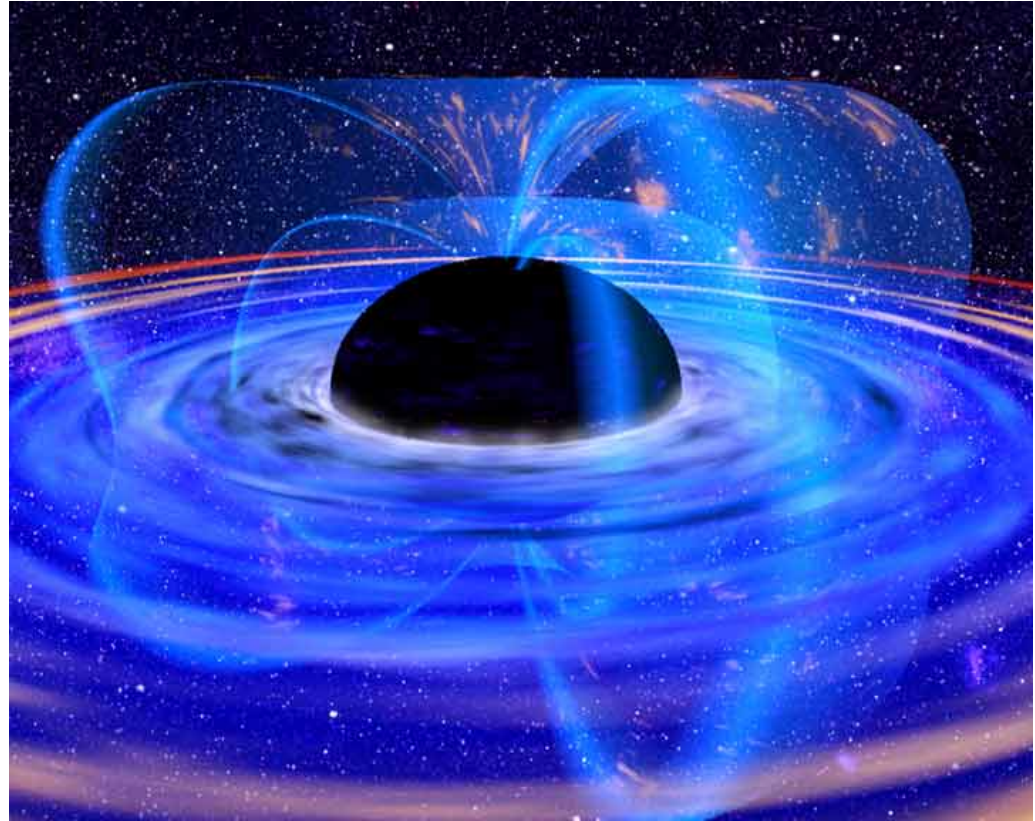
$$t_{evap} = 10^{10} \text{ yrs} \times \left(\frac{M}{10^{12} \text{ kg}} \right)^3$$

- Solar-mass black hole would take 10^{65} years to evaporate!
- Mini-black holes that could evaporate are not known to exist now, but possibly existed in early Universe

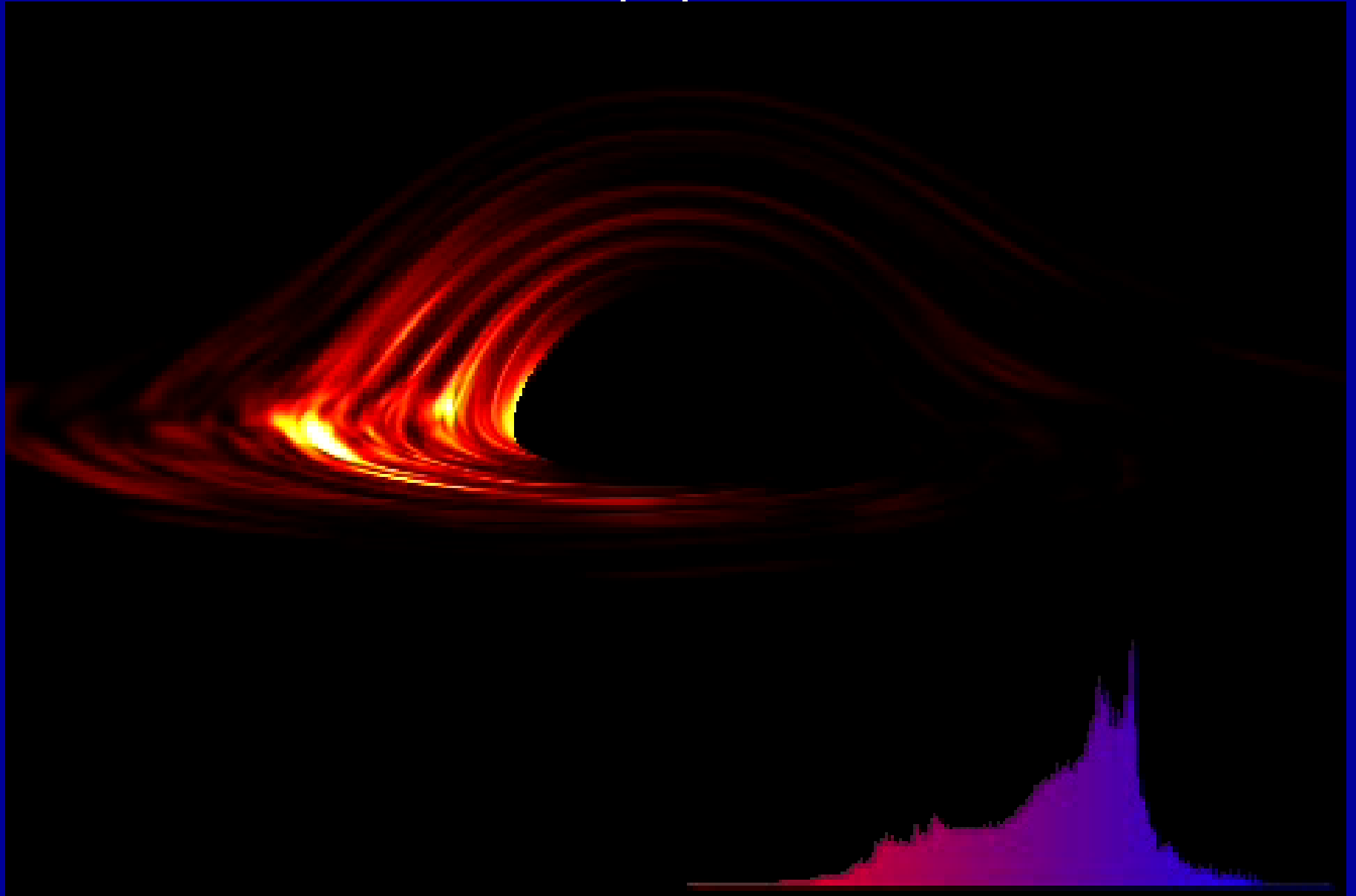
Active galaxies (e.g. radiating supermassive black holes) emit radio, IR, optical, UV, x-ray and γ -ray radiation !

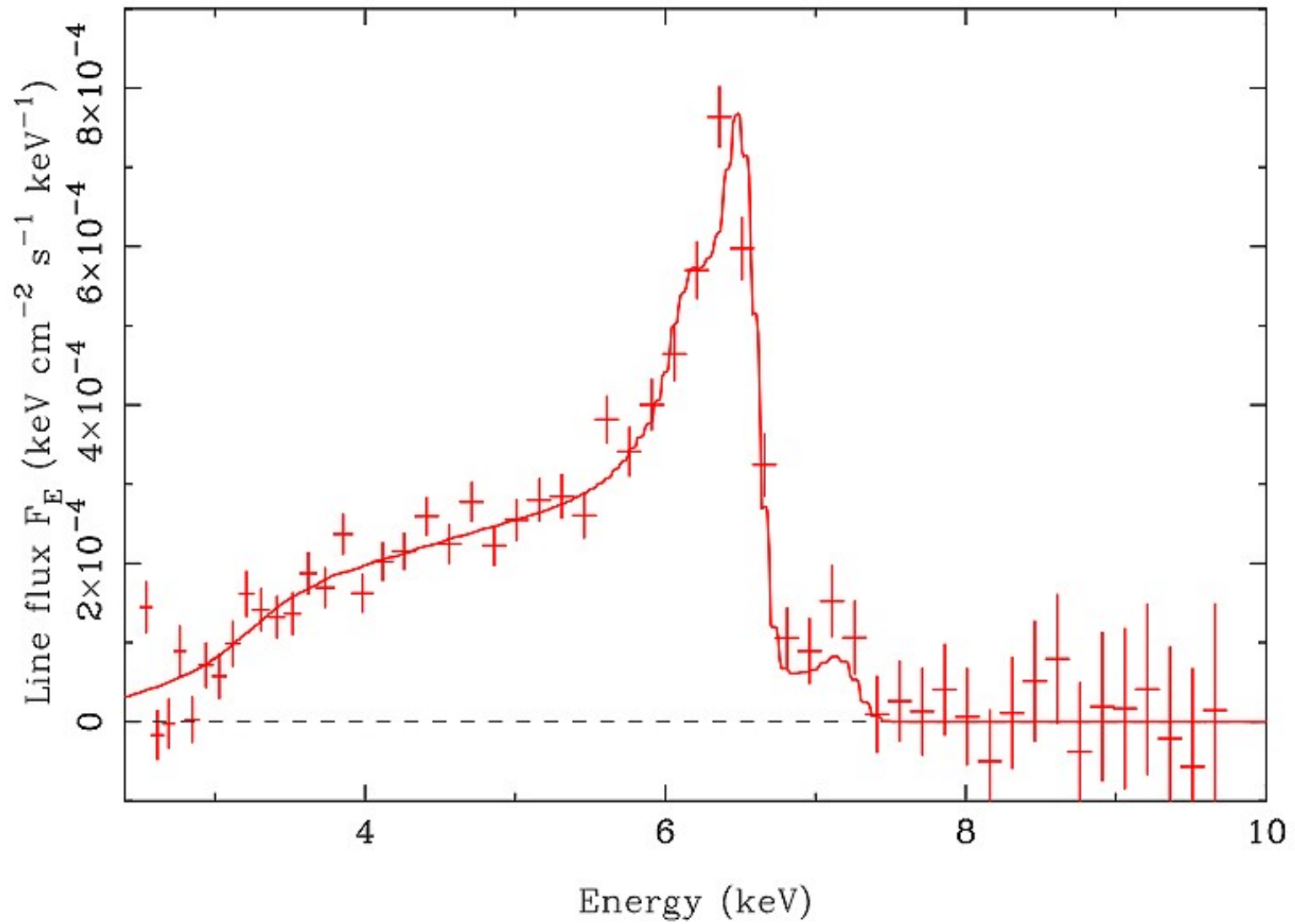
Broad band spectrum very different than stars

In the x-ray band there is a signature of the physics very close to the event horizon- Fe K emission from innermost part of disk



Numerical Simulation of Gas Accreting Onto a Black





Broad iron line in MCG-6-30-15 (Fabian et al. 2002)