Chemical Evolution of the Universe

- A major area of astrophysical research is understanding when stars and galaxies formed and how the elements are produced
- With the exception of H and He (which are produced in the big bang) all the other elements (called metals in astrophysical jargon) are "cooked" in the centers of massive stars and supernova and then "ejected" by explosions or winds
- The gas in these explosions is moving very fast (1000 km/sec) and can easily escape a galaxy.
- Clusters are essentially giant "boxes" which can hold onto all their material

•Measurement of the amount and change of metals with time in clusters directly measures their production

•In the hot gas elements such as silicon and iron have only 1 or 2 electrons
These ions produce strong H,He like x-ray emission lines.
The strengths of these lines is 'simply' related to the amount of silicon or iron in the cluster

Why are Clusters Interesting or Important

- Laboratory to study
  - Dark matter
    - Can study in detail the distribution and amount of dark matter and baryons
  - Chemical evolution
    - Most of the 'heavy' elements are in the hot x-ray emitting gas
  - Formation and evolution of cosmic structure
    - Feedback
    - Galaxy formation and evolution
    - Mergers
  - Cosmological constraints
    - Evolution of clusters is a strong function of cosmological parameters
  - Plasma physics on the largest scales
  - Numerical simulations
  - Particle acceleration

Each one of these issues Leads to a host of topics

Dark matter:

How to study it

lensing

Velocity and density distribution of galaxies

Temperature and density distribution of the hot gas

Chemical Evolution

Hot and when where the elements created

Why are most of the baryons in the hot gas

Does the chemical composition of the hot gas and stars differ?

X-rays from Clusters of Galaxies

- The baryons thermalize to $> 10^6$ K making clusters strong X-ray sources- the potential energy of infall is converted into kinetic energy of the gas.

- Most of the baryons in a cluster are in the X-ray emitting plasma - only 10-20% are in the galaxies.

- Clusters of galaxies are self-gravitating accumulations of dark matter which have trapped hot plasma (intracluster medium - ICM) and galaxies. (the galaxies are the least important constituent)

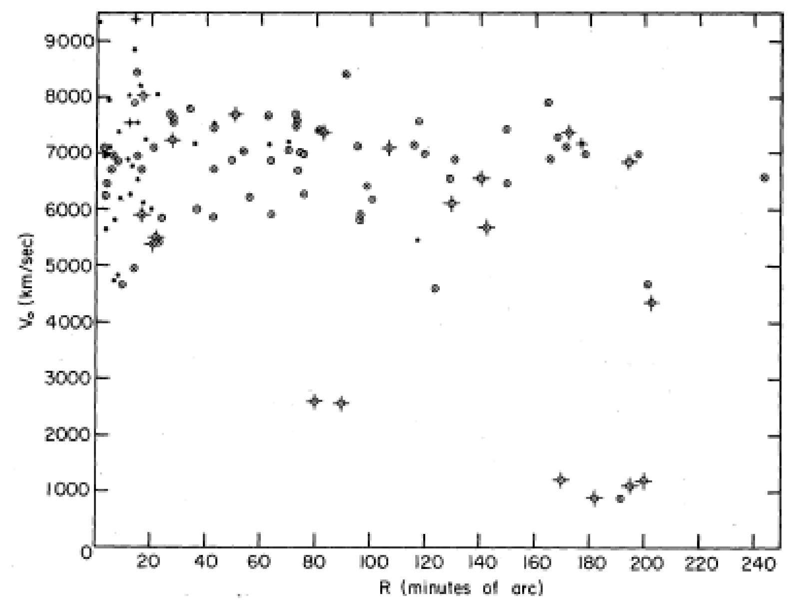
Todays Material

- How do we know that clusters are massive
  - Virial theorem
  - Lensing
  - -X-ray Hydrostatic equilibrium (but first we will discuss x-ray spectra) Equation of hydrostatic equilibrium (*)
- What do x-ray spectra of clusters look like

- *$\nabla P=-\rho_g \nabla \phi(r)$ where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter) P is gas pressure and ρ_g is the gas density ($\nabla f=(\partial f/x_1, \partial f/x_2 \ldots \partial f/x_n$)

The First Detailed Analysis

- Rood et al used the King (1969) analytic models of potentials (developed for globular clusters) and the velocity data and surface density of galaxies to infer a very high mass to light ratio of ~230.
- Since "no" stellar system had M/L>12 dark matter was necessary



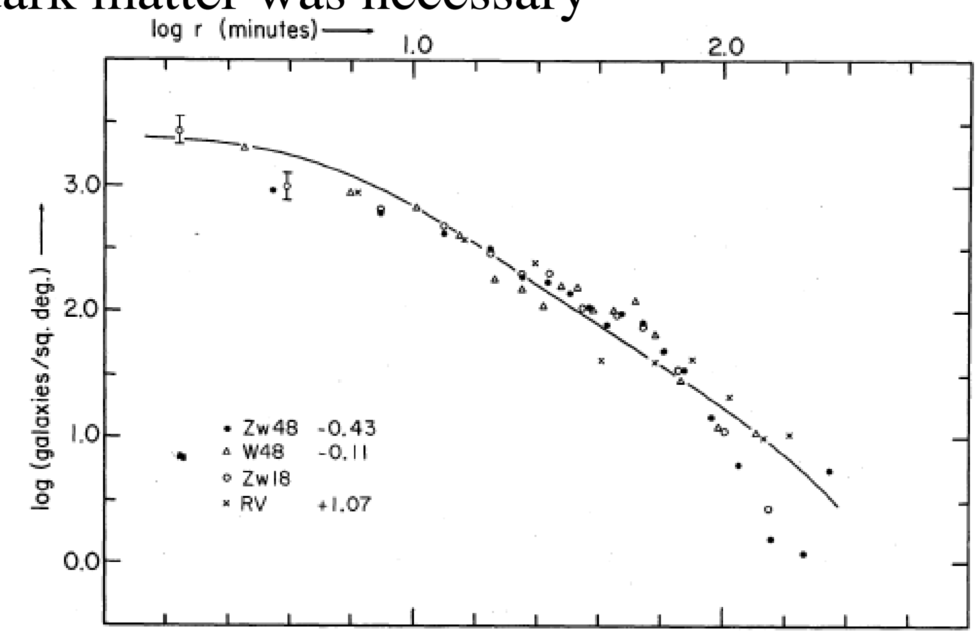


FIG. 5.—Surface densities, corrected for backgrounds given in table 2. For this fitting, logarithms of

Rood 1972- velocity vs position of galaxies in Coma
Surface density of galaxies

Paper is worth reading ApJ 175,627

Virial Theorem (Kaiser sec 26.3)

- The virial theorem states that, for a stable, self-gravitating, spherical distribution of equal mass objects (stars, galaxies, etc), the total kinetic energy of the objects is equal to -1/2 times the total gravitational potential energy.

$2<T>=-<W_{TOT}>$

T is the time average of the Kinetic energy and W is the time overage of the potential energy

- In other words, the potential energy must equal 1/2 the kinetic energy. Consider a system of N particles with mass m and velocity v.
- kinetic energy of the total system is K.E.(system) = $1/2\ m\ N\ v^2 = 1/2\ M_{tot}\ \ v^2$

•$PE \sim 1/2GN^2m^2/R_{tot} = 1/2GM^2_{tot}/R_{tot}$

(dimensional analysis)

If the orbits are random KE=1/2PE (virial theorm)

$M_{tot} \sim 2R_{tot}v^2_{tot}/G$

Binney, J. and Tremaine, S. "The Virial Equations." §4.3 in Galactic Dynamics. Princeton, NJ: Princeton University Press, pp. 211-219, 1987

Virial Theorem Actual Use (Kaiser 26.4.2)

- Photometric observations provide the surface brightness Σ_{light} of a cluster. On the other hand, measurements of the velocity dispersion σ^2_v together with the virial theorem give $\sigma^2_v \sim W/M \sim GM/R \sim G\Sigma_{mass} R$

Σ_{mass} is the projected mass density.

At a distance D the mass to light ratio (M/L)can be estimated as
$M/L = \Sigma_{mass}/\Sigma_{light} = \sigma^2_v / GD\Theta\Sigma_{light}$
where Θ is the angular size of the cluster.

Applying this technique, Zwicky found that clusters have M/L ~300 in solar units

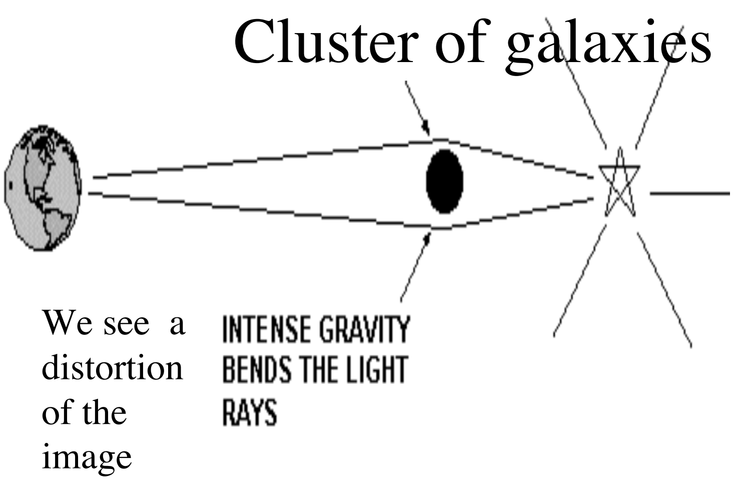
- The virial theorem is exact, but requires that the light traces the mass-it will fail if the dark matter has a different profile from the luminous particles.

Mass Estimates

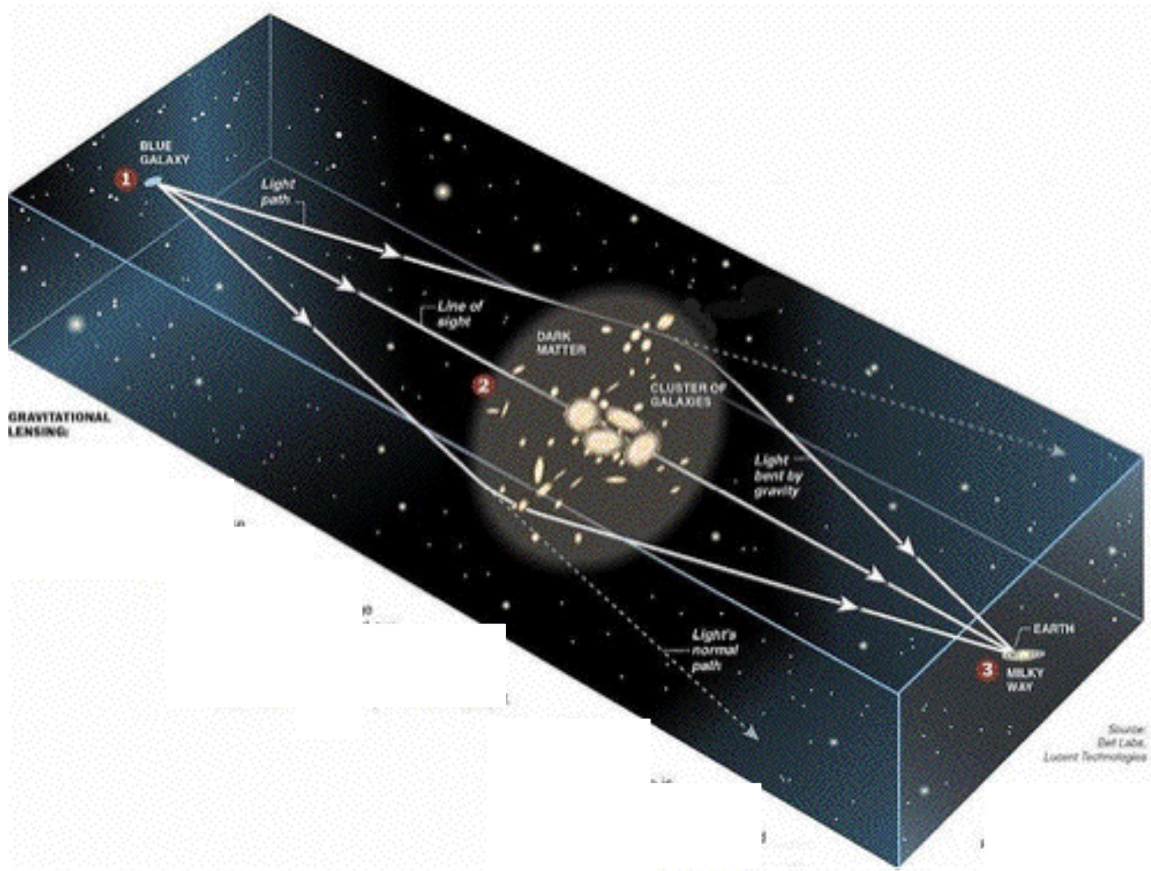
- While the virial theorem is fine it depends on knowing the time averaged orbits, the distribution of particles etc etc- a fair amount of systematic errors
- Would like better techniques
  - Gravitational lensing
  - Use of spatially resolved x-ray spectra

Light Can Be Bent by Gravity

The more mass-
the more the
light is bent



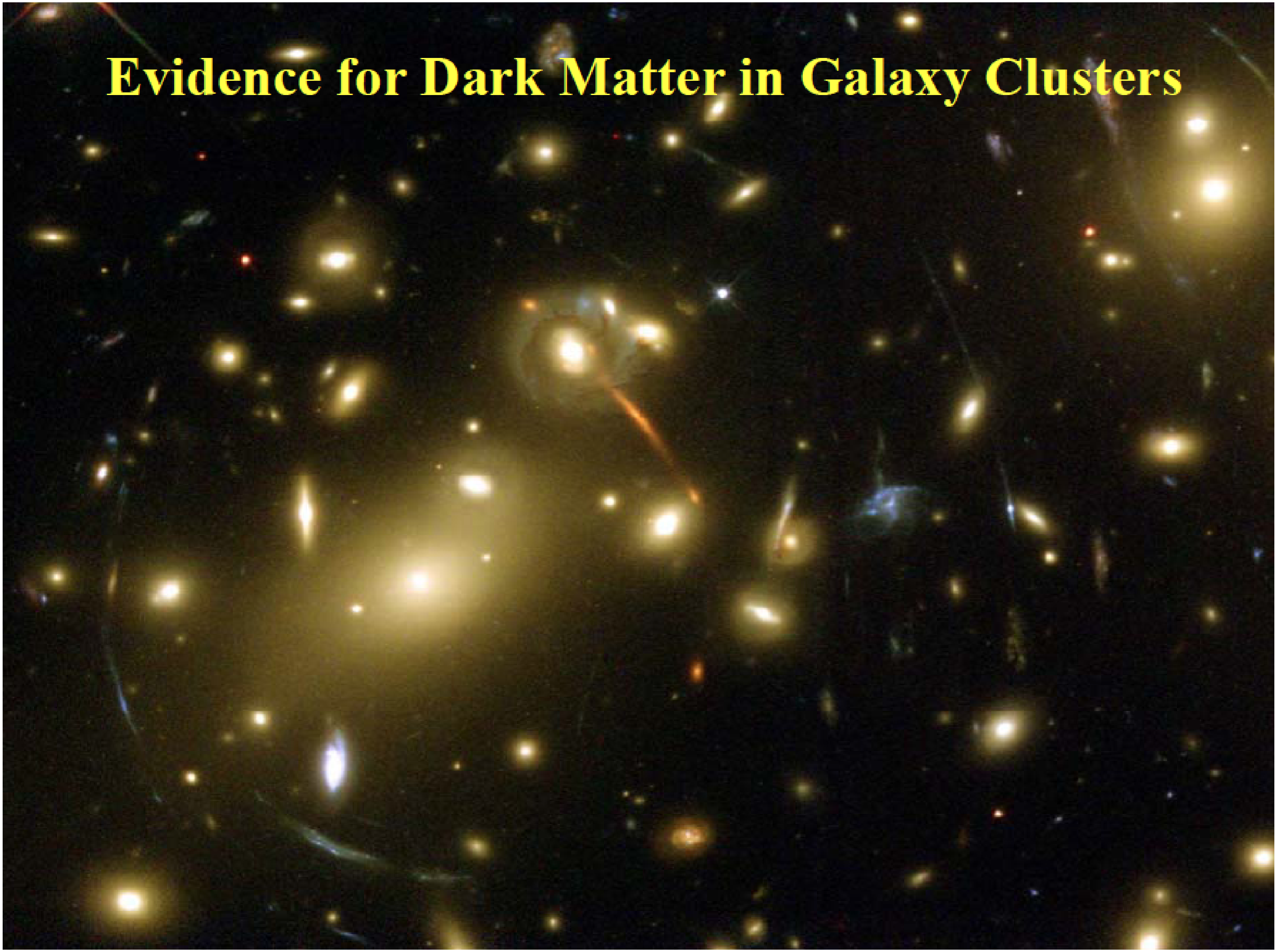
Gravitational Lensing



faculty.lsmsa.edu

Amount and type of distortion is related to
amount and distribution of mass in gravitational lens

Evidence for Dark Matter in Galaxy Clusters



Basics of Gravitational Lensing

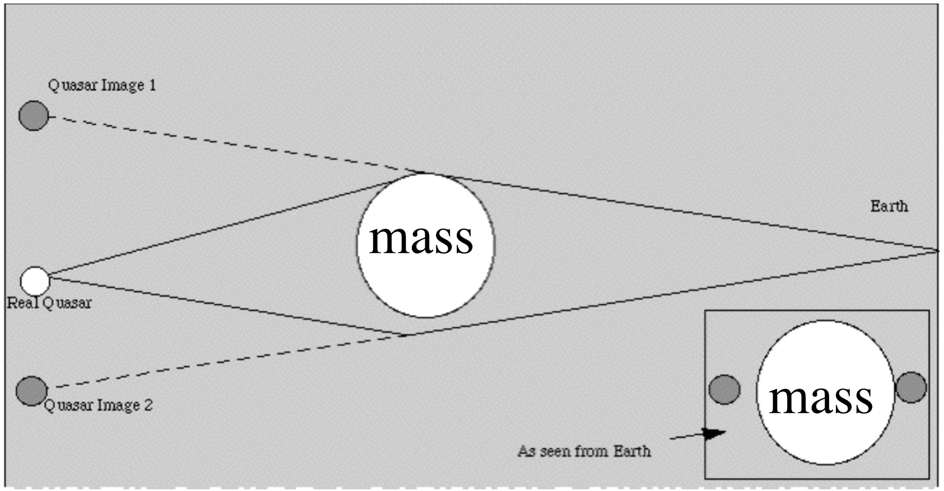
- See Lectures on Gravitational Lensing by

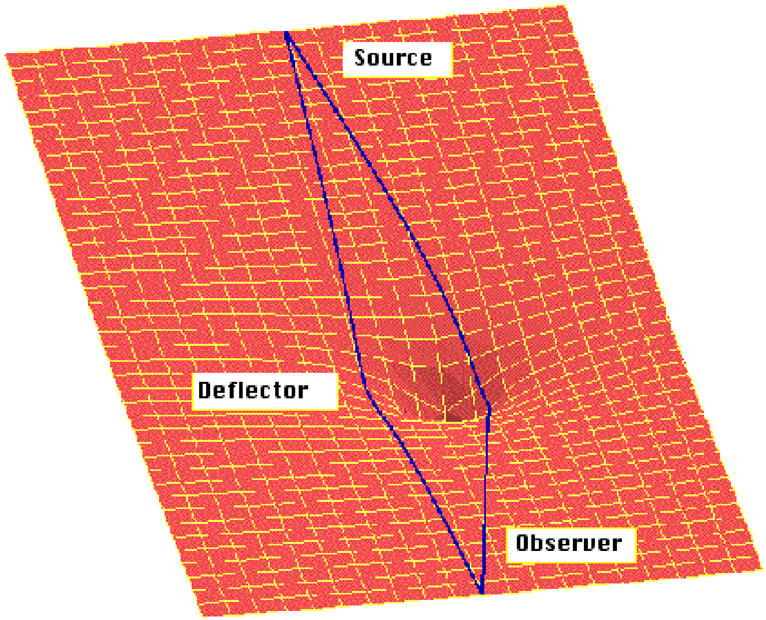
Ramesh Narayan Matthias Bartelmann or
http://www.pgss.mcs.cmu.edu/1997/Volume16/physics/GL/GL-II.html

For a detailed discussion of the problem

- Rich centrally condensed clusters occasionally produce giant arcs when a background galaxy happens to be aligned with one of the cluster caustics.
- Every cluster produces weakly distorted images of large numbers of background galaxies.
  - These images are called arclets and the phenomenon is referred to as weak lensing.
- The deflection of a light ray that passes a point mass M at impact parameter b is

$$\Theta_{def} = 4GM/c^2 b$$





Lensing

- assume -

the overall geometry of the universe is Friedmann--Robertson- Walker metric

matter inhomogeneities which cause the lensing are local perturbations.

- Light paths propagating from the source past the lens 3 regimes
- 1)light travels from the source to a point close to the lens through unperturbed spacetime.
- 2)near the lens, light is deflected.
- 3) light again travels through unperturbed spacetime.

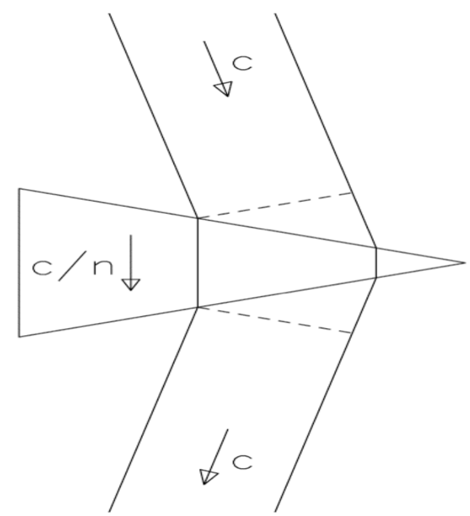
The effect of spacetime curvature on the light paths can be expressed in terms of an effective index of refraction, n, (e.g. Schneider et al. 1992)

$n = 1 -(2/c^2)\ \phi$;$\phi(r)$ the Newtonian gravitational potential

As in normal optics, for refractive index $n > 1$ light travels slower than in free vacuum.

effective speed of a ray of light in a gravitational field is

$v = c/n \sim c-(2/c)\phi$



Lensing

- Due to slower speed of light the signal is delayed by

$$\Delta t = \int_{\text{source}}^{\text{observer}} \frac{2}{c^3} |\Phi| \, dl \; .$$

This is called the Shapiro delay and has been used to obtain the orbits of neutron stars as well

As an example, we now evaluate the deflection angle of a point mass M (cf. Fig. 3). The Newtonian potential of the lens is

$$\Phi(b,z) = -\frac{GM}{(b^2+z^2)^{1/2}}\;, \tag{5}$$

where b is the impact parameter of the unperturbed light ray, and z indicates distance along the unperturbed light ray from the point of closest approach. We therefore have

$$\vec{\nabla}_\perp\Phi(b,z) = \frac{GM\,\vec{b}}{(b^2+z^2)^{3/2}}\;, \tag{6}$$

where $\vec{b}$ is orthogonal to the unperturbed ray and points toward the point mass. Equation (6) then yields the deflection angle

$$\hat{\alpha} = \frac{2}{c^2}\int\vec{\nabla}_\perp\Phi\,dz = \frac{4GM}{c^2 b}\;. \tag{7}$$

Note that the Schwarzschild radius of a point mass is

$$R_\mathrm{S} = \frac{2GM}{c^2}\;, \tag{8}$$

so that the deflection angle is simply twice the inverse of the impact parameter in units of the Schwarzschild radius. As an example, the Schwarzschild radius of the Sun is 2.95 km, and the solar radius is 6.96×10^5 km. A light ray grazing the limb of the Sun is therefore deflected by an angle $(5.9/7.0)\times10^{-5}\,\mathrm{radians} = 1\overset{''}{.}7$.

Narayan and Bartellman 1996

Lensing

Einstein radius is the scale of lensing

- For a point mass it is

$\theta_E = ((4GM/c^2)(Dds/DdDs))^{1/2}$

- or in more useful units

$\theta_E = (0.9'')M_{11}^{1/2}\, D_{Gpc}^{-1/2}$

- Lens eq

$\beta \;= \theta-(D_{ds}/D_dD_s)4GM/\theta c^2.$

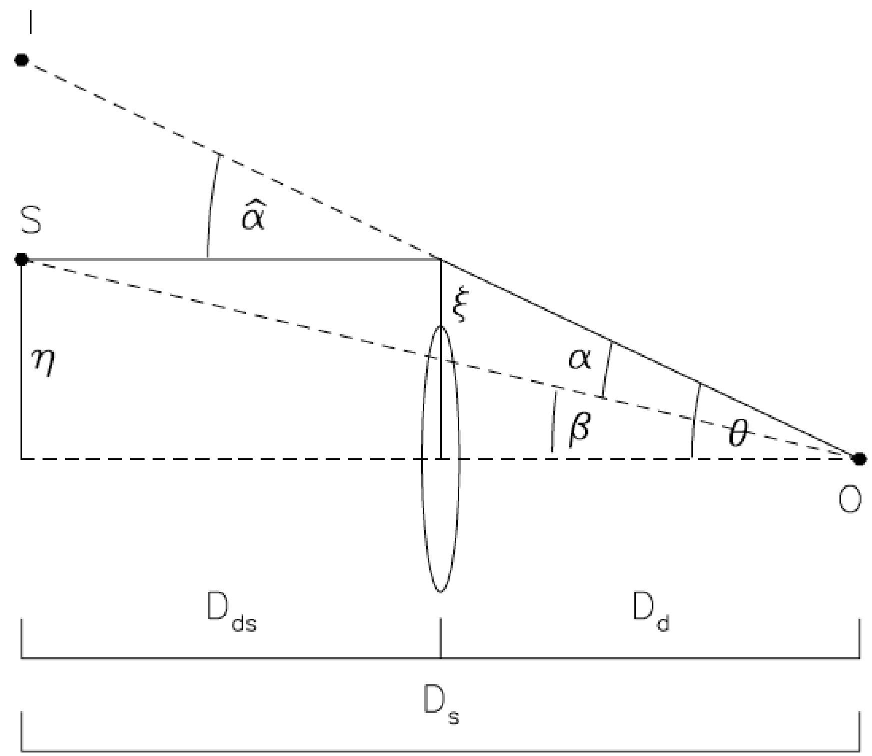
or

$\beta \;=\theta-\theta^2_E/\theta$

β 2 solutions

Any source is imaged twice by a point mass lens

Gravitational light deflection preserves surface brightness because of the Liouville theorm



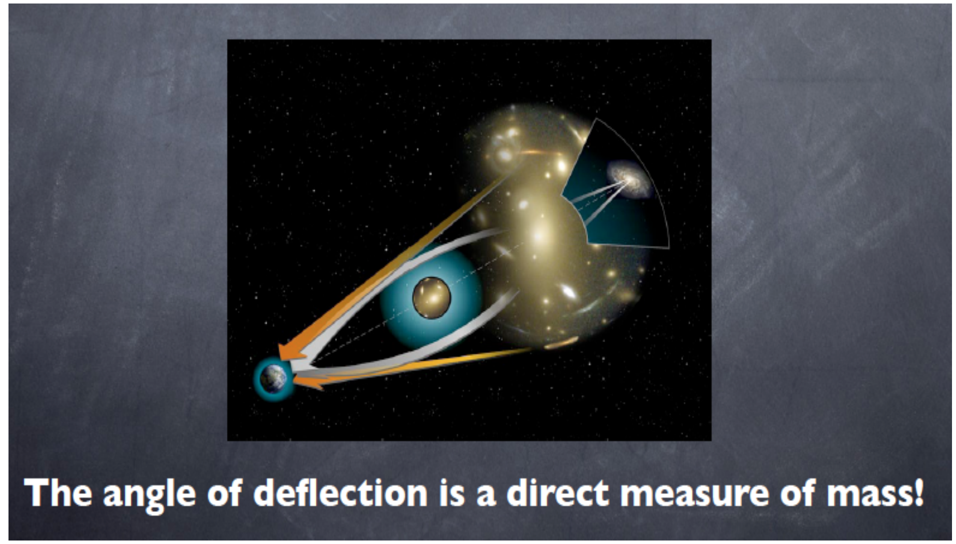
Ways of Thinking About Lensing (Kaiser sec 33.5)

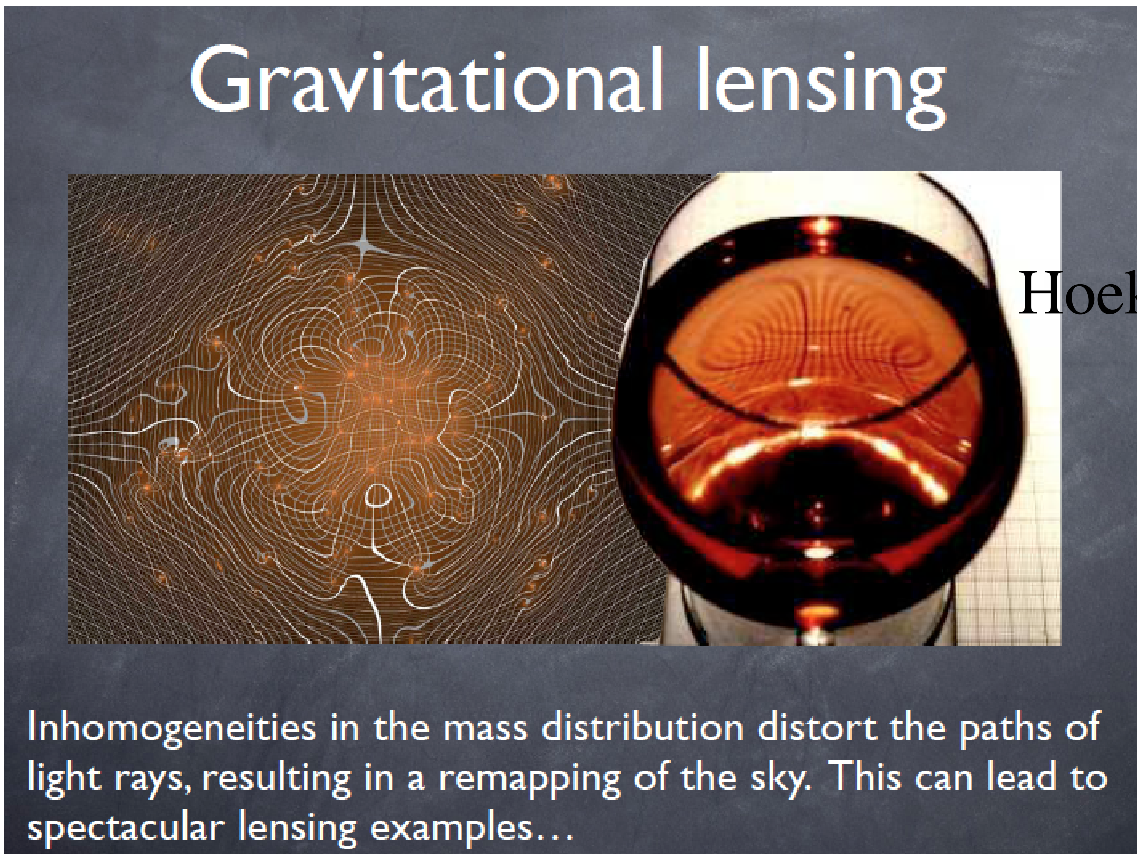
- This deflection is just twice what Newtonian theory would give for the deflection of a test particle moving at v = c where we can imagine the radiation to be test particles being pulled by a gravitational acceleration.
- another way to look at this using wave-optics; the inhomogeneity of the mass distribution causes space-time to become curved. The space in an over-dense region is positively curved.

light rays propagating through the over-density have to go a slightly greater distance than they would in the absence of the density perturbation.

Consequently the wave-fronts get retarded slightly in passing through the over-density and this results in focusing of rays.

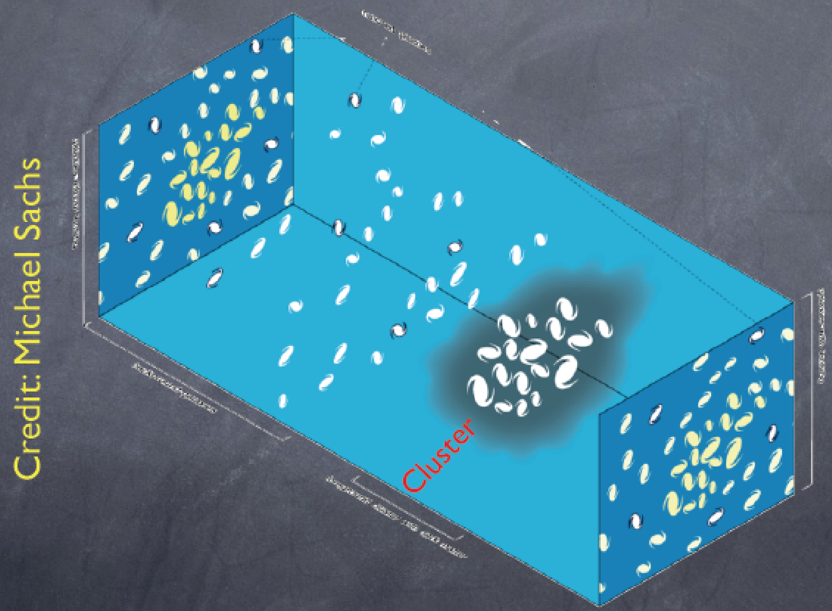
- Another way : The optical properties of a lumpy universe are, in fact, essentially identical to that of a block of glass of inhomogeneous density where the refractive index is $n(r) = (1- 2\phi(r)/c^2)$ with $\phi(r)$ the Newtonian gravitational potential. In an over-dense region, ϕ is negative, so n is slightly greater than unity. In this picture we think of space as being flat, but that the speed of light is slightly retarded in the over-dense region.
- All three of the above pictures give identical results





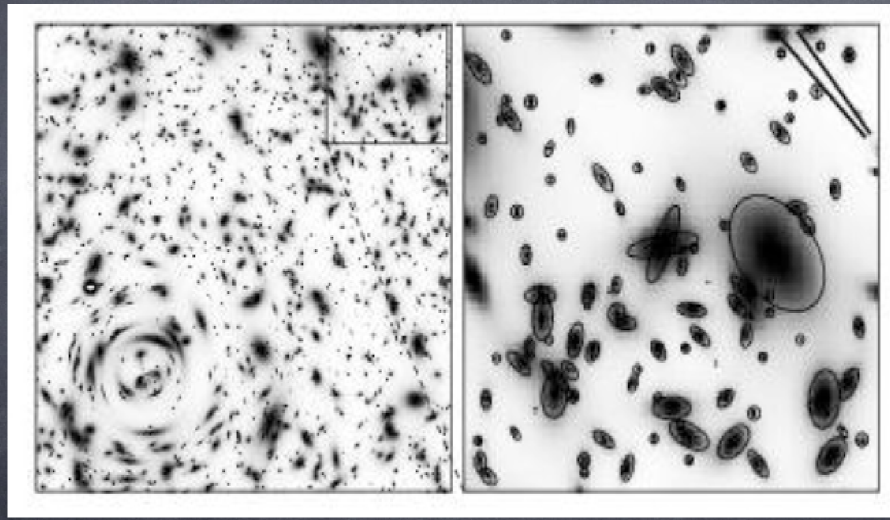
Hoekstra 2008 Texas Conference

Weak gravitational lensing



In the absence of noise we would be able to map the matter distribution in the universe (even “dark” clusters).

Weak gravitational lensing



A measurement of the ellipticity of a galaxy provides an unbiased but noisy measurement of the shear

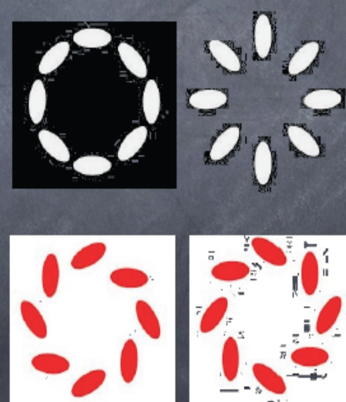
Hoekstra 2008 Texas Conference

Diagnostics

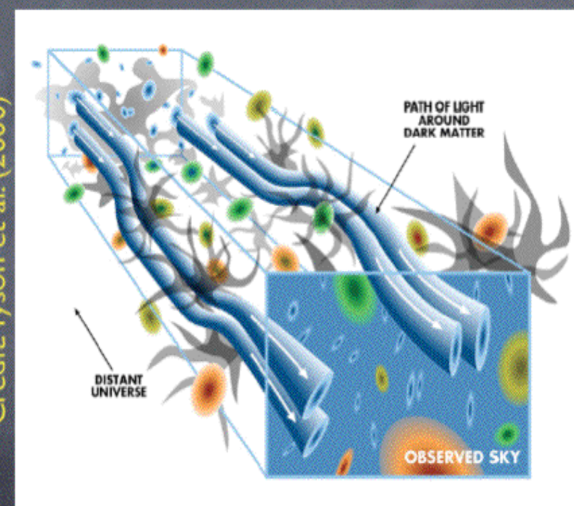
The lensing signal should be curl-free. We can project the correlation functions into one that measures the divergence and one that measures the curl: *E-B mode decomposition*. We can also look for correlations between the corrected galaxy shapes and the PSF anisotropy.

E-mode (curl-free)

B-mode (curl)



Cosmic shear



Credit: Tyson et al. (2000)

Cosmic shear is the lensing of distant galaxies by the overall distribution of matter in the universe: it is the most "common" lensing phenomenon.

What we try to measure with X-ray Spectra

- From the x-ray spectrum of the gas we can measure a mean temperature, a redshift, and abundances of the most common elements (heavier than He).
- With good S/N we can determine whether the spectrum is consistent with a single temperature or is a sum of emission from plasma at different temperatures.
- Using symmetry assumptions the X-ray surface brightness can be converted to a measure of the ICM density.

What we try to measure II

If we can measure the temperature and density at different positions in the cluster then assuming the plasma is in hydrostatic equilibrium we can derive the gravitational potential and hence the amount and distribution of the dark matter.

There are two other ways to get the gravitational potential :

- The galaxies act as test particles moving in the potential so their redshift distribution provides a measure of total mass.
- The gravitational potential acts as a lens on light from background galaxies.

Why do we care ?

Cosmological simulations predict distributions of masses.

If we want to use X-ray selected samples of clusters of galaxies to measure cosmological parameters then we must be able to relate the observables (X-ray luminosity and temperature) to the theoretical masses.

Theoretical Tools

- Physics of hot plasmas
  - Bremmstrahlung
  - Collisional equilibrium
  - Atomic physics

Physical Processes

- Continuum emission
  - Thermal bremsstrahlung, ~exp(-hν/kT)
  - Bound-free (recombination)
  - Two Photon
- Line Emission

  (line emission)

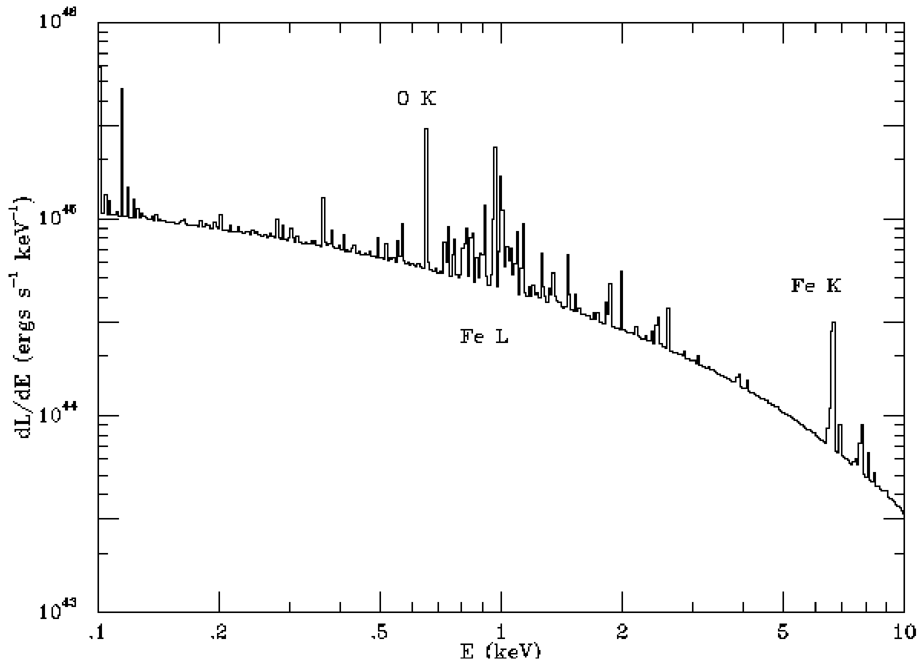
  $L_\nu \sim \varepsilon_\nu$ (T, abund) $(n_e^2\, V)$

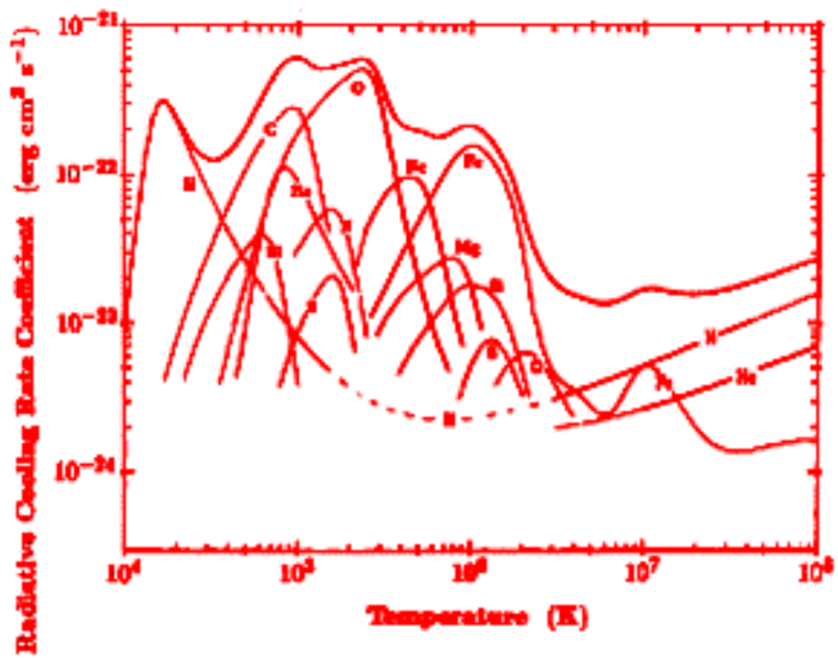
  $I_\nu \sim \varepsilon_\nu$ (T, abund) $(n_e^2\, l)$

  Line emission dominates cooling at $T<10^7$ K

  Bremmstrahlung at higher temperatures

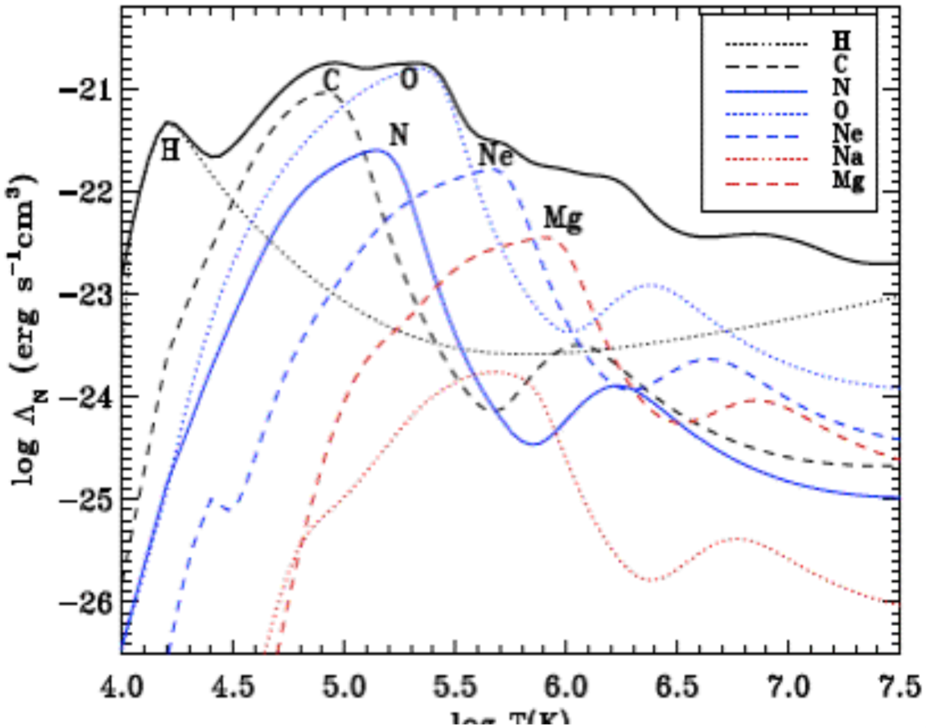
$$\epsilon(\nu) = \frac{16\, e^6}{3\, m_e\, c^2}\left(\frac{2\pi}{3 m_e\, k_B T_X}\right)^{1/2} n_e n_i\, Z^2\, g_{ff}(Z, T_X, \nu)\, \exp\left(\frac{-h\nu}{k_B T_X}\right),$$





ooling rate of hot plasma as a function of the plasma temperature. The contribution to the cooling
s of different important abundant elements is indicated (Böhringer and Hensler 1989). Most of

Cooling Function



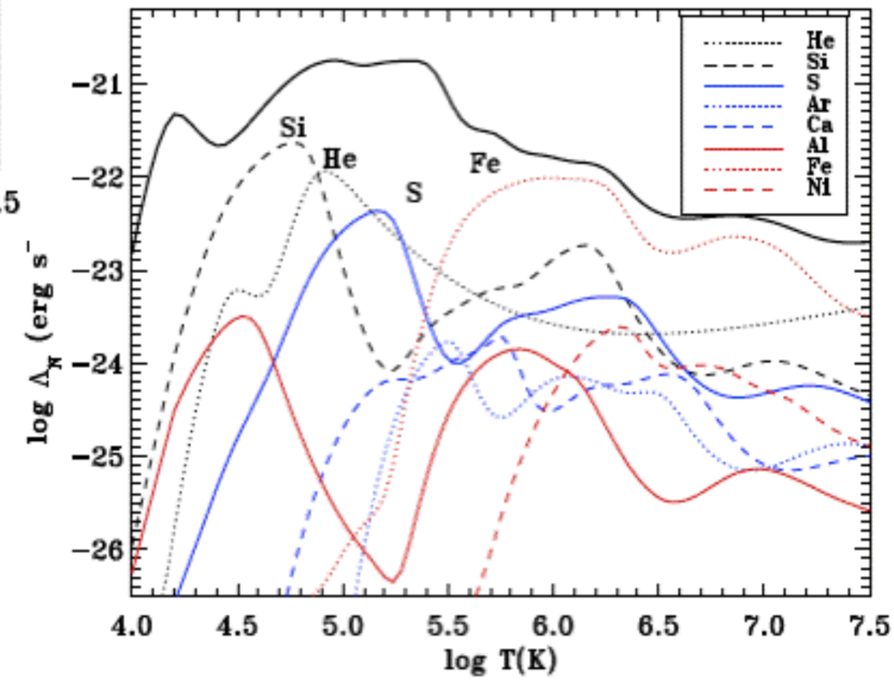
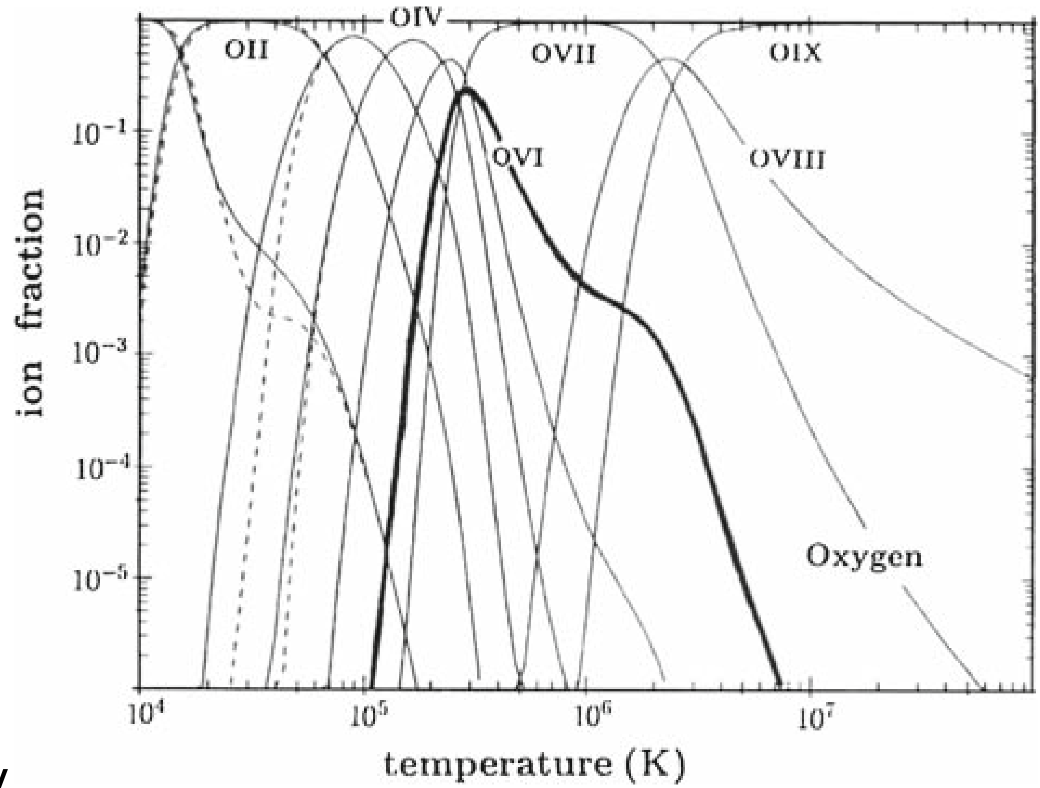


Fig. 2. Contributions of different elements to the cooling curve are given. Each of the plots shows a different set of elements. Important peaks are labelled with the name of the element. The total cooling curve (black solid line) is an addition of the individual elemental contributions.

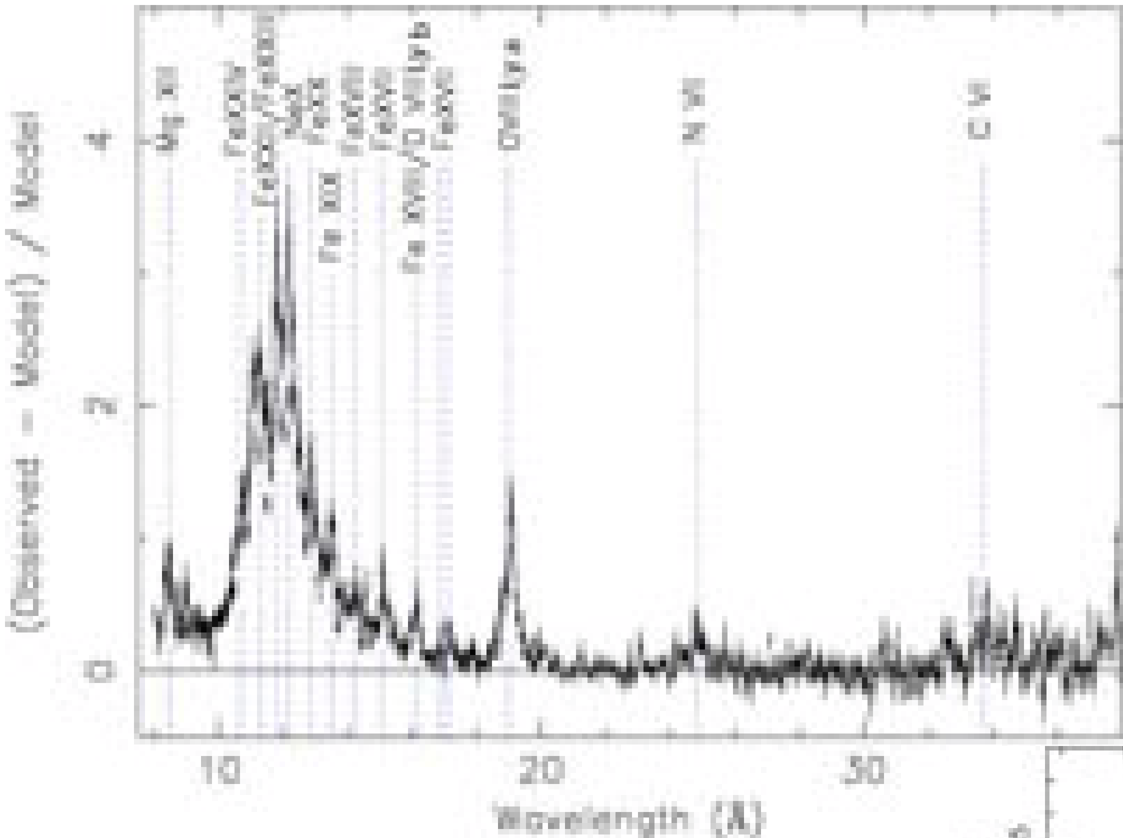
Plasma Parameters

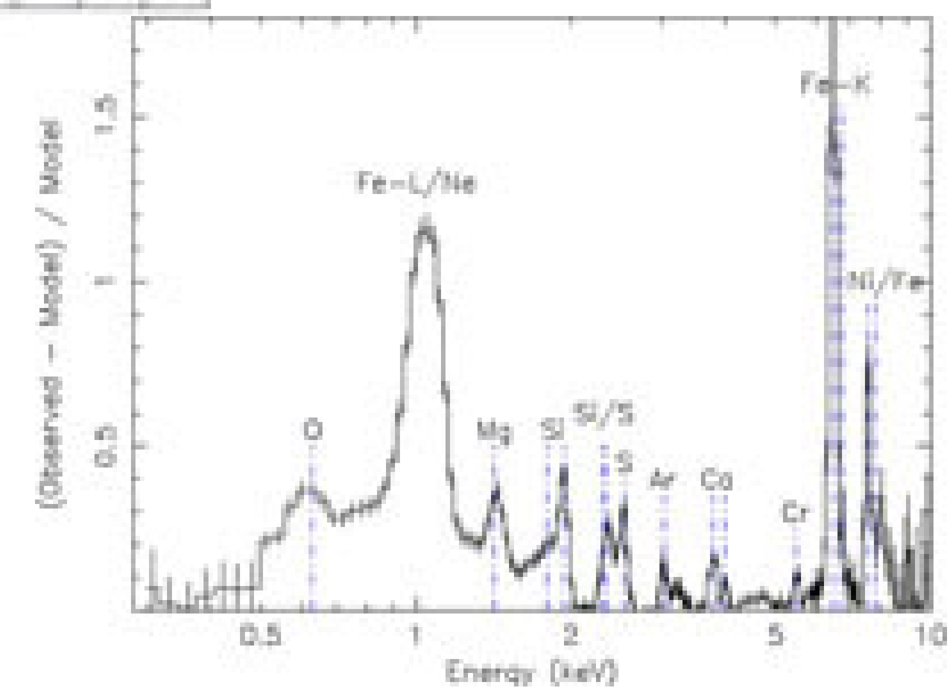
- Electron number density $n_e \sim 10^{-3}$ cm^{-3} in the center with density decreasing as $n_e \sim r^{-2}$
- $10^6 < T < 10^8$ k
- Mainly H, He, but with heavy elements (O, Fe, ..)
- Mainly emits X-rays
- $10^{42} L_X < 10^{45.3}$ erg/s, most luminous extended X-ray sources in Universe
- Age ~ 2-10 Gyr
- Mainly ionized, but not completely e.g. He and H-like ions of the abundant elements (O…Fe) exist in thermal equilibrium



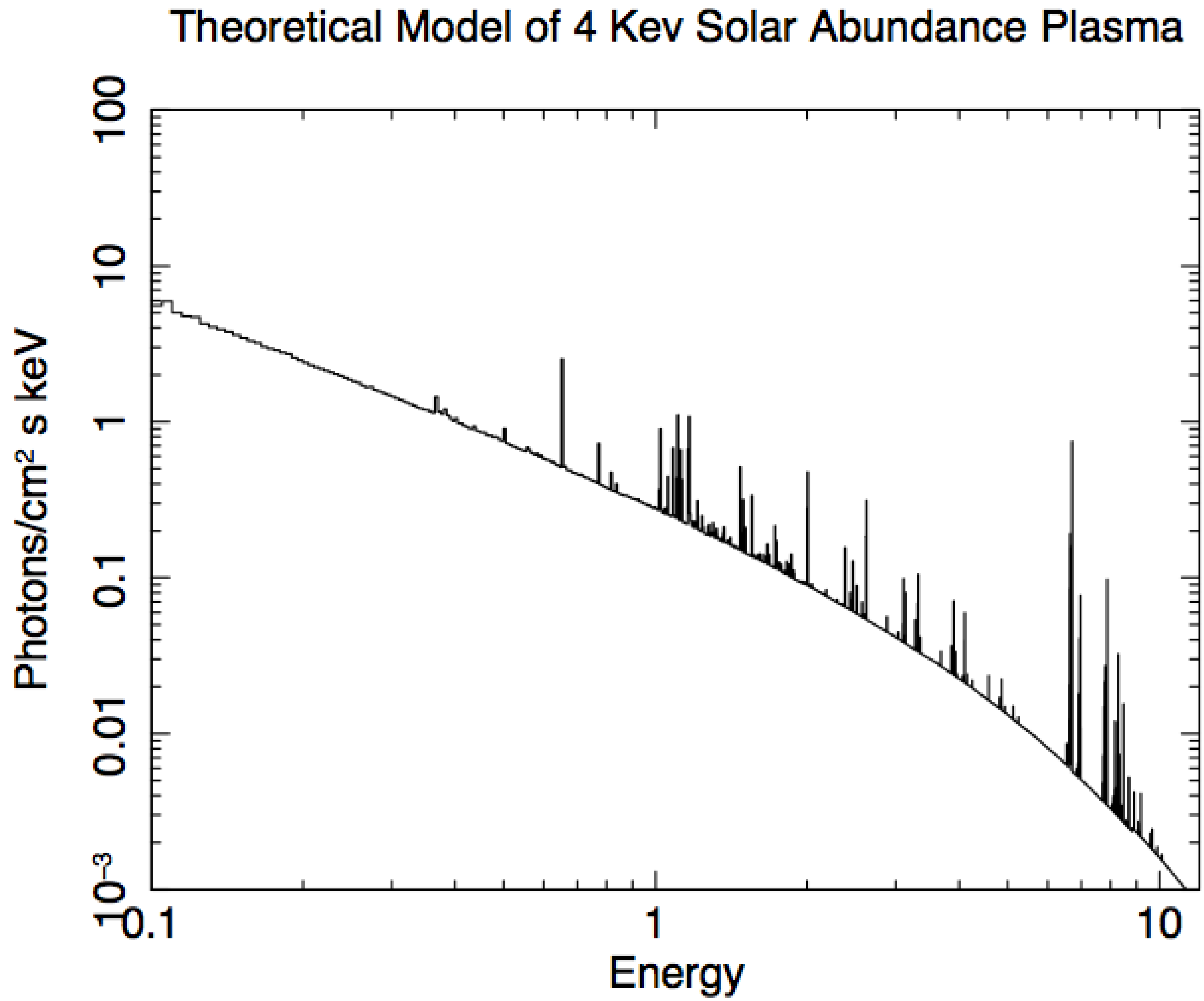
Ion fraction for oxygen vs electron temperature

X-ray Spectra of Clusters



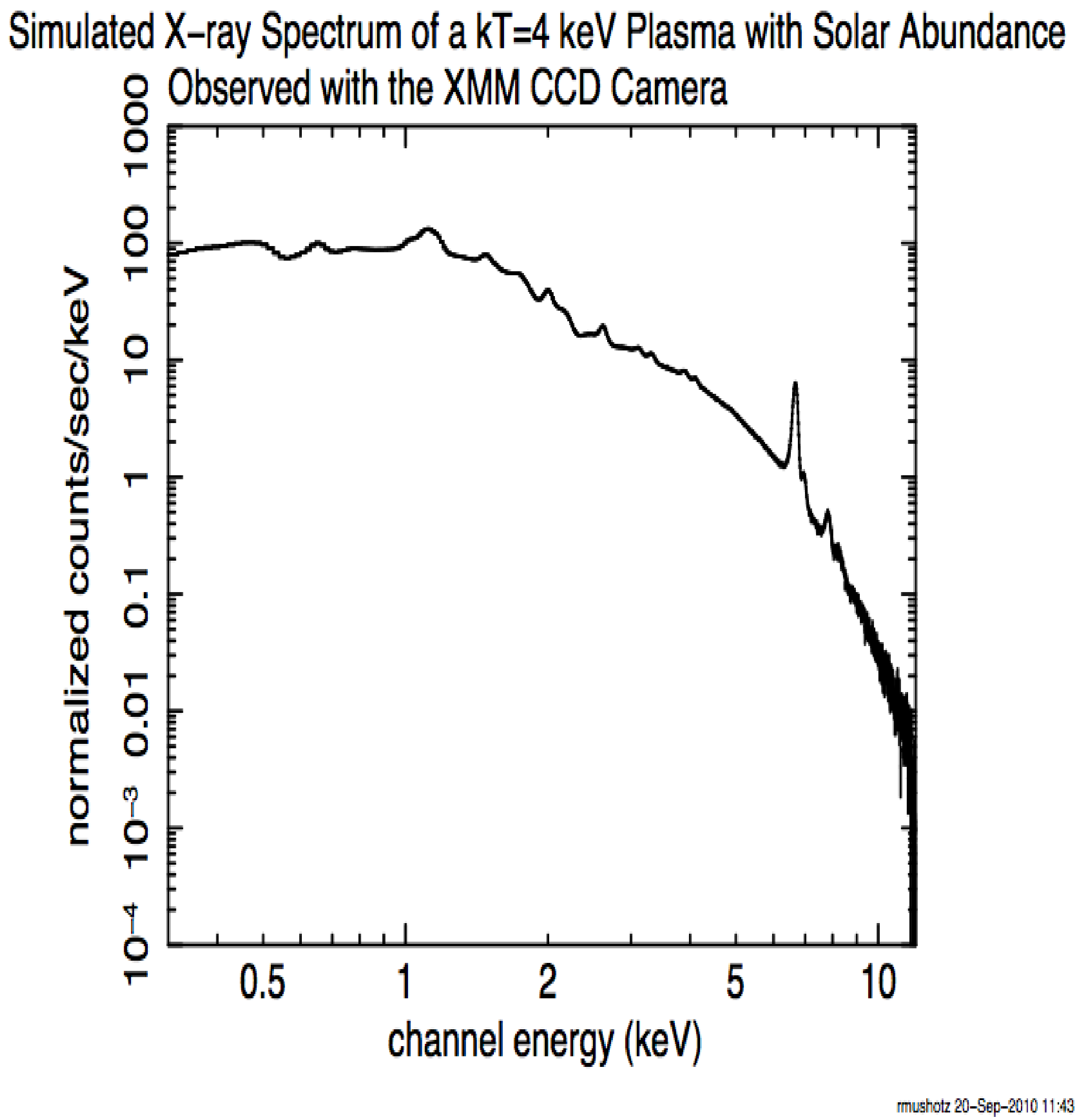


- Theoretical model of a collisionally ionized plasma kT=4 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^4



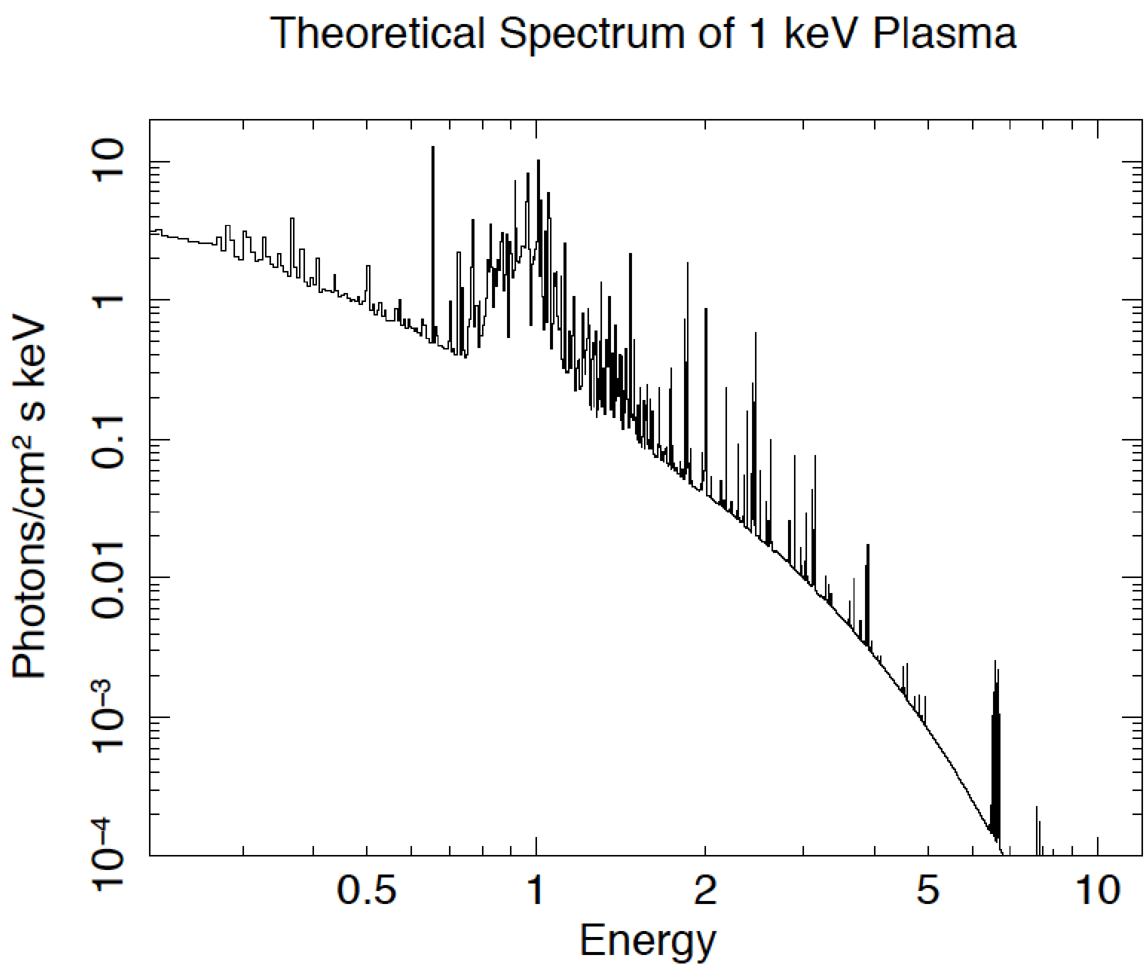
X-ray Spectra Data

- For hot ($kT > 3x10^7k$) plasmas the spectra are continuum dominated- most of the energy is radiated in the continuum
- (lines broadened by the detector resolution)

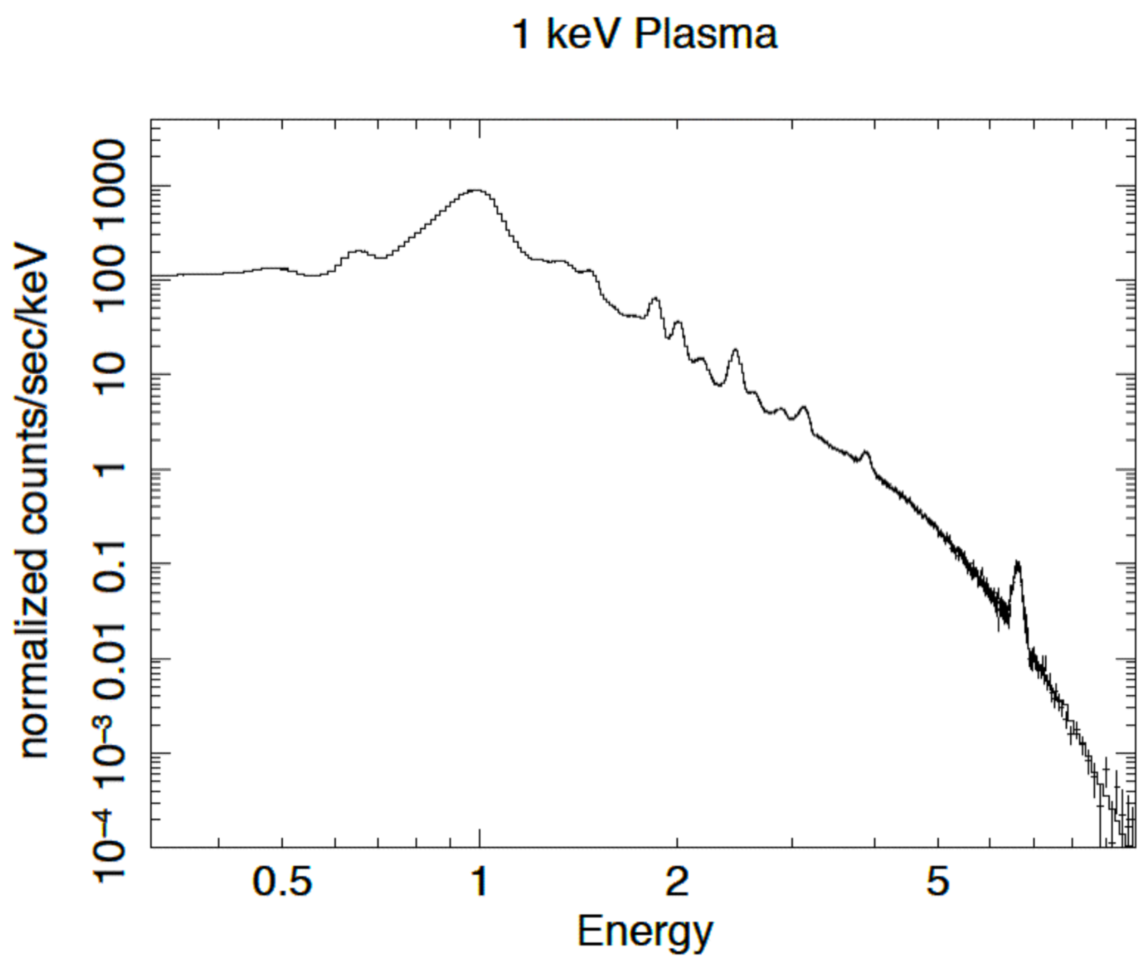


1 keV Plasma

- Theoretical model of a collisionally ionized plasma kT=1 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^5

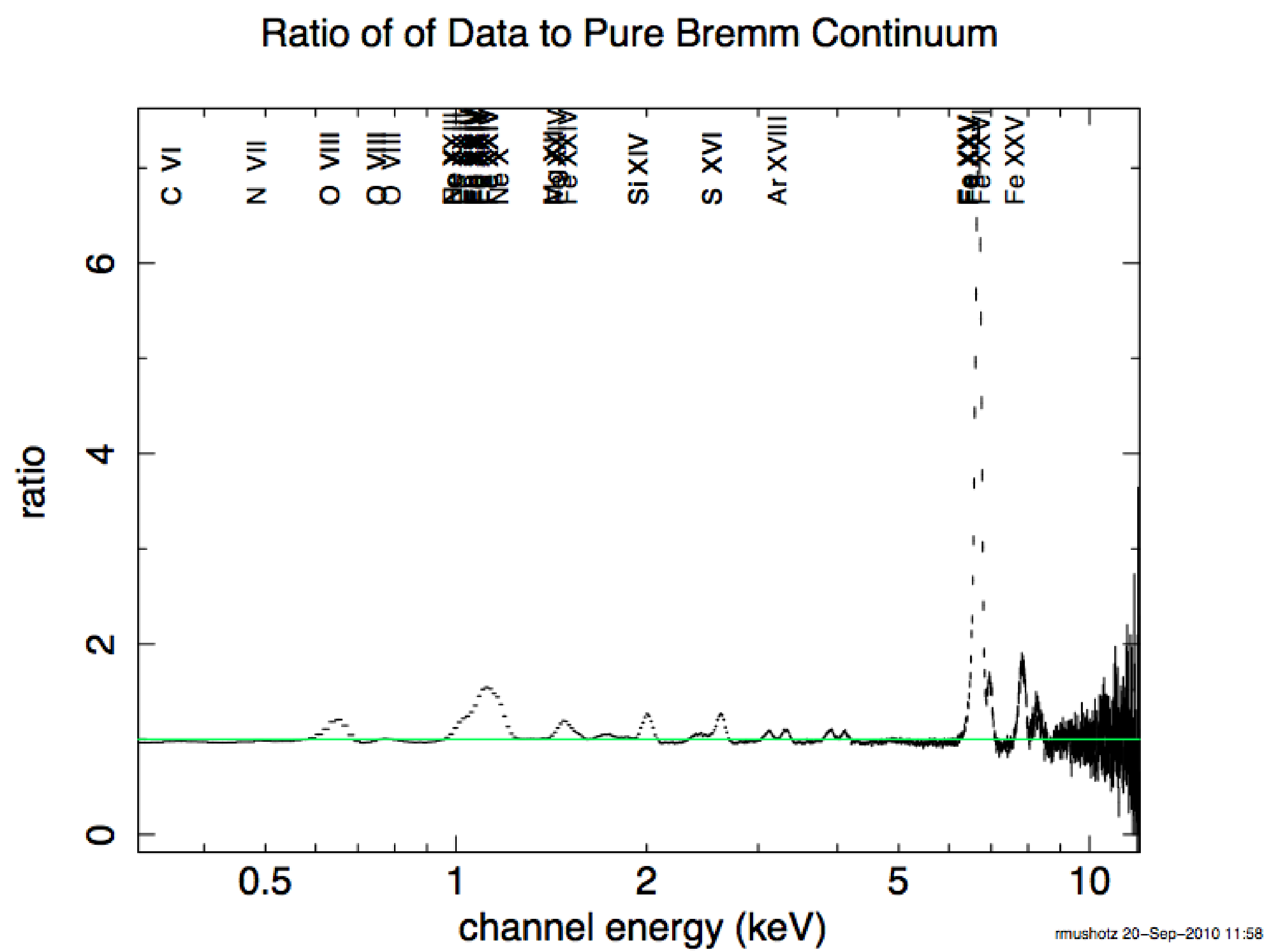


- Observational data for a collisionally ionized plasma kT=1 keV with solar abundances
- Notice the very large blend of lines near 1 keV- L shell lines of Fe
- Notice dynamic range of 10^7



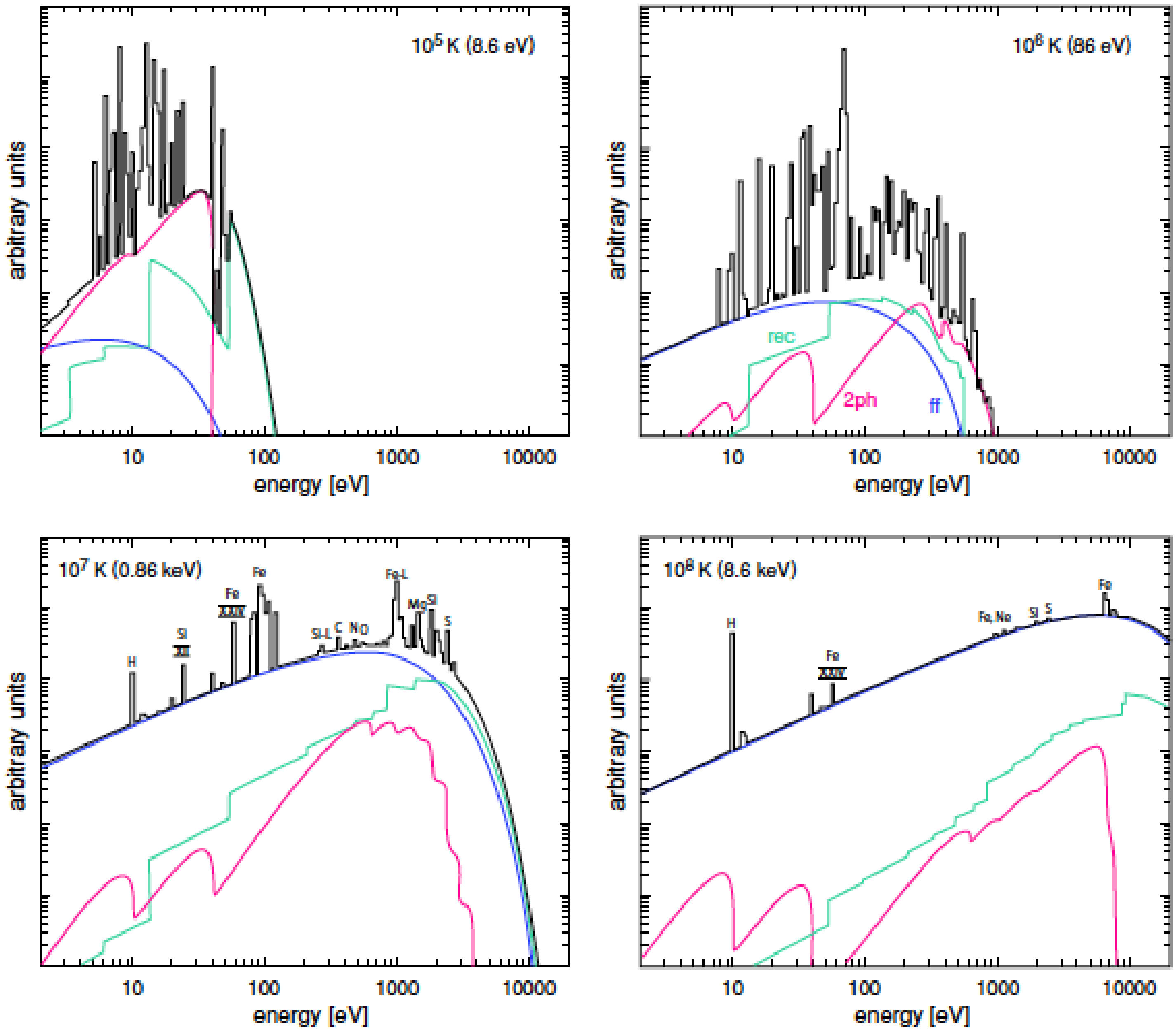
Collsionally Ionized Equilibrium Plasma

- Ratio of model to a 'pure' H/He plasma
- This plot is designed to show the lines



Strong Temperature Dependence of Spectra

- Line emission
- Bremms (black)
- Recombination (red)
- 2 photon green



Relevant Time Scales

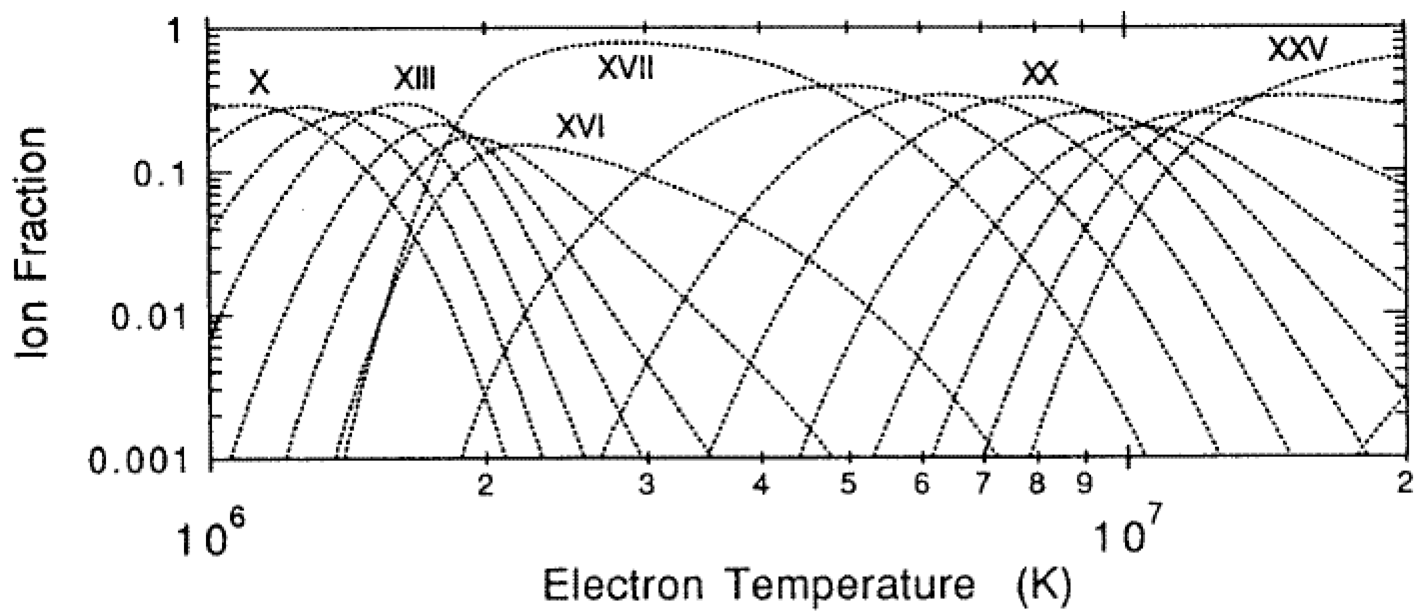
- The equilibration timescales between protons and electrons is t(p,e) ~ 2 x 10^8 yr at an 'average' location
- **In collisional ionization equilibrium population of ions is directly related to temperature**

$$\tau(1,2) = \frac{3m_1\sqrt{2\pi}(kT)^{3/2}}{8\pi\sqrt{m_2}n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}$$

$$\ln\Lambda \equiv \ln(b_{\max}/b_{\min}) \approx 40$$

$$\boxed{\begin{aligned}\tau(e,e) &\approx 3\times10^5\left(\frac{T}{10^8\,\mathrm{K}}\right)^{3/2}\left(\frac{n_e}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{yr}\\ \tau(p,p) &= \sqrt{m_p/m_e}\,\tau(e,e) \approx 43\tau(e,e)\\ \tau(p,e) &= (m_p/m_e)\tau(e,e) \approx 1800\tau(e,e)\end{aligned}}$$

Ion fraction for Fe vs electron temperature



How Did I Know This??

- Why do we think that the emission is thermal bremmstrahlung?
  - X-ray spectra are consistent with model
  - X-ray 'image' is also consistent
  - Derived physical parameters 'make sense'
  - Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)

Mean Free Path for Collisions/ Energy

- Mean-free-path $\lambda_e \sim 20$ kpc < 1% of cluster size $\lambda_p \approx \lambda_e = \dfrac{3^{3/2}(kT)^2}{8\sqrt{\pi}\, n_e e^4 \ln\Lambda}$

$$\approx 23 \left(\frac{T}{10^8\ \mathrm{K}}\right)^2 \left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1} \mathrm{kpc}$$

At $T>3\times10^7$ K the major form of energy emission is thermal bremmstrahlung continuum

$\varepsilon \sim 3\times10^{-27}\ T^{1/2}\ n^2$ ergs/cm^3/sec; emissivity of gas

- how long does it take a parcel of gas to lose its energy?

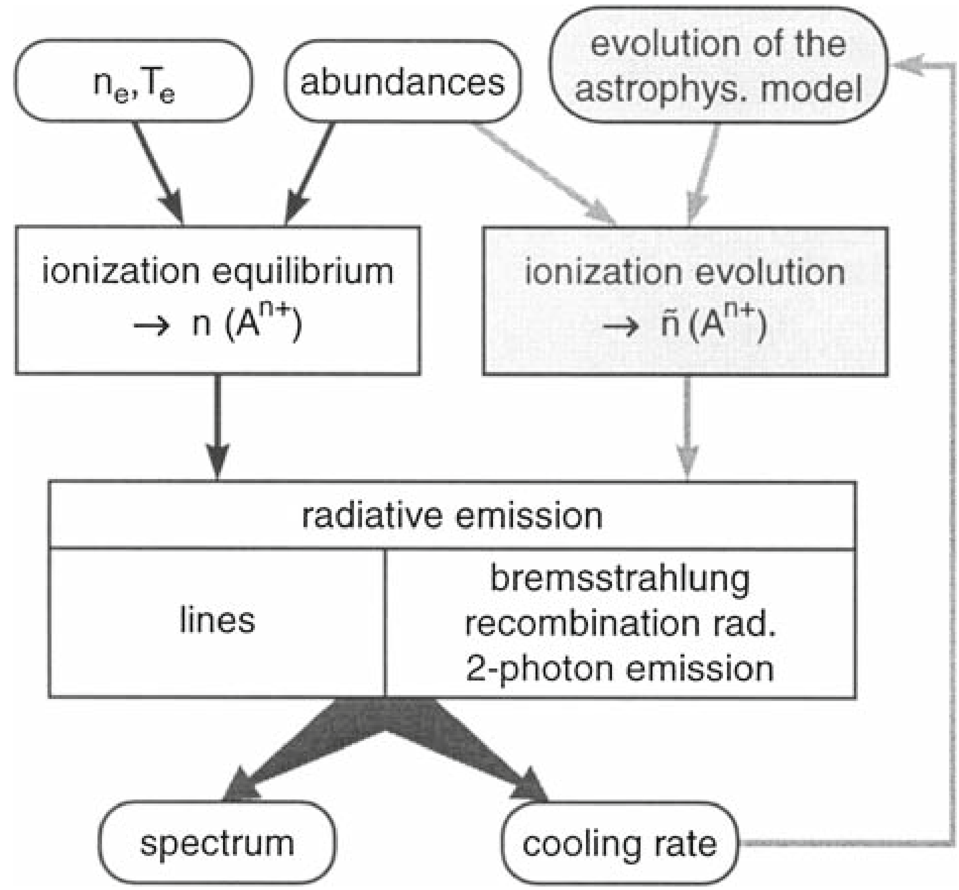
$\tau \sim nkT/\varepsilon \sim 8.5\times10^{10}$ yrs$(n/10^{-3})^{-1}\ T_8^{\ 1/2}$

At lower temperatures line emission is important

Why is Gas Hot

- To first order if the gas were cooler it would fall to the center of the potential well and heat up
- If it were hotter it would be a wind and gas would leave cluster
- Idea is that gas shocks as it 'falls into' the cluster potential well from the IGM
  - Is it 'merger' shocks (e.g. collapsed objects merging)
  - Or in fall (e.g. rain)

  BOTH



Physical Conditions in the Gas

- the elastic collision times for ions and electrons) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 \times 10^8 \mathrm{yr}\ (T_{gas}/10^8)^{1/2} (D/\mathrm{Mpc})$
- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ (P is the pressure, ρ_g is the gas density, $\gamma=5/3$ is the adiabatic index for a monoatomic ideal gas)

Hydrostatic Equilibrium Kaiser 19.2

- Equation of hydrostatic equil

$$\nabla P = -\rho_g \nabla \phi(r)$$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter)

P is the gas pressure

ρ_g **is the gas density**

Hydrodynamics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad \text{mass conservation (continuity)}$$

$$\rho \frac{Dv}{Dt} + \nabla P + \rho \nabla \phi = 0 \quad \text{momentum conservation (Euler)}$$

$$\rho T \frac{Ds}{Dt} = H - L \quad \text{entropy (heating \& cooling)}$$

$$P = \frac{\rho k T}{\mu m_p} \quad \text{equation of state}$$

Add viscosity, thermal conduction, …

Add magnetic fields (MHD) and cosmic rays

Gravitational potential ϕ from DM, gas, galaxies

- density and potential are related by Poisson's equation

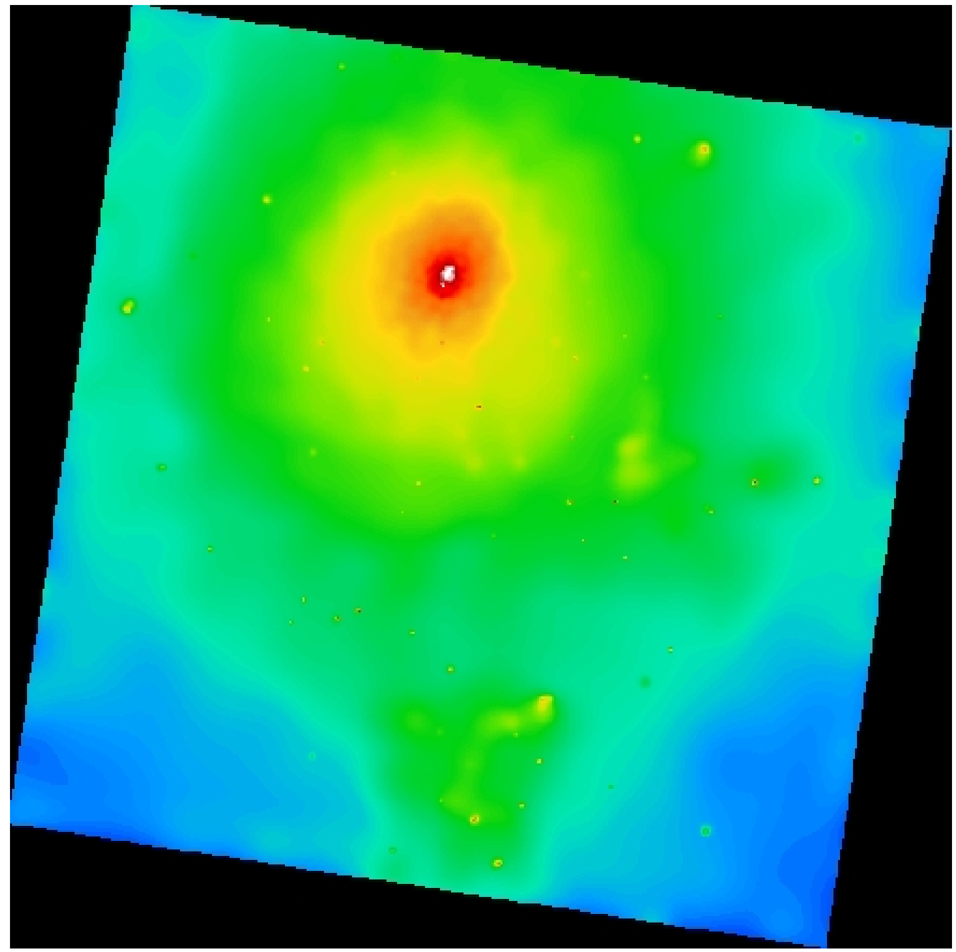
$\nabla^2 \phi = 4\pi\rho G$

- and combining this with the equation of hydrostaic equil

- $\nabla \cdot (1/\rho \nabla P) = -\nabla^2 \phi = -4\pi G\rho$

- or, for a spherically symmetric system

$1/r^2\, d/dr\, (r^2/\rho\; dP/dr) = -4\pi\rho G\rho$



Deriving the Mass from X-ray Spectra

For spherical symmetry this reduces to

$(1/\rho_g)\, dP/dr = -d\phi(r)/dr = GM(r)/r^2$

With a little algebra and the definition of pressure - the total cluster mass can be expressed as

$$\boxed{GM(r) = kT_g(r)/\mu G m_p)\mathbf{r}\ (d\ln T/dr + d\ln \rho_g/dr)}$$

k is Boltzmans const, μ is the mean mass of a particle and m_H is the mass of a hydrogen atom

Every thing is observable

The temperature T_g from the spatially resolved spectrum

The density ρ_g from the knowledge that the emission is due to bremmstrahlung

And the scale size, **r**, from the conversion of angles to distance

- The emission measure along the line of sight at radius r, EM(r), can be deduced from the X-ray surface brightness, $S(\Theta)$:

$EM(r) = 4\pi (1 + z)4\, S(\Theta)/\Lambda(T, z)$; $r = dA(z)\, \Theta$

where $\Lambda(T, z)$ is the emissivity in the detector band, taking into account the instrument spectral response,

dA(z) is the angular distance at redshift z.

The emission measure is linked to the gas density ρ_g by:

- $EM(r) = \int_r^\infty \rho_g^2(R)\, R\,dr/\sqrt{(R^2 - r^2)}$
- The shape of the surface brightness profile is thus governed by the form of the gas distribution, whereas its normalization depends also on the cluster overall gas content.

Density Profile

- a simple model(the β model) fits the surface brightness well
  - $S(r)=S(0)(1/r/a)^2)^{-3\beta+1/2}$ cts/cm^2/sec/solid angle
- Is analytically invertible (inverse Abel transform) to the density profile

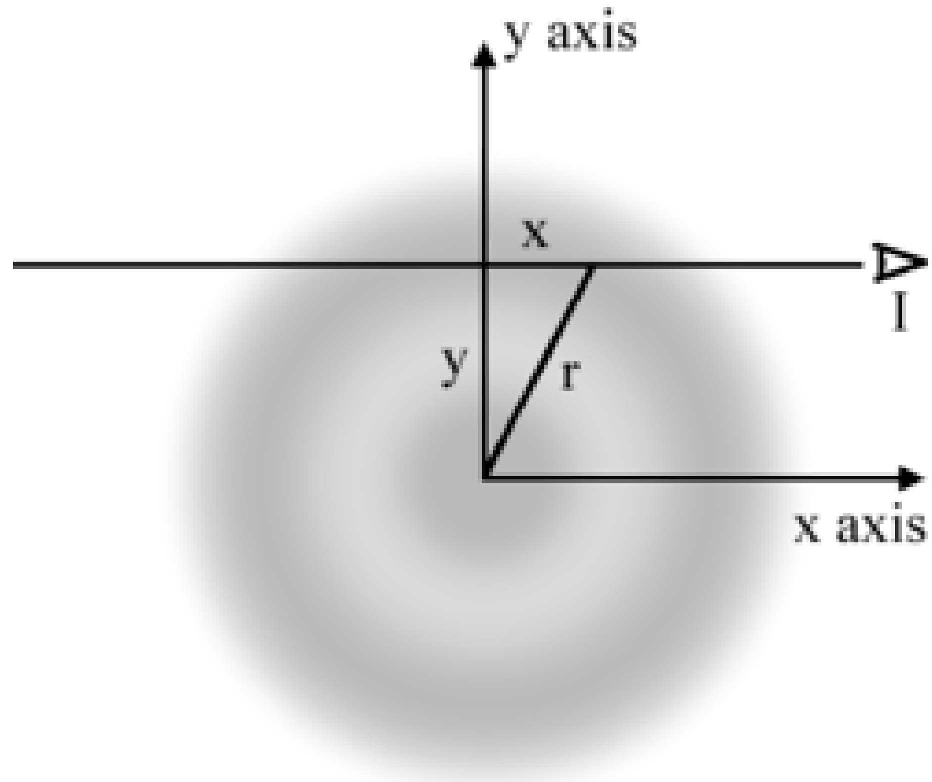
  $\rho(r)=\rho(0)(1/r/a)^2)^{-3\beta/2}$

  The conversion function from S(0) to ρ(0) depends on the detector

  The quantity 'a' is a scale factor- sometimes called the core radius

  ----
- The Abel transform, , is an integral transform used in the analysis of spherically symmetric or axially symmetric functions. The Abel transform of a function f(r) is given by:
- $f(r)=1/p\int_r^\infty dF/dy\ dy/\sqrt{(y^{2-}r^2)}$
- In image analysis tthe reverse Abel transform is used to calculate the emission function given a projection (i.e. a scan or a photograph) of that emission function.
- In general the integral is not analytic which makes the

A geometrical interpretation of the Abel transform in two dimensions. An observer (I) looks along a line parallel to the x-axis a distance y above the origin. What the observer sees is the projection (i.e. the integral) of the circularly symmetric function f(r) along the line of sight. The function f(r) is represented in gray in this figure. The observer is assumed to be located infinitely far from the origin so that the limits of integration are ±∞



Sarazin sec 5

The gas distributions in clusters can be derived directly from observations of the X-ray surface brightness of the cluster, if the shape of the cluster is known and if the X-ray observations are sufficiently detailed and accurate. This method of analysis also leads to a method for determining cluster masses (Section 5.5.5). The X-ray surface brightness at a photon frequency ν and at a projected distance b from the center of a spherical cluster is

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\epsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}, \qquad (5.80)$$

where ϵ_ν is the X-ray emissivity. This Abel integral can be inverted to give the emissivity as a function of radius,

$$\epsilon_\nu = -\frac{1}{2\pi r}\frac{d}{dr}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}}. \qquad (5.81)$$

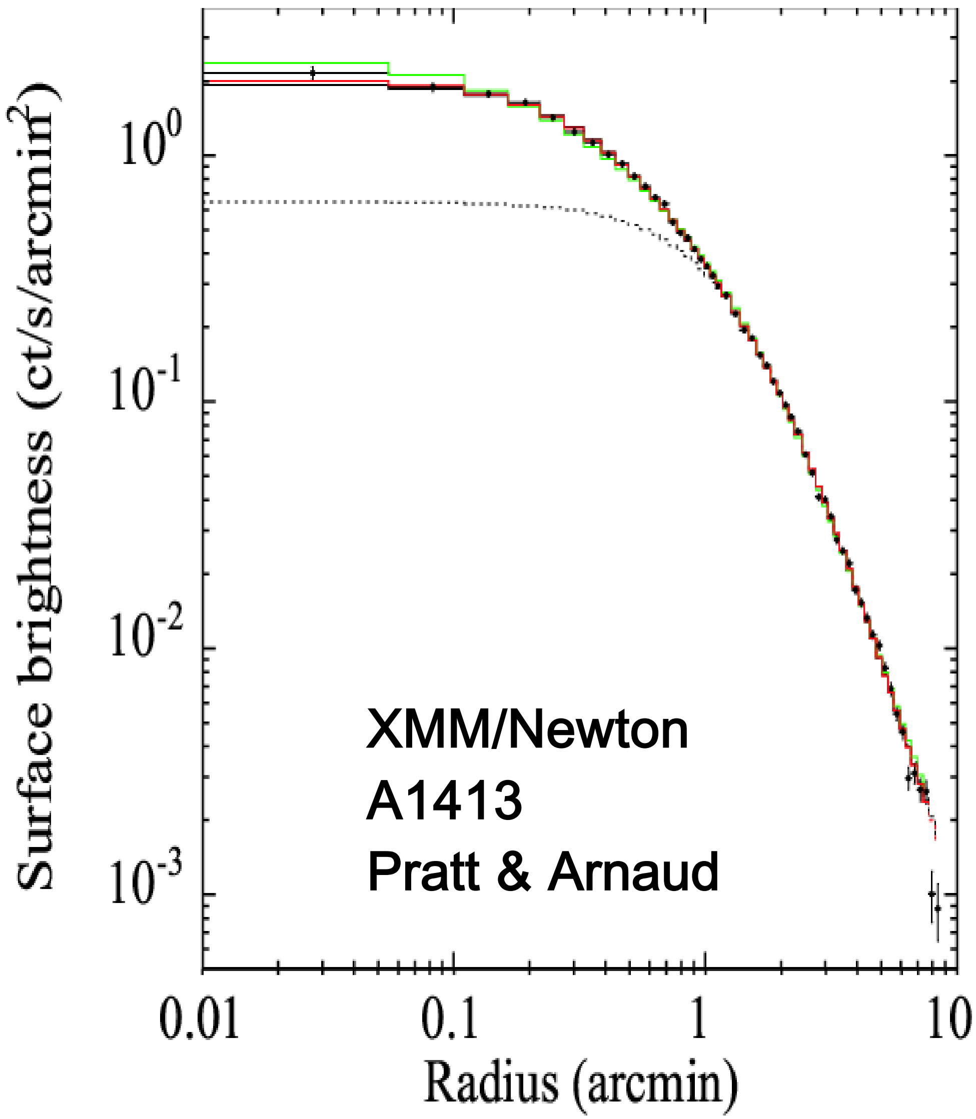
Surface Brightness Profiles

- It has become customary to use a 'β' model (Cavaliere and Fesco-Fumiano)'
- clusters have <β>~2/3

$$\rho(r) = \frac{\rho_0}{\left[1+\left(\frac{r}{r_c}\right)^2\right]^{3\beta/2}}$$

$$\beta \equiv \frac{\mu m_p \sigma_{gal}^2}{kT}$$ but treat as fitting parameter

$$I_X(r) \propto \left[1+\left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2}$$



'Two' Types of Surface Brightness Profiles

- 'Cored'
- Central Excess
- Range of core radii and β