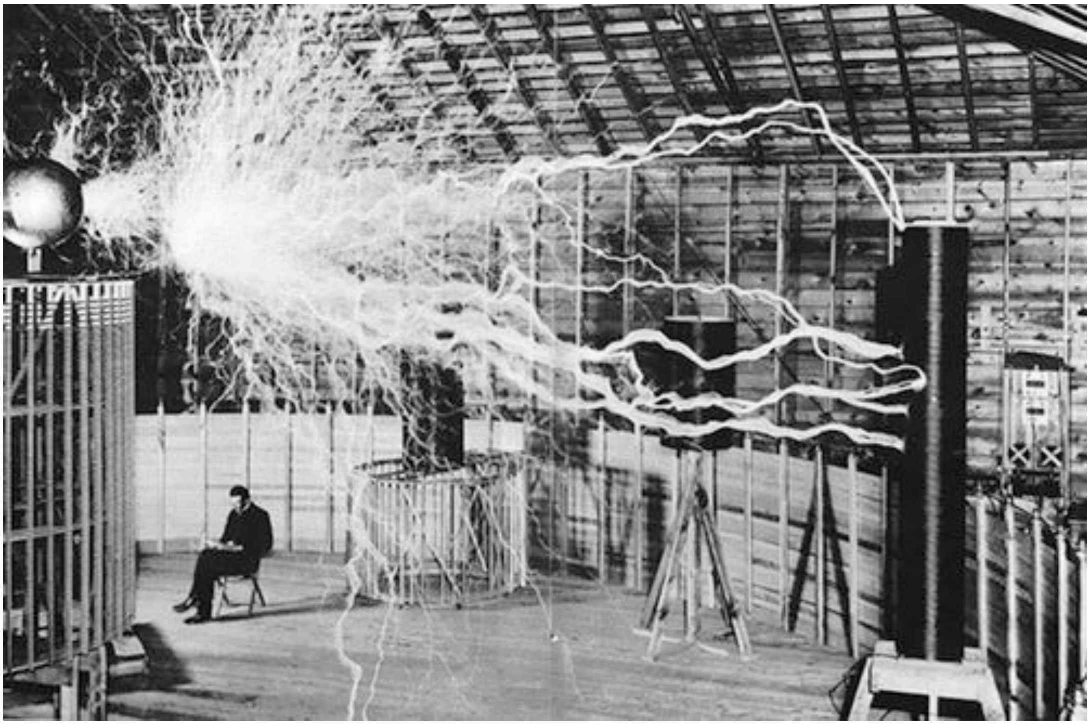
Back from Japan



Clusters of Galaxies X-ray Overview

Probes of the history of structure formation
Dynamical timescales are not much shorter than the age of the universe
• Studies of their evolution, temperature and luminosity function can place strong constraints on all theories of large scale structure
• and determine precise values for many of the cosmological parameters

Provide a record of nucleosynthesis in the universe- as opposed to galaxies, clusters probably retain all the enriched material created in them

•Measurement of the elemental abundances and their evolution provide fundamental data for the origin of the elements

•The distribution of the elements in the clusters reveals how the metals were removed from stellar systems into the IGM

Clusters should be "fair" samples of the universe"

•Studies of their mass and their baryon fraction reveal the "gross" properties of the universe as a whole

•Much of the entropy of the gas in low mass systems is produced by processes other than shocks-

- a major source of energy in the universe ?
- a indication of the importance of non-gravitational processes in structure formation ?

X-rays from Clusters of Galaxies

• The baryons *thermalize* to > 10^6 K making clusters strong X-ray sources- the potential energy of infall is converted into kinetic energy of the gas.

• Most of the baryons in a cluster are in the X-ray emitting plasma - only 10-20% are in the galaxies.

• Clusters of galaxies are self-gravitating accumulations of dark matter which have trapped hot plasma (intracluster medium - ICM) and galaxies: (the galaxies are the least massive constituent)

Todays Material

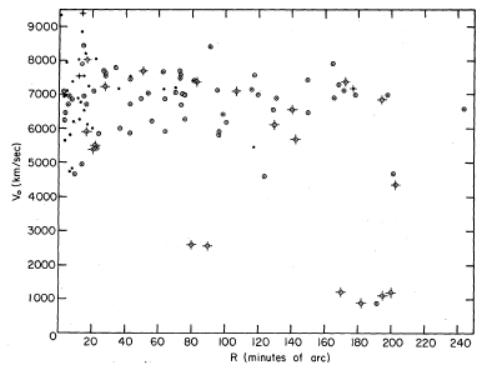
- How do we know that clusters are massive
  - Virial theorem
  - Lensing
  - -X-ray Hydrostatic equilibrium (but first we will discuss x-ray spectra) Equation of hydrostatic equilibrium (*)
- What do x-ray spectra of clusters look like

*Hydrostatic equilibrium

$\nabla \mathbf{P}=-\rho_g \nabla \phi(\mathbf{r})$ where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter) P is gas pressure and ρ_g is the gas density ($\nabla \mathbf{f}=(\partial f/x_1, \partial f/x_2 \ldots \partial f/x_n$)

The First Detailed Analysis

- Rood et al used the King (1969) analytic models of potentials (developed for globular clusters) and the velocity data and surface density of galaxies to infer a very high mass to light ratio of ~230.
- Since "no" stellar system had M/L>12 dark matter was necessary



Rood 1972- velocity vs position of galaxies in Coma

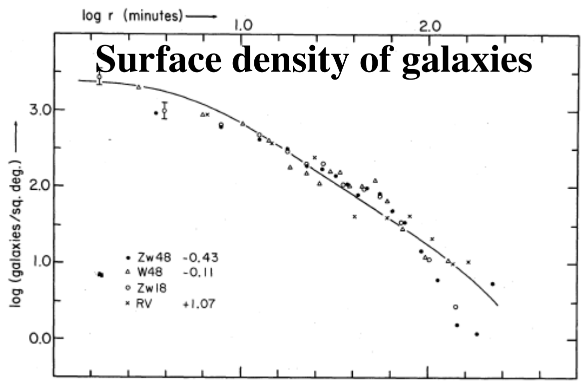


FIG. 5.—Surface densities, corrected for backgrounds given in table 2. For this fitting, logarithms of

Paper is worth reading ApJ 175,627

Virial Theorem (Read Longair 3.5.1; see also Kaiser sec 26.3)

- The virial theorem states that, for a stable, self-gravitating, spherical distribution of equal mass objects (stars, galaxies, etc), the total kinetic energy of the objects is equal to -1/2 times the total gravitational potential energy.

$2\langle T\rangle=-\langle U_{TOT}\rangle$

T is the time average of the Kinetic energy and U is the time overage of the potential energy

- In other words, the potential energy must equal 1/2 the kinetic energy. Consider a system of **N** particles with mass m and velocity v.
- kinetic energy of the total system is K.E. (system) = $1/2\ m\ N\ v^2 = 1/2\ M_{tot}v^2$

•PE~$1/2GN^2m^2/R_{tot} = 1/2GM^2_{tot}/R_{tot}$ (dimensional analysis)

If the orbits are random KE=1/2PE (virial theorm)

$M_{tot} \sim 2R_{tot}v^2_{tot}/G$

No assumptions been made about the orbits or velocity distributions of the particles.

The virial theorem applies to all cases provided the system is in dynamical equilibrium- "virialized"

Virial Theorem Actual Use (Kaiser 26.4.2)

- Photometric observations provide the surface brightness Σ_{light} of a cluster. Measurements of the velocity dispersion σ^2_v together with the virial theorem give $\sigma^2_v \sim U/M \sim GM/R \sim G\Sigma_{mass} R$

  Σ_{mass} is the projected mass density.

At a distance D the mass to light ratio (M/L) can be estimated as
$M/L = \Sigma_{mass}/\Sigma_{light} = \sigma^2_v / GD\Theta\Sigma_{light}$
where Θ is the angular size of the cluster.

Applying this technique, Zwicky found that clusters have M/L ~300 in solar units

- The virial theorem is exact, *but requires that the light traces the mass-it will fail if the dark matter has a different profile from the luminous particles*.

Mass Estimates

- While the virial theorem is fine it depends on knowing the time averaged orbits, the distribution of particles etc etc- a fair amount of systematic errors
- If the system is spherically symmetric, a suitably weighted mean separation

R_{cl} can be estimated from the observed surface distribution of stars or galaxies and so the gravitational potential energy can be written $|U| = GM^2/R_{cl}$.

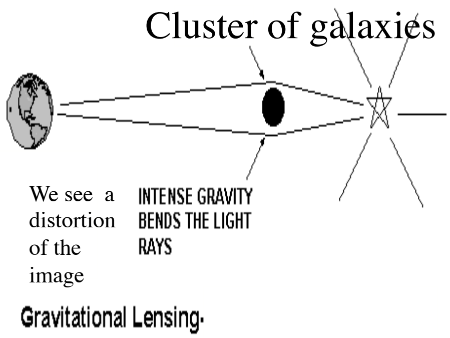
- The mass of the system is using $T = ½ |U|$
- $M = 3v^2R/G$.

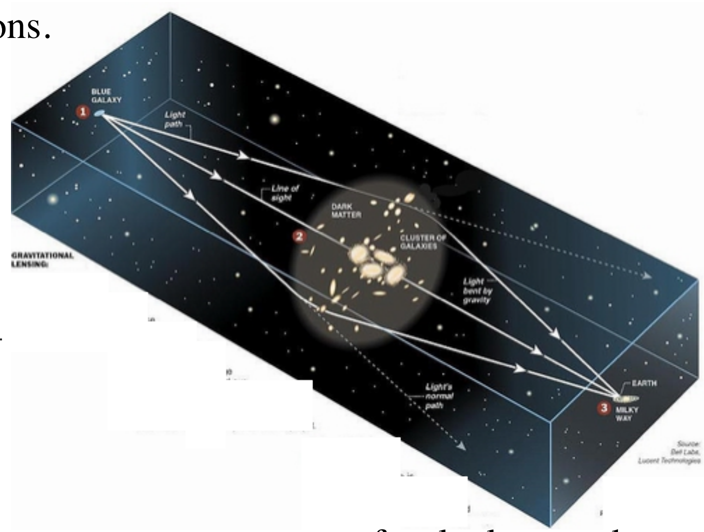
- Would like better techniques
  - Gravitational lensing
  - Use of spatially resolved x-ray spectra

Light Can Be Bent by Gravity- read sec 4.7 Longair

weak gravitational lensing. Light rays propagating to us through the inhomogeneous universe get tugged from side to side by mass concentrations.

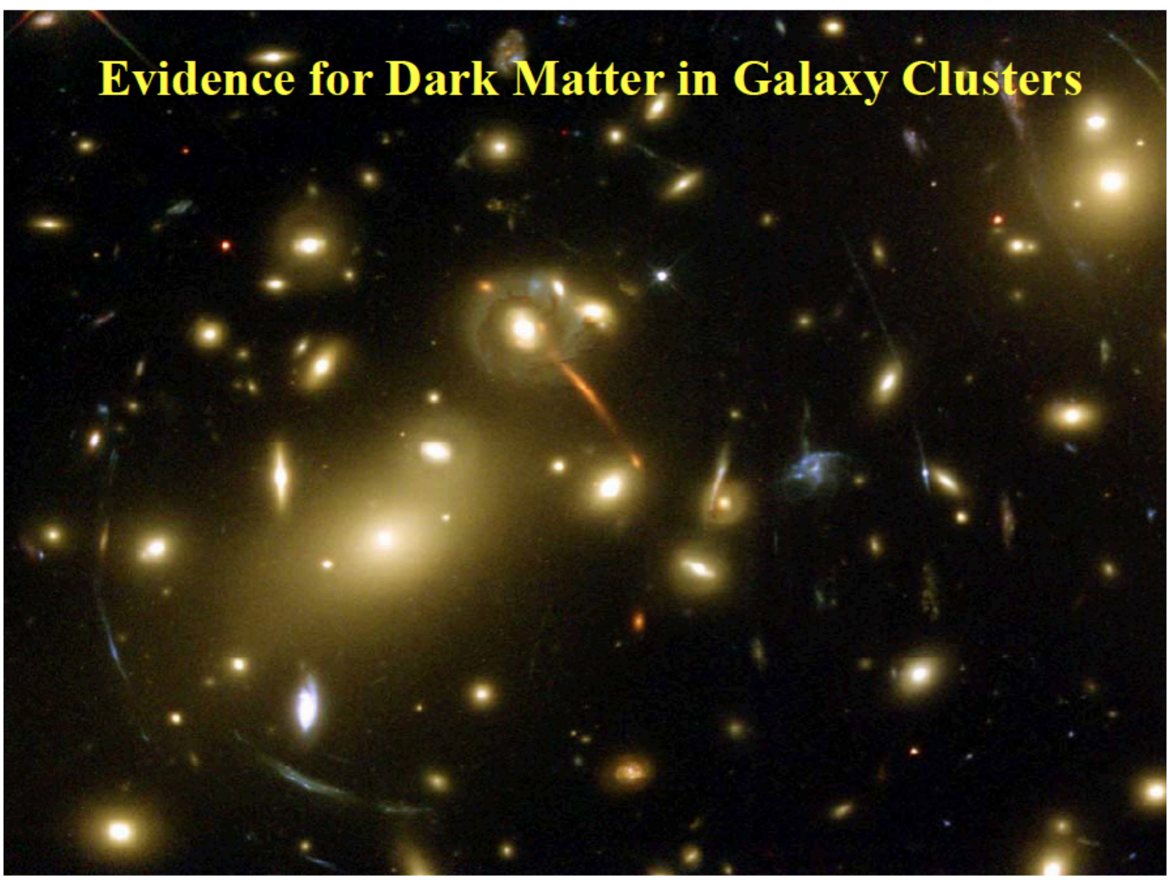
The more mass- the more the light is bent





faculty.lsmsa.edu

Amount and type of distortion is related to amount and distribution of mass in gravitational lens



Basics of Gravitational Lensing Sec 4.7 Longair

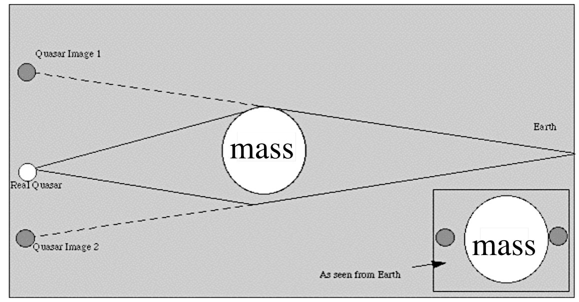
- See Lectures on Gravitational Lensing by

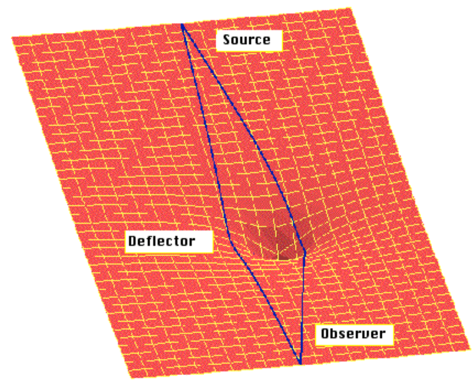
Ramesh Narayan Matthias Bartelmann or
    http://www.pgss.mcs.cmu.edu/1997/Volume16/physics/GL/GL-II.html

For a detailed discussion of the problem

- Rich centrally condensed clusters occasionally produce giant arcs when a background galaxy happens to be aligned with one of the cluster caustics.
- Every cluster produces weakly distorted images of large numbers of background galaxies.
  - These images are called arclets and the phenomenon is referred to as weak lensing.
- The deflection of a light ray that passes a point mass M at *impact parameter b* is eq 4.28

$$\Theta_{def} = 4GM/c^2b$$





Lensing

- assume -

the overall geometry of the universe is Friedmann--Robertson- Walker metric

matter inhomogeneities which cause the lensing are local perturbations.

- Light paths propagating from the source past the lens 3 regimes
- 1)light travels from the source to a point close to the lens through unperturbed spacetime.
- 2)near the lens, light is deflected.
- 3) light again travels through unperturbed spacetime.

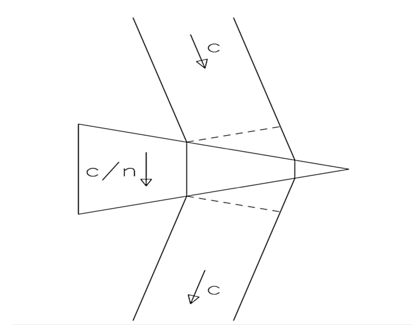
The effect of spacetime curvature on the light paths can be expressed in terms of an effective index of refraction, n, (e.g. Schneider et al. 1992)

$n = 1 - (2/c^2)\,\phi(r)$;$\phi(r)$ the Newtonian gravitational potential

As in normal optics, for refractive index n > 1 light travels slower than in free vacuum.

effective speed of a ray of light in a gravitational field is

$v = c/n \sim c - (2/c)\phi$



As an example, we now evaluate the deflection angle of a point mass M (cf. Fig. 3). The Newtonian potential of the lens is

$$\Phi(b,z) = -\frac{GM}{(b^2+z^2)^{1/2}}\,, \tag{5}$$

where b is the impact parameter of the unperturbed light ray, and z indicates distance along the unperturbed light ray from the point of closest approach. We therefore have

$$\vec{\nabla}_\perp \Phi(b,z) = \frac{GM\,\vec{b}}{(b^2+z^2)^{3/2}}\,, \tag{6}$$

where $\vec{b}$ is orthogonal to the unperturbed ray and points toward the point mass. Equation (6) then yields the deflection angle

$$\hat{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi\, dz = \frac{4GM}{c^2 b}\,. \tag{7}$$

Note that the Schwarzschild radius of a point mass is

$$R_S = \frac{2GM}{c^2}\,, \tag{8}$$

so that the deflection angle is simply twice the inverse of the impact parameter in units of the Schwarzschild radius. As an example, the Schwarzschild radius of the Sun is 2.95 km, and the solar radius is 6.96×10^5 km. A light ray grazing the limb of the Sun is therefore deflected by an angle $(5.9/7.0) \times 10^{-5}$ radians = 1.″7.

Narayan and Bartellman 1996

Einstein radius is the scale of lensing

- For a point mass it is

$\theta_E = ((4GM/c^2)(D_{ds}/D_d D_s))^{1/2}$ (4.29)

- or in more useful units

$\theta_E = (0.9")M_{11}^{1/2}\,D_{Gpc}^{-1/2}$ (4.32)

- Lens eq

$\beta\ = \theta-(D_{ds}/D_d D_s)4GM/\theta c^2$.

or

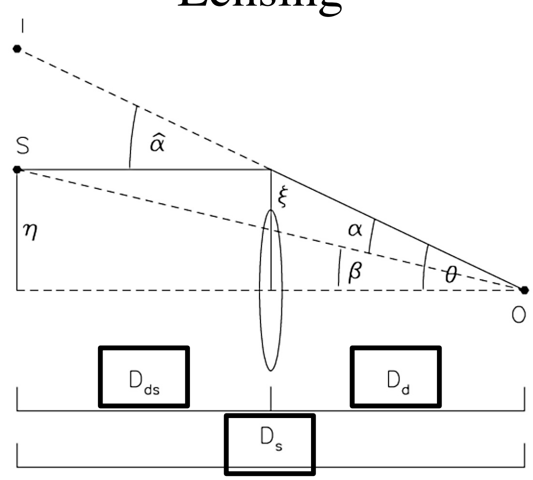
$\beta\ = \theta-\theta^2_E/\theta$

β 2 solutions

Any source is imaged twice by a point mass lens

Gravitational light deflection preserves surface brightness because of the Liouville theorm

Lensing



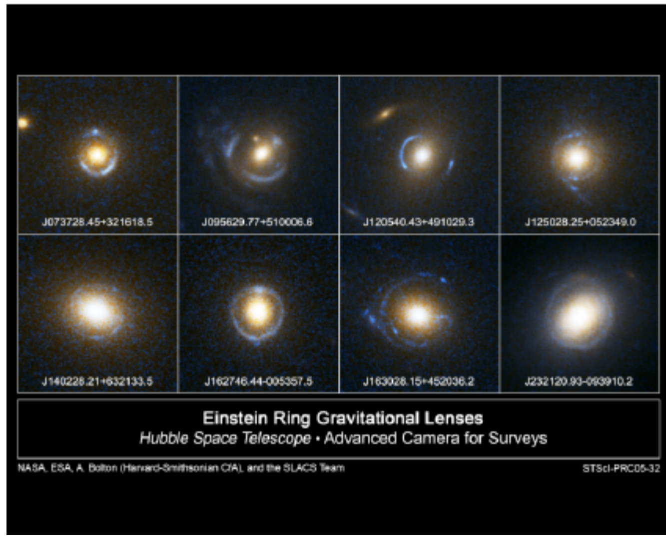
Condition for formation of lensed image (Σ is surface mass density)

$\Sigma_{cr} > [c^2/4\pi G]D_s/D_d D_{ds} =$
0.35 g cm^{-2}[D/Gpc^{-1}]; $D = D_d D_{ds}/D_s$
see 4.35-4.39

Lensing

- Gravitational deflection of the light ray $\alpha=4\pi v^2/c^2$ (4.43)
- This is exact for a single isothermal sphere model of the mass of the lensing object and the Einstein radius is
- $\theta_E=28.8\ V^2_{1000}\ D_{LS/}D_s$ "

Examples of strong lenses



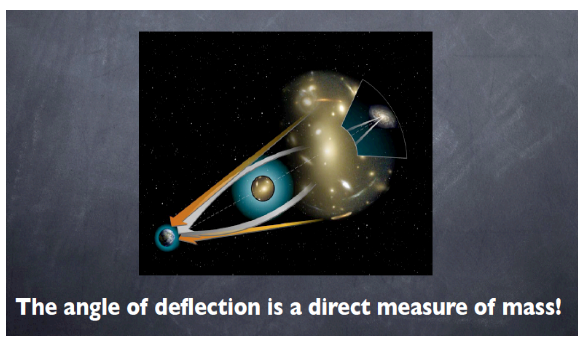
Ways of Thinking About Lensing (Kaiser sec 33.5)

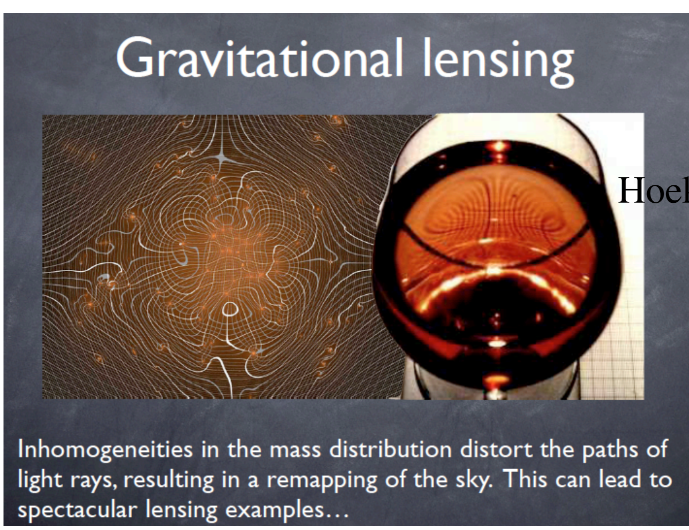
- This deflection is just twice what Newtonian theory would give for the deflection of a test particle moving at v = c where we can imagine the radiation to be test particles being pulled by a gravitational acceleration.

- another way to look at this using wave-optics; the inhomogeneity of the mass distribution causes space-time to become curved. The space in an over-dense region is positively curved.
  light rays propagating through the over-density have to go a slightly greater distance than they would in the absence of the density perturbation.
  Consequently the wave-fronts get retarded slightly in passing through the over-density and this results in focusing of rays.

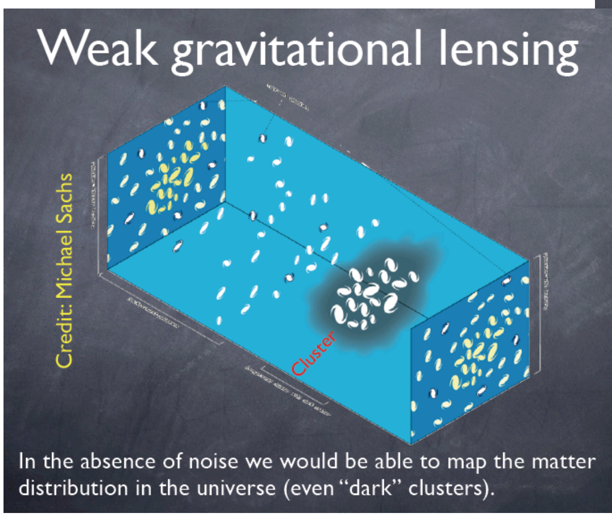
- Another way : The optical properties of a lumpy universe are, in fact, essentially identical to that of a block of glass of inhomogeneous density where the refractive index is $n(r) = (1- 2\phi(r)/c^2)$ with $\phi(r)$ the Newtonian gravitational potential. In an over-dense region, ϕ is negative, so n is slightly greater than unity. In this picture we think of space as being flat, but that the speed of light is slightly retarded in the over-dense region.

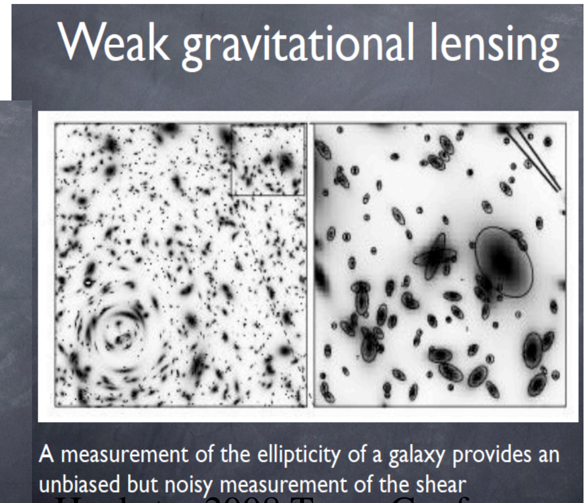
- All three of the above pictures give identical results



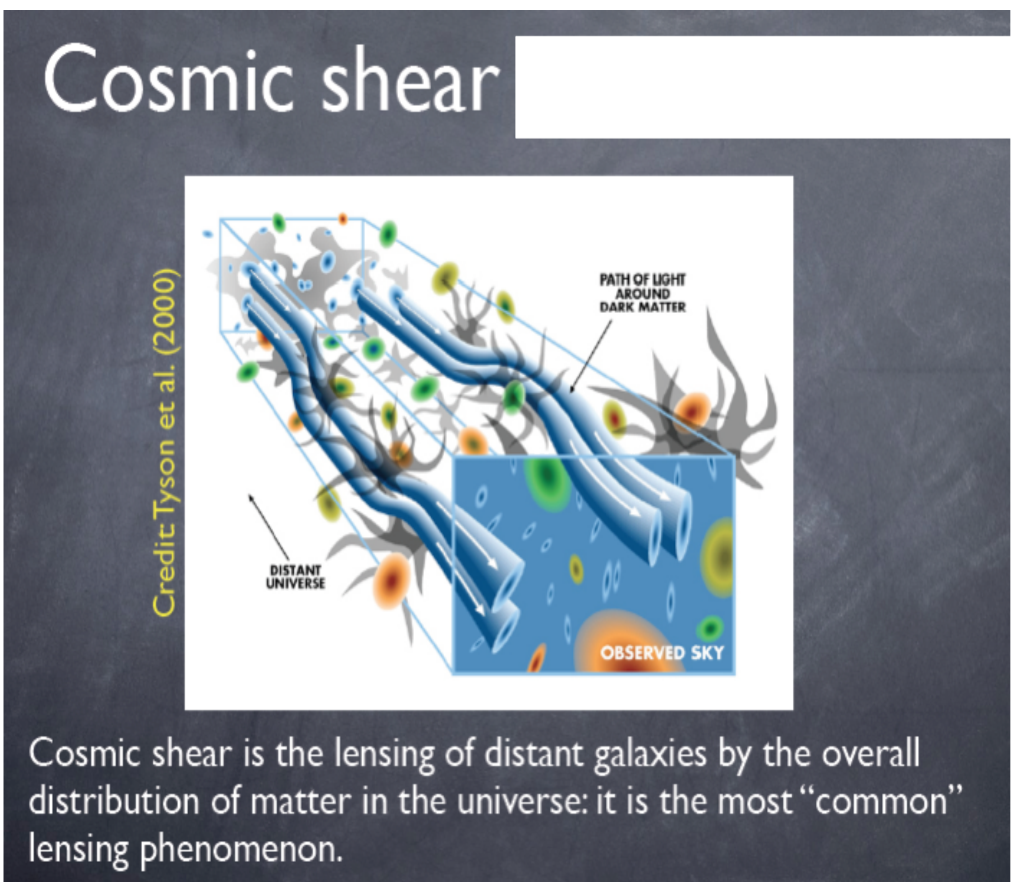


Hoekstra 2008 Texas Conference

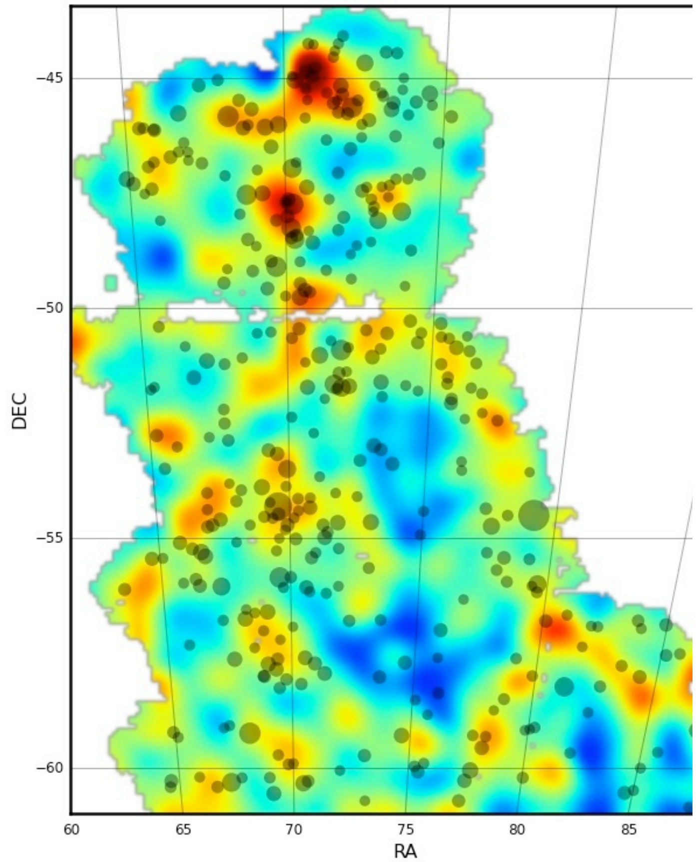




Hoekstra 2008 Texas Conference



- The detailed distribution of dark matter traced across a large area of sky: yellow and red represent relatively dense regions of dark matter and the black circles represent galaxy clusters (Chang et al 2015)



What we try to measure with X-ray Spectra

- From the x-ray spectrum of the gas we can measure a mean temperature, a redshift, and abundances of the most common elements (heavier than He).
- With good S/N we can determine whether the spectrum is consistent with a single temperature or is a sum of emission from plasma at different temperatures.
- Using symmetry assumptions the X-ray surface brightness can be converted to a measure of the ICM density.

What we try to measure II

If we can measure the temperature and density at different positions in the cluster then assuming the plasma is in hydrostatic equilibrium we can derive the gravitational potential and hence the amount and distribution of the dark matter.

There are two other ways to get the gravitational potential :

- The galaxies act as test particles moving in the potential so their velocities and poitional distribution provides a measure of total mass (Viral theorm)
- The gravitational potential acts as a lens on light from background galaxies.

Why do we care ?

Cosmological simulations predict distributions of masses.

If we want to use X-ray selected samples of clusters of galaxies to measure cosmological parameters then we must be able to relate the observables (X-ray luminosity and temperature) to the theoretical masses.

Theoretical Tools

- Physics of hot plasmas
  - Bremmstrahlung
  - Collisional equilibrium
  - Atomic physics

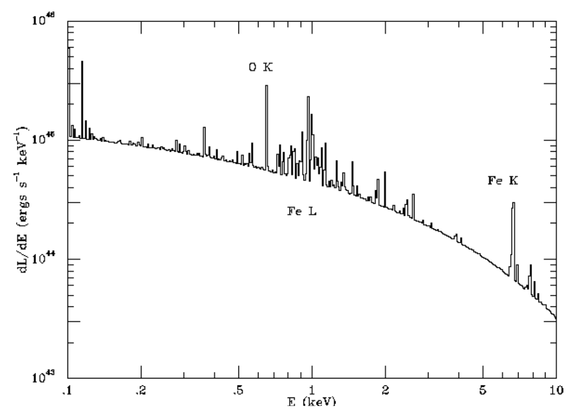
Physical Processes

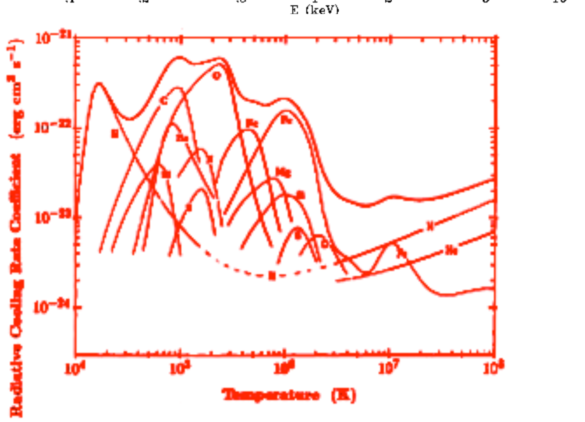
- Continuum emission
  - Thermal bremsstrahlung, ~exp(-hν/kT)
  - Bound-free (recombination)

- Line Emission
  (line emission)
  $L_\nu \sim \varepsilon_\nu$ (T, abund) (n_e^2 V)

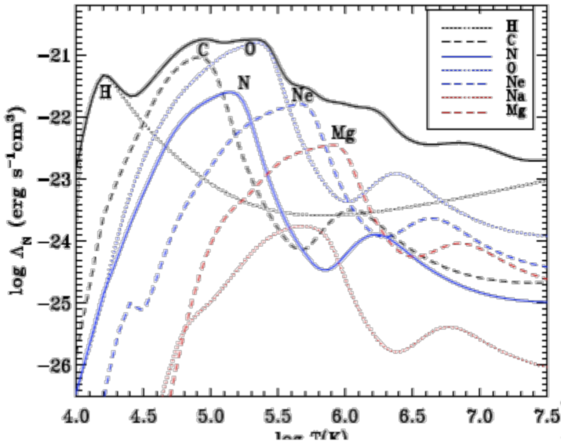
  Line emission dominates cooling at T<10^7 K
  Bremmstrahlung at higher temperatures

$$\epsilon(\nu) = \frac{16\, e^6}{3\, m_e\, c^2}\left(\frac{2\pi}{3 m_e\, k_B T_X}\right)^{1/2} n_e n_i\, Z^2\, g_{ff}(Z, T_X, \nu)\, \exp\left(\frac{-h\nu}{k_B T_X}\right),$$





oling rate of hot plasma as a function of the plasma temperature. The contribution to the cooling
s of different important abundant elements is indicated (Böhringer and Hensler 1989). Most of



Cooling Function Λ

Notice the 5 order of magnitude range in the various components of Λ in as a function of kT

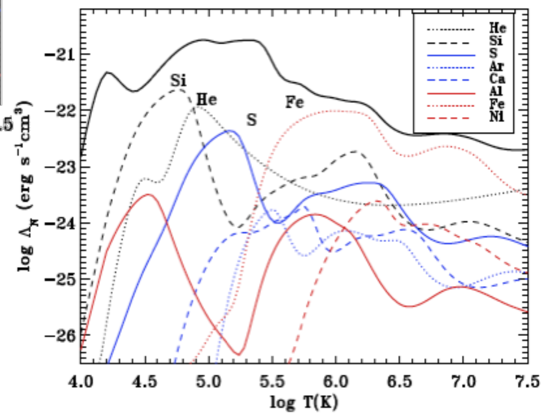
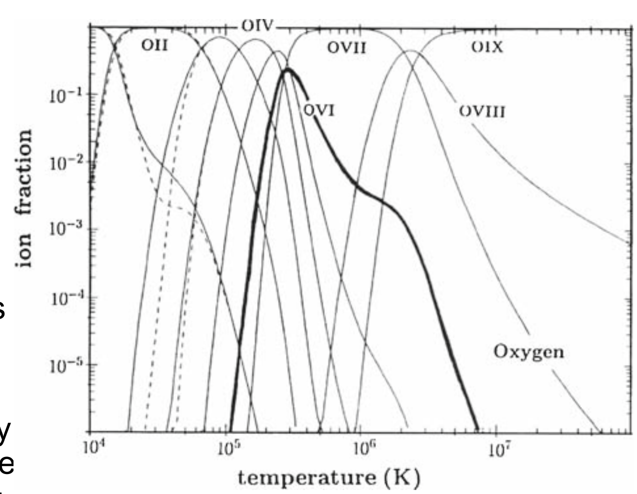


Fig. 2. Contributions of different elements to the cooling curve are given. Each of the plots shows a different set of elements. Important peaks are labelled with the name of the element. The total cooling curve (black solid line) is an addition of the individual elemental contributions.

- Cooling function is the emissivity per unit volume at a fixed density as a function of temperature
- The two panels show a different set of elements
- The black curve is the sum of line and continuum cooling

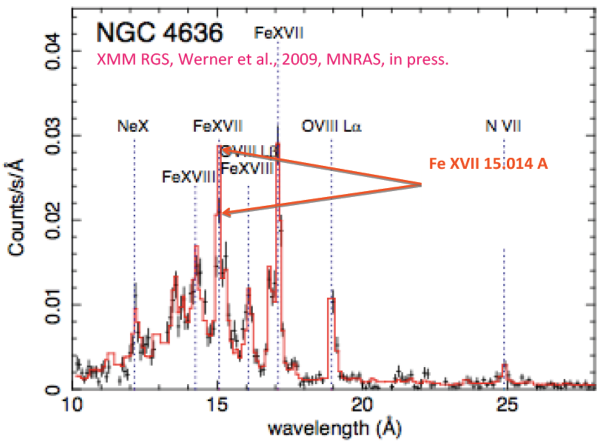
Cluster Plasma Parameters

- Electron number density $n_e \sim 10^{-3}$ cm^{-3} in the center with density decreasing as $n_e \sim r^{-2}$
- $10^6 < T < 10^8$ k
- Mainly H, He, but with heavy elements (O, Fe, ..)
- Mainly emits X-rays
- $10^{42} < L_X < 10^{45.3}$ erg/s, most luminous extended X-ray sources in Universe
- Age ~ 2-10 Gyr
- Mainly ionized, but not completely e.g. He and H-like ions of the abundant elements (O…Fe) exist in thermal equilibrium



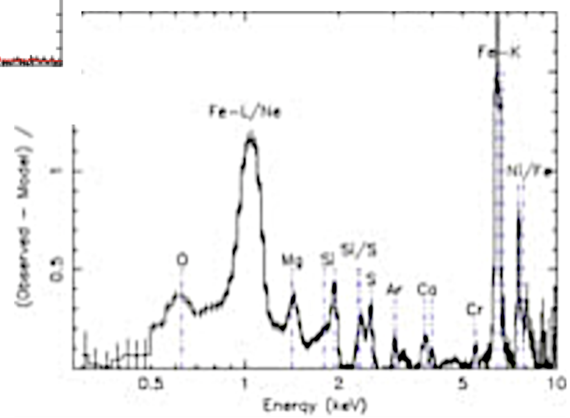
Ion fraction for oxygen vs electron temperature

X-ray Spectra of Clusters



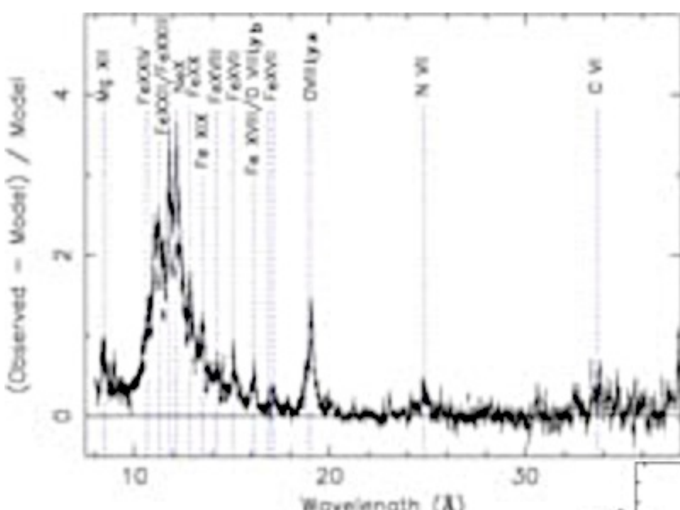
The ratio of the line strengths to that of the continuum is a measure of the abundance of a given ion

To derive the elemental abundance one has to take into account the ionization balance



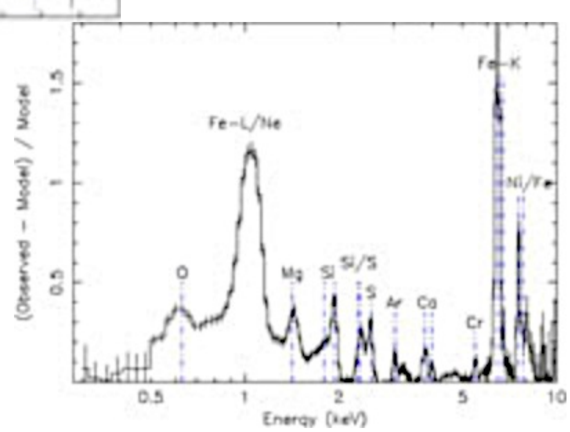
Top panel in wavelength space bottom in energy space kT~10^7K

X-ray Spectra of Clusters

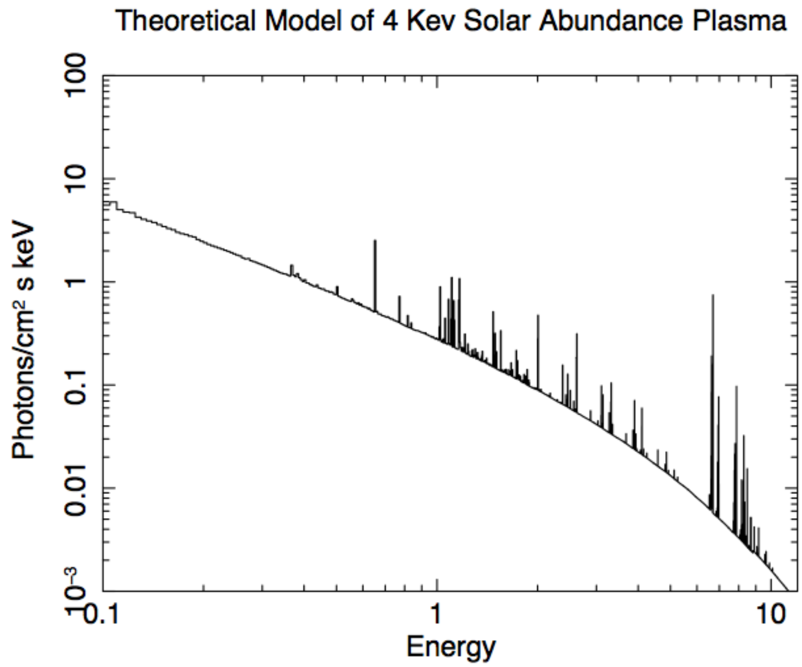


Grating Spectrum

- CCD Spectrum

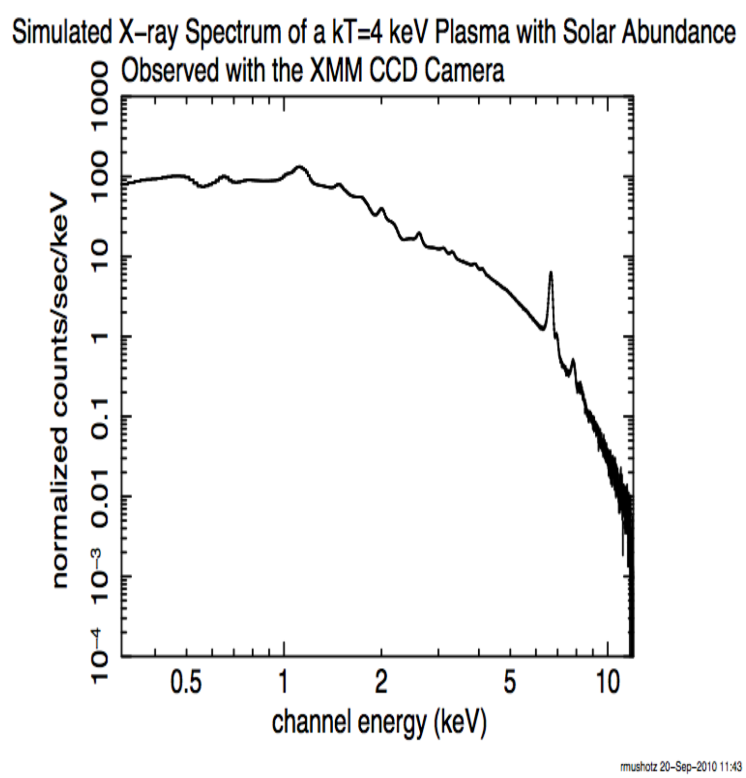


- Theoretical model of a collisionally ionized plasma kT=4 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^4



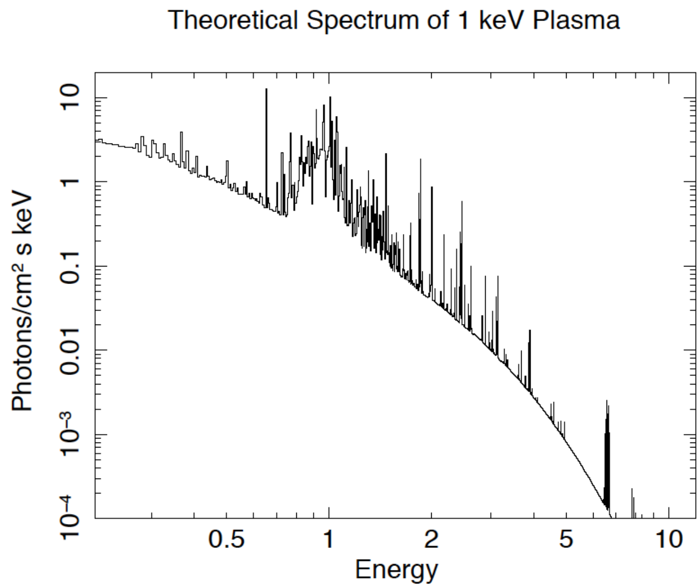
X-ray Spectra Data

- For hot ($kT>3x10^7$k) plasmas the spectra are continuum dominated- most of the energy is radiated in the continuum
- (lines broadened by the detector resolution)

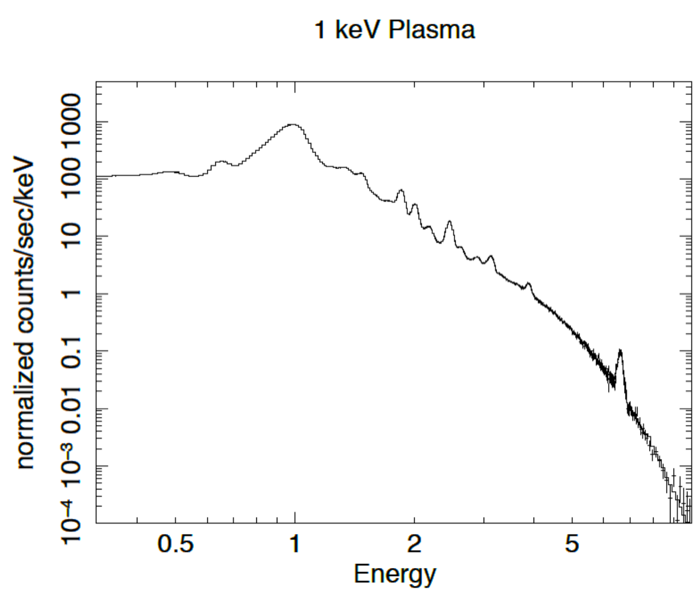


1 keV Plasma

- Theoretical model of a collisionally ionized plasma kT=1 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^5

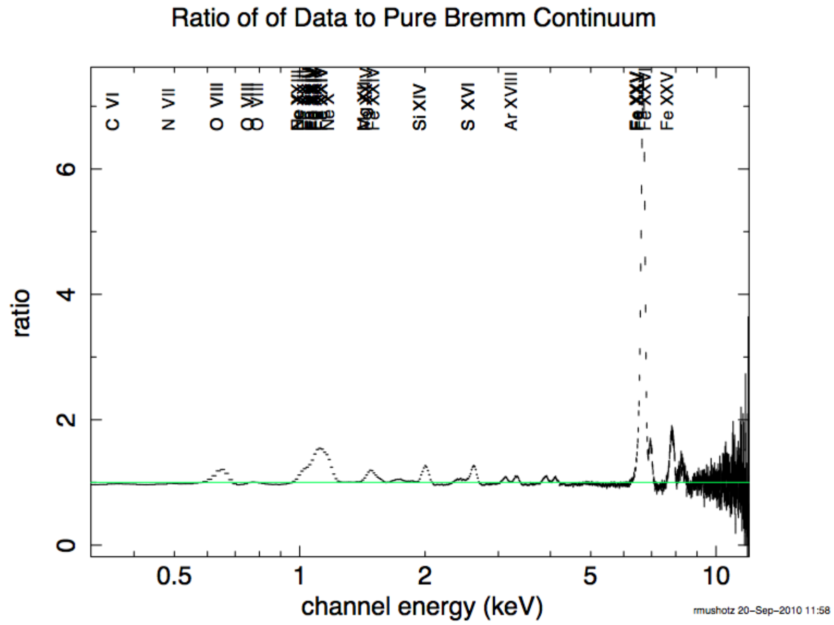


- Observational data for a collisionally ionized plasma kT=1 keV with solar abundances
- Notice the very large blend of lines near 1 keV- L shell lines of Fe
- Notice dynamic range of 10^7



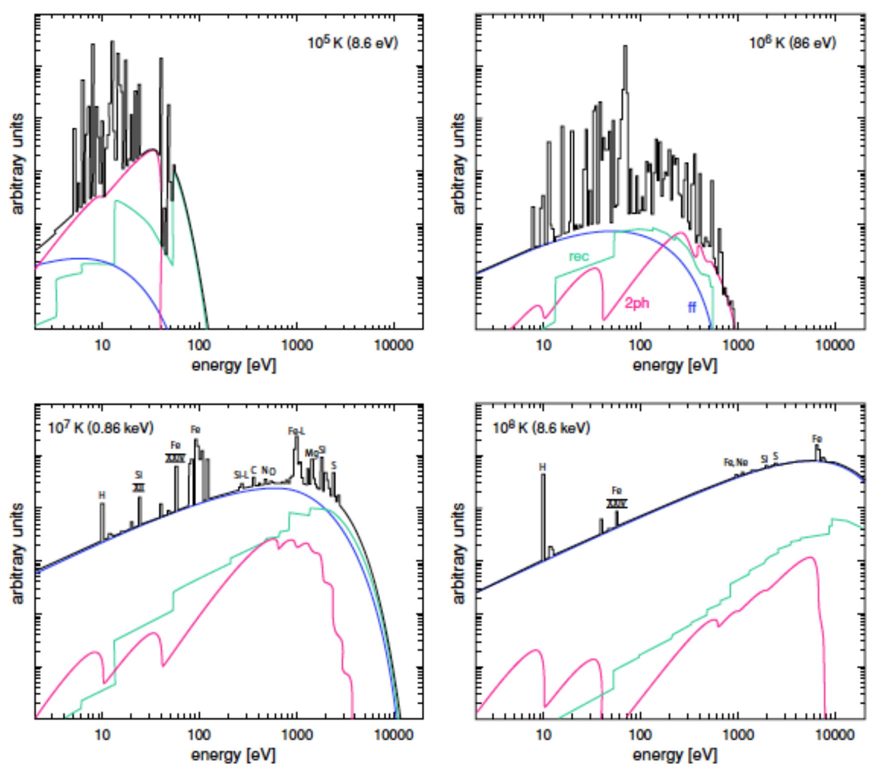
Collsionally Ionized Equilibrium Plasma

- Ratio of model to a 'pure' H/He plasma
- This plot is designed to show the lines



Strong Temperature Dependence of Spectra

- Line emission
- Bremms (black)
- Recombin ation (red)
- 2 photon green



Relevant Time Scales- see Longair pg 301

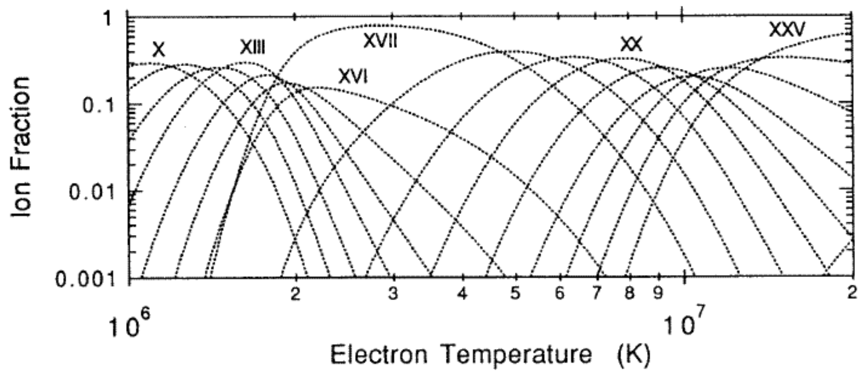
- The equilibration timescales between protons and electrons is t(p,e) ~ 2 x 10^8 yr at an 'average' location
- **In collisional ionization equilibrium population of ions is directly related to temperature**

$$\tau(1,2) = \frac{3m_1\sqrt{2\pi}(kT)^{3/2}}{8\pi\sqrt{m_2}n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}$$

$$\ln\Lambda \equiv \ln(b_{\max}/b_{\min}) \approx 40$$

$$\boxed{\begin{aligned} \tau(e,e) &\approx 3\times10^5\left(\frac{T}{10^8\,\mathrm{K}}\right)^{3/2}\left(\frac{n_e}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1}\ \mathrm{yr} \\ \tau(p,p) &= \sqrt{m_p/m_e}\,\tau(e,e) \approx 43\tau(e,e) \\ \tau(p,e) &= (m_p/m_e)\tau(e,e) \approx 1800\tau(e,e) \end{aligned}}$$

Ion fraction for Fe vs electron temperature



How Did I Know This??

- Why do we think that the emission is thermal bremmstrahlung?
  - X-ray spectra are consistent with model
  - X-ray 'image' is also consistent
  - Derived physical parameters 'make sense'
  - Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)

Mean Free Path for Collisions/ Energy

- Mean-free-path λ_e ~ 20 kpc < 1% of cluster size

$$\lambda_p \approx \lambda_e = \frac{3^{3/2}(kT)^2}{8\sqrt{\pi}\, n_e e^4 \ln\Lambda}$$

$$\approx 23 \left(\frac{T}{10^8\,\mathrm{K}}\right)^2 \left(\frac{n_e}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1} \mathrm{kpc}$$

At T>3x10^7 K the major form of energy emission is thermal bremmstrahlung continuum

$\varepsilon \sim 3\times10^{-27}\, T^{1/2}\, n^2$ ergs/cm^3/sec; emissivity of gas
- how long does it take a parcel of gas to lose its energy?

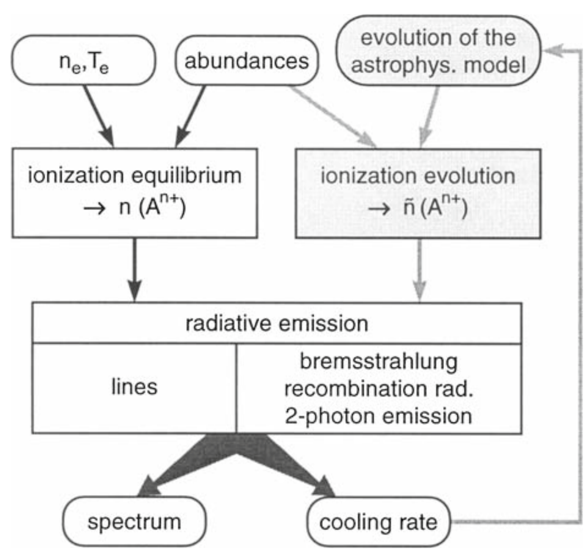
$\tau \sim nkT/\varepsilon \sim 8.5\times10^{10}\mathrm{yrs}(n/10^{-3})^{-1}\, T_8^{\ 1/2}$

At lower temperatures line emission is important

Why is Gas Hot

- To first order if the gas were cooler it would fall to the center of the potential well and heat up
- If it were hotter it would be a wind and gas would leave cluster
- Idea is that gas shocks as it 'falls into' the cluster potential well from the IGM
  - Is it 'merger' shocks (e.g. collapsed objects merging)
  - Or in fall (e.g. rain)

  BOTH



Physical Conditions in the Gas

- the elastic collision times for (ions and electrons) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 \times 10^8 \text{yr} \, (T_{gas}/10^8)^{1/2} (D/\text{Mpc})$
- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ (P is the pressure, ρ_g is the gas density, $\gamma = 5/3$ is the adiabatic index for a monoatomic ideal gas)