What we try to measure with X-ray Spectra

- From the x-ray spectrum of the gas we can measure a mean temperature, a redshift, and abundances of the most common elements (heavier than He).

- With good S/N we can determine whether the spectrum is consistent with a single temperature or is a sum of emission from plasma at different temperatures.

- Using symmetry assumptions the X-ray surface brightness can be converted to a measure of the ICM density.

What we try to measure II

If we can measure the temperature and density at different positions in the cluster then assuming the plasma is in hydrostatic equilibrium we can derive the gravitational potential and hence the amount and distribution of the dark matter.

There are two other ways to get the gravitational potential :we discussed them earlier

- The galaxies act as test particles moving in the potential so their redshift distribution provides a measure of total mass.
- The gravitational potential acts as a lens on light from background galaxies.

Why do we care ?

Cosmological simulations predict distributions of masses.

If we want to use X-ray selected samples of clusters of galaxies to measure cosmological parameters then we must be able to relate the observables (X-ray luminosity and temperature) to the theoretical masses.

Theoretical Tools

- Physics of hot plasmas
  - Bremmstrahlung
  - Collisional equilibrium
  - Atomic physics

Physical Processes

- Continuum emission
  - Thermal bremsstrahlung, ~exp(-hν/kT)
  - Bound-free (recombination)
  - Two Photon
- Line Emission

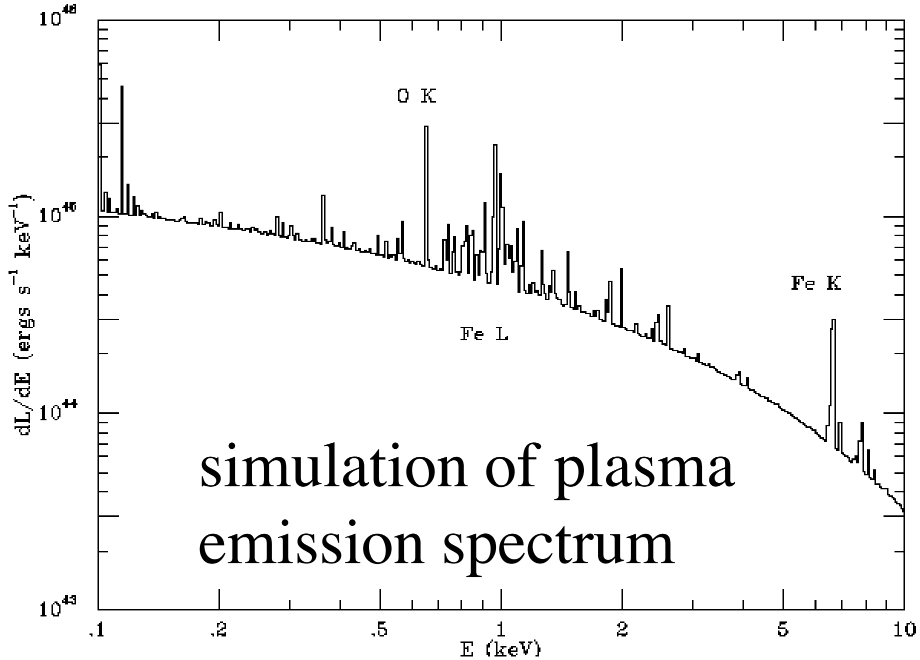
  (line emission)
- Continuum luminosity

  $L_\nu \sim \varepsilon_\nu$ (T, abund) (n_e^2 V)
- Line luminosity

  Line emission dominates cooling at T<10^7 K

  Bremmstrahlung dominates at higher temperatures

$$\epsilon(\nu) = \frac{16\, e^6}{3\, m_e\, c^2} \left(\frac{2\pi}{3 m_e\, k_B T_X} \right)^{1/2} n_e n_i\, Z^2\, g_{ff}(Z, T_X, \nu)\, \exp\left(\frac{-h\nu}{k_B T_X} \right),$$



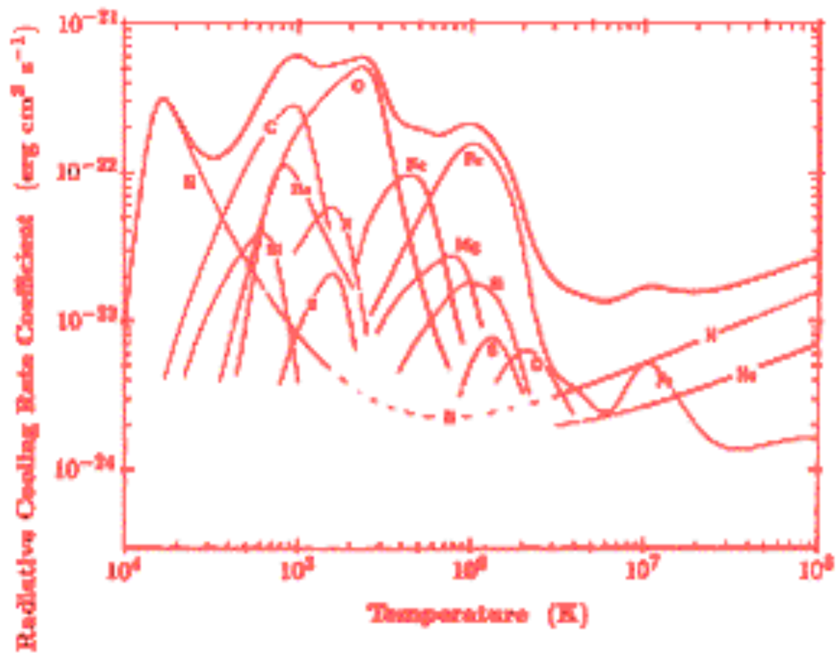
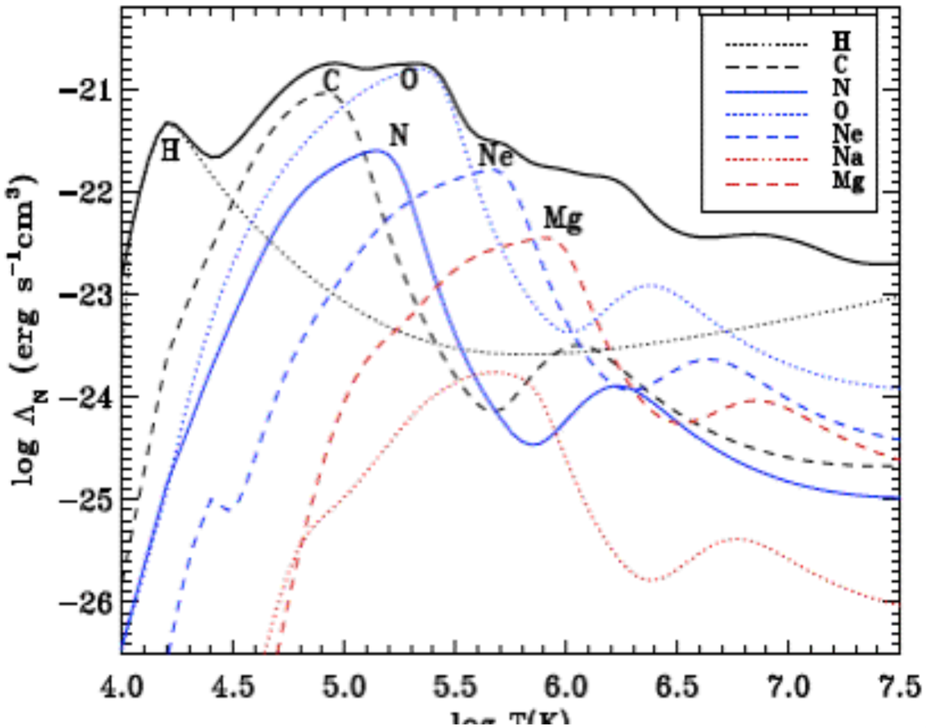


Fig. 5 Cooling rate of hot plasma as a function of the plasma temperature. The contribution to the cooling by the ions of different important abundant elements is indicated (Böhringer and Hensler 1989). Most of

Cooling Function Λ



Notice the 5 order of magnitude range in the various components of Λ in as a function of kT

- Cooling function is the emissivity per unit volume at a fixed density as a function of temperature
- The two panels show a different set of elements
- The black curve is the sum of line and continuum cooling

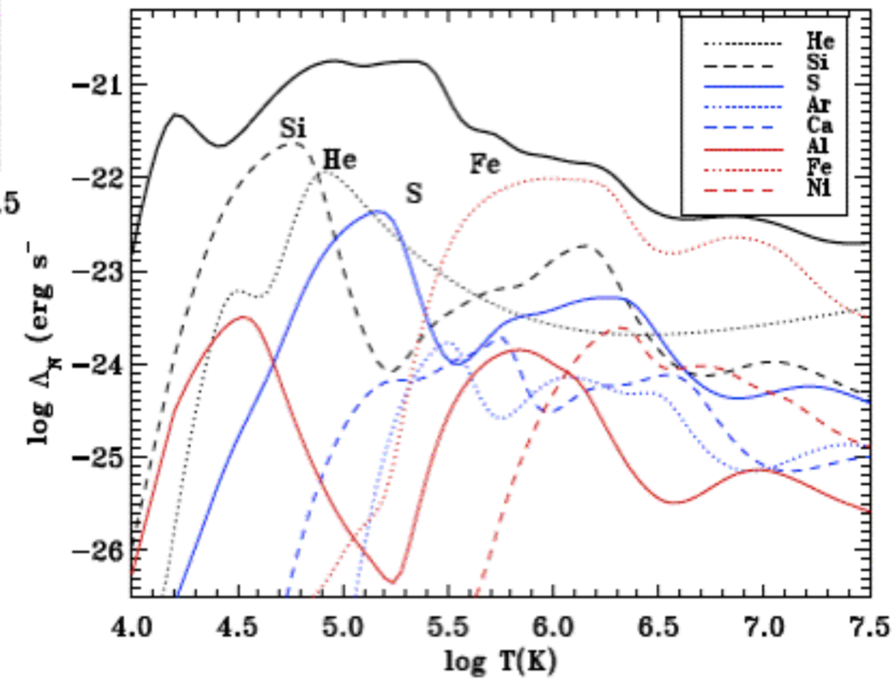
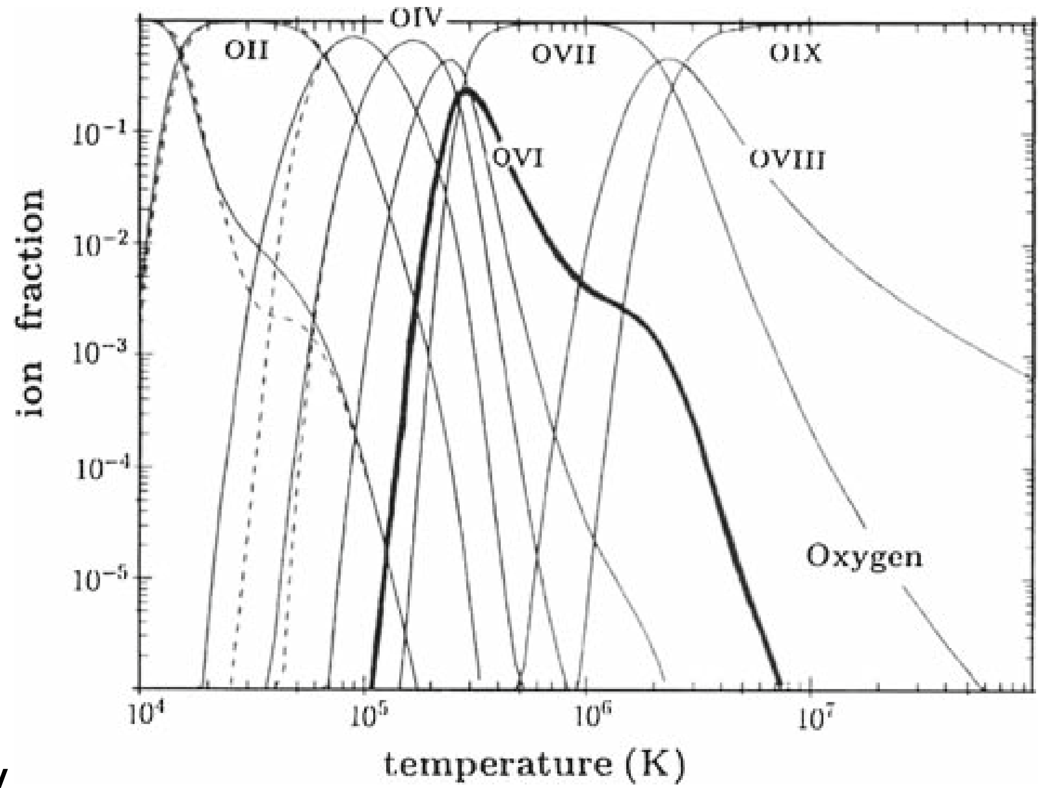


Fig. 2. Contributions of different elements to the cooling curve are given. Each of the plots shows a different set of elements. Important peaks are labelled with the name of the element. The total cooling curve (black solid line) is an addition of the individual elemental contributions.

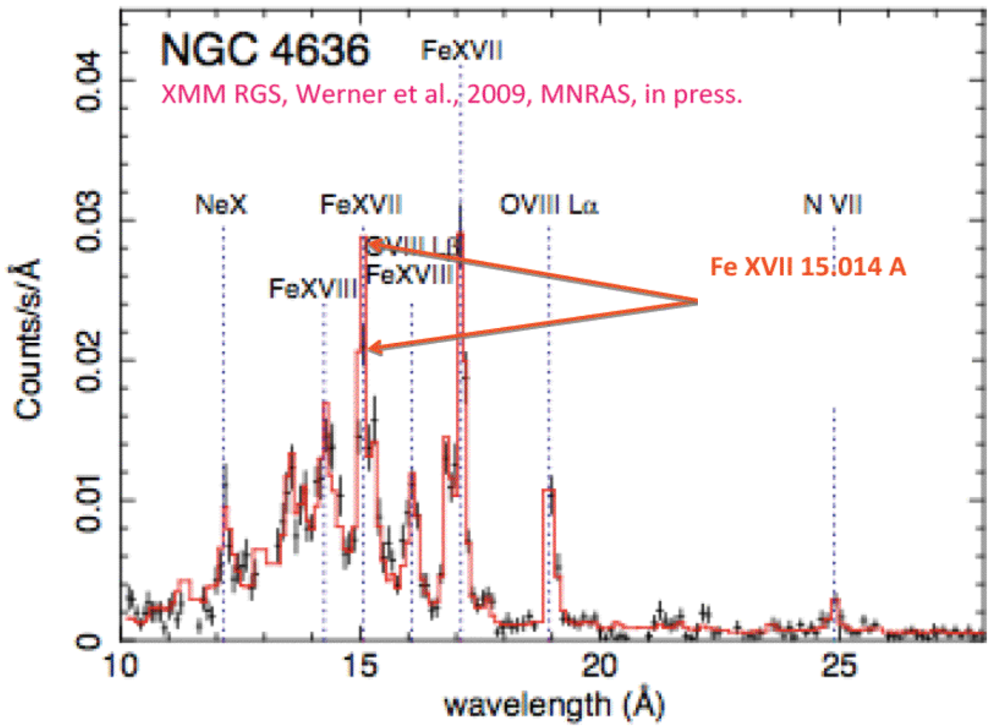
Plasma Parameters

- Electron number density $n_e \sim 10^{-3}$ cm^{-3} in the center with density decreasing as $n_e \sim r^{-2}$
- $10^6 < T < 10^8$ k
- Mainly H, He, but with heavy elements (O, Fe, ..)
- Mainly emits X-rays
- $10^{42} L_X < 10^{45.3}$ erg/s, most luminous extended X-ray sources in Universe
- Age ~ 2-10 Gyr
- Mainly ionized, but not completely
  e.g. He and H-like ions of the abundant elements (O…Fe) exist in thermal equilibrium



Ion fraction for oxygen vs electron temperature

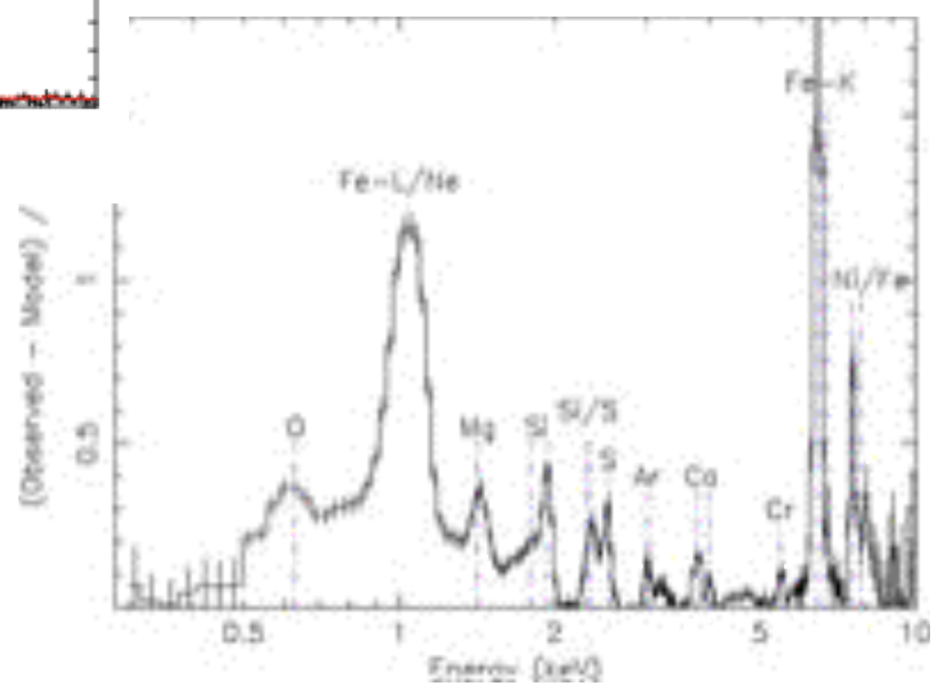
X-ray Spectra of Clusters



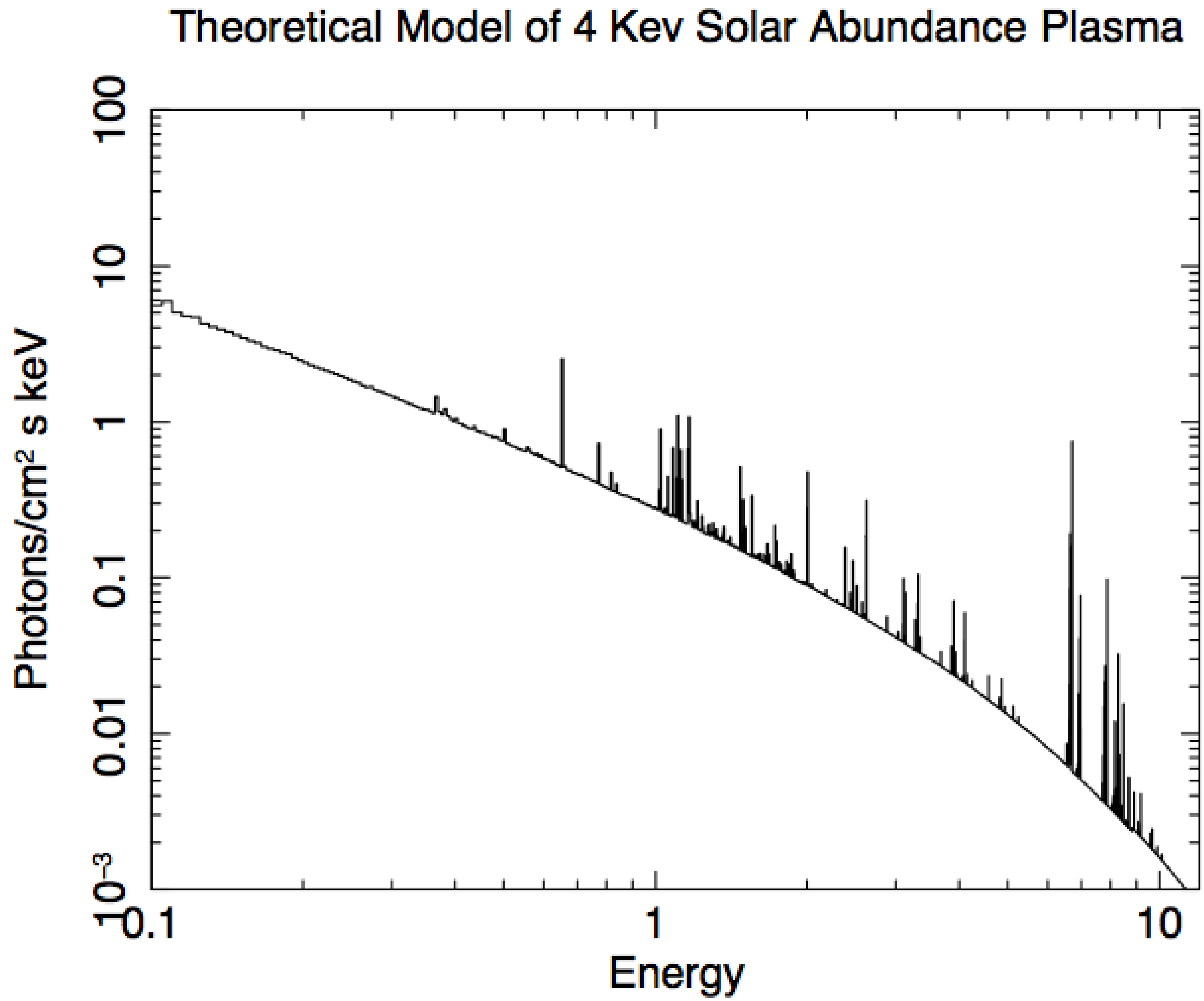
The ratio of the line strengths to that of the continuum is a measure of the abundance of a given ion

To derive the elemental abundance one has to take into account the ionization balance

Top panel in wavelength space bottom in energy space kT~10^7K

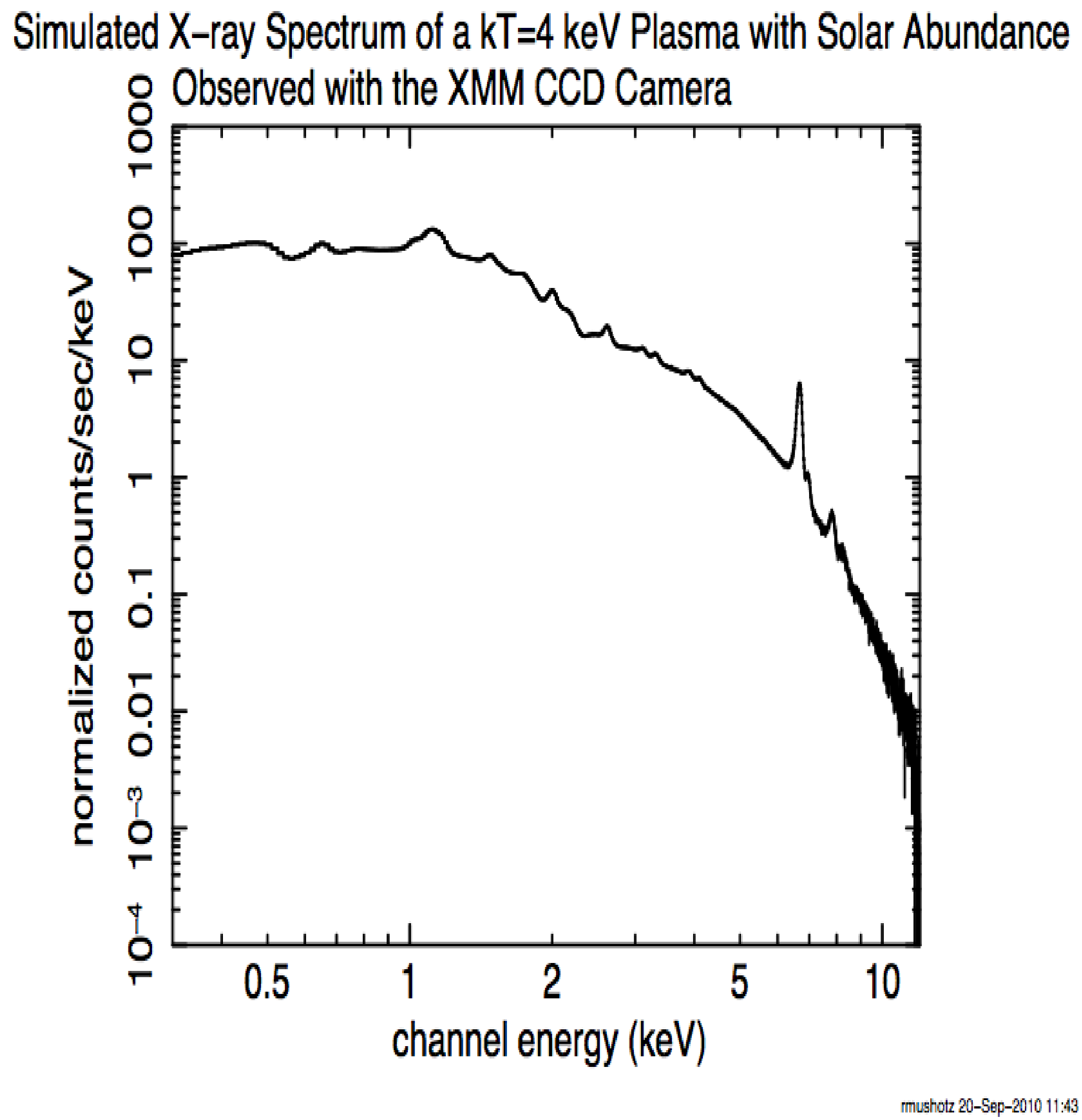


- Theoretical model of a collisionally ionized plasma kT=4 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^4

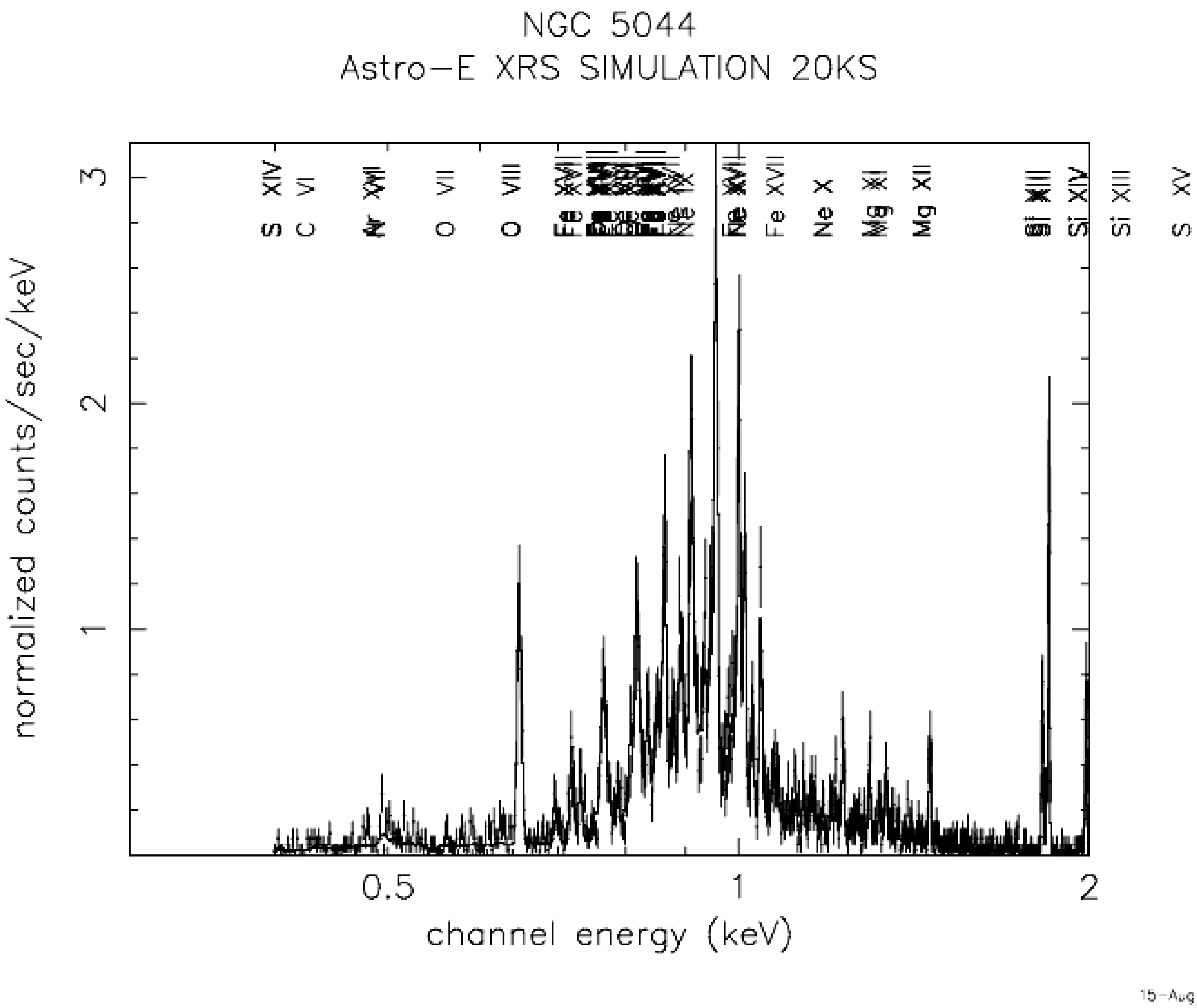


X-ray Spectra Data

- For hot (kT>3x10^{7}k) plasmas the spectra are continuum dominated- most of the energy is radiated in the continuum
- (lines broadened by the detector resolution)

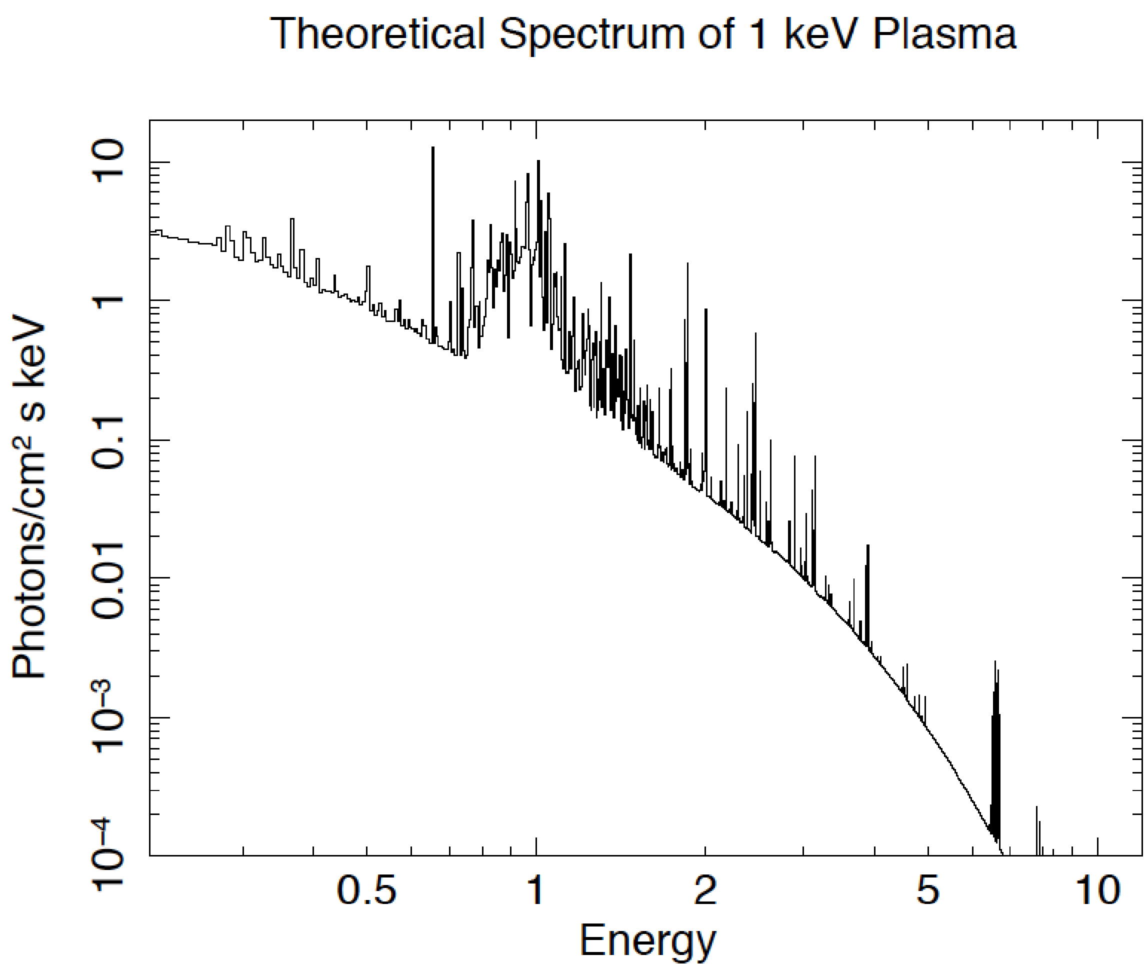


Astro-H Cluster Simulation

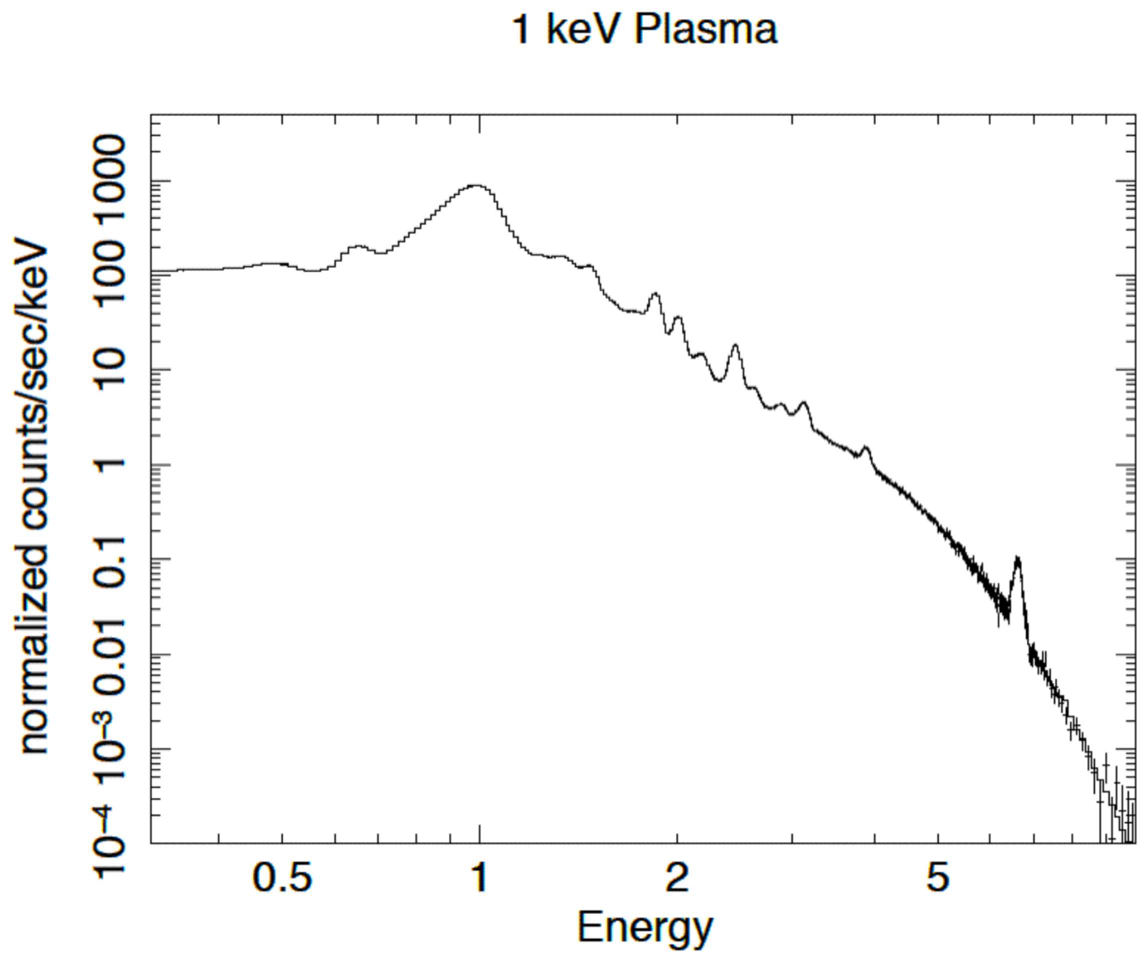


1 keV Plasma

- Theoretical model of a collisionally ionized plasma kT=1 keV with solar abundances
- The lines are 'narrow'
- Notice dynamic range of 10^5

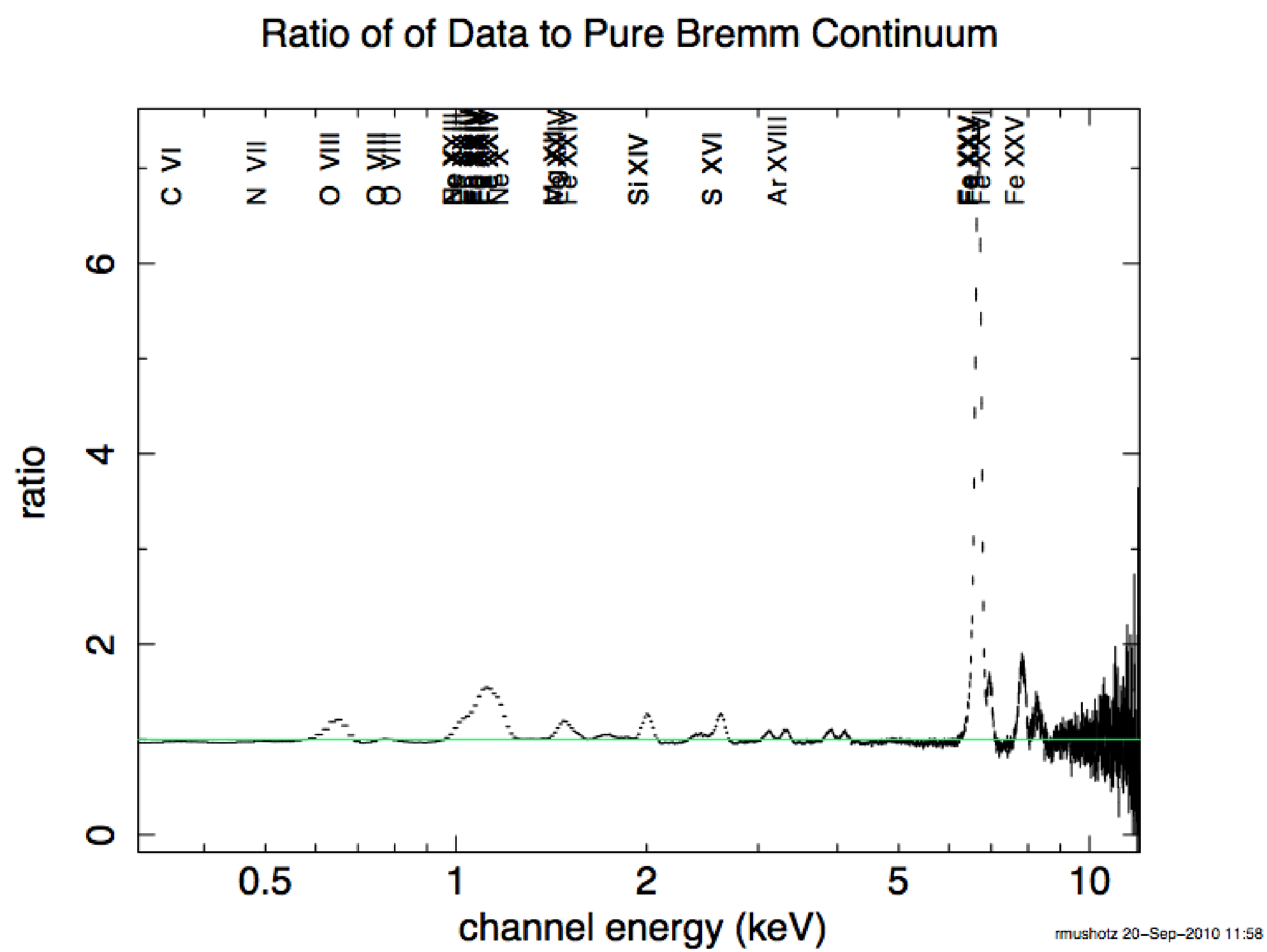


- Observational data for a collisionally ionize plasma kT=1 keV wit solar abundances
- Notice the very large blend of lines near 1 keV- L shell lines of F
- Notice dynamic range of 10^7



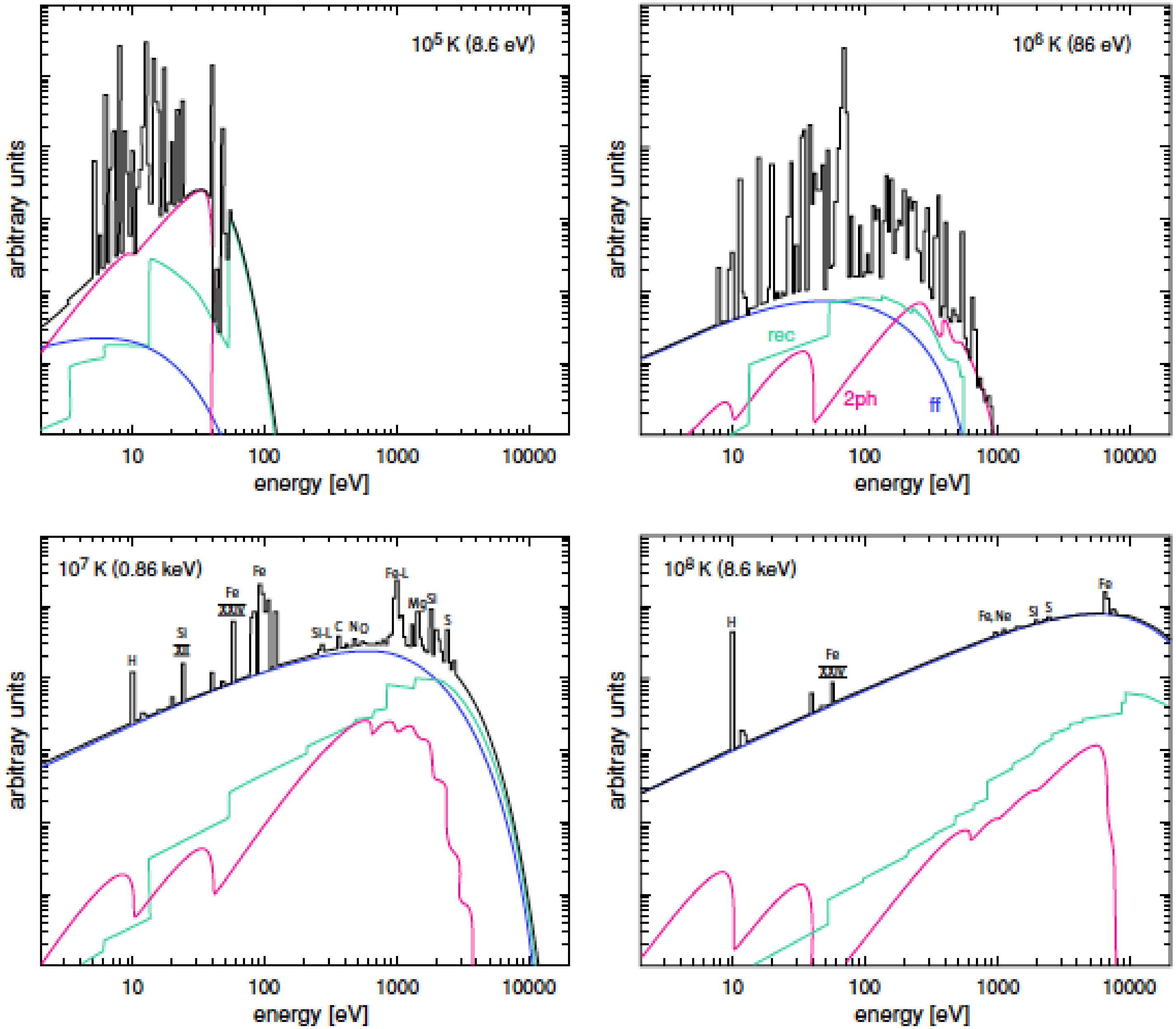
Collsionally Ionized Equilibrium Plasma

- Ratio of model to a 'pure' H/He plasma
- This plot is designed to show the lines



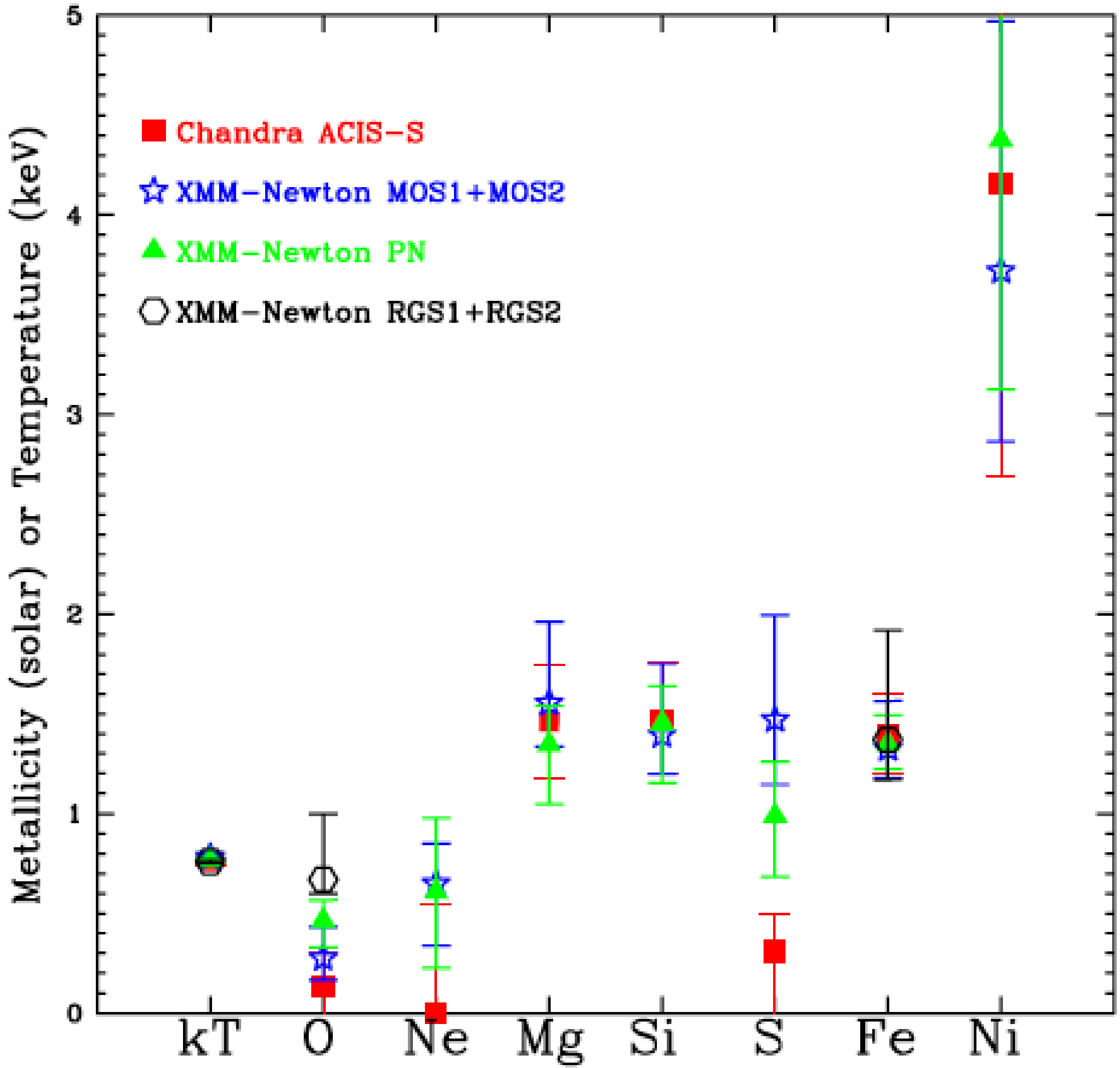
Strong Temperature Dependence of Spectra

- Line emission
- Bremms (black)
- Recombination (red)
- 2 photon green



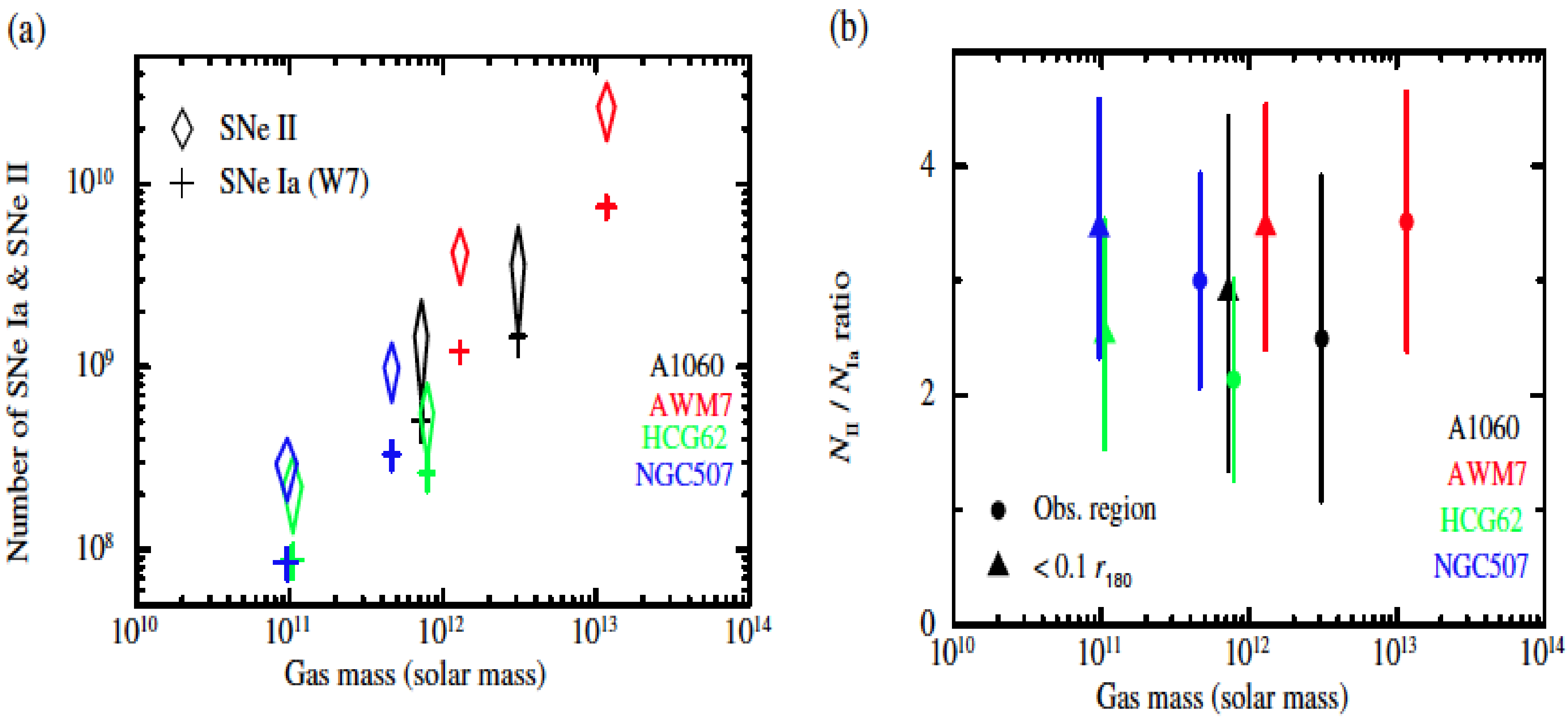
Elemental Abundances in a Group

- Li et al compare the elemental abundances with respect to solar for Oxygen thru Ni for the gas in the center of NGC4636 a nearby low mass group



Numbers of Type I and II Supernova

- As we will discuss later the two types of SN produce a very different mix of heavy elements
- This allows a decomposition into their relative numbers and absolute numbers (Sato et al 2008) - (~10^9-10^{10} SN per cluster)



Relevant Time Scales

- The equilibration timescales between protons and electrons is τ(p,e) ~ 2 x 10^8 yr at an 'average' location
- **In collisional ionization equilibrium population of ions is directly related to temperature**

$$\tau(1,2) = \frac{3m_1\sqrt{2\pi}(kT)^{3/2}}{8\pi\sqrt{m_2}n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}$$

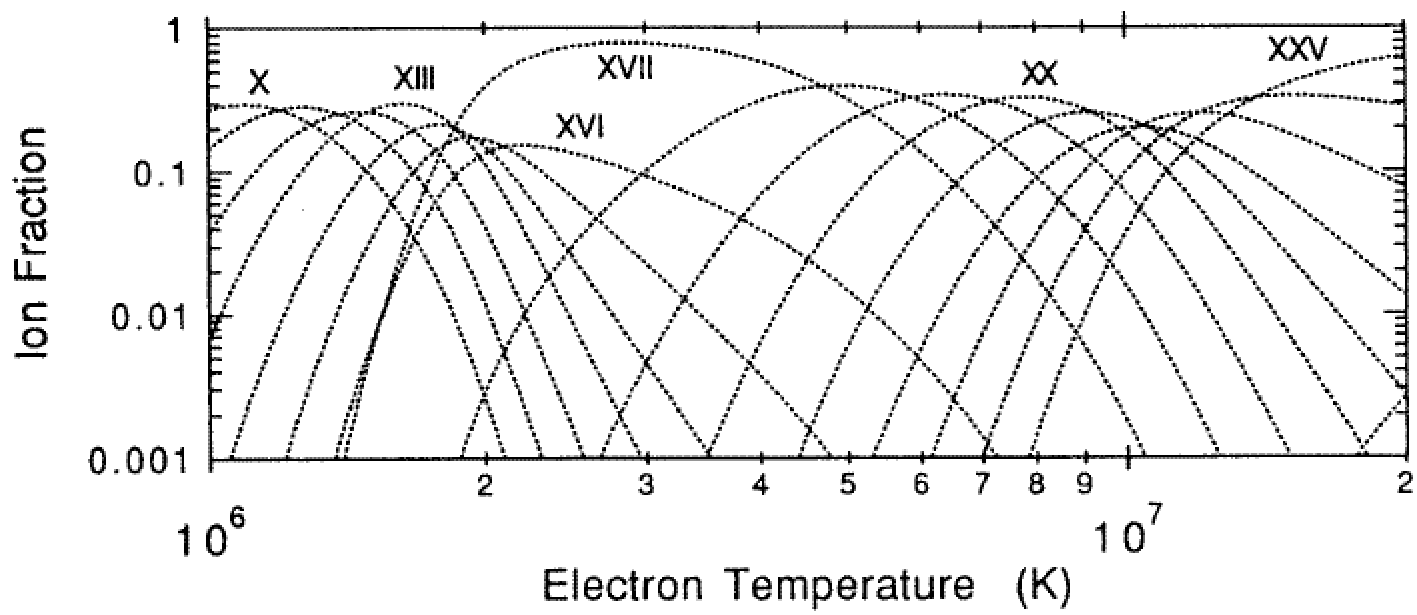
$$\ln\Lambda \equiv \ln(b_{\max}/b_{\min}) \approx 40$$

$$\tau(e,e) \approx 3\times10^5 \left(\frac{T}{10^8\,\mathrm{K}}\right)^{3/2} \left(\frac{n_e}{10^{-3}\,\mathrm{cm}^{-3}}\right)^{-1} \mathrm{yr}$$

$$\tau(p,p) = \sqrt{m_p/m_e}\,\tau(e,e) \approx 43\tau(e,e)$$

$$\tau(p,e) = (m_p/m_e)\tau(e,e) \approx 1800\tau(e,e)$$

Ion fraction for Fe vs electron temperature



Why do we think that the emission is thermal bremmstrahlung?

- X-ray spectra are consistent with model
- X-ray 'image' is also consistent
- Derived physical parameters 'make sense'
- Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)

Mean Free Path for Collisions/ Energy

- Mean-free-path $\lambda_e \sim 20$ kpc < 1% of cluster size $\lambda_p \approx \lambda_e = \dfrac{3^{3/2}(kT)^2}{8\sqrt{\pi}\, n_e e^4 \ln\Lambda}$

$$\approx 23\left(\frac{T}{10^8\ \mathrm{K}}\right)^2\left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1}\mathrm{kpc}$$

At $T>3\times10^7$ K the major form of energy emission is thermal bremmstrahlung continuum

$\varepsilon \sim 3\times10^{-27}\ T^{1/2}\ n^2$ ergs/cm^3/sec- how long does it take a parcel of gas to lose its energy?

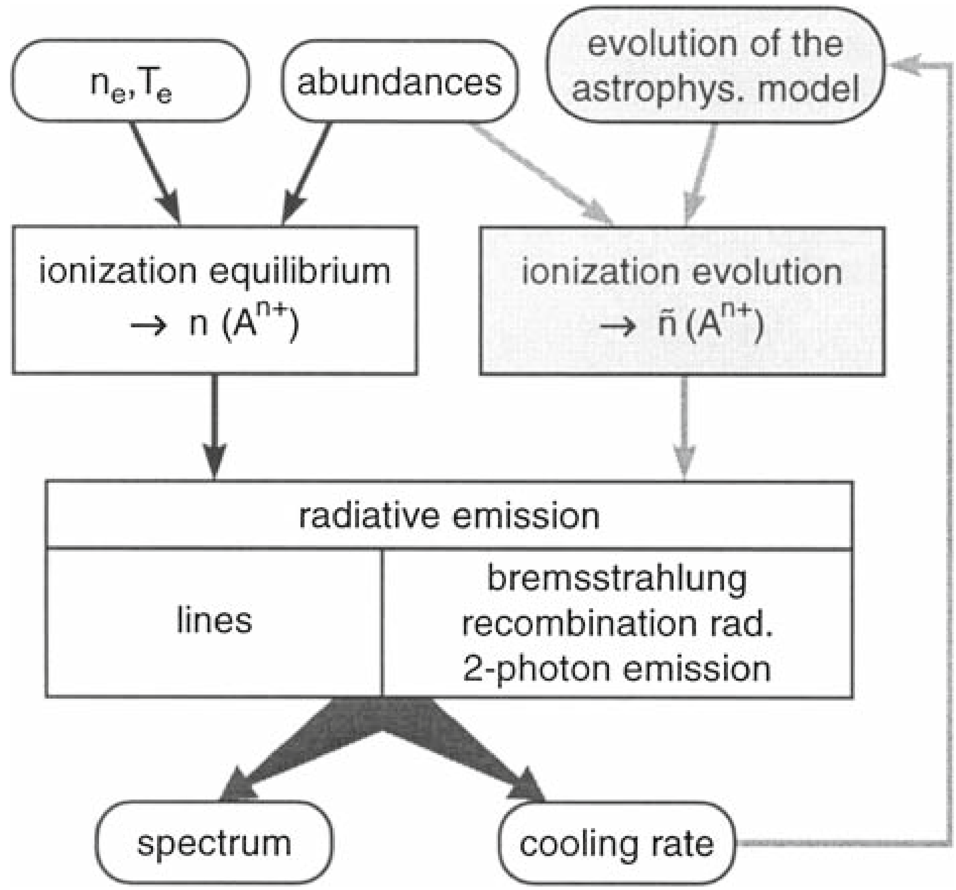
$\tau \sim nkT/\varepsilon \sim 8.5\times10^{10}$ yrs$(n/10^{-3})^{-1}\ T_8^{\ 1/2}$

At lower temperatures line emission is important

Why is Gas Hot

- To first order if the gas were cooler it would fall to the center of the potential well and heat up
- If it were hotter it would be a wind and gas would leave cluster
- Idea is that gas shocks as it 'falls into' the cluster potential well from the IGM
  - Is it 'merger' shocks (e.g. collapsed objects merging)
  - Or in fall (e.g. rain)

  BOTH



Physical Conditions in the Gas

- the elastic collision times for ions and electrons) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 \times 10^8 \mathrm{yr}\, (T_{gas}/10^8)^{1/2} (D/\mathrm{Mpc})$
- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ (P is the pressure, ρ_g is the gas density, $\gamma=5/3$ is the adiabatic index for a monoatomic ideal gas)

Hydrostatic Equilibrium Kaiser 19.2

- Equation of hydrostatic equil

$$\nabla \mathbf{P} = -\rho_g \nabla \phi(\mathbf{r})$$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter)

P is the gas pressure

ρ_g **is the gas density**

Hydrodynamics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \text{ mass conservation (continuity)}$$

$$\rho \frac{Dv}{Dt} + \nabla P + \rho \nabla \phi = 0 \text{ momentum conservation (Euler)}$$

$$\rho T \frac{Ds}{Dt} = H - L \text{ entropy (heating \& cooling)}$$

$$P = \frac{\rho k T}{\mu m_p} \text{ equation of state}$$

Add viscosity, thermal conduction, …

Add magnetic fields (MHD) and cosmic rays

Gravitational potential ϕ from DM, gas, galaxies

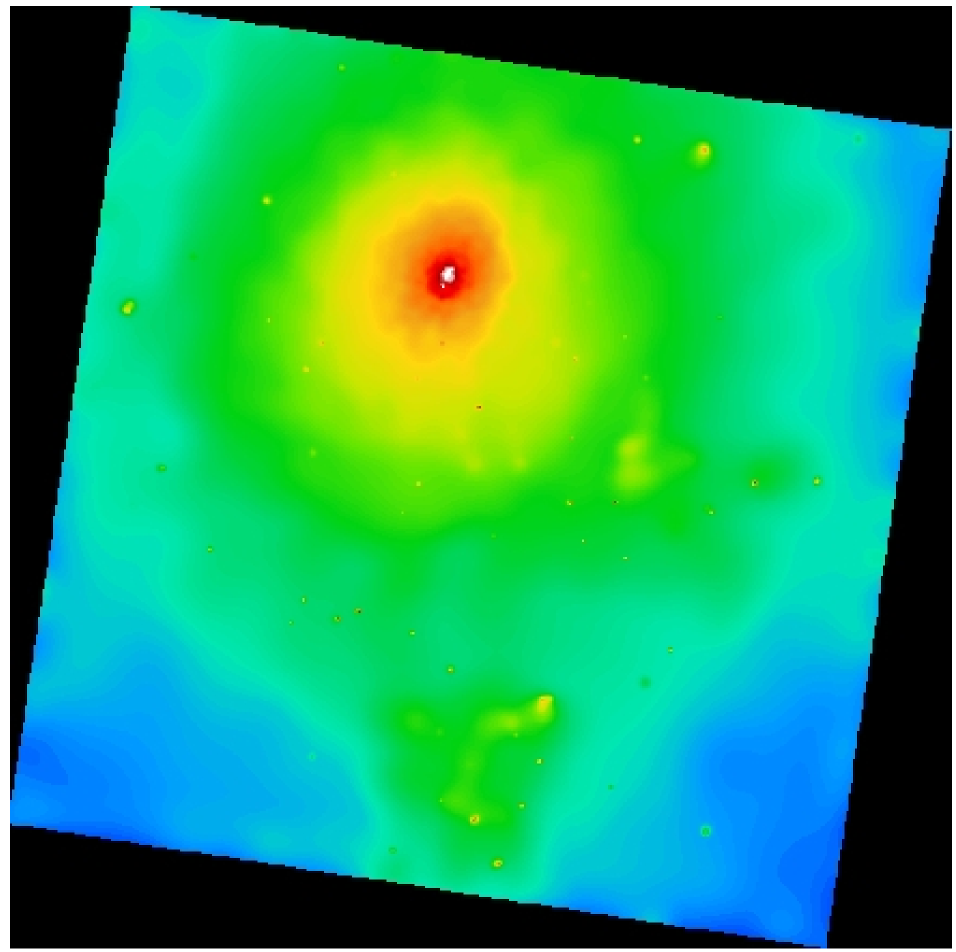
- density and potential are related by Poisson's equation

$$\nabla^2\phi = 4\pi\rho G$$

- and combining this with the equation of hydrostaic equil
- $\nabla \cdot (1/\rho \nabla P) = -\nabla^2\phi = -4\pi G\rho$

- or, for a spherically symmetric system

$$1/r^2\, d/dr\, (r^2/\rho\; dP/dr) = -4\pi\rho G\rho$$



Deriving the Mass from X-ray Spectra

For spherical symmetry this reduces to

$(1/\rho_g)\, dP/dr = -d\phi(r)/dr = GM(r)/r^2$

With a little algebra and the definition of pressure - the total cluster mass can be expressed as

$$\boxed{GM(r) = kT_g(r)/\mu G m_p)\mathbf{r}\ (d\ln T/dr + d\ln \rho_g/dr)}$$

k is Boltzmans const, μ is the mean mass of a particle and m_H is the mass of a hydrogen atom

Every thing is observable

The temperature T_g from the spatially resolved spectrum

The density ρ_g from the knowledge that the emission is due to bremmstrahlung

And the scale size, **r**, from the conversion of angles to distance

- The emission measure along the line of sight at radius r, EM(r), can be deduced from the X-ray surface brightness, $S(\Theta)$:

$EM(r) = 4\pi (1 + z)4\ S(\Theta)/\Lambda(T, z)$; $r = dA(z)\ \Theta$

where $\Lambda(T, z)$is the emissivity in the detector band, taking into account the instrument spectral response,

dA(z) is the angular distance at redshift z.

The emission measure is linked to the gas density ρ_g by:

- $EM(r) = \int_r^{\infty} \rho^2_g(R)\ R dr/\sqrt{(R^2 - r^2)}$
- The shape of the surface brightness profile is thus governed by the form of the gas distribution, whereas its normalization depends also on the cluster overall gas content.

Density Profile

- a simple model(the β model) fits the surface brightness well
  - $S(r)=S(0)(1/r/a)^2)^{-3\beta+1/2}$ cts/cm^2/sec/solid angle
- Is analytically invertible (inverse Abel transform) to the density profile

  $\rho(r)=\rho(0)(1/r/a)^2)^{-3\beta/2}$

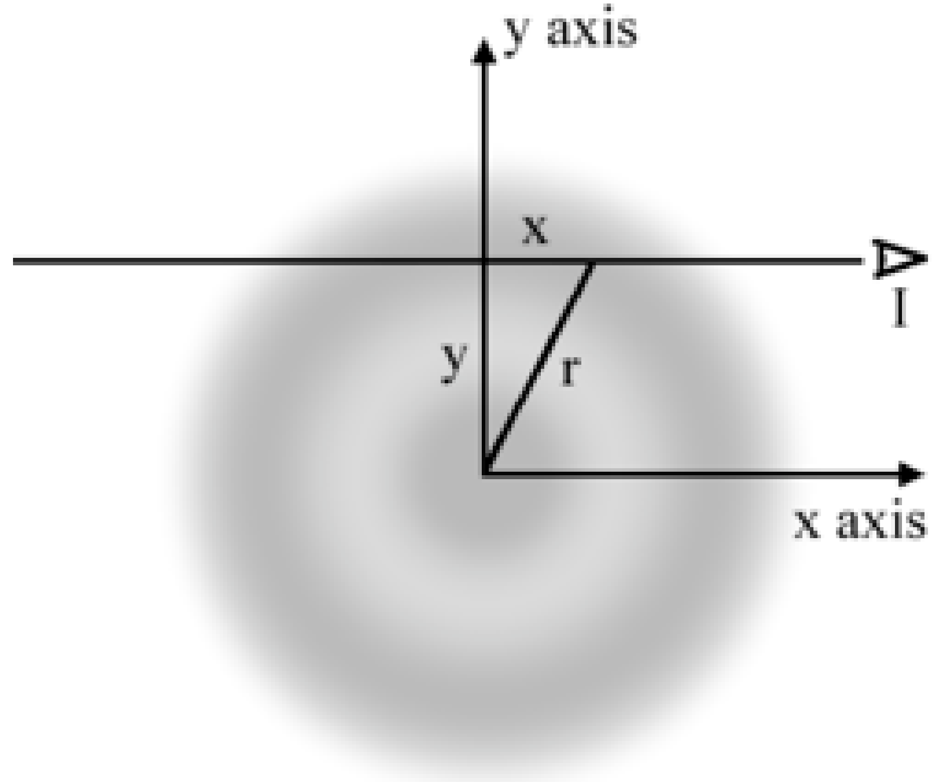
  The conversion function from S(0) to $\rho(0)$ depends on the detector

  The quantity 'a' is a scale factor- sometimes called the core radius

  ----

- The Abel transform, , is an integral transform used in the analysis of spherically symmetric or axially symmetric functions. The Abel transform of a function f(r) is given by:
- $f(r)=1/p\int_r^\infty dF/dy\ dy/\sqrt{(y^{2-}r^2)}$
- In image analysis tthe reverse Abel transform is used to calculate the emission function given a projection (i.e. a scan or a photograph) of that emission function.
- In general the integral is not analytic which makes the

A geometrical interpretation of the Abel transform in two dimensions. An observer (I) looks along a line parallel to the x-axis a distance y above the origin. What the observer sees is the projection (i.e. the integral) of the circularly symmetric function f(r) along the line of sight. The function f(r) is represented in gray in this figure. The observer is assumed to be located infinitely far from the origin so that the limits of integration are ±∞



Sarazin sec 5

The gas distributions in clusters can be derived directly from observations of the X-ray surface brightness of the cluster, if the shape of the cluster is known and if the X-ray observations are sufficiently detailed and accurate. This method of analysis also leads to a method for determining cluster masses (Section 5.5.5). The X-ray surface brightness at a photon frequency ν and at a projected distance b from the center of a spherical cluster is

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\epsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}, \tag{5.80}$$

where ϵ_ν is the X-ray emissivity. This Abel integral can be inverted to give the emissivity as a function of radius,

$$\epsilon_\nu = -\frac{1}{2\pi r}\frac{d}{dr}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}}. \tag{5.81}$$

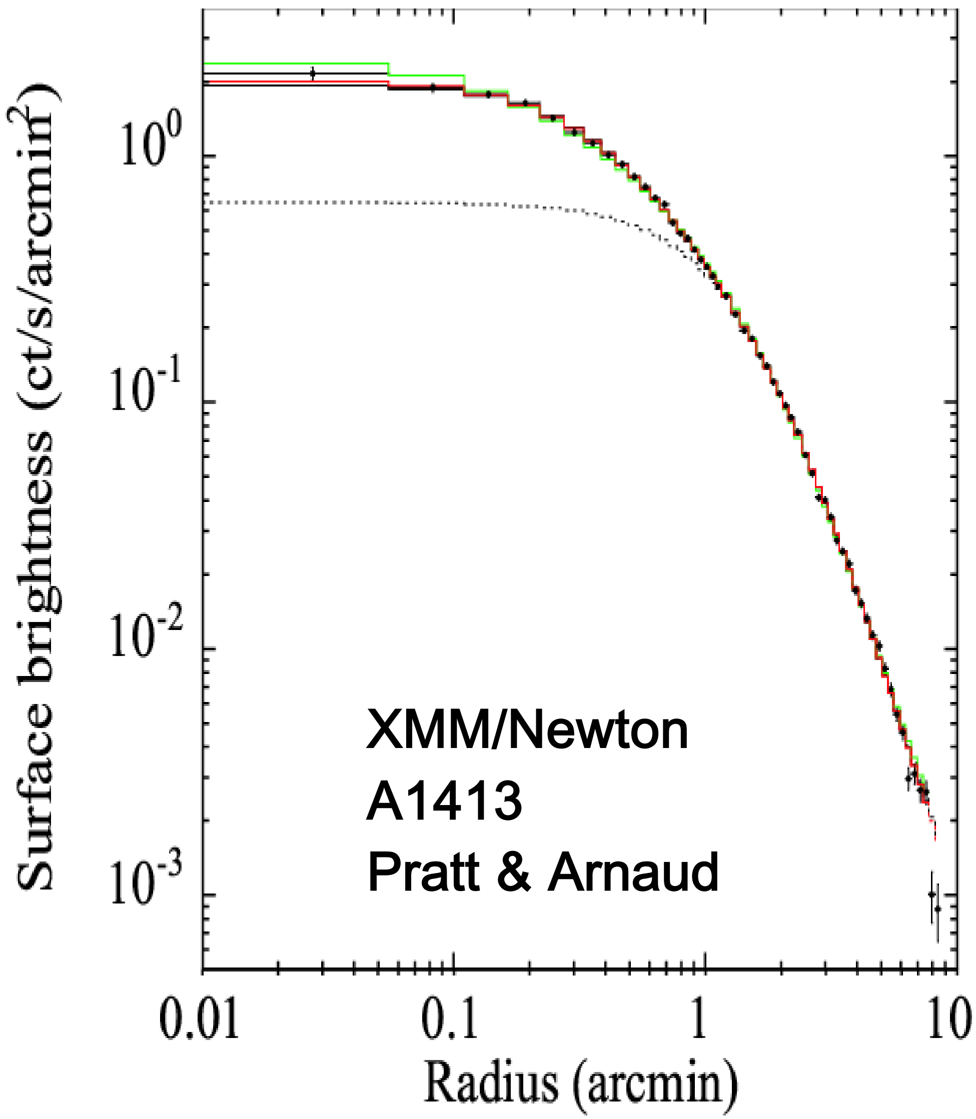
Surface Brightness Profiles

- It has become customary to use a 'β' model (Cavaliere and Fesco-Fumiano)'
- clusters have <β>~2/3

$$\rho(r) = \frac{\rho_0}{\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{3\beta/2}}$$

$$\beta \equiv \frac{\mu m_p \sigma_{gal}^2}{kT}$$ but treat as fitting parameter

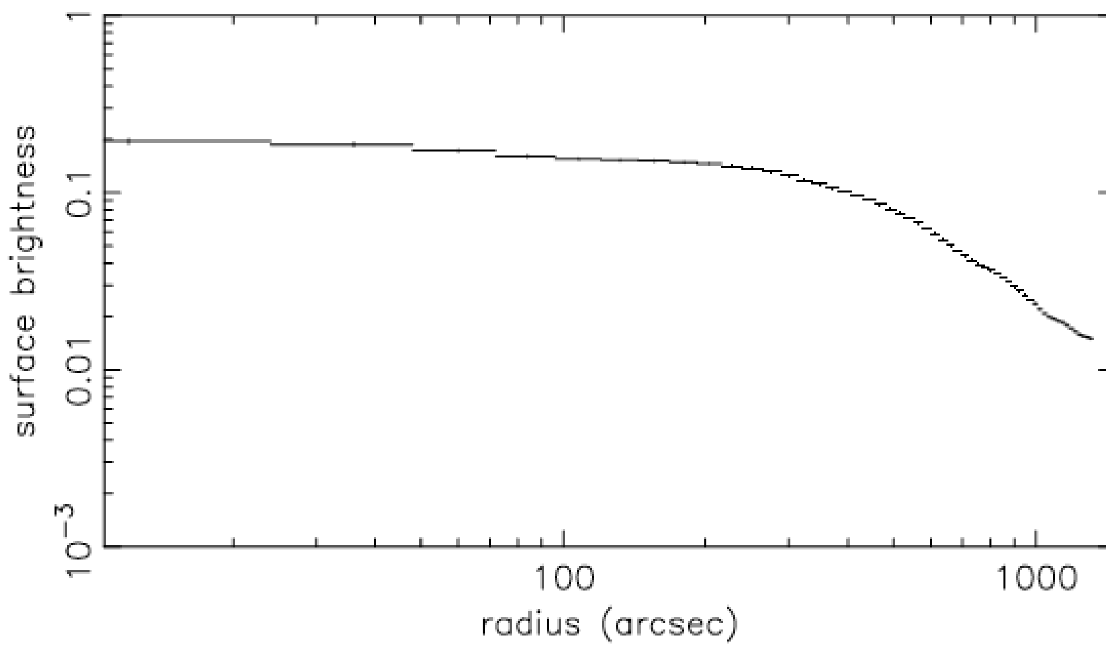
$$I_X(r) \propto \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2}$$

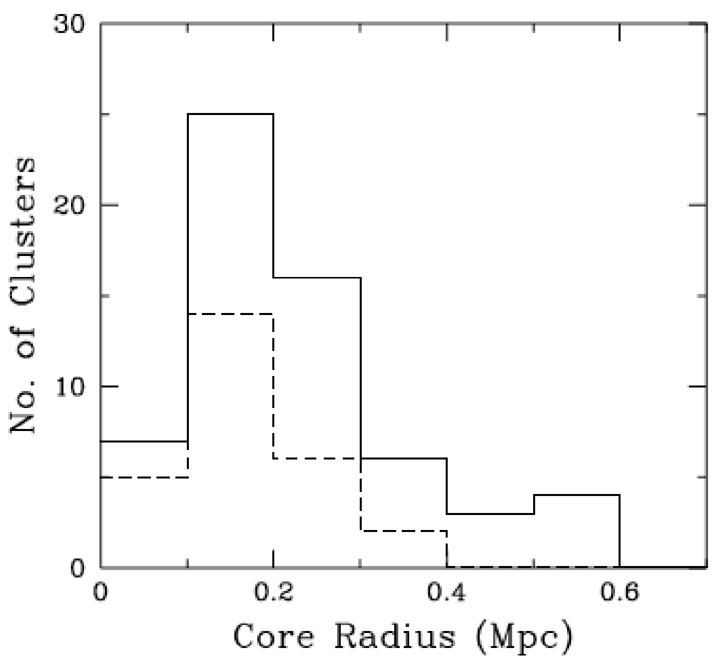


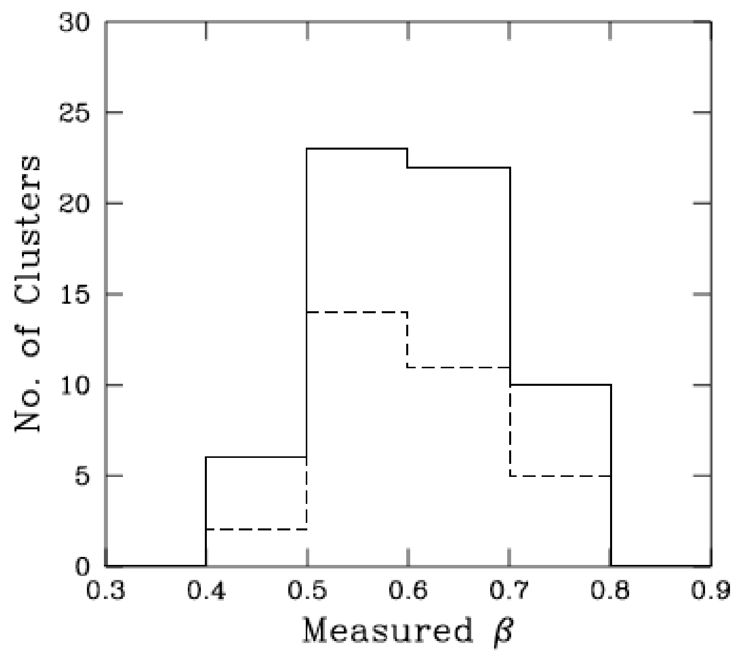
'Two' Types of Surface Brightness Profiles

- 'Cored'

- Central Excess
- Range of core radii and β







X-ray Emissivity

- The observed x-ray emissivity is a projection of the density profile

A large set of clusters over a wide range in redshift