Physical Conditions in the Gas

- the elastic collision times for (ions and electrons) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 \times 10^8$ yr $(T_{gas}/10^8)^{1/2}(D/\mathrm{Mpc})$
- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ (P is the pressure, ρ_g is the gas density, $\gamma = 5/3$ is the adiabatic index for a monoatomic ideal gas)

Hydrostatic Equilibrium Kaiser 19.2

- Equation of hydrostatic equil

$$\nabla P = -\rho_g \nabla \phi(r)$$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter)

P is the gas pressure

ρ_g **is the gas density**

How do Clusters Form- Mergers

- As time progresses more and more objects come together- merge

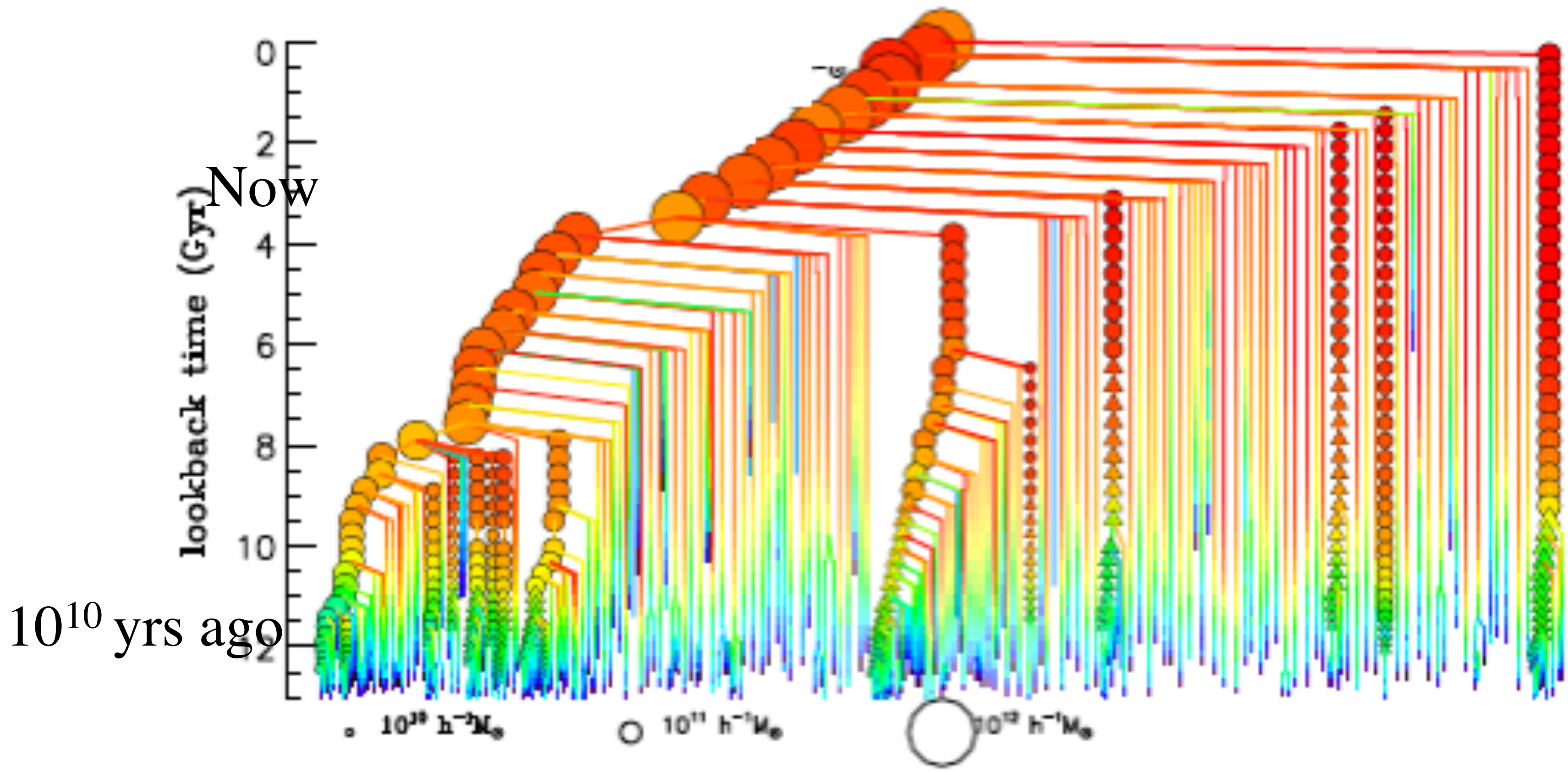
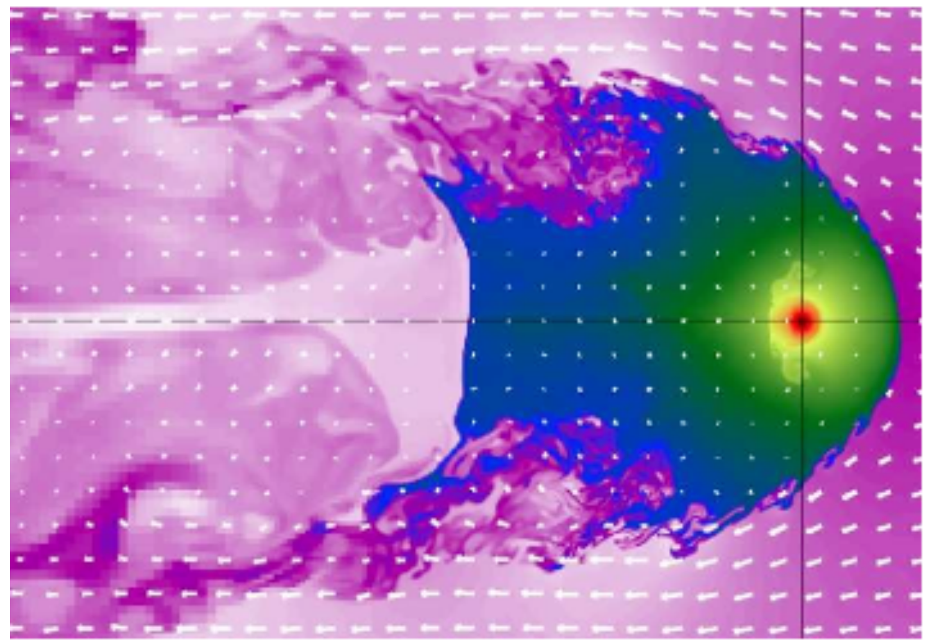
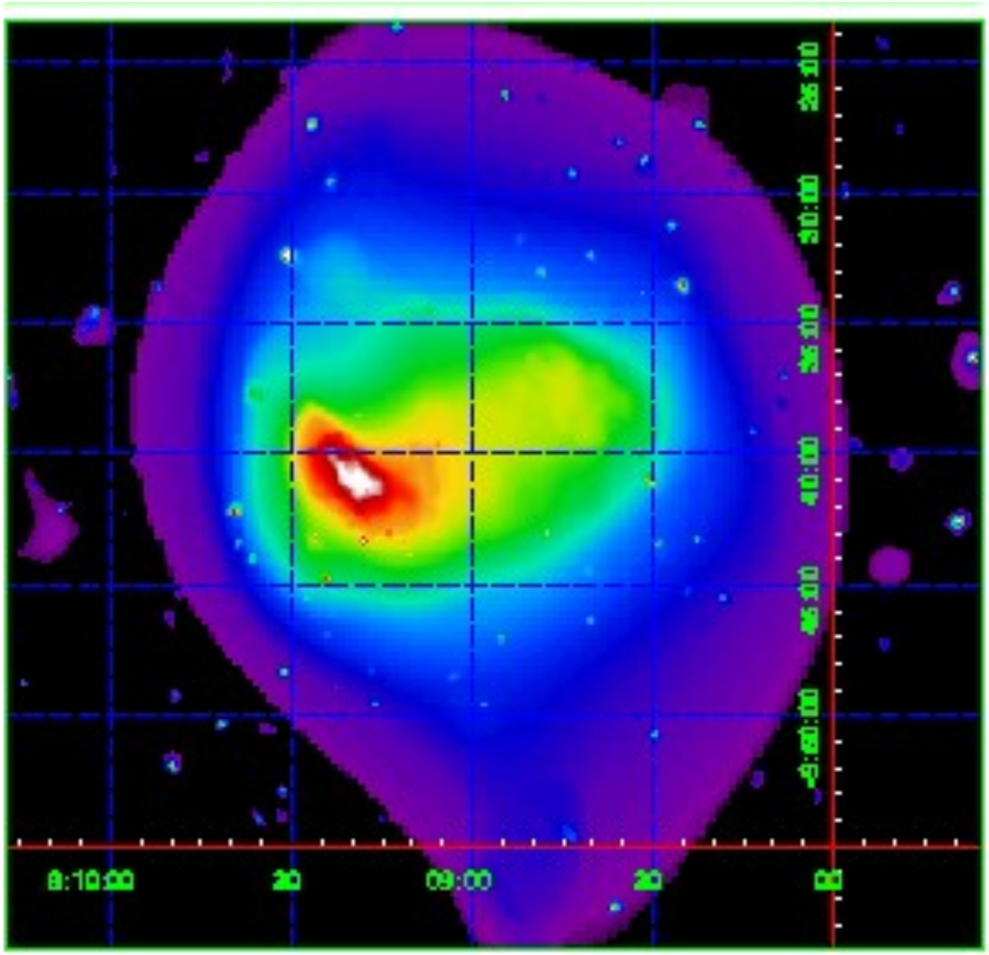


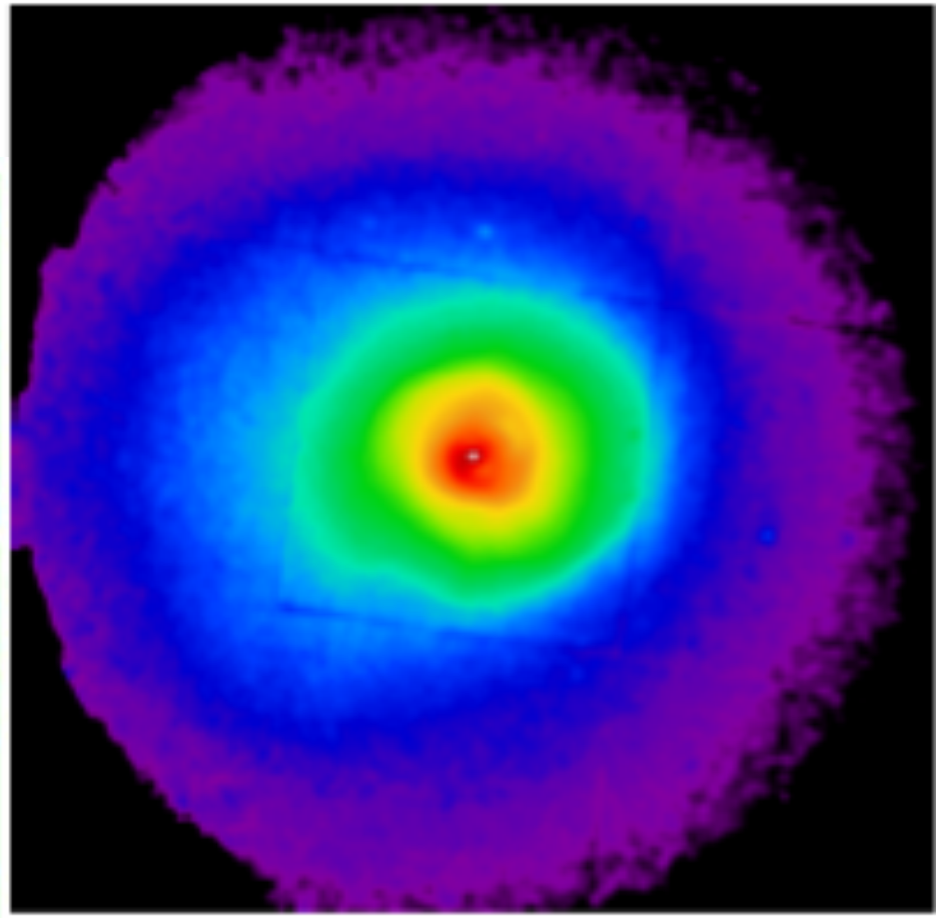
Figure 1. BCG merger tree. Symbols are colour–coded as a function of B - V colour and their area scales with the stellar mass. Only progenitors more massive than 10^{10} $M_{\odot}$ h^{-1} are shown with symbols. Circles are used for galaxies that reside in the FOF group inhabited by the main branch. Triangles show galaxies that have not yet joined this FOF group.



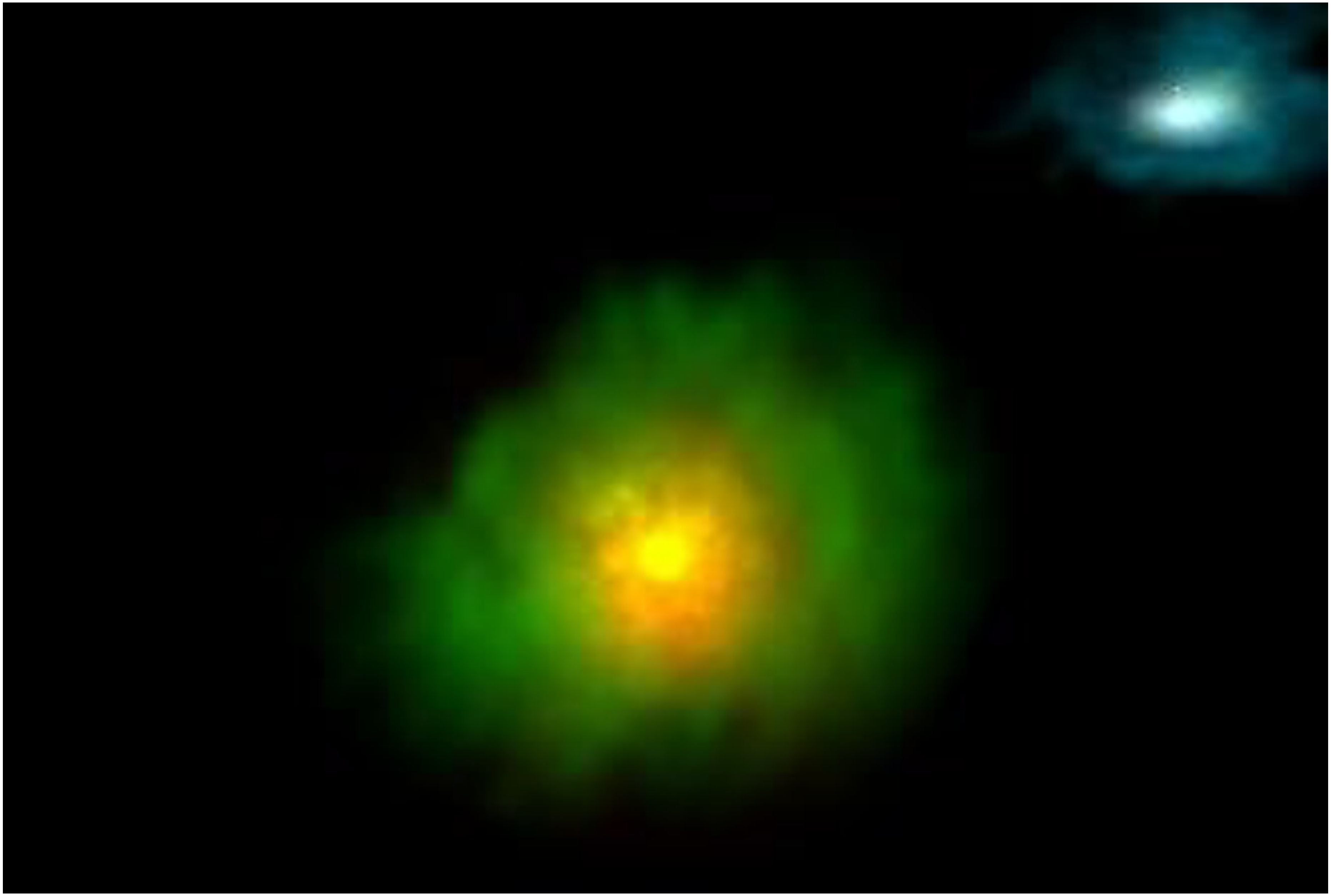
ig. 16. From Roediger et al. (2015a). Snapshot of a high-
solution simulation of the hot atmosphere of an elliptical
alaxy falling into a galaxy cluster. The bottom of the po-



Merger

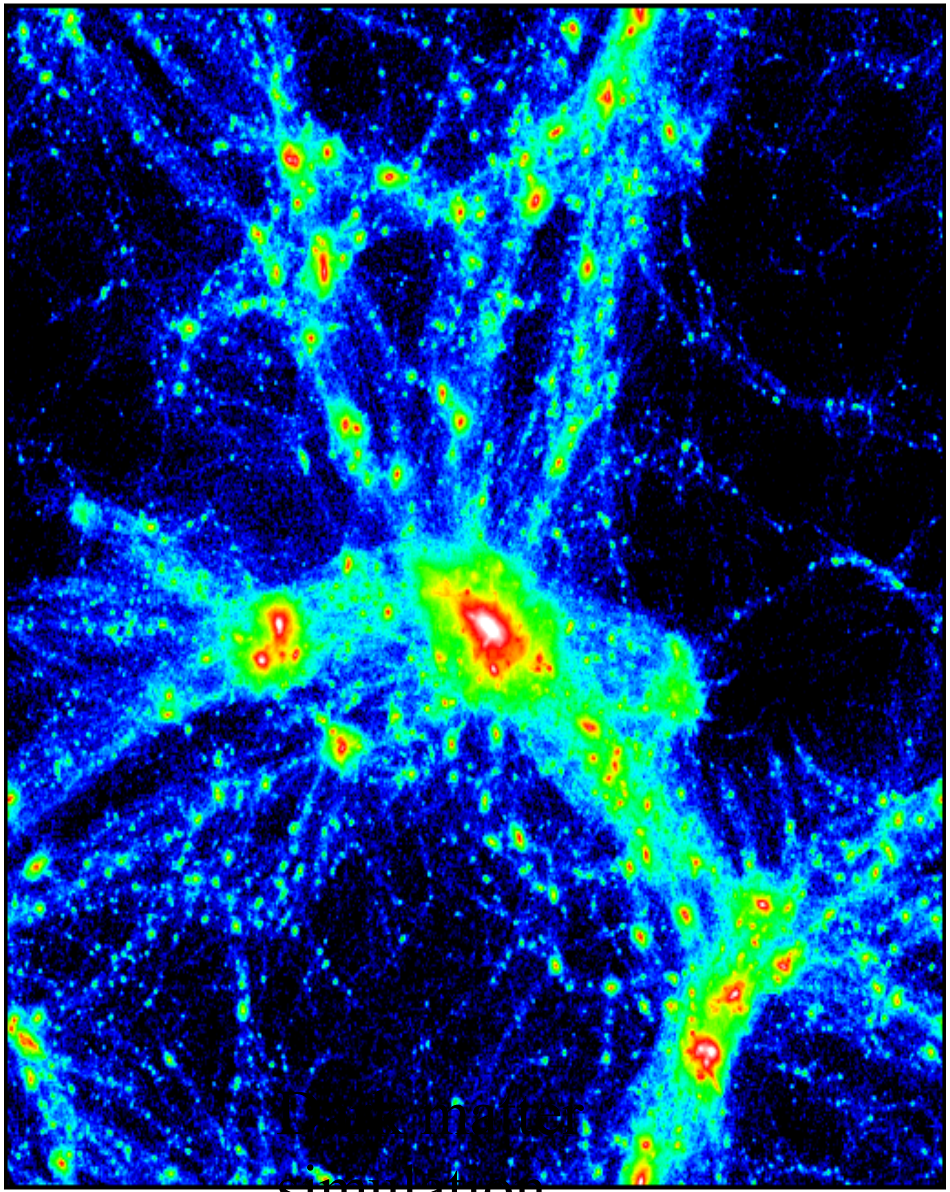


NOT A
MERGER

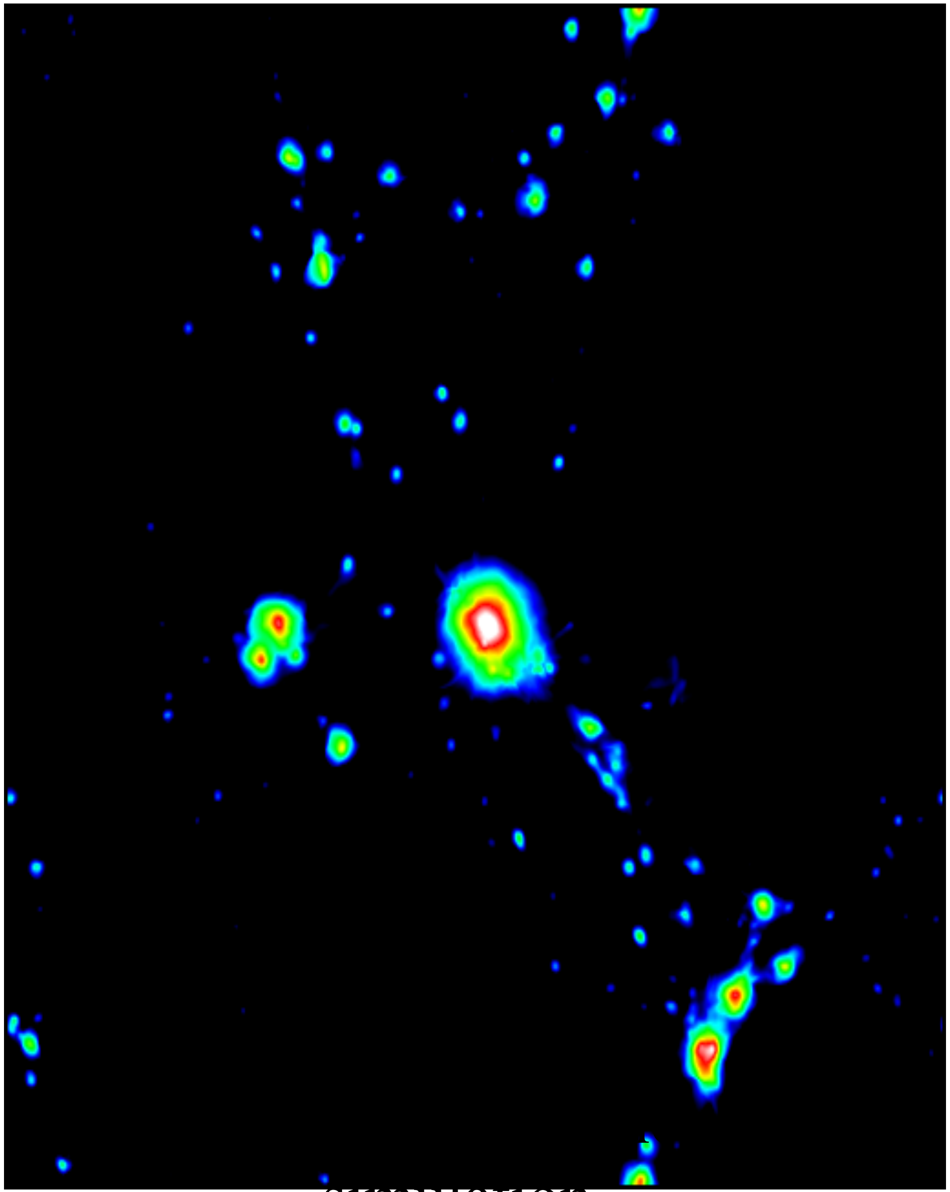


Comparison of dark matter and x-ray cluster and group distribution

every bound system visible in the numerical simulation is detected in the x-ray band - bright regions are massive clusters, dimmer regions groups,



simulation



simulation

Extreme Merger

- Bullet cluster (1E0657)
- Allen and Million

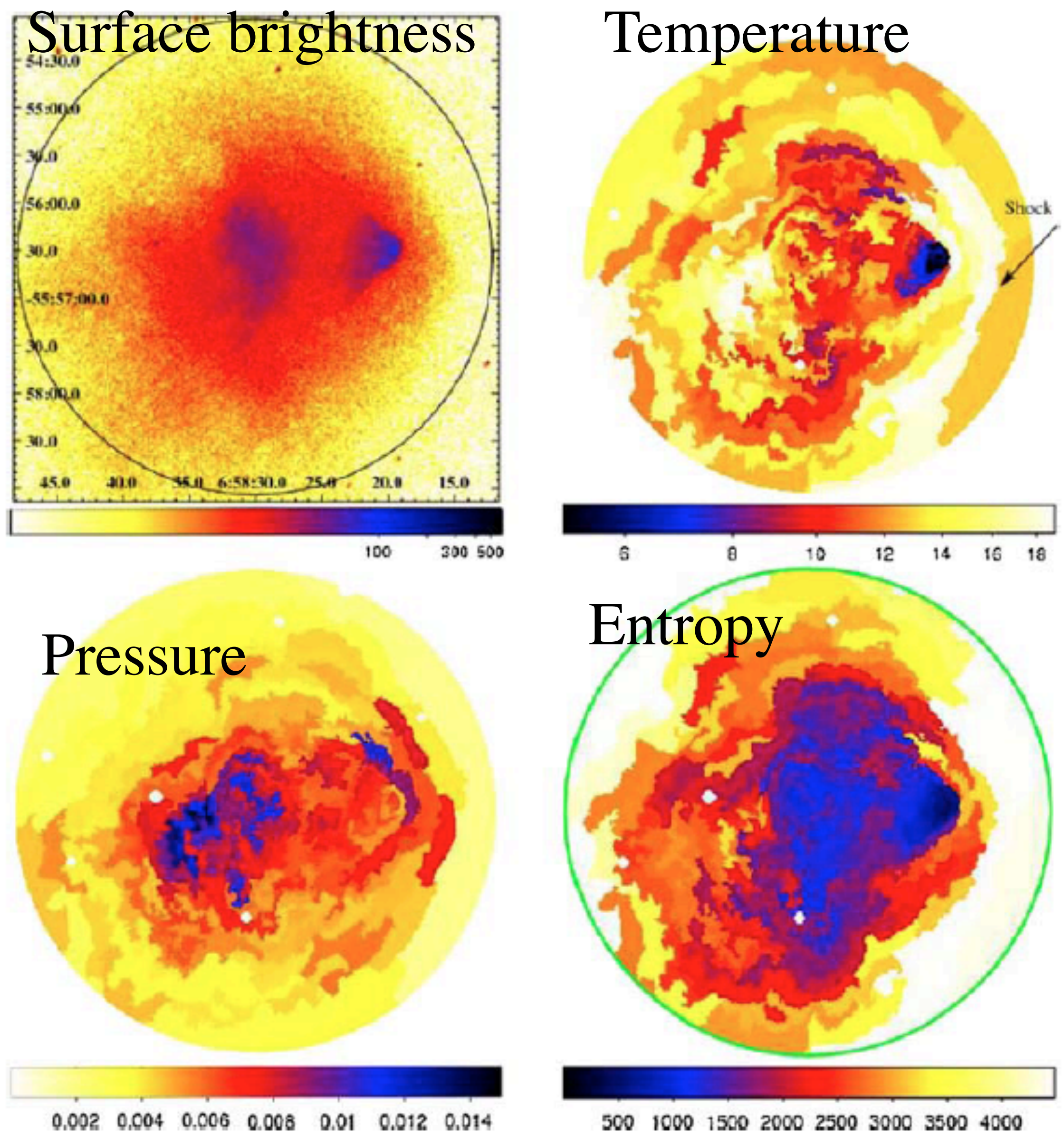


Fig. 16 Thermodynamic maps for the ICM of the “bullet cluster”, 1E0657-56 (Million and Allen 2008)

Use of X-rays to Determine Mass

- X-ray emission is due to the combination of thermal bremmstrahlung and line emission from hot gas
- The gas "should be" in equilibrium with the gravitational potential (otherwise flow out or in)
- density and potential are related by Poisson's equation

$$\nabla^2\phi = 4\pi\rho G$$

- and combining this with the equation of hydrostaic equil

$$\nabla\cdot(1/\rho\nabla P)=-\nabla^2\phi$$
$$=-4\pi G\rho$$

gives for for a spherically symmetric system

$$(1/\rho_g)\, dP/dr=-d\phi(r)/dr=GM(r)/r^2$$

With a little algebra and the definition of pressure - the total cluster mass (dark and baryonic) can be expressed as

$$M(r)=-(kT_g(r)/\mu G m_p)\mathbf{r}\ (d\ln T/dr+d\ln\rho_g/dr)$$

k is Boltzmans const, μ is the mean mass of a particle and m_H is the mass of a hydrogen atom
Every thing is observable
The temperature T_g from the spatially resolved spectrum
The density ρ_g from the knowledge that the emission is due to bremmstrahlung
And the scale size, **r**, from the conversion of angles to distance

see Longair eq 4.2-4.6

- density and potential are related by Poisson's equation

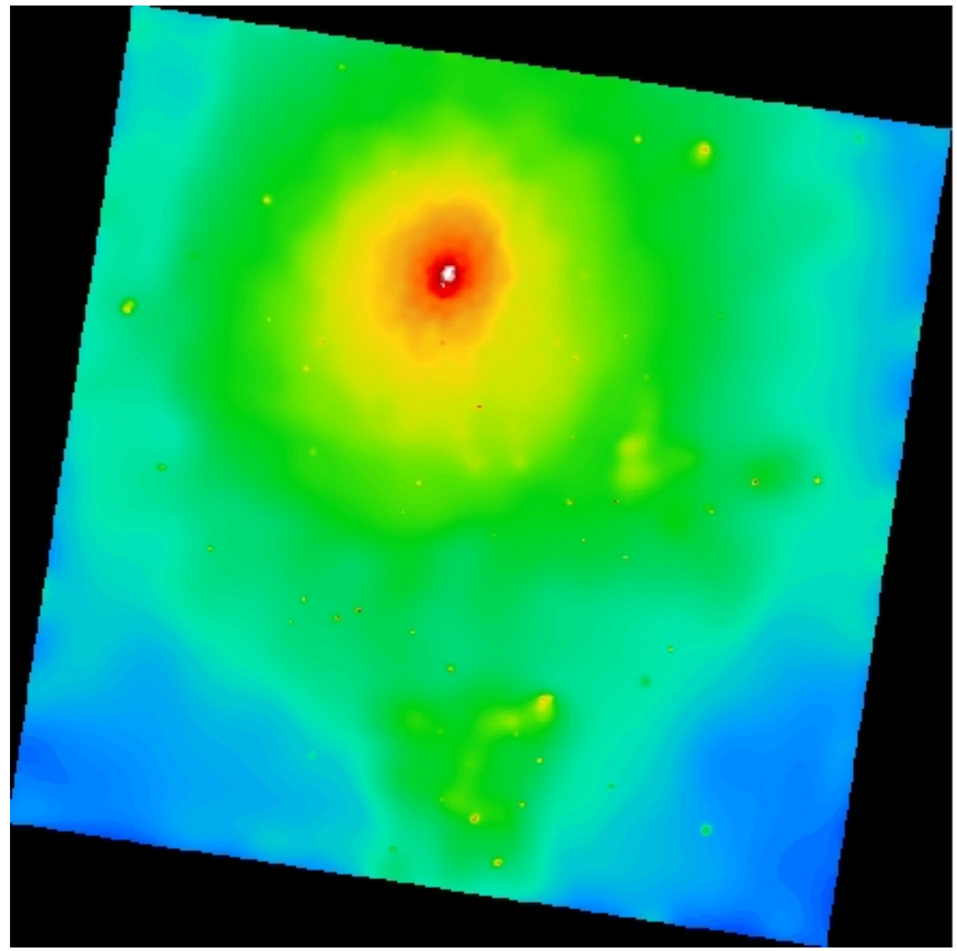
$\nabla^2\phi = 4\pi\rho G$

- and combining this with the equation of hydrostaic equil

- $\nabla \cdot ((1/\rho)\nabla P) = -\nabla^2\phi$ $= -4\pi G\rho$ (eq 4.3 in Longair)

- or, for a spherically symmetric system

$1/r^2 \, d/dr \, (r^2/\rho \; dP/dr) = -4\pi\rho G\rho$



Hydrodynamics--see Longair ch 4.2

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \ \text{ mass conservation (continuity)}$$

$$\rho \frac{Dv}{Dt} + \nabla P + \rho \nabla \phi = 0 \ \text{ momentum conservation (Euler)}$$

$$\rho T \frac{Ds}{Dt} = H - L \ \text{ entropy (heating \& cooling)}$$

$$P = \frac{\rho k T}{\mu m_p} \ \text{ equation of state}$$

Add viscosity, thermal conduction, …
Add magnetic fields (MHD) and cosmic rays
Gravitational potential ϕ from DM, gas, galaxies

Deriving the Mass from X-ray Spectra
Ch 4.4 Longair

For spherical symmetry this reduces to

$(1/\rho_g)\, dP/dr = -d\phi(r)/dr = GM(r)/r^2$

With a little algebra and the definition of pressure - the total cluster mass can be expressed as (eqs 4.17-4.19 in Longair)

$$M(r) = (kT_g(r)/\mu G m_p)\mathbf{r}\ (d\ln T/dr + d\ln\rho_g/dr)$$

k is Boltzmans const, μ is the mean mass of a particle and m_H is the mass of a hydrogen atom

Every thing is observable

The temperature T_g from the spatially resolved spectrum

The density ρ_g from the knowledge that the emission is due to bremmstrahlung and the x-ray image

And the scale size, **r**, from the conversion of angles to distance

- And the mass in gas is
- $M_{gas}(< r) = 4\pi\int^{r}_{0} r^2 dr\, \varrho_{gas}(r)$

How to Obtain Gas Density

- De-project X-ray surface brightness profile I(R) to obtain gas density vs. radius, $\rho(r)$

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\varepsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}$$

$$\varepsilon_\nu(r) = -\frac{1}{\pi}\frac{d}{dr^2}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}} = \Lambda_\nu[T(r)]n_e^2(r)$$

- Where Λ is the cooling function and n_e is the gas density (subtle difference between gas density and electron density because the gas is not pure hydrogen
- De-project X-ray spectra in annuli T(r)
- Pressure P = $\rho kT/(\mu m_p)$

X-ray Mass Estimates

- use the equation of hydrostatic equilibirum
-

$$\frac{dP_{\rm gas}}{dr} = \frac{-G\mathfrak{M}_*(r)\rho_{\rm gas}}{r^2} \qquad (3)$$

where $P_{\rm gas}$ is the gas pressure, $\rho_{\rm gas}$ is the density, G is the gravitational constant, and $\mathfrak{M}_*(r)$ is the mass of M87 interior to the radius r.

$$P_{\rm gas} = \frac{\rho_{\rm gas} KT_{\rm gas}}{\mu \mathfrak{M}_{\rm H}} \qquad (4)$$

where μ is the mean molecular weight (taken to be 0.6), and $\mathfrak{M}_{\rm H}$ is the mass of hydrogen atom.

$$\frac{KT_{\rm gas}}{\mu\mathfrak{M}_{\rm H}}\left(\frac{d\rho_{\rm gas}}{\rho_{\rm gas}} + \frac{dT_{\rm gas}}{T_{\rm gas}}\right) = \frac{-G\mathfrak{M}_*(r)}{r^2}dr, \qquad (5)$$

which may be rewritten as:

$$-\frac{KT_{\rm gas}}{G\mu\mathfrak{M}_{\rm H}}\left(\frac{d\log\rho_{\rm gas}}{d\log r} + \frac{d\log T_{\rm gas}}{d\log r}\right)r = \mathfrak{M}_*(r) \qquad (6)$$

Putting numbers in gives

$$M(r) = -3.71\times10^{13}\,M_\odot\,T(r)\,r\left(\frac{d\log\rho_g}{d\log r} + \frac{d\log T}{d\log r}\right),$$

ere T is in units of keV and r is in units of Mpc. (

VIKHLININ ET AL.

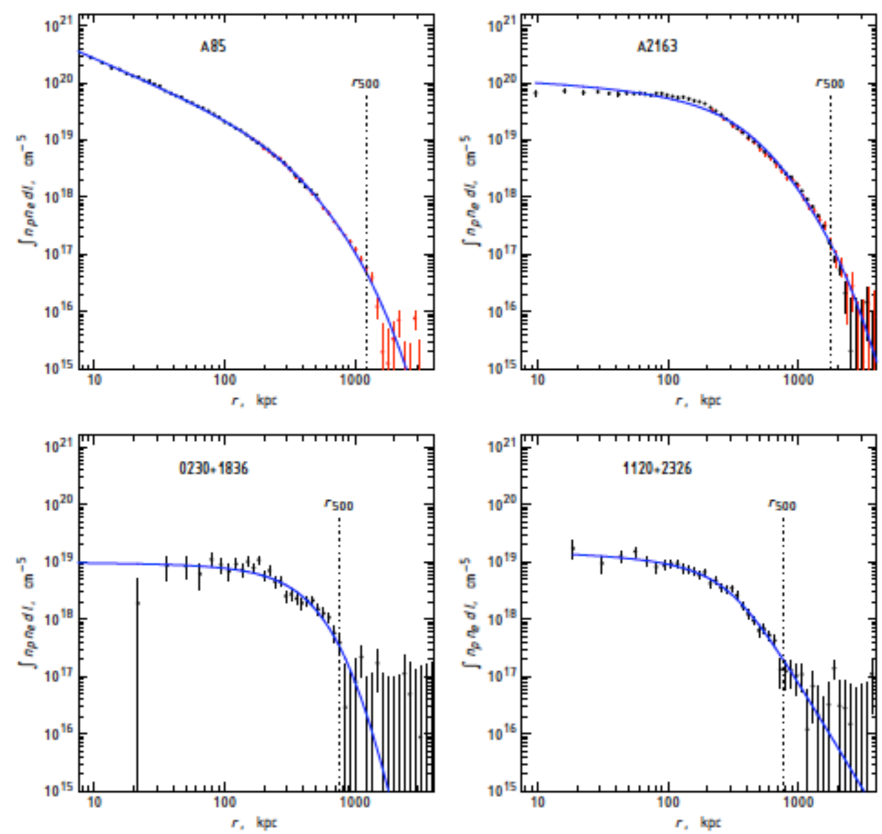


FIG. 5.— Examples of the surface brightness profile modeling for clusters shown in Fig. 3 and 4. The observed X-ray count rates are converted to the projected emission measure integral (see § 3.4 and V06). The black and red data points show the *Chandra* and *ROSAT* measurements, respectively. The best fit models (the projected emission measure integral for the three-dimensional distribution given by eq. 2) are shown by solid lines. The dashed lines indicate the estimated r_{500}

Density Profile

- a simple model(the β model) fits the *surface brightness* well

  $S(r)=S(0)(1+(r/a)^2)^{-3\beta+1/2}$ cts/cm^2/sec/solid angle
- Is analytically invertible (inverse Abel transform) to the *density profile*

  $\rho(r)=\rho(0)(1+(r/a)^2)^{-3\beta/2}$

  The conversion function from S(0) to ρ(0) depends on the detector

  The quantity 'a' is a scale factor- sometimes called the core radius

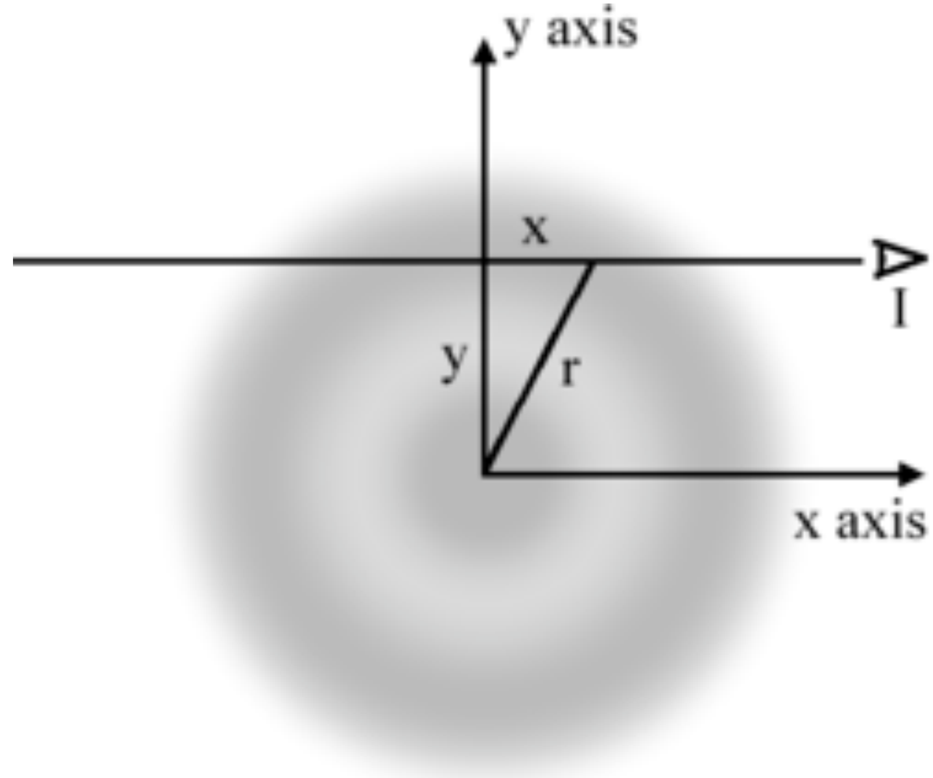
  ----

- The Abel transform, , is an integral transform used in the analysis of spherically symmetric or axially symmetric functions. The Abel transform of a function **f(r)** is given by:

$$f(r)=1/p\int_r^\infty dF/dy\, dy/\sqrt{(y^{2}-r^2)}$$

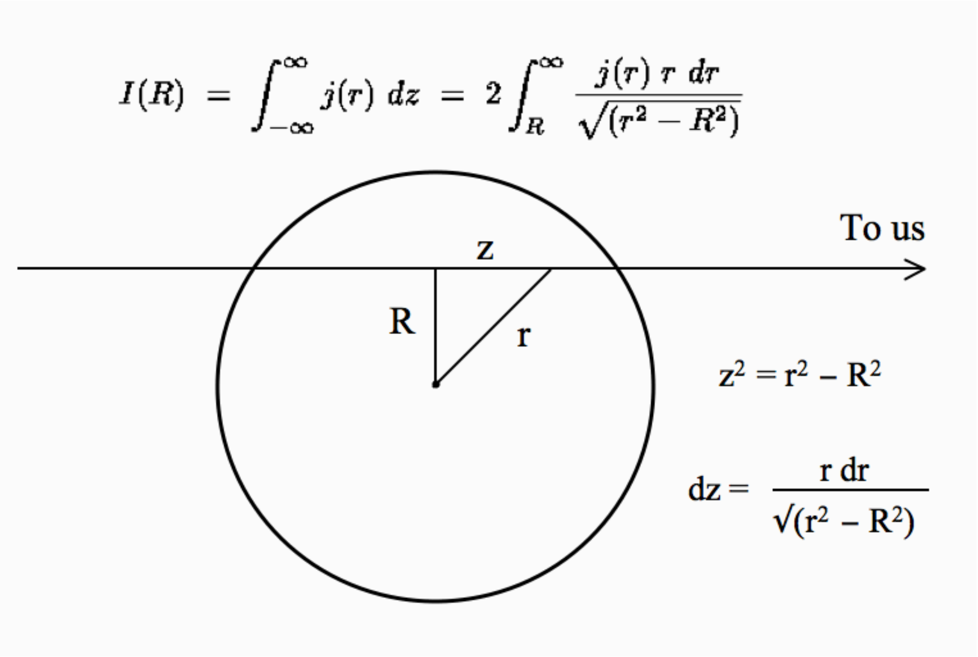
- In image analysis the reverse Abel transform is used to calculate the emission function given a projection (i.e. a scan or a photograph) of that emission function.
- In general the integral is not analytic

A geometrical interpretation of the Abel transform in two dimensions. An observer (I) looks along a line parallel to the x-axis a distance y above the origin. What the observer sees is the projection (i.e. the integral) of the circularly symmetric function f(r) along the line of sight. The function f(r) is represented in gray in this figure. The observer is assumed to be located infinitely far from the origin so that the limits of integration are ±∞



Density Profile- Longair 4.22

- I(R) is the projected luminosity surface brightness, j(r) is the 3-D luminosity density (circular images- if image is elliptical no general solution)



$j(r)=-1/\pi \int^{\infty}_{R} dI/dR/\mathrm{sqrt}(R^2-r^2)$

this is an Abel integral which has only a few analytic solutions

Simple power law models $I(R)=r^{-\alpha}$

then $j(r)=r^{-\alpha-1}$

generalized King profile with surface brightness

$I(r)=I(0)(1+(r/r_c)^2)^{-5/2}$

gives a density law $\rho(r)=\rho(0)(1+(r/r_c)^2)^{-3/2}$ where $r_c=3\sigma/\mathrm{sqrt}(4\pi G\rho_c)$

Sarazin sec 5

The gas distributions in clusters can be derived directly from observations of the X-ray surface brightness of the cluster, if the shape of the cluster is known and if the X-ray observations are sufficiently detailed and accurate. This method of analysis also leads to a method for determining cluster masses (Section 5.5.5). The X-ray surface brightness at a photon frequency ν and at a projected distance b from the center of a spherical cluster is

$$I_\nu(b) = \int_{b^2}^{\infty} \frac{\epsilon_\nu(r)\,dr^2}{\sqrt{r^2 - b^2}}, \qquad (5.80)$$

where ϵ_ν is the X-ray emissivity. This Abel integral can be inverted to give the emissivity as a function of radius,

$$\epsilon_\nu = -\frac{1}{2\pi r}\frac{d}{dr}\int_{r^2}^{\infty} \frac{I_\nu(b)\,db^2}{\sqrt{b^2 - r^2}}. \qquad (5.81)$$

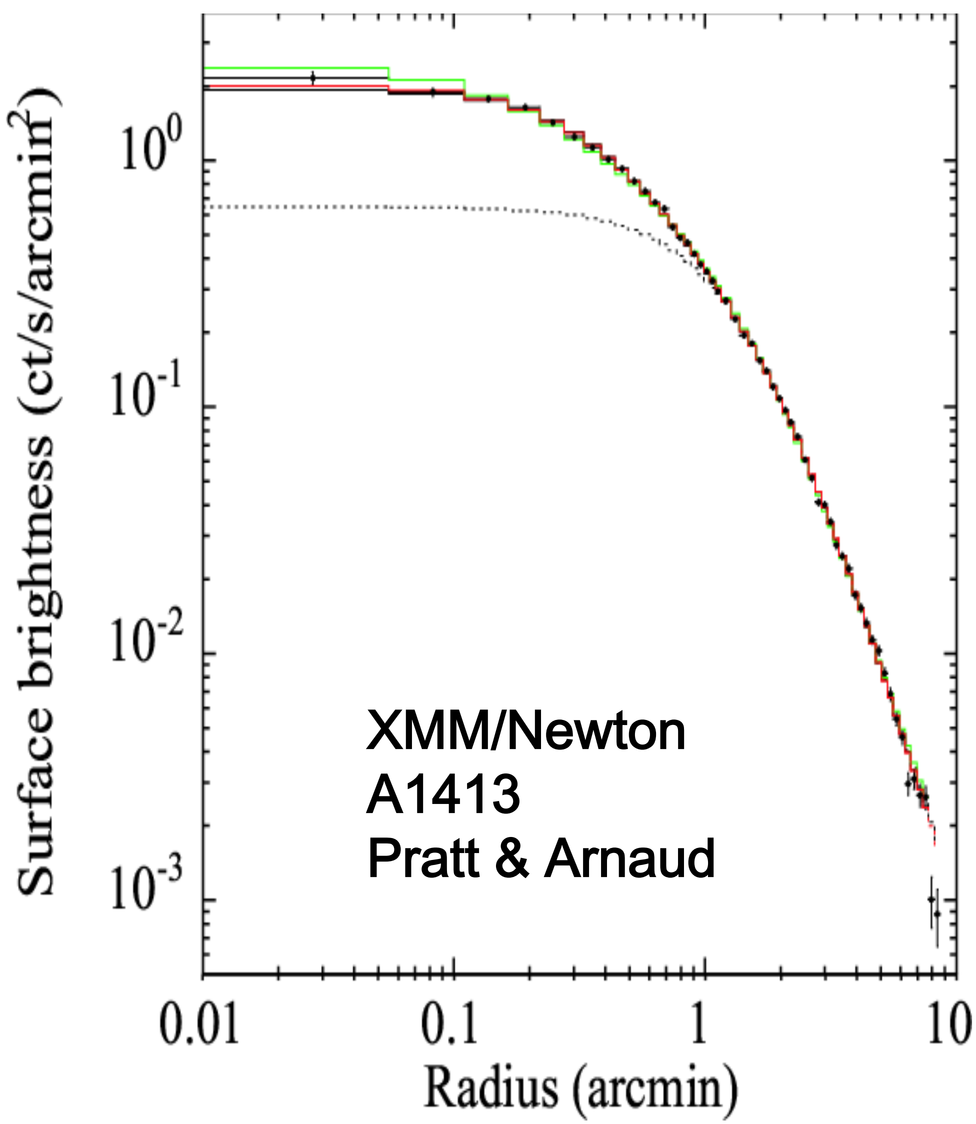
Surface Brightness Profiles

- It has become customary to use a 'β' model (Cavaliere and Fesco-Fumiano)'
- clusters have <β>~2/3

$$\rho(r) = \frac{\rho_0}{\left[1 + \left(\frac{r}{r_c}\right)^2\right]^{3\beta/2}}$$

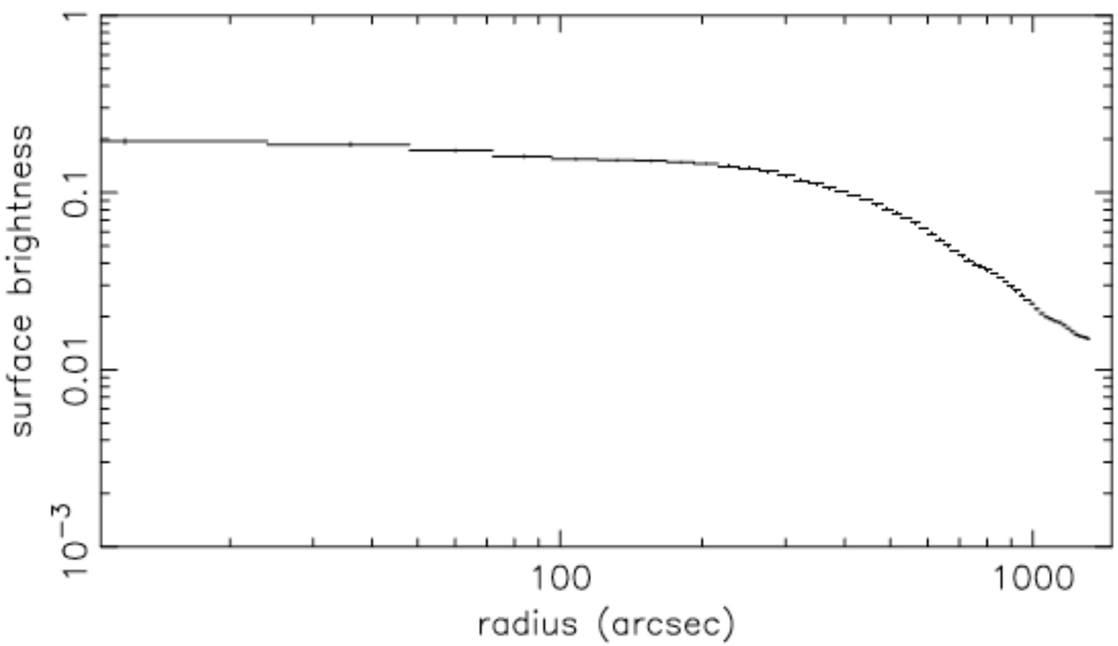
$$\beta \equiv \frac{\mu m_p \sigma_{gal}^2}{kT}$$ but treat as fitting parameter

$$I_X(r) \propto \left[1 + \left(\frac{r}{r_c}\right)^2\right]^{-3\beta+1/2}$$

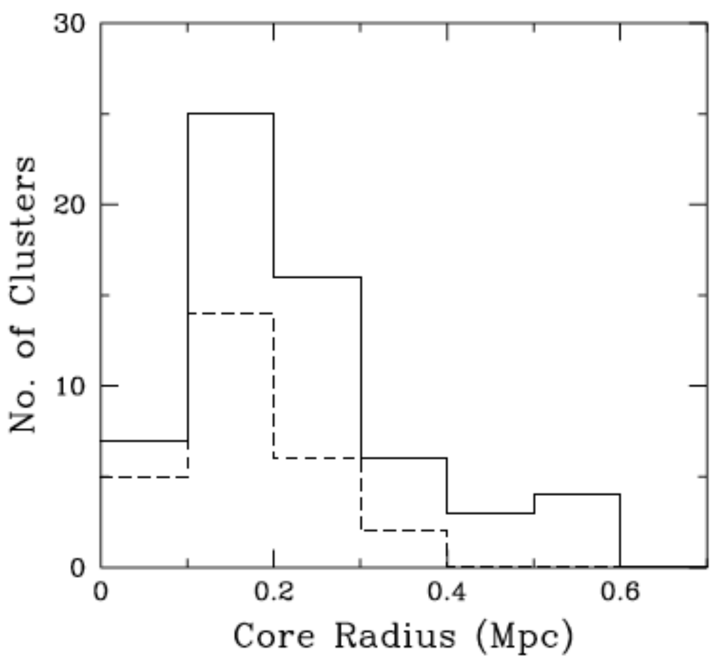


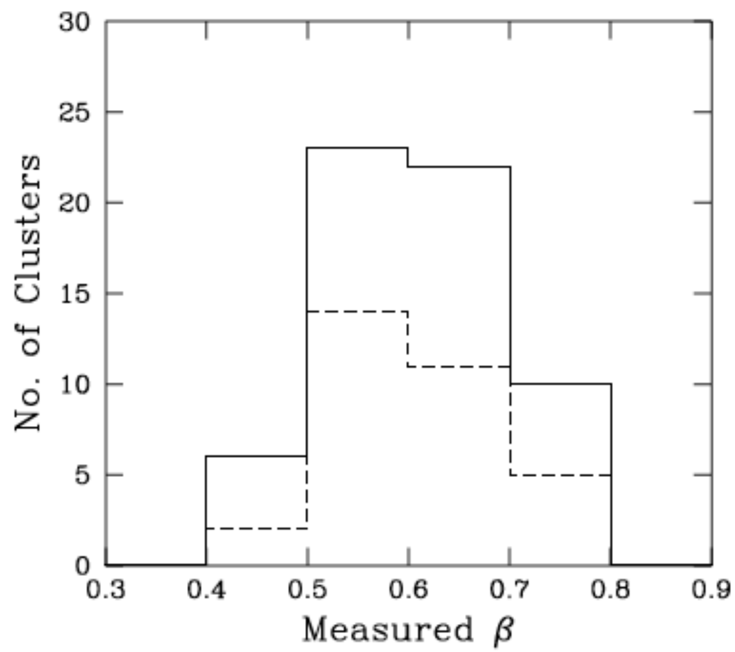
'Two' Types of Surface Brightness Profiles

- 'Cored'



- Central Excess
- Range of core radii and β

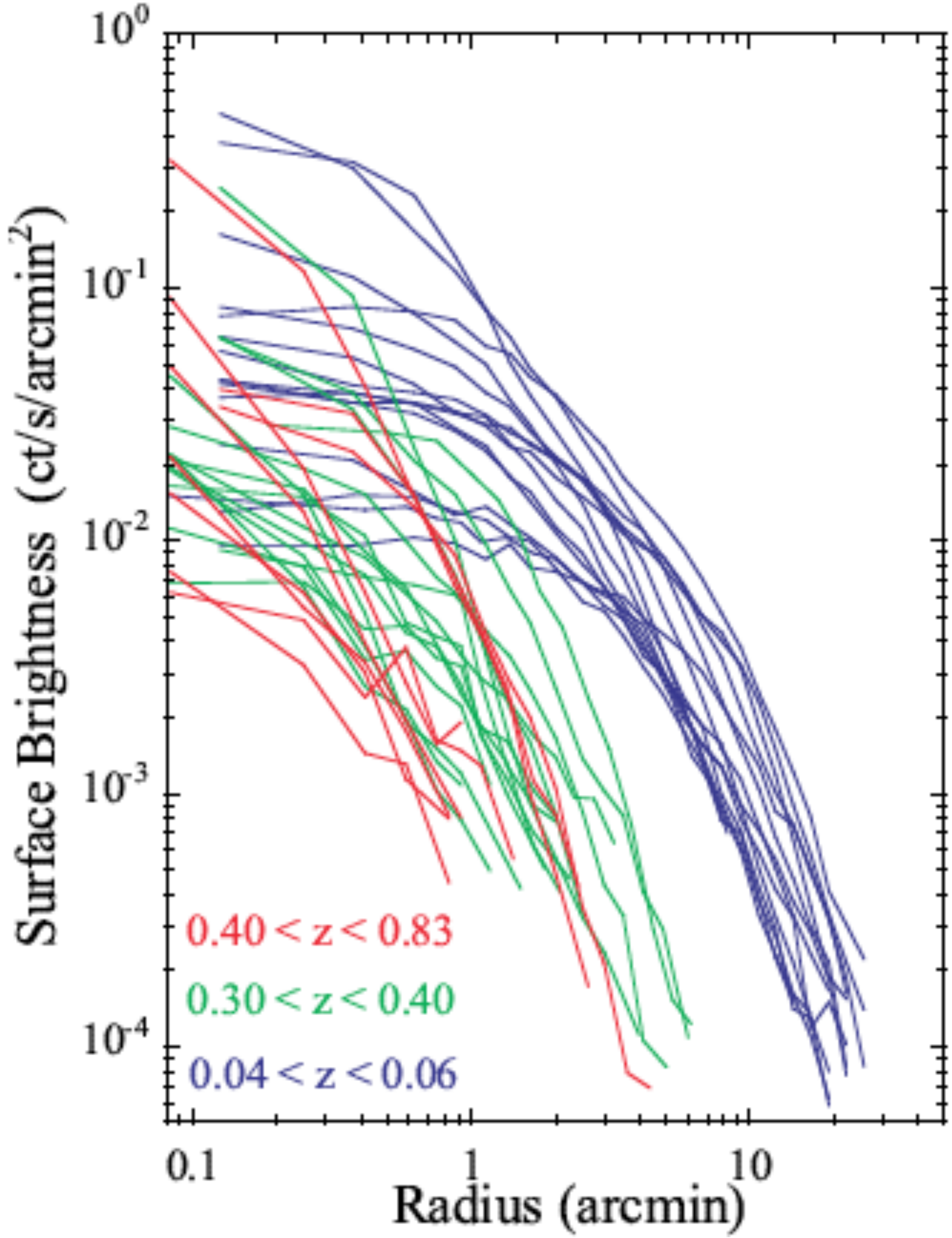




X-ray Emissivity

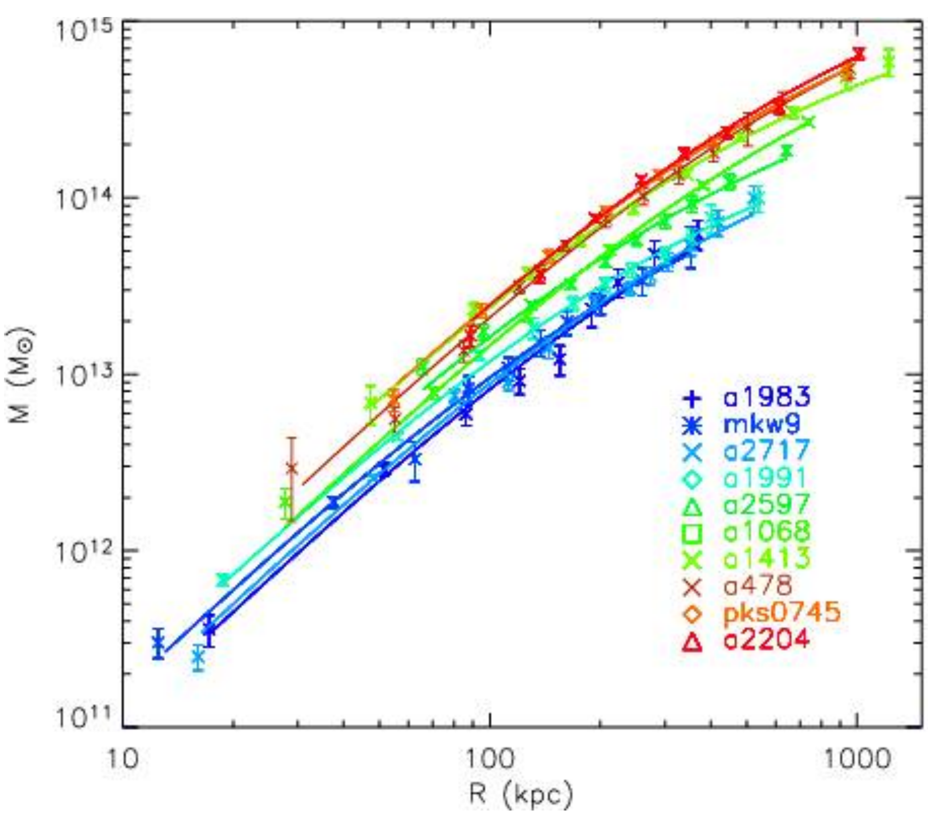
- The observed x-ray emissivity is a projection of the density profile

A large set of clusters over a wide range in redshift

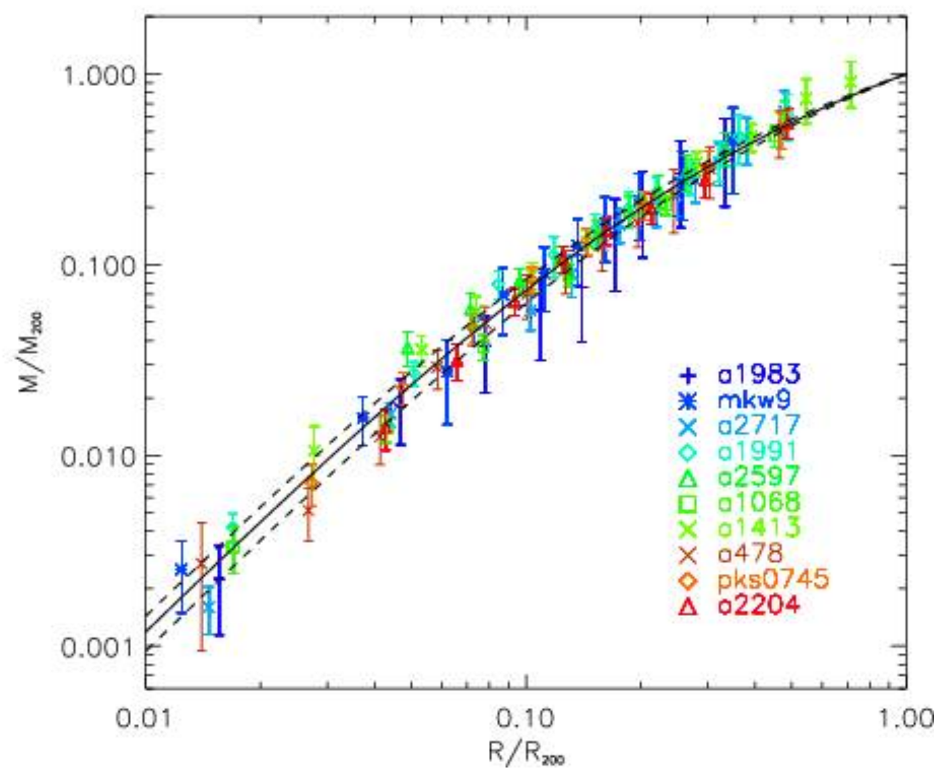


Mass Profiles from Use of Hydrostatic Equilibrium

- Use temperature and density profiles +hydrostatic equilibrium to determine masses



Physical units



Scaled units

Checking that X-ray Properties Trace Mass

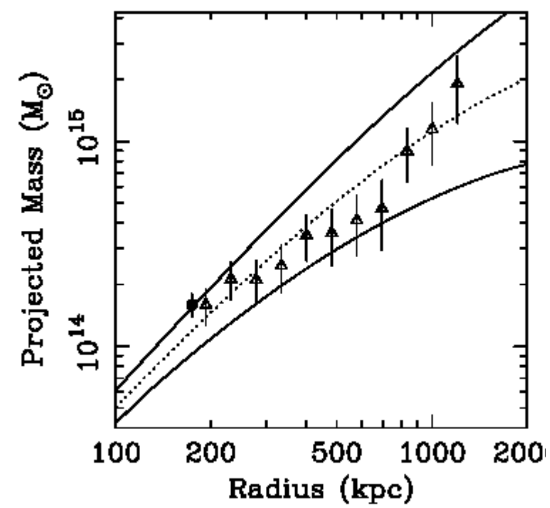
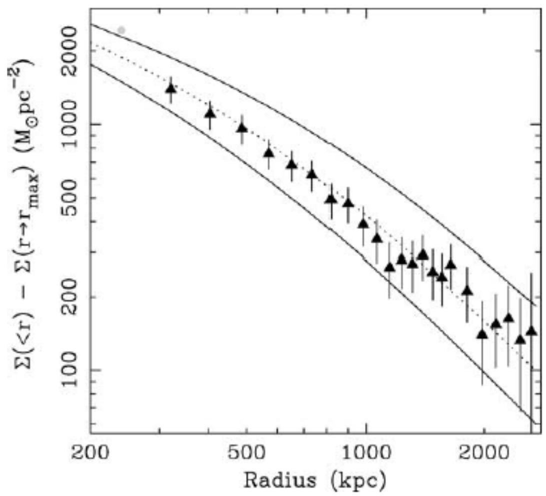
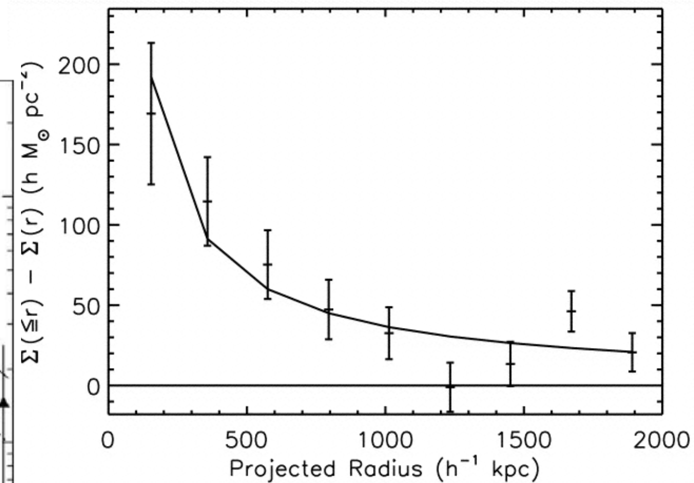


Figure 8. A comparison of the projected total mass determined from *Chandra* X-ray data (Section 5) with the strong lensing result of Pierre e (1996; filled circle) and the weak lensing results of Squires et al. (19



projected surface mass density contrast determined from the *Chandra* X-ray data (Section 5) w



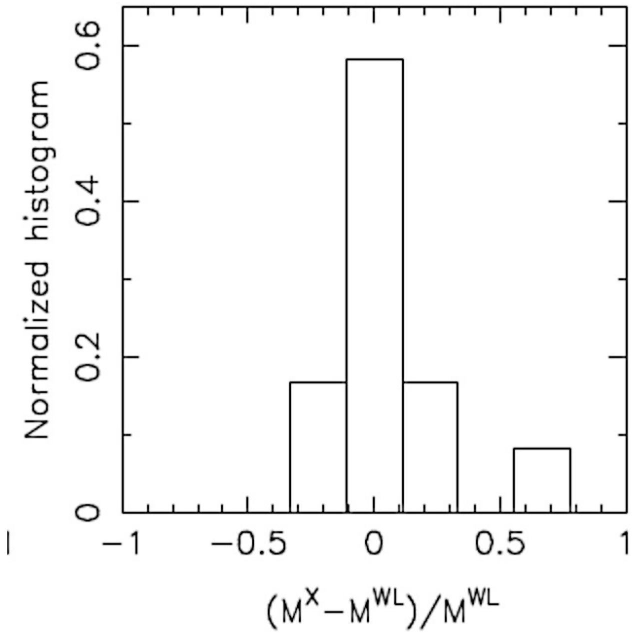
Surface mass density for 42 Rosat selected clusters from Sloan lensing analysis

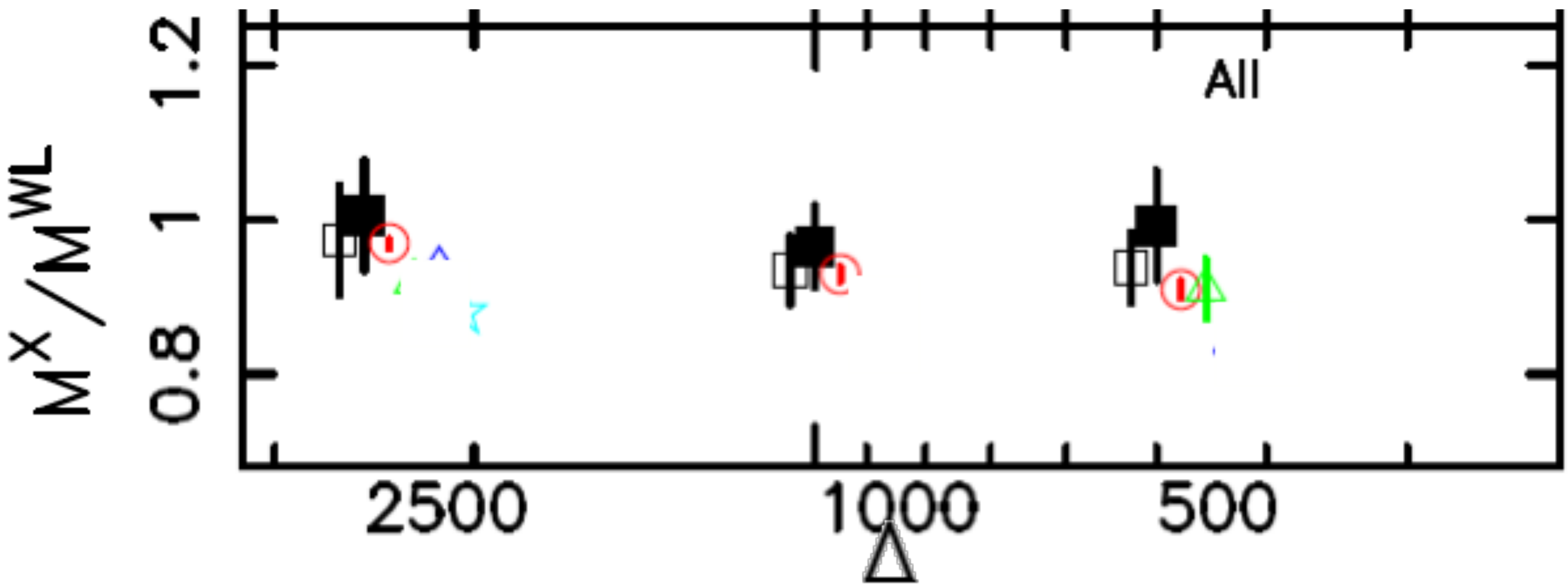
Comparison of cluster mass from lensing and x-ray hydrostatic equilibrium for A2390 and RXJ1340 (Allen et al 2001)

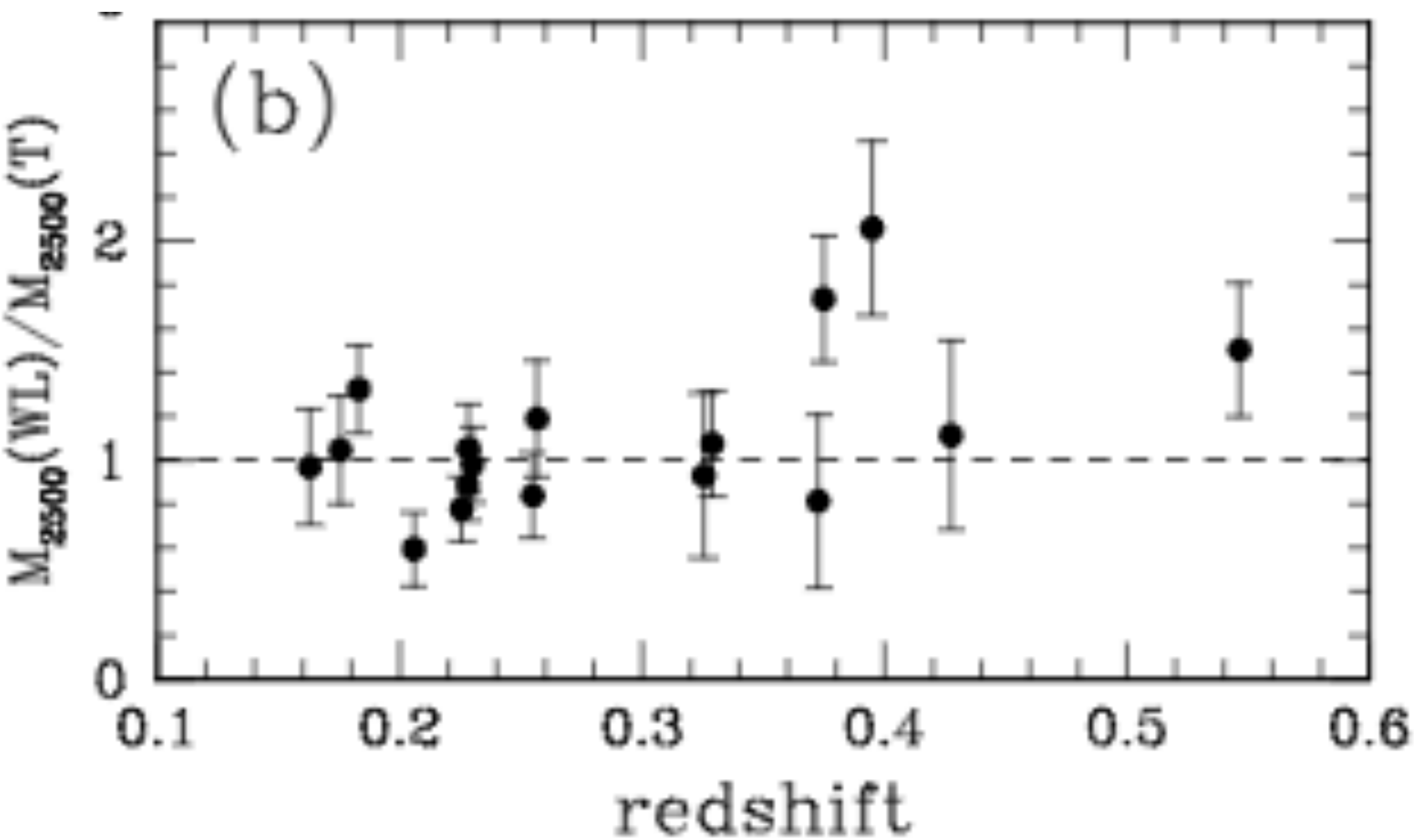
At the relative level of accuracy for smooth relaxed systems the x-ray and lensing mass estimators agree

Comparison of Lensing to X-ray Masses

- Δ is the overdensity of the part of the cluster used for the observations of the cluster mass compared to the critical density of the universe at the redshift of the cluster
- M_x is the mass from x-ray observations and assumption of hydrostatic equilibrium
- M_L is the mass from weak lensing



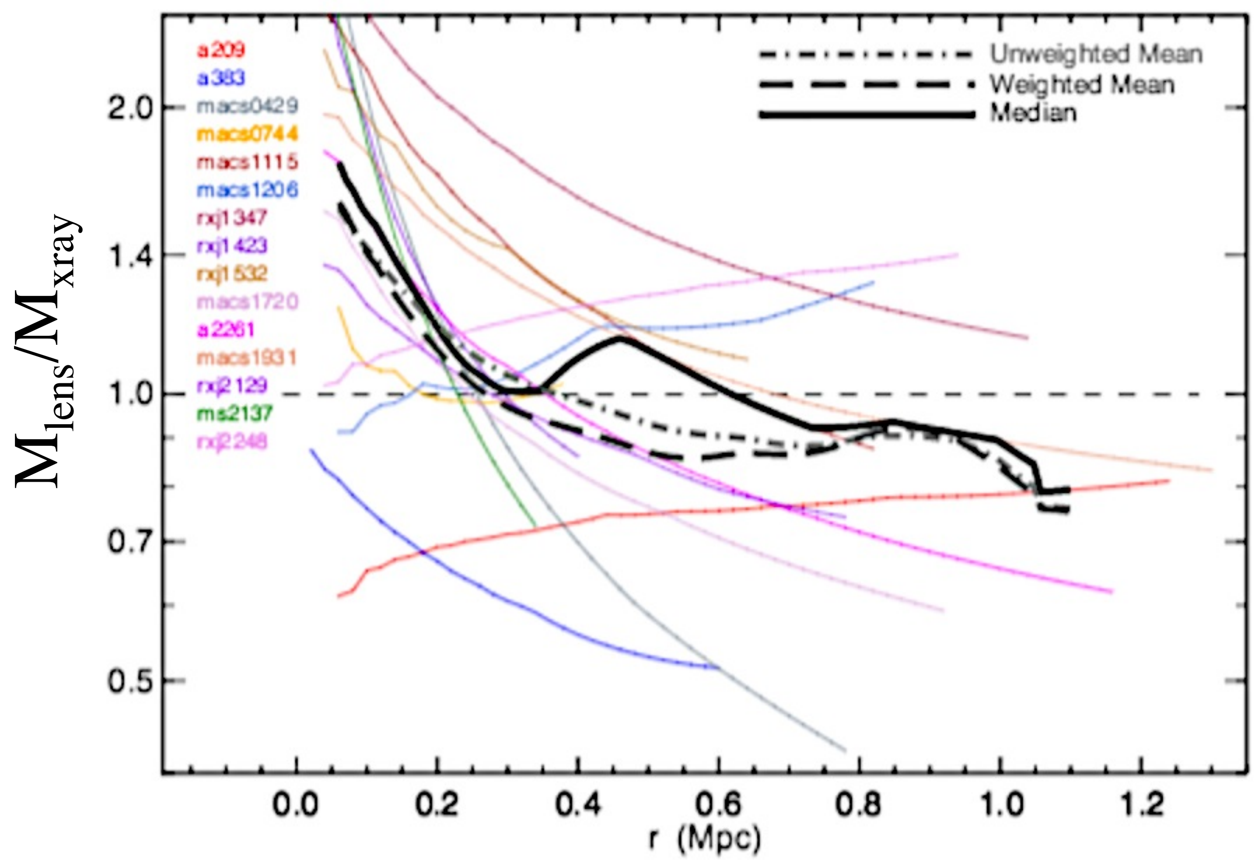




Hoekstra 2007

Lensing vs Hydro

- See Clash-x: A Comparison Of Lensing And X-ray Techniques For Measuring The Mass Profiles Of Galaxy Clusters- Donahue et al 2015



Why are Clusters Interesting or Important

- Laboratory to study
  - Dark matter
    - Can study in detail the distribution and amount of dark matter and baryons
  - Chemical evolution
    - Most of the 'heavy' elements are in the hot x-ray emitting gas
  - Formation and evolution of cosmic structure
    - Feedback
    - Galaxy formation and evolution
    - Mergers
  - Cosmological constraints
    - Evolution of clusters is a strong function of cosmological parameters
  - Plasma physics on the largest scales
  - Numerical simulations
  - Particle acceleration

Each one of these issues Leads to a host of topics

Dark matter:

How to study it

lensing

Velocity and density distribution of galaxies

Temperature and density distribution of the hot gas

Chemical Evolution

Hot and when where the elements created

Why are most of the baryons in the hot gas

Does the chemical composition of the hot gas and stars differ?

Chemical Evolution of the Universe

- A major area of astrophysical research is understanding when stars and galaxies formed and how the elements are produced

- With the exception of H and He (which are produced in the big bang) all the other elements (called metals in astrophysical jargon) are "cooked" in the centers of massive stars and supernova and then "ejected" by explosions or winds

- The gas in these explosions is moving very fast (1000 km/sec) and can easily escape a galaxy.

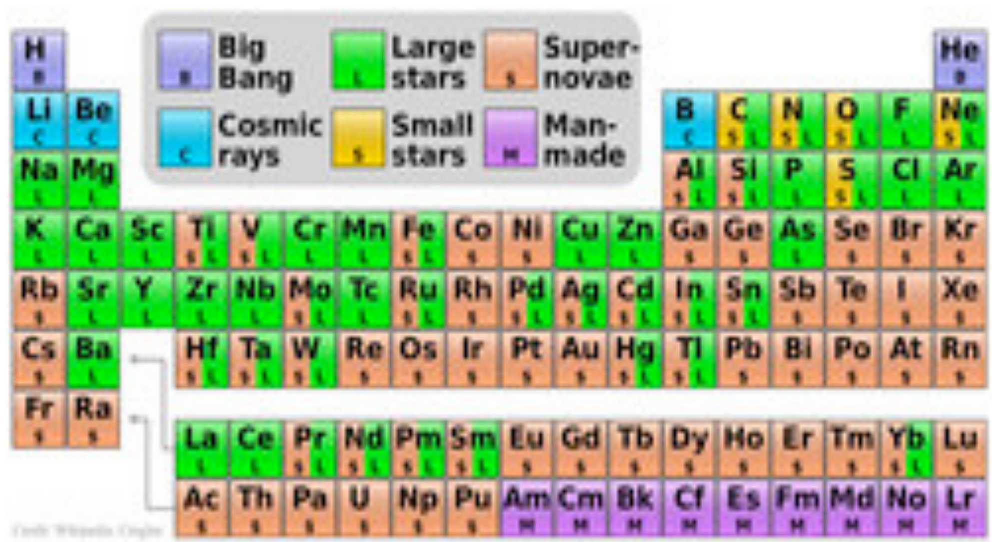
- Clusters are essentially giant "boxes" which can hold onto all their material

•Measurement of the amount and change of metals with time in clusters directly measures their production

•In the hot gas elements such as silicon and iron have only 1 or 2 electrons

These ions produce strong H, He like x-ray emission lines.

The strengths of these lines is 'simply' related to the amount of silicon or iron in the cluster



Origin of 'Metals'

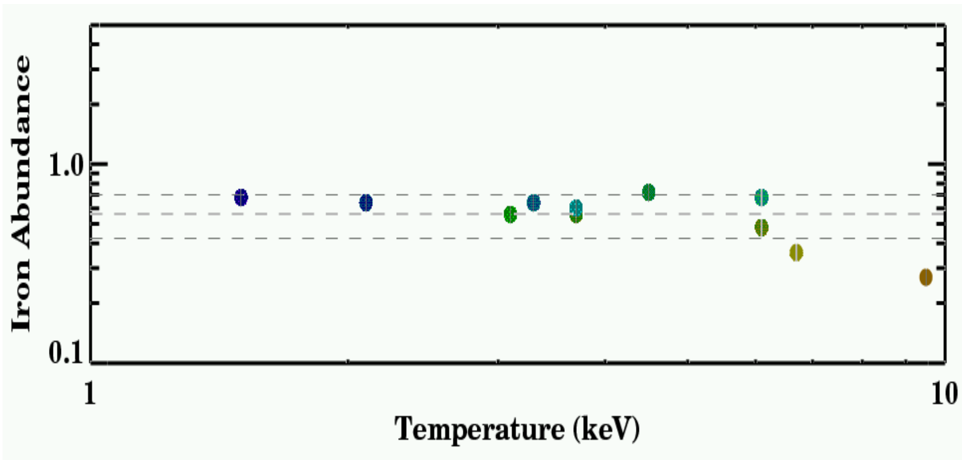
- Metal production is dominated for (0…Ni) by supernova.
- Type II (core collapse) produce most of the O and Type I produce most of the Fe.
- The fraction of other elements (e.g. Si,S) that are produced by the SN depend on the IMF and the (poorly understood) yields of the SN.
- If the observed cluster galaxies are the source of the metals and 'standard' SN rates and IMF are assumed they produce only 1/3 of the the observed metals

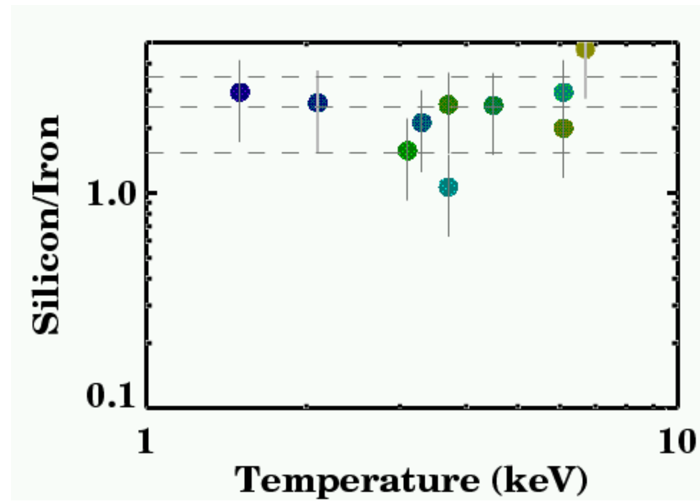
Since most of the metals are in the gas>70% of the metals generated in galaxies has to be 'lost' from galaxies (where the stars live) to the ICM

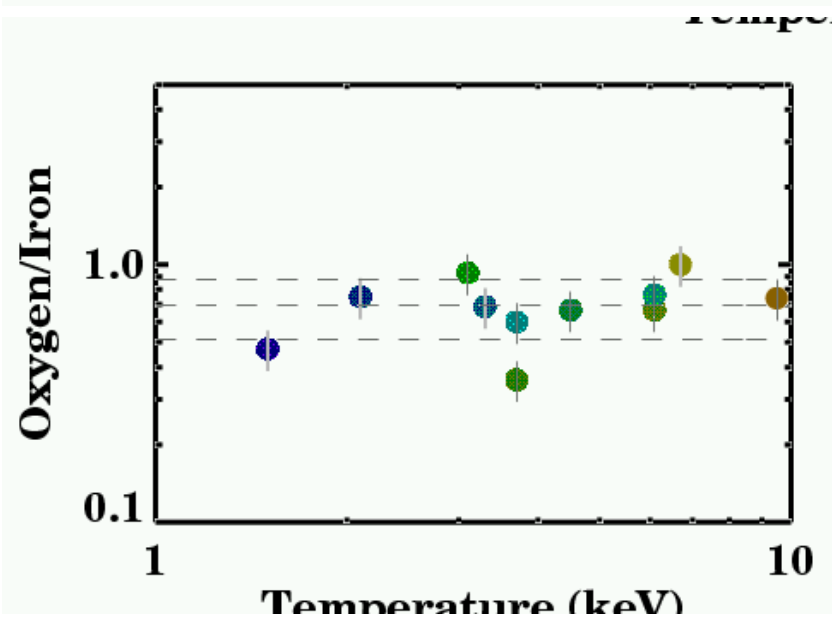
Virialized systems- Clusters, Groups and Big galaxies

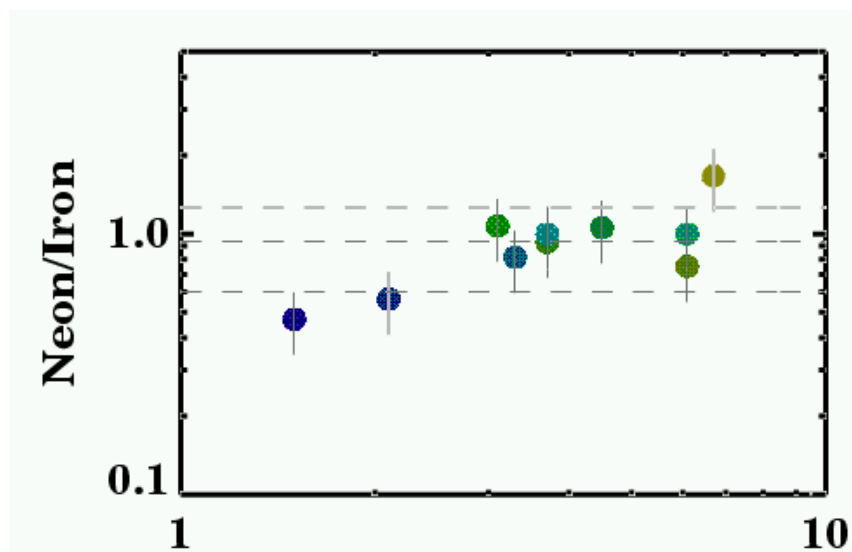
XMM Grating Results- J. Peterson et al

these abundances are normalized to solar values



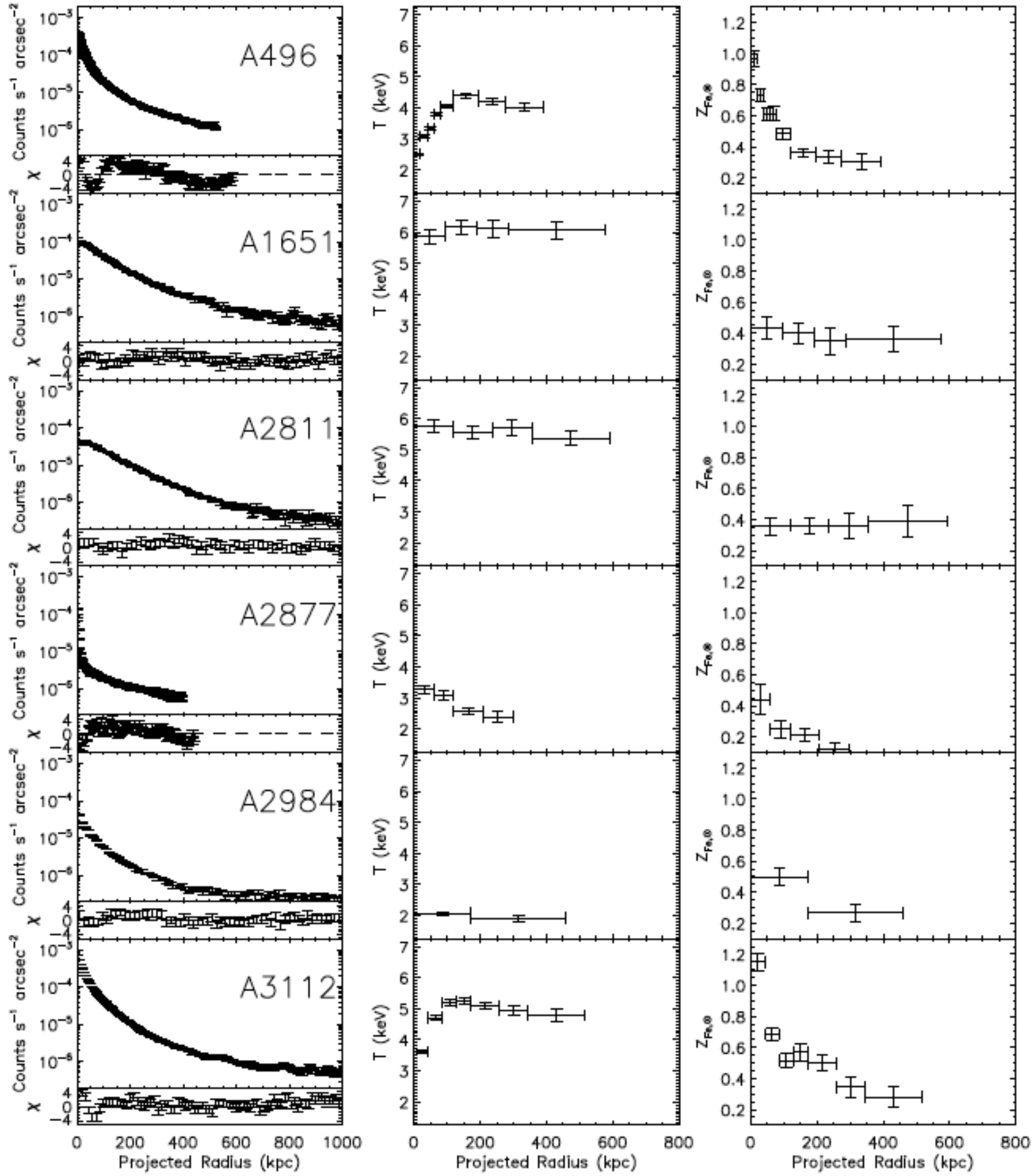






Cluster Metallicity

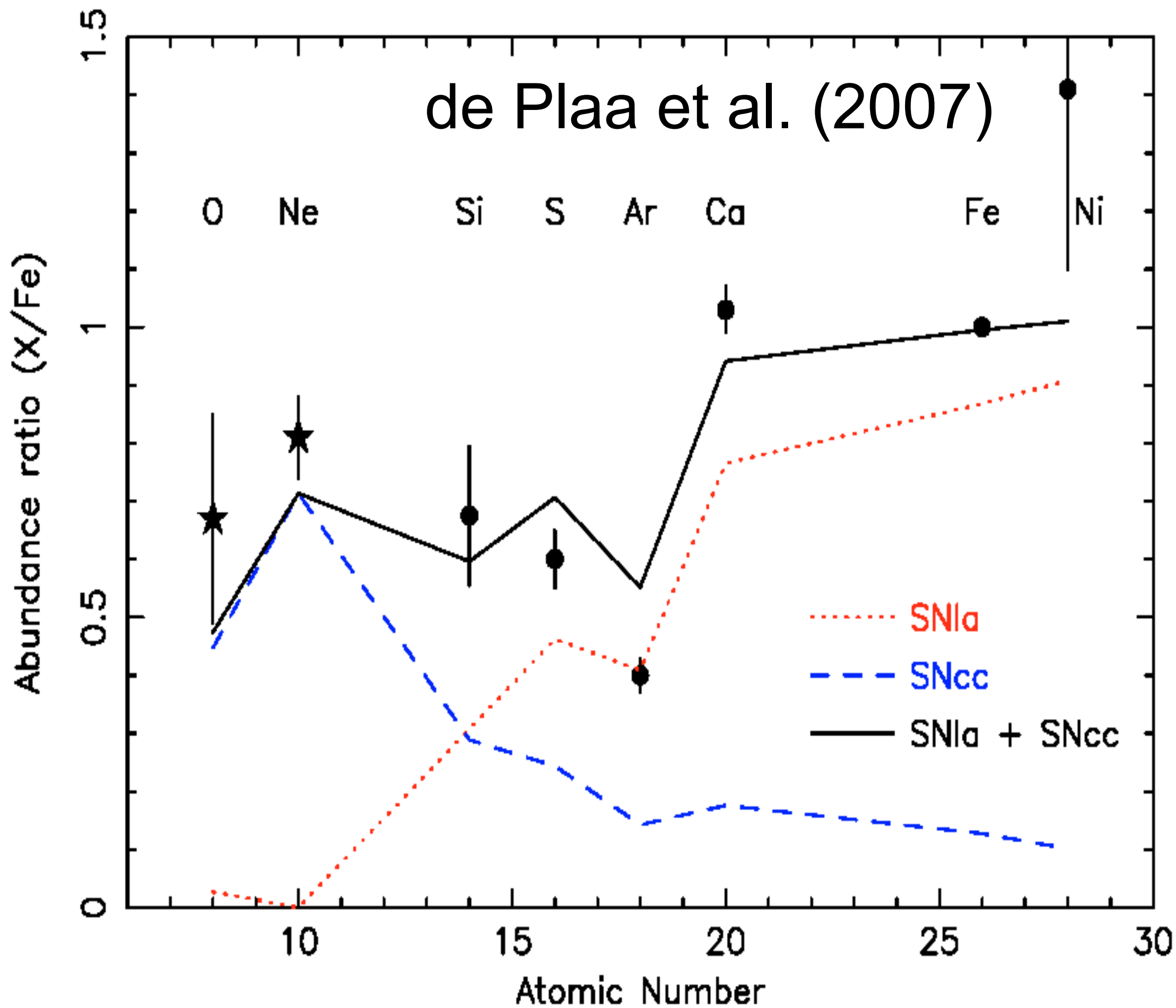
- The abundances are not uniform in the cluster but can be higher in the center



Temperature Abundance

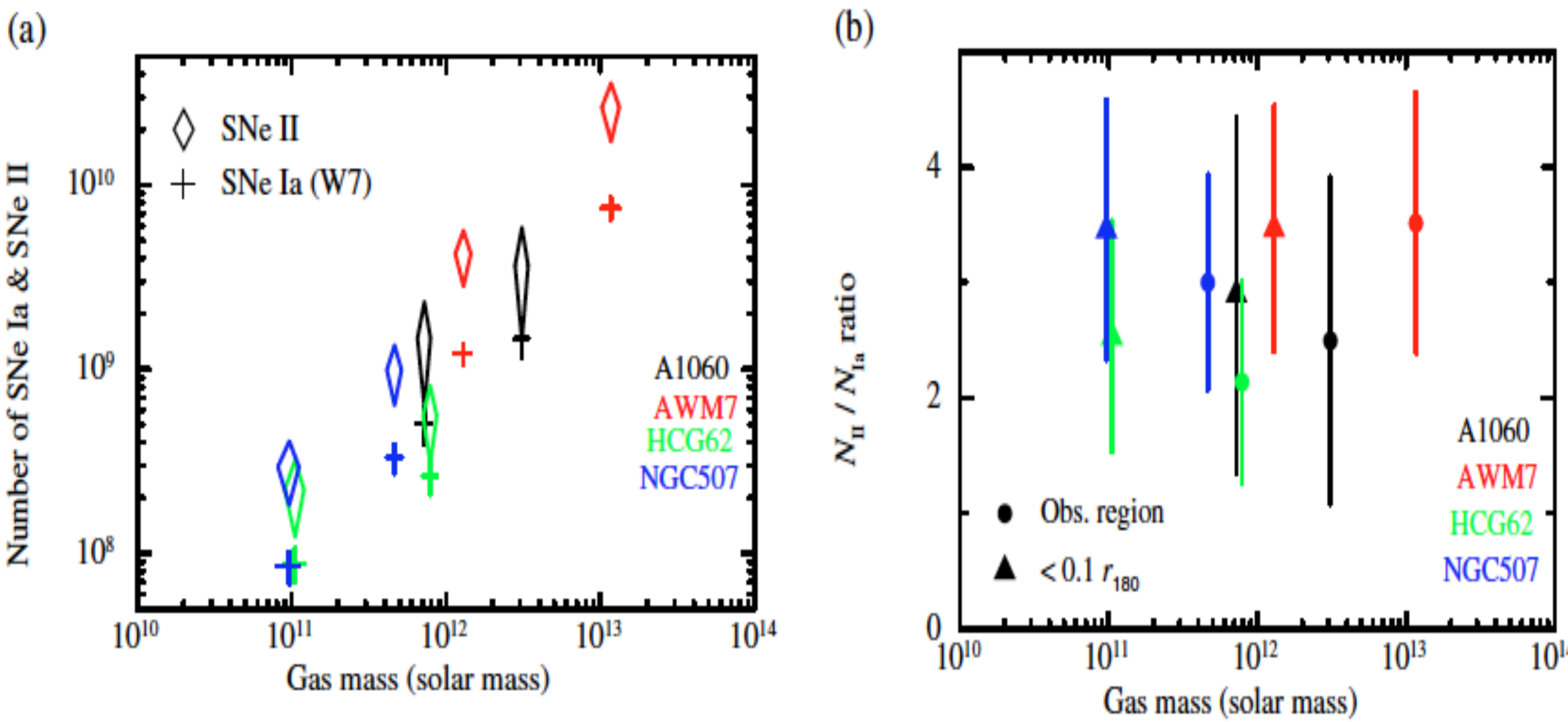
Relative Abundance of Different Elements

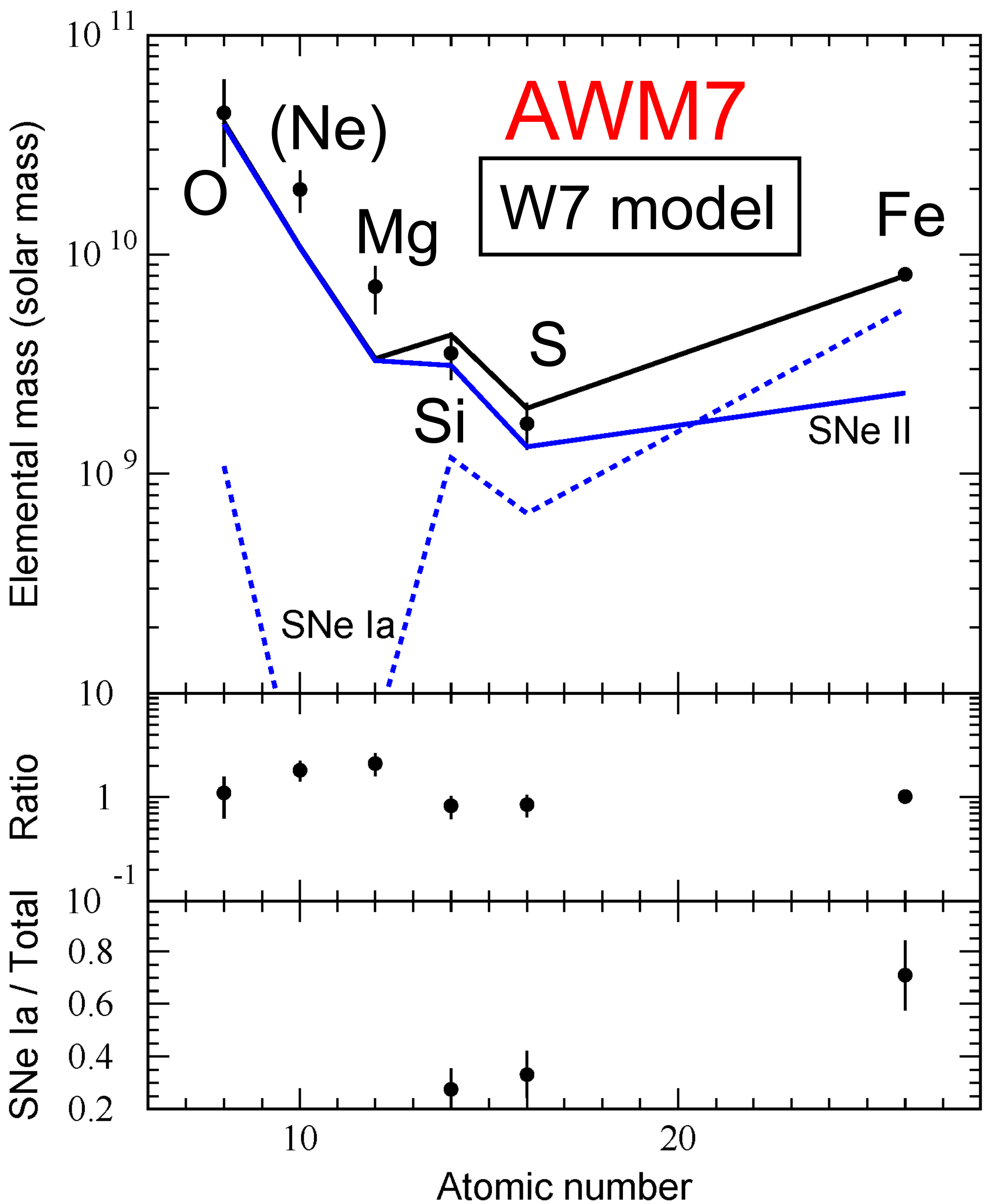
- The relative abundance of differenet elements is related to the processes that produce them
- Fe and Ni are mostly made (we think) in type I supernova (the explosion of a white dwarf)
- Oxygen and Neon are made mostly in a type II SN- the explosion of a massive star
- The relative and absolute number of SN is related to the distribution of the masses of the stars and other interesting things



Numbers of Type I and II Supernova

- As we will discuss later the two types of SN produce a very different mix of heavy elements
- This allows a decomposition into their relative numbers and absolute numbers (Sato et al 2008) - (~10^9-10^{10} SN per cluster)





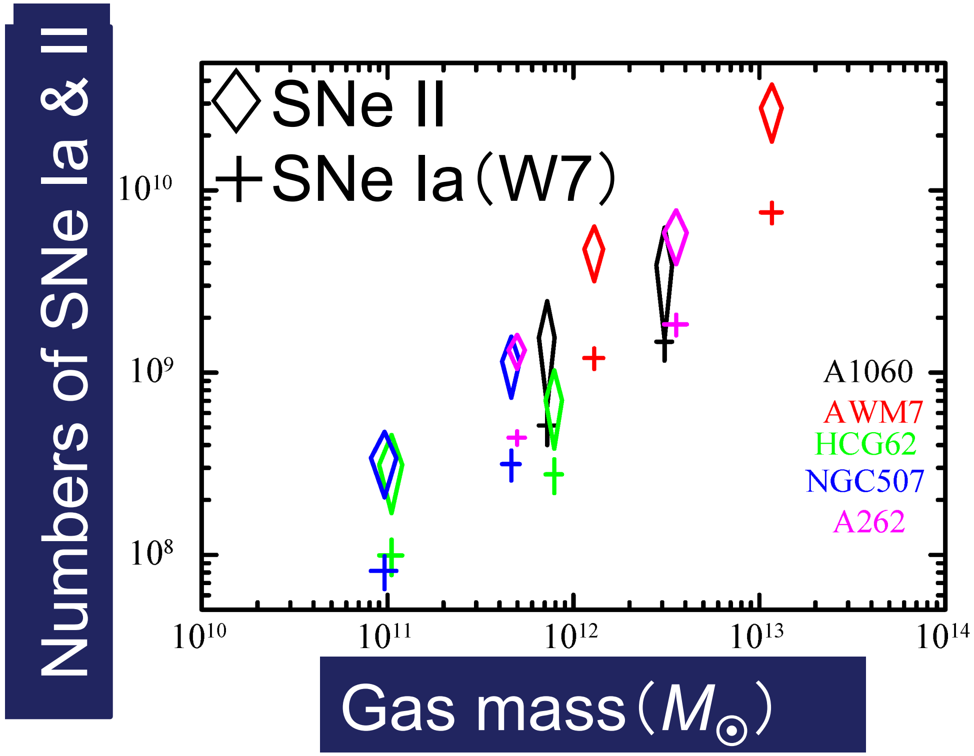
Ratio of the number of each type of SN

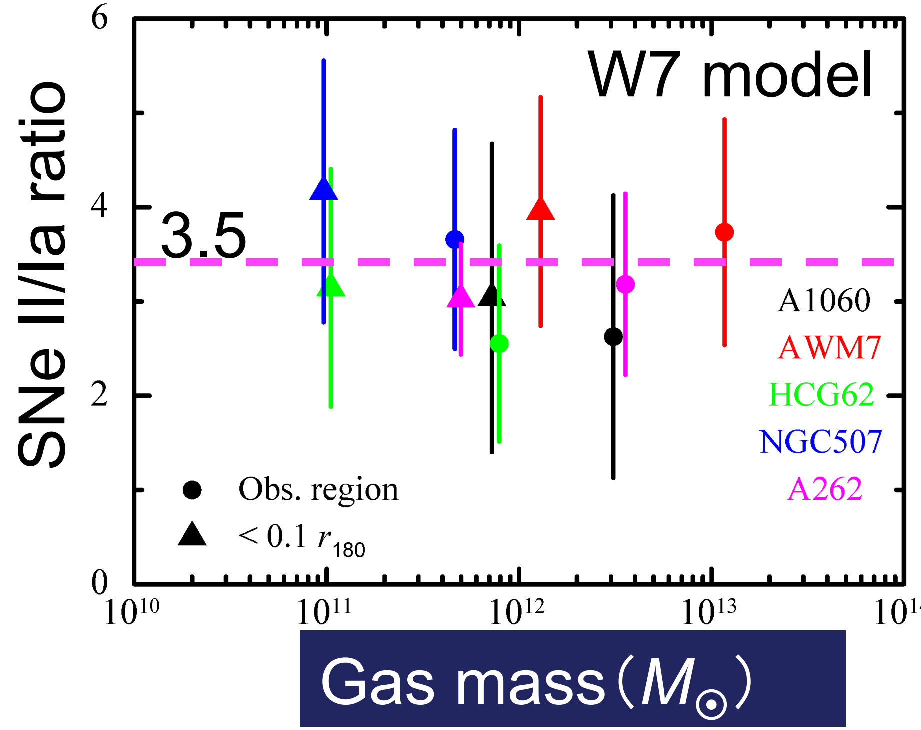
$N_{\mathrm{SNe\ II}} / N_{\mathrm{SNe\ Ia}} = 4.0 \pm 1.2$

- ~75% of Fe, ~40% of Si and S from SNe Ia

Sato et al

Numbers and Ratio of SNe Ia &



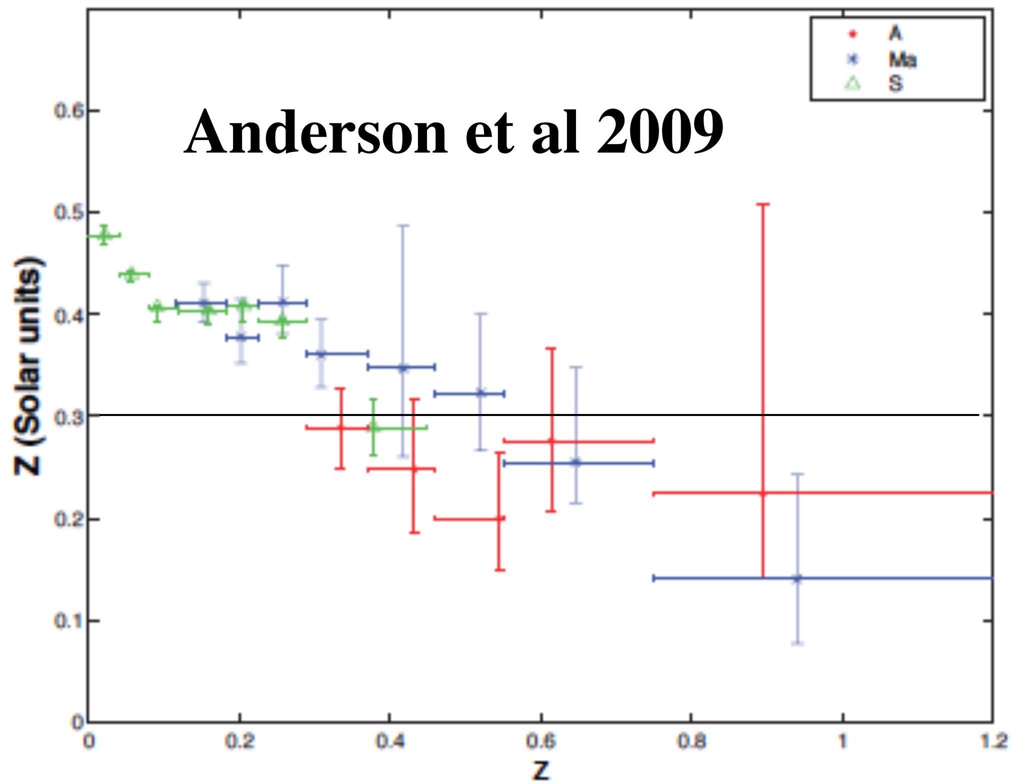


- Numbers of SNe Ia &
  SNe II/Ia Ratio: ~3.5 (W7 and WDD2), ~2.5 (WDD1)

cf. Clusters (*XMM* ; de Plaa et al. 2007): ~3.5
Our Galaxy (Tsujimoto et al. 1995): ~6.7
LMC & SMC (Tsujimoto et al 1995): 3.3 – 5

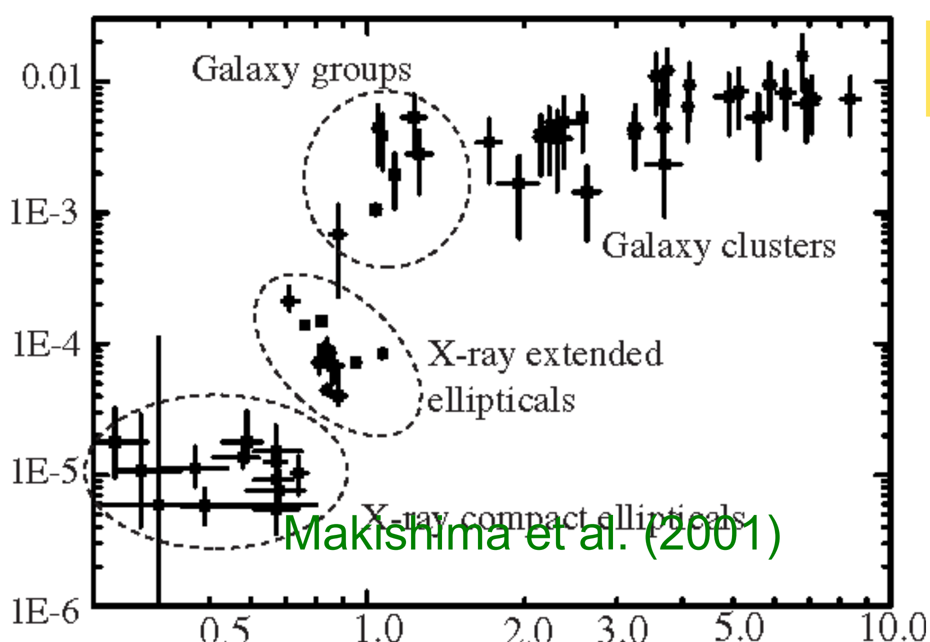
Metallicity Evolution

- There is weak evidence for cluster metallicity evolution- when were the metals produced
- Most of the metals were in place at z~0.5 and maybe at z~1



Ehlert and Ulmer 200

Metals are synthesized in stars (galaxies):
Compare the mass of metals $M_{metal, < R}$ (in units of $M_\odot$)
with B-band luminosity of stars
(similar to mass to light ratio)

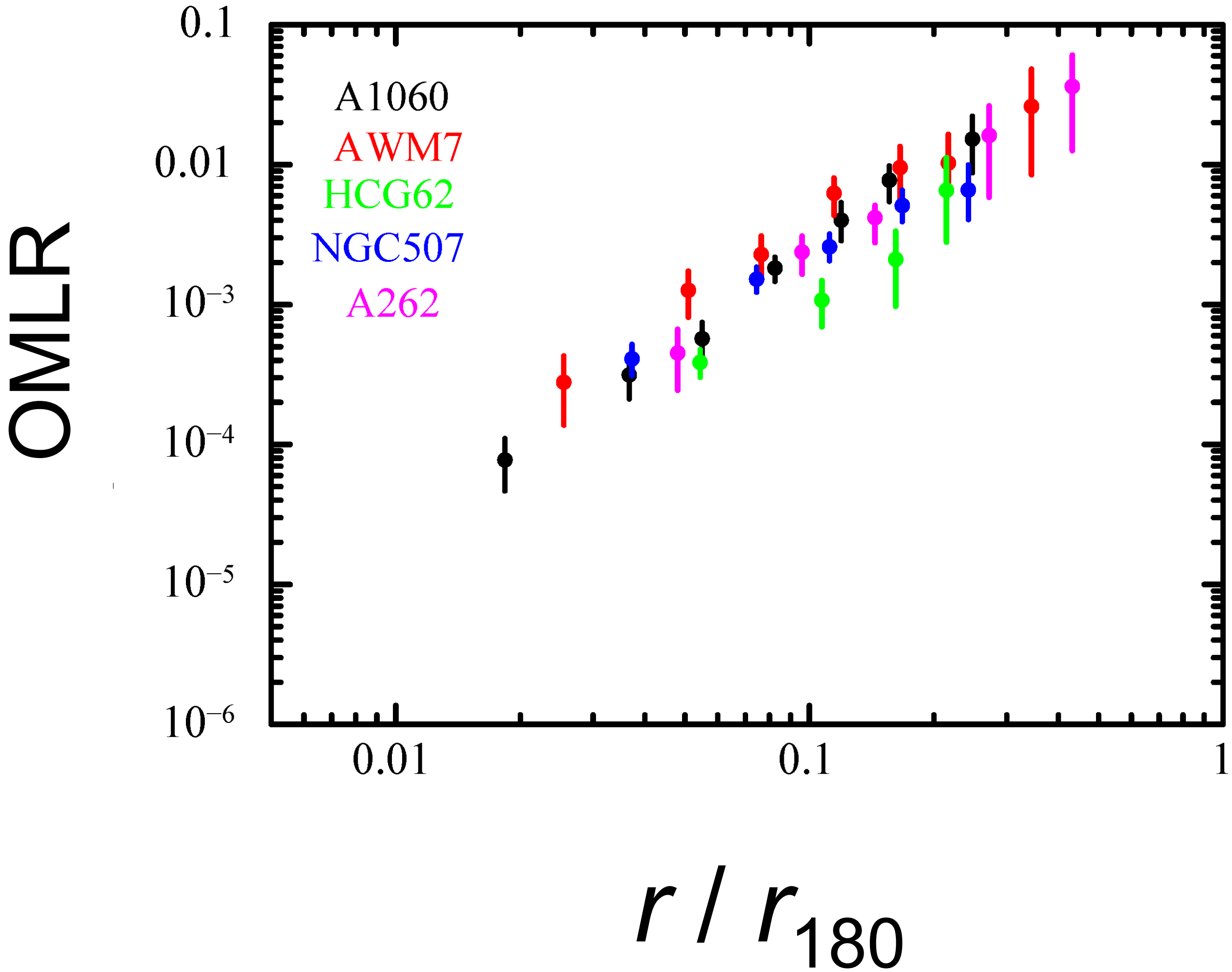


Temperature (keV) ∝ mass of system

$$MLR = \frac{M_{metal, < R}}{L_{B\ or\ K, < R}} \quad \frac{M_\odot}{L_\odot}$$

Oxygen Mass-to-Light Ratio: OMLR
Magnesium Mass-to-Light Ratio: MMLR
Iron Mass-to-Light Ratio: IMLR

Metal enrichment process in the ICM
- shows factor of several variation



Elemental Abundances in a Group

- Li et al compare the elemental abundances with respect to solar for Oxygen thru Ni for the gas in the center of NGC4636 a nearby low mass group