Spectacularly rapid and regular X-ray eruptions observed from an active galaxy





How to measure the mass of clusters? Longair sec 4.4



Assumptions

• Gas is in equilibrium- what are the relevant time scales

Another good review article is Astron Astrophys Rev (2010) 18:127–196 X-ray spectroscopy of galaxy clusters: studying astrophysical processes in the largest celestial laboratories Hans Böhringer and Norbert Werner

Relevant Time Scales- see Longair pg 301

 $\tau(1,2) = \frac{3m_1\sqrt{2\pi}(kT)^{3/2}}{8\pi\sqrt{m_2}n_2Z_1^2Z_2^2e^4\ln\Lambda}$ The equilibration timescales ٠ between protons and electrons is $t(p,e) \sim 2 \ge 10^8$ yr at an $\ln \Lambda = \ln(b_{\max} / b_{\min}) \approx 40$ 'average' location $\tau(e,e) \approx 3 \times 10^{5} \left(\frac{T}{10^{8} \text{ K}}\right)^{3/2} \left(\frac{n_{e}}{10^{-3} \text{ cm}^{-3}}\right)^{-1} \text{ yr}$ In collisional ionization ٠ equilibrium population of ions $\tau(p,p) = \sqrt{m_p / m_e} \tau(e,e) \approx 43 \tau(e,e)$ is directly related to temperature $\tau(p,e) = (m_p / m_e)\tau(e,e) \approx 1800\tau(e,e)$ 1 XXV XVII XIII xx х Ion fraction XVI on Fraction 0.1 for Fe vs electron 0.01 temperature 0.001 8 ġ ò 5 6 10^{7} 10^{6} Electron Temperature (K)

Mean Free Path for Collisions/ Energy

• Mean-free-path $\lambda_{e} \sim 20 \text{ kpc}$ < 1% of cluster size $\lambda_{p} \approx \lambda_{e} = \frac{3^{3/2} (kT)^{2}}{8\sqrt{\pi} n_{e} e^{4} \ln \Lambda}$ $\approx 23 \left(\frac{T}{10^{8} \text{ K}}\right)^{2} \left(\frac{n_{e}}{10^{-3} \text{ cm}^{-3}}\right)^{-1} \text{ kpc}$

Assumptions

- At T>3x10⁷ K the major form of energy emission is thermal bremmstrahlung continuum
- $\varepsilon \sim 3x10^{-27} T^{1/2} n^2 ergs/cm^3/sec$; emissivity of gas
- - how long does it take a parcel of gas to lose its energy?
- $\tau \sim nkT/\epsilon \sim 8.5 \times 10^{-10} yrs(n/10^{-3})^{-1} T_8^{-1/2}$
- At lower temperatures line emission is important

How Did I Know This??

- Why do we think that the emission is thermal bremmstrahlung?
 - X-ray spectra are consistent with model
 - X-ray 'image' is also consistent
 - Derived physical parameters 'make sense'
 - Other mechanisms 'do not work' (e.g. spectral form not consistent with black body, synchrotron from a power law: presence of x-ray spectral lines of identifiable energy argues for collisional process; ratio of line strengths (e.g. He to H-like) is a measure of temperature which agrees with the fit to the continuum)=precision measurement from Hitomi



• The physical conditions are consistent with the assumptions

Physical Conditions in the Gas

- the collision times for (ions and electrons) in the intracluster gas are much shorter than the time scales for heating or cooling , and the gas can be treated as a fluid. The time required for a sound wave in the intracluster gas to cross a cluster is given by
- $T_s \sim 6.6 \times 10^8 \text{yr} (T_{gas}/10^8)^{1/2} (D/Mpc)$
 - thus on timescale of Gyr system should be in pressure equilbrium "if nothing happens"
- (remember that for an ideal gas $v_{sound} = \sqrt{(\gamma P/\rho_g)}$ a sqrt (P is the pressure, ρ_g is the gas density, $\gamma = 5/3$ is the adiabatic index for a monoatomic ideal gas) α sqrt(T γ) independent of the density $\sim c_s \sim 1500(T_g/10^8 \text{ K})^{1/2} \text{ km/s}$ is the *sound speed* in the gas.

Why is Gas Hot

- To first order if the gas were cooler it would fall to the center of the potential well and heat up
- If it were hotter it would be a wind and gas would leave cluster
- Idea is that gas shocks as it 'falls into' the cluster potential well from the IGM
 - Is it 'merger' shocks (e.g. collapsed objects merging)
 - Or in fall (e.g. rain)

BOTH



How do Clusters Form- Mergers

• As time progresses more and more objects come together- merge



Figure 1. BOG merger tree. Symbols are colour-coded as a function of B - V colour and their area scales with the stellar mass. Only progenitors more massive than $10^{10} M_{\odot} h^{-1}$ are shown with symbols. Circles are used for galaxies that reside in the FOF group inhabited by the main branch. Triangles show galaxies that have not yet joined this FOF group.





NOT A MERGER

Merger

Galaxy Cluster Merger

Rubens Machado, Gastão Lima Neto IAG-USP 2012

Comparison of dark matter and x-ray cluster and group distribution every bound system visible in the numerical simulation is detected in the x-ray band - bright regions are massive clusters, dimmer regions groups,



Formation

- Galaxy clusters form through gravitational collapse, driven by dark matter (~80% of their total mass)
- In the hierarchical scenario more massive objects form at later times: clusters of galaxies are produced by the gravitational merger of smaller systems, such as groups and subclusters





Extreme Merger

- Bullet cluster (1E0657) Allen and Million
- The assumptions we have given are NOT satisfied



Hydrostatic Equilibrium Kaiser 19.2

• Equation of hydrostatic equil

∇P =- $\rho_g \nabla \phi(r)$

where $\phi(r)$ is the gravitational potential of the cluster (which is set by the distribution of matter)

P is the gas pressure

ρ_g is the gas density

However the fundamental assumption is that the total pressure IS gas pressure and that the contribution of turbulence, magnetic fields, cosmic rays etc is negligible

Use of X-rays to Determine Mass

- X-ray emission is due to the combination of thermal bremmstrahlung and line emission from hot gas
- The gas "should be" in equilibrium with the gravitational potential (otherwise flow out or in)
- density and potential are related by Poisson's equation

$$\nabla^2 \mathbf{\phi} = 4\pi\rho \mathbf{G}$$

• and combining this with the equation of hydrostaic equil

$\nabla \cdot (1/\rho \nabla P) = -\nabla^2 \phi$ $= -4\pi G\rho$

gives for for a spherically symmetric system (1/ρ_a) dP/dr=-dφ(r)/dr=GM(r)/r² With a little algebra and the definition of pressure - the total cluster mass (dark and baryonic) can be expressed as

$M(r)=-(kT_g(r)/\mu Gm_p)r (dlnT/dr+dln\rho_g/dr)$

k is Boltzmans const, μ is the mean mass of a particle and $m_{\rm H}$ is the mass of a hydrogen atom Every thing is observable

The temperature T_g from the spatially resolved spectrum

The density ρ_g from the knowledge that the emission is due to bremmstrahlung And the scale size, **r**, from the conversion of angles to distance

see Longair eq 4.2-4.6

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• density and potential are related by Poisson's equation

$\nabla^2 \mathbf{\phi} = 4\pi\rho G$

- and combining this with the equation of hydrostaic equil
- $\nabla \cdot ((1/\rho) \nabla P) = -\nabla^2 \phi$ =-4 $\pi G \rho$ (eq 4.3 in Longair)



Hydrodynamics--see Longair ch 4.2

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \text{ mass conservation (continuity)}$$

$$\rho \frac{D v}{D t} + \nabla P + \rho \nabla \phi = 0 \text{ momentum conservation (Euler)}$$

$$\rho T \frac{D s}{D t} = H - L \text{ entropy (heating & cooling)}$$

$$P = \frac{\rho k T}{\mu m_p} \text{ equation of state}$$

Add viscosity, thermal conduction, ... Add magnetic fields (MHD) and cosmic rays Gravitational potential ϕ from DM, gas, galaxies

Deriving the Mass from X-ray Spectra Ch 4.4 Longair

For spherical symmetry this reduces to $(1/\rho_g) dP/dr=-d\phi(r)/dr=GM(r)/r^2$

With a little algebra and the definition of pressure - the total cluster mass can be expressed as (eqs 4.17-4.19 in Longair)

$GM(r) = kT_g(r)/\mu Gm_p)r (dlnT/dlnr+dln\rho_g/dlnr)$

k is Boltzmans const, μ is the mean mass of a particle and $m_{\rm H}$ is the mass of a hydrogen atom

Every thing is observable

The temperature T_g from the spatially resolved spectrum

The density ρ_g from the knowledge that the emission is due to bremmstrahlung and the x-ray image

And the scale size, \mathbf{r} , from the conversion of angles to distance



Figure 6: Left:Combined EPIC/MOS1&2 image of A 1795 in the [0.3-10]keV energy band. The circles define the

1 Mpc=14' X-ray spectra of a Clusterelements are detected via emission lines

How to Obtain Gas Density

 De-project X-ray surface brightness profile I(R) to obtain gas density vs. radius, ρ(r)

$$I_{v}(b) = \int_{b^{2}}^{\infty} \frac{\varepsilon_{v}(r)dr^{2}}{\sqrt{r^{2} - b^{2}}}$$

$$\varepsilon_{v}(r) = -\frac{1}{\pi} \frac{d}{dr^{2}} \int_{r^{2}}^{\infty} \frac{I_{v}(b)db^{2}}{\sqrt{b^{2} - r^{2}}} = \Lambda_{v}[T(r)]n_{e}^{2}(r)$$

- Where Λ is the cooling function and n_e is the gas density (subtle difference between gas density and electron density because the gas is not pure hydrogen
- De-project X-ray spectra in annuli T(r)
- Pressure P = $\rho kT/(\mu m_p)$
- The mass in gas is $M_{gas}(< r) = 4\pi \int_0^r r^2 dr \ \varrho_{gas}(r)$

A geometrical interpretation of the Abel transform in two dimensions. An observer (I) looks along a line parallel to the x-axis a distance y above the origin. What the observer sees is the projection (i.e. the integral) of the circularly symmetric function f(r) along the line of sight. The function f(r) is represented in gray in this figure. The observer is assumed to be located infinitely far from the origin so that the limits of integration are $\pm \infty$



• I(R) is the projected luminosity surface brightness, j(r) is the 3-D luminosity density (circular images- if image is elliptical no general solution)

 $j(r)=-1/\pi \int_{R}^{\infty} dI/dR/sqrt(R^2-r^2)$ this is an <u>Abel integral</u> which has only a few analytic solutions Simple power law models I(R)=r^{- α} then j(r)=r^{- α -1}

Density Profile- Longair 4.22



generalized King profile with surface brightness $I(r)=I(0)(1+(r/r_c)^2)^{-5/2}$ gives a density law $\rho(r)=\rho(0)(1+(r/r_c)^2)^{-3/2}$ where $r_c=3\sigma/sqrt(4\pi G\rho_c)$

Sarazin sec 5

The gas distributions in clusters can be derived directly from observations of the X-ray surface brightness of the cluster, if the shape of the cluster is known and if the X-ray observations are sufficiently detailed and accurate. This method of analysis also leads to a method for determining cluster masses (Section 5.5.5). The X-ray surface brightness at a photon frequency ν and at a projected distance b from the center of a spherical cluster is

$$I_{\nu}(b) = \int_{b^2}^{\infty} \frac{\epsilon_{\nu}(r)dr^2}{\sqrt{r^2 - b^2}},$$
(5.80)

where ϵ_{ν} is the X-ray emissivity. This Abel integral can be inverted to give the emissivity as a function of radius,

$$\epsilon_{\nu} = -\frac{1}{2\pi r} \frac{d}{dr} \int_{r^2}^{\infty} \frac{I_{\nu}(b)db^2}{\sqrt{b^2 - r^2}}.$$
(5.81)

Density Profile

• a simple model(the β model) fits the *surface brightness* well outside the core

- $S(r)=S(0)(1/r/a)^2$) $-3\beta+1/2$ ph/cm²/sec/solid angle

• Is analytically invertible (inverse Abel transform) to the *density profile* $\rho(r)=\rho(0)(1+(r/a)^2)^{-3\beta/2}$

The conversion function from S(0) to $\rho(0)$ depends on the detector

The quantity 'a' is a scale factor- sometimes called the core radius

β is a free parameter

• The Abel transform, , is an integral transform used in the analysis of spherically symmetric or axially symmetric functions. The Abel transform of a function **f**(**r**) is given by:

$f(r)=1/p\int_r^{\infty} dF/dy \, dy/\sqrt{(y^2-r^2)}$

- In image analysis the reverse Abel transform is used to calculate the emission function given a projection (i.e. a scan or a photograph) of that emission function.
- In general the integral is not analytic

Surface Brightness Profiles



X-ray Mass Estimates equation of hydrostatic equilibrum

$$\frac{dP_{\rm gas}}{dr} = \frac{-G\mathfrak{M}_*(r)\rho_{\rm gas}}{r^2} \tag{3}$$

where P_{gas} is the gas pressure, ρ_{gas} is the density, G is the gravitational constant, and $\mathfrak{M}_{\bullet}(r)$ is the mass of M87 interior to the radius r.

$$P_{\rm gas} = \frac{\rho_{\rm gas} K T_{\rm gas}}{\mu \mathfrak{M}_{\rm H}} \tag{4}$$

where μ is the mean molecular weight (taken to be 0.6), and \mathfrak{M}_{H} is the mass of hydrogen atom.

$$\frac{KT_{\text{gas}}}{\mu\mathfrak{M}_{\text{H}}}\left(\frac{d\rho_{\text{gas}}}{\rho_{\text{gas}}} + \frac{dT_{\text{gas}}}{T_{\text{gas}}}\right) = \frac{-G\mathfrak{M}_{*}(r)}{r^{2}}dr, \quad (5)$$

which may be rewritten as:

$$-\frac{KT_{\text{gas}}}{G\mu\mathfrak{M}_{\text{H}}}\left(\frac{d\log\rho_{\text{gas}}}{d\log r} + \frac{d\log T_{\text{gas}}}{d\log r}\right)r = \mathfrak{M}_{*}(r) \quad (6)$$

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X-ray Mass Estimates

• use the equation of hydrostatic equilibirum

$$\frac{dP_{gas}}{dr} = \frac{-G\mathfrak{M}_{*}(r)\rho_{gas}}{r^{2}}$$
(3)

where $P_{\rm gas}$ is the gas pressure, $\rho_{\rm gas}$ is the density, G is the gravitational constant, and $\mathfrak{M}_{*}(r)$ is the mass of M87 interior to the radius r.

$$P_{\rm gas} = \frac{\rho_{\rm gas} K T_{\rm gas}}{\mu \mathfrak{M}_{\rm H}} \tag{4}$$

where μ is the mean molecular weight (taken to be 0.6), and \mathfrak{M}_{H} is the mass of hydrogen atom.

$$\frac{KT_{gas}}{\mu\mathfrak{M}_{H}}\left(\frac{d\rho_{gas}}{\rho_{gas}} + \frac{dT_{gas}}{T_{gas}}\right) = \frac{-G\mathfrak{M}_{\bullet}(r)}{r^{2}}dr, \qquad (5)$$

which may be rewritten as:

4

$$-\frac{KT_{gas}}{G\mu\mathfrak{M}_{H}}\left(\frac{d\log\rho_{gas}}{d\log r} + \frac{d\log T_{gas}}{d\log r}\right)r = \mathfrak{M}_{\bullet}(r) \quad (6)$$

Putting numbers in gives

$$M(r) = -3.71 \times 10^{13} M_{\odot} T(r) r \left(\frac{d \log \rho_g}{d \log r} + \frac{d \log T}{d \log r} \right)$$

ere T is in units of keV and r is in units of Mpc



Hitomi to the

Rescue

- Hitomi observations of the Perseus cluster measured both the mass motion and 'turbulence' of the gas (Hitomi Collaboration, 2016 Nature,535,117 and 2017a, arXiv:1711.00240)
- The width of a variety of lines is consistent with σ=148+/-6 km/sec
- Energy content due to small scale motions (turbulence) ~4% and mass motion is even less

 $V_{sound} \sim 1000 \text{km/sec } T_{4\text{kev}}^{1/2}$



Hitomi to the

Rescue

• Indicating the reason for the "quiescent" nature of the plasma is the lack of strong drivers of gas motions in the core despite the presence of a strong AGN and x-ray structure









X-ray Temperature Profiles

• Comparison of two satellites



Mass Profiles from Use of Hydrostatic Equilibrium

• Use temperature and density profiles +hydrostatic equilibrium to determine masses



- Scaled total density and gas density for a sample of clusters- yellow line is the NFW model
- Vikhlinin ApJ 640:691–709



Fio. 17.—Scaled density profiles. Total density profiles are plotted within the radial range covered by the temperature profile. Gas density profiles are extended to r_{den} (see Table 2). The thick yellow line shows the NFW model with c₅₀₀ = 3, a typical value for CDM halos in our mass range (§ 6.1; see Fig. 18).



Arnaud et al 2010

Checking that X-ray Properties Trace Mass





Surface mass density for 42 Rosat selected clusters from Sloan lensing analysis fitted with NFW profile

Comparison of cluster mass from lensing and x-ray hydrostatic equilibrium for A2390 and RXJ1340 (Allen et al 2001)

At the relative level of accuracy for smooth relaxed systems the x-ray and lensing mass estimators agree



Strong Lensing vs Weak Lensing



• A2261 Coe et al 2012



 Mass derived from different observational techniquesA2261 Coe et al. (Ap 757 19 2012)- x-ray mass (red curve)dynamical mass (green point)- gas masses (brown and yellow from x-ray and S-Z



4 low mass (~1014M) systems- Sasaki et al 2014

Cluster Temperature Structure

• 2-D cluster temperature maps Lagan'a, Durret & Lopes 1901.03851.pdf



Observed Temperature Profiles

- If the gas is in equilbrium with the potential (of the NFW form) it should be hotter in the center
- But in many clusters it is cooler in the center- additional physics





Left panel (from Burns et al 2010) shows the theoretical temperature profile if a NFW potential (in grey) compared to an set of actual cluster temperature profiles

Chemical Evolution of the

Universe

- A major area of astrophysical research is understanding when stars and galaxies formed and how the elements are produced
- With the exception of H and He (which are produced in the big bang) all the other elements (called metals in astrophysical jargon) are "cooked" in the centers of massive stars and supernova and then "ejected" by explosions or winds
- The gas in these explosions is moving very fast (1000 km/sec) and can easily escape a galaxy.
- Clusters are essentially giant "boxes" which can hold onto all their material

•Measurement of the amount and change of metals with time in clusters directly measures their production

In the hot gas elements such as silicon and iron have only 1 or 2 electrons
These ions produce strong H, He like xray emission lines.
The strengths of these lines is 'simply' related to the amount of silicon or iron in the cluster

