Today's Lecture

- First crack at accretion disks
- EHT
- Black holes

Oral Presentations

10 students have not presented or signed up yet...after today we have only 8 lectures; the math is obvious

If no one volunteers I will assign talks in reverse alphabetical order; e.g. **Teal** would be next, then **Jongwon, Sergio** etc. Aiming for 2 per lecture.

This will start April 16 and then April 18, April 23, April 30, May 2, May 7 and May 9 and the 'last class'

I will consider changing this if the next person in line agrees.

**Red** has given talk, **green** signed up
The winner is Friday May 10 (17 votes) next are Mon May 6 and Wed May 8 with 13 votes- 21/23 total votes.

Now we need to pick a time
I will send out another poll later today

- Ken K. will present 2014MNRAS.437.1698 X-ray emission from star-forming galaxies – III.Calibration of the LX-SFR relation up to redshift z ≈ 1.3 Mineo, S.et al on April 16
- Zhiyu Yin will be presenting.... 2014MNRAS.437.1698 X-ray emission April 1
- Yvette will be presenting on Tuesday on Done, Gierliński & Kubota 2007A&ARv..15....1D Sec 1,2 April16

- Next paper arXiv:1312.6698 Observational Evidence for Black Holes Ramesh Narayan, Jeffrey E. McClintock
Downwards to Black Holes! - Longair 13.11

- a neutron star has a maximum mass
- If this mass is exceeded on has a complete gravitational collapse to a black hole

- Basic anatomy of a black hole
- Observational discovery of black holes

How much energy is released by accretion onto a compact object?

- Consider matter in an accretion disk assume that...
  - The matter orbits in circular paths (will always be approximately true)
  - Centripetal acceleration is mainly due to gravity of central object (i.e., radial pressure forces are negligible... will be true if the disk is thin)

\[ \frac{v^2}{r} = \frac{GM}{r^2} \]

- Energy is...

\[ E = \frac{1}{2} v^2 - \frac{GM}{r} = -\frac{GM}{2r} \]
The total luminosity liberated by accreting a flow of matter is
- Longair 14.48

\[ L = \left[ 0 - \left( -\frac{GM}{2r_{in}} \right) \right] \dot{M} = \frac{GM\dot{M}}{2r_{in}} \]

- Initial energy (at infinity)
- Final energy
- Mass flow rate

- Total luminosity of disk depends on inner radius of dissipative part of accretion disk

- The matter falling in from infinity passes through a series of bound Keplerian orbits for which the kinetic energy is equal to half the gravitational potential energy.
- The matter dissipates half its potential energy in falling from infinity to radius \( r \) and this is the source of the luminosity of the disc.
- When the matter reaches the boundary layer, it has only liberated half its gravitational potential energy. If the matter is then brought to rest on the surface of the star, the rest of the gravitational potential energy can be dissipated.
- Thus, the boundary layer can be just as important a source of luminosity as the disc itself. Of course black holes do not have a surface so the energy 'disappears'
Thin accretion disks

Accretion disks form due to angular-momentum of incoming gas

Once in circular orbit, specific angular momentum (i.e., per unit mass) is

\[ J = vr = \sqrt{GMr} \]

So, gas must shed its angular momentum for it to actually accrete...

Releases gravitational potential energy in the process!

Matter goes in, angular momentum goes out!

The First Physical Disk Model- Longair 14.3.3

- The first physical model of a disk was developed by Shakura and Sunyaev in 1973
- They made a set of assumptions which have proved to be reasonable.
- The disk is optically thick
- The local emission should consist of a sum of quasi–blackbody spectra
Condition for Thin Disks 14.3.1 Longair

It is assumed that the mass of the disc is much less than the mass of the central star; $M_{\text{disk}} \ll M_{\text{star}}$.

Conditions for thin accretion discs

$$\frac{\partial p}{\partial z} = -\frac{GM_x \rho \sin \theta}{r^2}$$

$$\sin \theta = \frac{z}{r}$$

$$\frac{\partial p}{\partial z} \approx \frac{p}{H}, \text{ where } H \text{ is the thickness of the disc, } p=\text{pressure, } \rho = \text{density}$$

Thus the condition for hydrostatic support in the $z$-direction is

$$\frac{p}{H} = \frac{GM_x \rho}{r^3}.$$

$$H/r \approx \frac{c_s}{v_\phi}; c_s \text{ is the sound speed } \sim \frac{P}{\rho} \text{ so } H/r \sim 1/\text{Mach number}$$

the thin disc approximation is 'ok' if the rotation velocity of the disc is very much greater than the sound speed.

A Simple Disk- see Longair 14.48

C. Done IAC winter school

• The underlying physics of a Shakura-Sunyaev accretion disc (a very simple derivation just conserving energy -rather than the proper derivation which conserves energy and angular momentum).

• A mass accretion rate $\dot{M}$ spiraling inwards from $R$ to $R-dR$ liberates potential energy at a rate $dE/dt = L_{\text{pot}} = (GM \dot{M}/R^2) dR$.

• The virial theorem says that only half of this can be radiated, so $dL_{\text{rad}} = GM \dot{M} dR/(2R^2)$. 
• If this thermalizes to a blackbody (locally) then \(dL = (dA)xk_BT^4\) where \(k_B\) is the Stephan-Boltzman constant and area of the annulus 
\(dA = 2 \times 2\pi R x dR\) (where the factor 2 comes from the fact that there is a top and bottom face of the ring).

• Then the luminosity from the annulus \(dL_{\text{rad}} = GM\dot{M}dR/(2R^2) = 4\pi dRkBT^4\) or 
  
  \[kT^4(R) = (GM\dot{M}/8\pi R^3)\]

• This is only out by a factor \(3(1-(R_{in}/R)^{1/2})\) which comes from a full analysis including angular momentum

• Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

### Temperature Structure of Accretion disk
Longair 14.3.5

• Energy released by an element of mass in going from \(r+dr\) to \(r\) 
Gravitational potential energy is 
\(E_p = -GMm/2r\) so energy released is 
\(E_g = -GMmdr/r^2\). 

the luminosity of this annulus, for an accretion rate \(\dot{M}\), is 
\(dL \sim G M \dot{M} dr/r^2\).

• \(dE/dt = (3G\dot{M} M_*/4\pi r^3) [1 -(r_*/r)^{1/2}]\) Longair 14.47 see derivation on pages 456,457

• The energy release rate is the half of the gravitational potential at \(r=r_*\). This is because the other half of the gravitational potential energy is converted to the kinematic energy (Keplerian motion. This is exactly what expected from the Viral theorem
Temperature Structure of Accretion disk
Longair14.3.5

assuming the annulus radiates its energy as a blackbody

For a

- blackbody, \( L = \sigma AT^4 \). The area of the annulus is \( 2\pi r dr \), and since
- \( L = MM^r dr/r^2 \) we have
- \( T^4 \sim \frac{MM}{r^3} \), or
- \( T \sim \left(\frac{MM}{r^3}\right)^{1/4} \); e.g. \( T \sim r^{-3/4} ; T \sim \frac{M^{1/4}}{r^{1/4}} \)
- If one puts in the constants \( T \sim \left(\frac{3GM}{8\pi\sigma r^3}\right)^{1/4} \).

Total Spectrum- see Longair eqs 14.54-14.57

- If each annulus radiates like a black body and the temperature scales as \( T \sim r^{-3/4} \)
- The emissivity scales over a wide range of energies as \( I(\nu) \sim \nu^{-1/3} \)
- At lower frequencies the spectrum has a Raleigh-Jeans \( \nu^2 \) shape and at higher energies has an exponential cutoff corresponding to the maximum temperature \( e^{-\hbar\nu/kT_{\text{inner}}} \)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.
Total Spectrum

• If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$ (Longair 14.54)

• The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{1/3}$

• At lower frequencies the spectrum has a Raleigh–Jeans $\nu^2$ shape and at higher energies has an exponential cutoff corresponding to the maximum temperature $(e^{-\hbar \nu / kT_{inner}})$

• Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

If the disk 'cuts off' at some radius $r_{inner}$ then the temperature profile is $T(r) = 3GM/M_8\pi\sigma r^3[1 - (r_{inner}/r)^{1/2}]^{1/4}$ eq in 14.7.1.

In more friendly units

• $T = 2 \times 10^7 (M_{NS}/M_\odot)^{1/4}(M/M_\odot/yr)^{-1/4}(r_*/10^{14} cm)^{-3/4} K$

• Notice this is close to the simple estimate of a NS temperature we had before and this maximum temperature is in the x-ray band for NS and stellar mass BHs.

• Also note that the 'effective' temperature should track the accretion rate and thus the luminosity

• or normalizing the accretion rate to the Eddington limit $m$:
  
  $m = L_{Edd}/\eta \approx 1.5 \times 10^{17}(M/M_\odot)gm/s$

• $T = 3.5 \times 10^7 m^{1/4}(M/M_\odot)^{-1/4}(r/r_g)^{-3/4}$

• or in 'x-ray' units

• $T_{in} = (1.58 \text{ keV}) (M/1.4 M_\odot)^{1/4}(M/10^{18} \text{ g/s})^{-1/4}$ if $R_{in} = 6 GM_\odot/c^2$. 

The emission spectrum of an optically thick accretion disc. The exponential cut-off at high energies occurs at frequency $\nu = kT/f$, where $T_1$ is the temperature of the innermost layers of the thin accretion disc. At lower frequencies the spectrum tends towards a Rayleigh–Jeans spectrum $I \propto \nu^2$. 

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Varying Parameters

- Kristin Salgado

Do They Really Look Like That

- X-ray spectrum of accreting Neutron star at various intensity levels
- Right panel is $T(r_{in})$ vs flux - follows the $T^4$ law (iff the radius is constant
Fit to Real Data

The data is of very high signal to noise
Simple spectral form fits well over a factor of 20 in energy
Emitted energy peaks over broad range from 2-6 keV

Actual Neutron Star X-ray Spectra

• Low Mass x-ray binaries (NS with a 'weak' magnetic field) have a 2 component spectrum
  – The low energy component is well described by a multi-color disk black body spectrum
  – And the hotter temperature black body is related to the boundary layer
• However the observed temperatures disagree with simple theory due to three effects
  – General relativity
  – The 'non-black body' nature of the radiation
  – Reprocessing of the radiator of the central regions by the outer regions and then re-emission
summary

• X-ray binaries exhibit a wide range of behaviors, but much of the interesting physics/astrophysics is common to all

• Understanding of accretion disks, accretion flows, X-ray induced winds, compact object evolution are all in a very active state of research.

This is a vast field - here are some references for further reading

• Dippers: Smale et al. 1988 MNRAS 232 647
• Black hole transient lmxbs: Remillard and McClintock, 2006 ARAA 44, 49
• Color-color diagrams for atoll/Z sources : Hasinger and VanderKlis 1989
• Microquasar GRS 1915+105: Mirabel and Rodriguez 1995 PNAS 92 11390
• Iron line from Cyg X-1: Miller et al. 2003 Ap. J. 578, 348
• Accretion disk corona modeling: Jimenez-Garate et al. 2002 Ap. J. 558, 458
• ‘Accretion power in Astrophysics’ Frank, King and Raine
• Catalog of X-ray Binaries, Liu Van Paradijs and Lewin 2007 A&A 469, 807
• GRO J1655 chandra spectrum: Miller et al., 2006 Nature 441, 953
Gravitational potential of spherically symmetric mass $M$ of radius $R$

$$\Phi = -\frac{GM}{r} \quad (r > R)$$

Acceleration of gravity

$$g = -\nabla \Phi = -\frac{GM}{r^2} \hat{r}$$

Particles freely falling from $r \to \infty$ to $r$:

$$E_K = \frac{1}{2} v^2$$

(kinetic energy per unit mass)

Energy conservation:

$$E_K + \Phi = E = \text{cst.}$$

At $r$:

$$v^2 = \frac{2GM}{r}$$

(free-fall or escape speed)

Viral temperature $T_{\text{viral}} = GM/kr$; for a NS $M \sim 1.4M_{\odot}$, $R \sim 10$ km

$T \sim 10^{12}$K

(H. Spruit)

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**Downwards to Black Holes!**

- The maximum mass of a neutron star
- Complete gravitational collapse to a black hole
- Basic anatomy of a black hole
- Observational discovery of black holes
How Can We Observe Black Holes

- If a black hole is a 'place' where radiation cannot escape to infinity, how can they be observed?

- Dynamical effects on 'nearby' material

- “Shining” black holes - a black hole can be a place where accretion occurs and as we have seen the process of accretion around a compact object can produce huge amounts of energy and radiation - making the black hole 'visible'.

Maximum Mass (Cont)

- None of the objects thought to be NS have a mass >2.4$M_{\text{sun}}$ but objects exist in the Milky Way way with a mass up to 19$M_{\text{sun}}$

Vertical lines are density in units of nuclear matter
Masses of Compact Objects

- Masses of neutron stars cluster around 1.4M (1 is near 2 M)
- Other sources are galactic black holes

Black Holes

- What do you mean 'black holes'?
- We know of objects whose mass (derived from observations of the lines from the companion objects and Newton's (Einstein) law) which are larger than possible for a NS or white dwarf.
- They have other unusual properties (related to their x-ray spectrum and timing behavior)
- Big differences-no surface, no magnetic field, higher mass, stronger GR effects.
General properties of emission from black hole systems

• Emission usually variable on wide variety of timescales
  – Galactic black hole binaries: millisecond and up
  – AGN: minutes and up
  – Most rapid variability approaches light-crossing timescale limit of physical size of object ($\tau \sim R/c$)

• Significant emission over very broad spectral range (radio to hard X-ray or gamma-rays)-NS and WDs tend to have 'thermal' like spectra (relatively narrow in wavelength)

• Lack of a signature of a surface - not a pulsar, no boundary layer emission (no x-ray bursts), no 'after glow' from cooling