

Basics of Accretion

Is there a limit on accretion?

If the accreting material is exposed to the radiation it is producing it receives a force due to radiation pressure

The minimum radiation pressure is
(Flux/c) $\times \kappa$ (κ is the relevant cross section)

Or

$L\sigma_T/4\pi r^2 m_p c$ (σ_T is the Thompson cross section ($6.6 \times 10^{-25} \text{ cm}^2$) m_p is the mass of the proton)

The gravitational force on the proton is

$$GM_x/R^2$$

Equating the two gives the **Eddington limit**

$$L_{\text{Edd}} = 4\pi M_x G m_p c / \sigma_T = 1.3 \times 10^{38} M_{\text{sun}} \text{ erg/sec}$$

Frank, King & Raine, "Accretion Power in Astrophysics",

Accretion -Basic idea

- Viscosity/friction moves angular momentum outward
 - allowing matter to spiral inward
 - Accreting onto the compact object at center

gravitational potential energy is converted by *friction* to heat

Some fraction is radiated as light

Very efficient process Energy

$$\sim GM/R = 1.7 \times 10^{16} (R/10\text{km})^{-1} \text{ J/kg}$$

$$\sim 1/2 mc^2$$

Nuclear burning releases $\sim 7 \times 10^{14} \text{ J/kg}$

(0.4% of mc^2)

Gravitational potential of spherically symmetric mass M of radius R

$$\Phi = -\frac{GM}{r} \quad (r > R)$$

Acceleration of gravity

$$\mathbf{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{r}$$

Particles freely falling from $r \rightarrow \infty$ to r :

$$E_K = \frac{1}{2}v^2 \quad (\text{kinetic energy per unit mass})$$

Energy conservation: $E_K + \Phi = E = \text{cst.}$

$$\text{At } r : \quad v^2 = \frac{2GM}{r} \quad (\text{free-fall or escape speed})$$

Viral temperature $T_{\text{viral}} = GM/kr$; for a NS $M \sim 1.4M_{\text{sun}}$, $R \sim 10 \text{ km}$

$T \sim 10^{12} \text{ K}$

(H. Spruit)

The Known Galactic Black holes

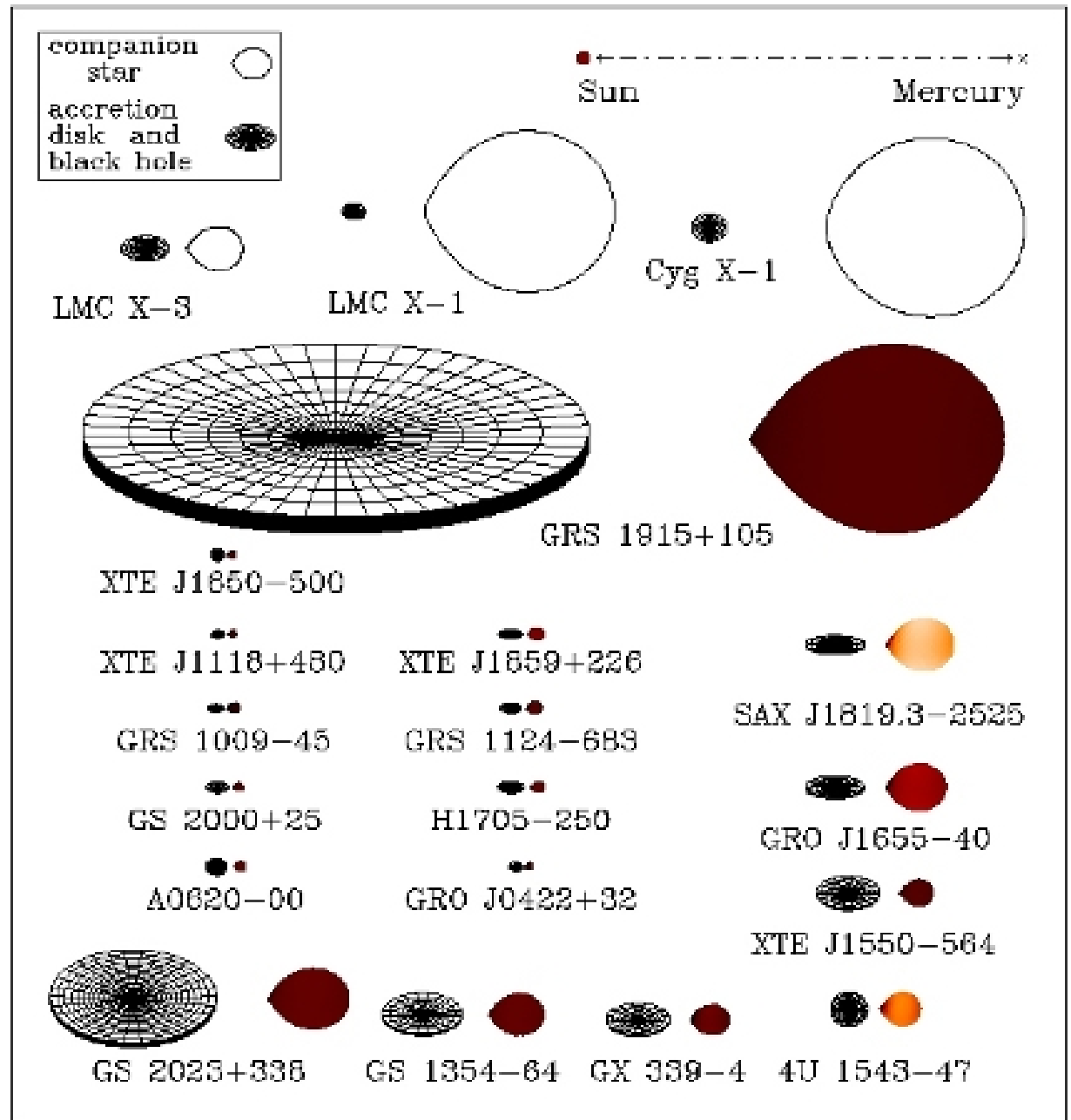
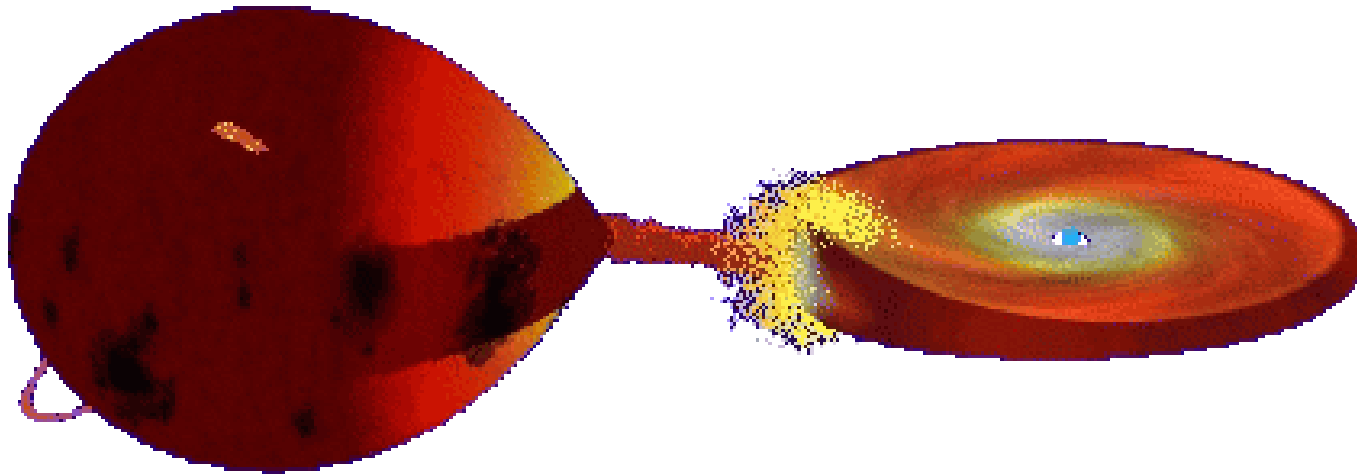


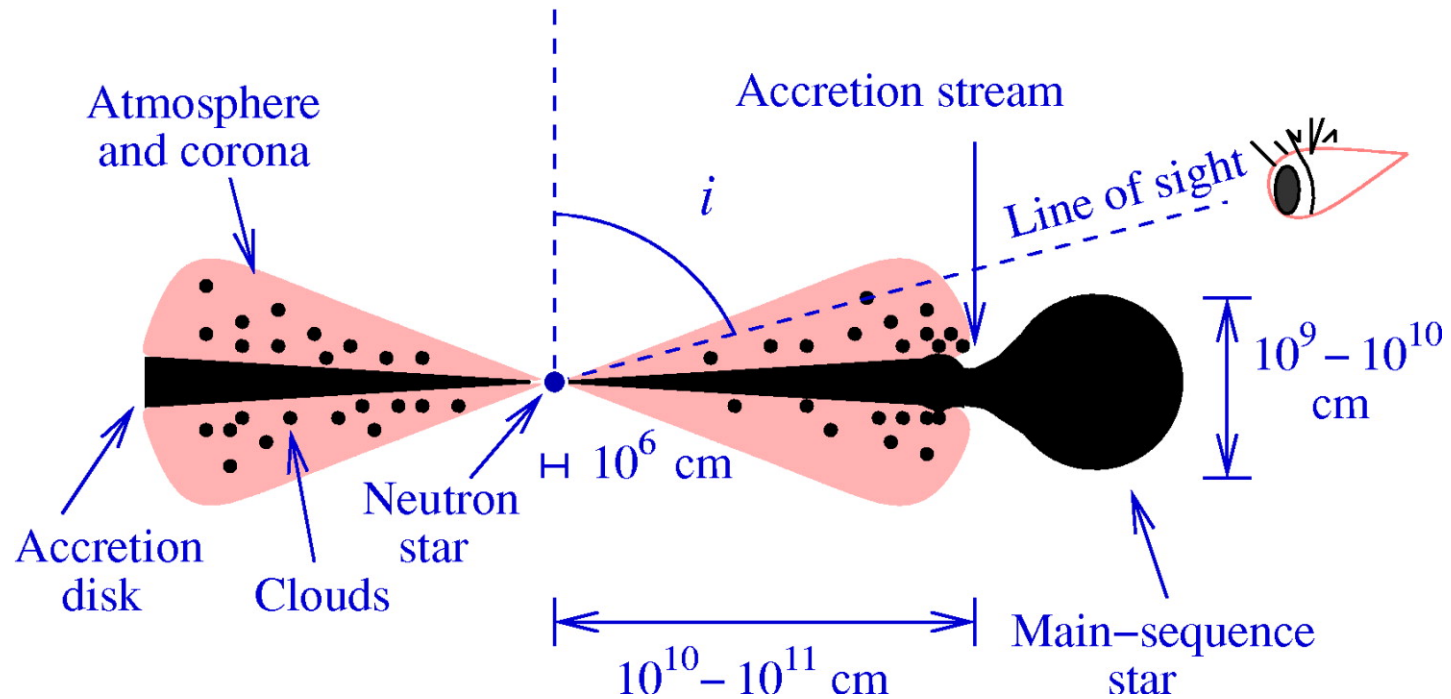
Figure by Jerome Arthur Orosz)

Accretion from a Dwarf Companion



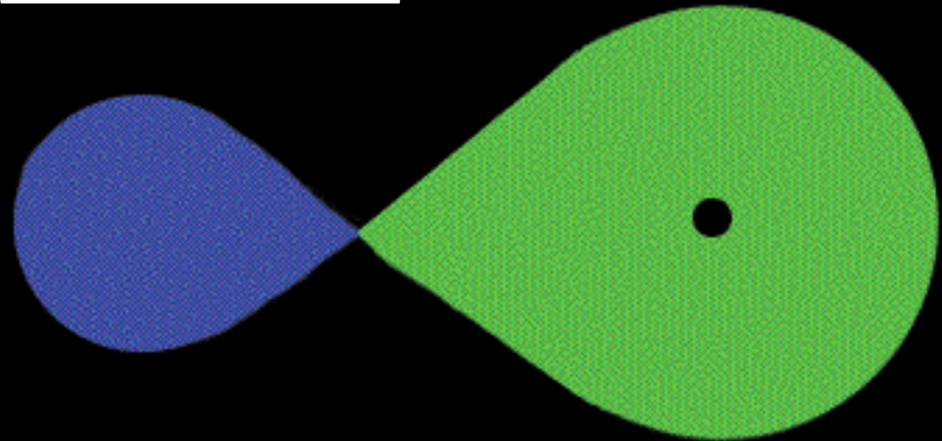
- http://physics.technion.ac.il/~astrogr/research/animation_cv_disc.gif

Geometry of heated accretion disk + coronal in LMXB



Jimenez-Garate et al. 2002

binary system's Roche lobes



The blue area is the Roche lobe for the companion.

The green area is the Roche lobe for the compact object (shown as a black dot).

Anything in the blue area is bound to the companion; anything in the green area is bound to the compact object. However, material found where the two lobes meet could find itself moving from one lobe to the other!

How much energy is released by accretion onto a black hole?

- Consider matter in the accretion disk assume that...
 - The matter orbits in circular paths (will always be approximately true)
 - Centripetal acceleration is mainly due to gravity of central object (i.e., radial pressure forces are negligible... will be true if the disk is thin)

- Energy is..

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

$$E = \frac{1}{2}v^2 - \frac{GM}{r} = -\frac{GM}{2r}$$

Pulsars

- The rate of change of the pulse period can
 - measure the orbital period of the source
 - The accreted angular momentum (e.g. the amount of material accreted)
 - $(dP/dt)/P \sim 3 \times 10^{-5} f (L/10_{37})^{6/7} \text{ yr}^{-1}$ (Ghost and Lamb 1978)

- So, the total luminosity liberated by accreting a flow of matter is

$$L = \left[0 - \left(-\frac{GM}{2r_{\text{in}}} \right) \right] \dot{M} = \frac{GM\dot{M}}{2r_{\text{in}}}$$

*Initial energy
(at infinity)*

Final energy

Mass flow rate

- Total luminosity of disk depends on inner radius of dissipative part of accretion disk

A Simple Disk

C. Done IAC winter school

- The underlying physics of a Shakura-Sunyaev accretion disc (a very simple derivation just conserving energy -rather than the proper derivation which conserves energy and angular momentum).
- A mass accretion rate \dot{M} spiraling inwards from R to $R-dR$ liberates potential energy at a rate $dE/dt = L_{pot} = (GM \dot{M}/R^2) \times dR$.
- The virial theorem says that only half of this can be radiated, so $dL_{rad} = GM \dot{M} dR/(2R^2)$.
- If this thermalises to a blackbody then $dL = (dA) \times kT^4$ where k is the Stephan-Boltzman constant and area of the annulus $dA = 2 \times 2 \pi R \times dR$ (where the factor 2 comes from the fact that there is a top and bottom face of the ring).
- Then the luminosity from the annulus $dL_{rad} = GM \dot{M} dR/(2R^2) = 4 \pi dR k dRT^4$ or $kT^4(R) = (GM \dot{M}/8 \pi R^3)$
- This is only out by a factor $3(1-(R_{in}/R)^{1/2})$ which comes from a full analysis including angular momentum
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

The First Physical Disk Model

- The first physical model of a disk was developed by Shakura and Sunyaev in 1973
- They made a set of reasonable assumptions which have proved to be reasonable.
- The disk is optically thick
- The local emission should consist of a sum of quasi-blackbody spectra

- Energy released by an element of mass in going from $r+dr$ to r

Gravitational potential energy is

$E_p = -GMm/2r$ so energy released is

$E_g = -GMm dr/r^2$.

the luminosity of this annulus, for an accretion rate \mathcal{M} , is

$dL \sim GM\mathcal{M} dr/r^2$.

assuming the annulus radiates its energy as a blackbody?

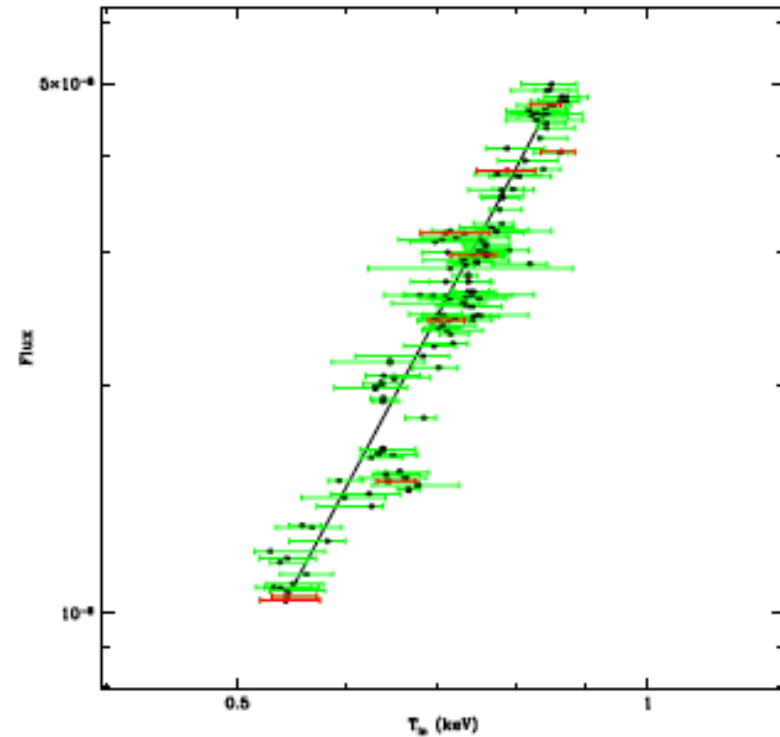
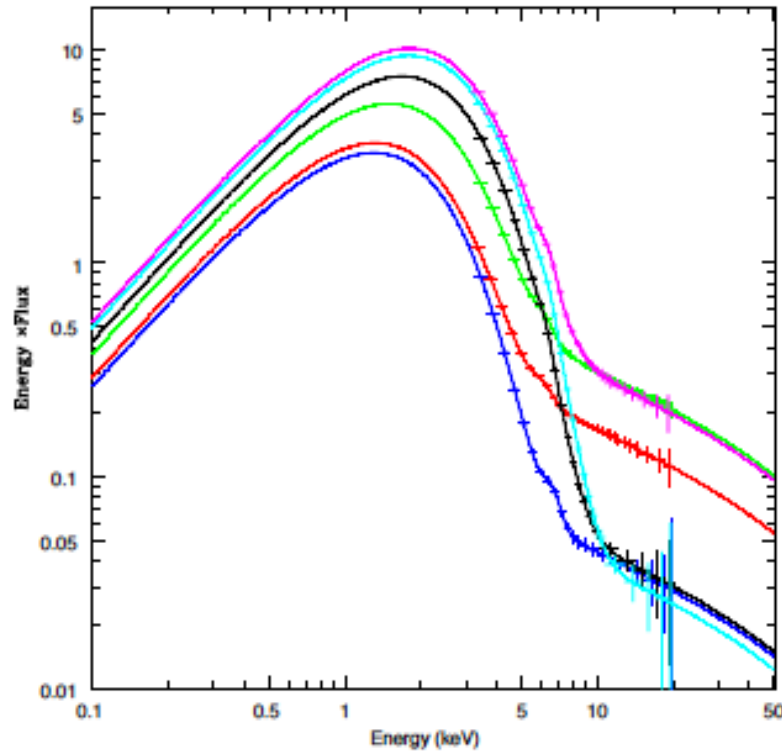
For a

- blackbody, $L = \sigma AT^4$. The area of the annulus is $2\pi r dr$, and since
- $L = M\mathcal{M} dr/r^2$ we have
- $T^4 \sim M\mathcal{M} r^{-3}$, or
- $T \sim (M\mathcal{M}/r^3)^{1/4}$

Total Spectrum

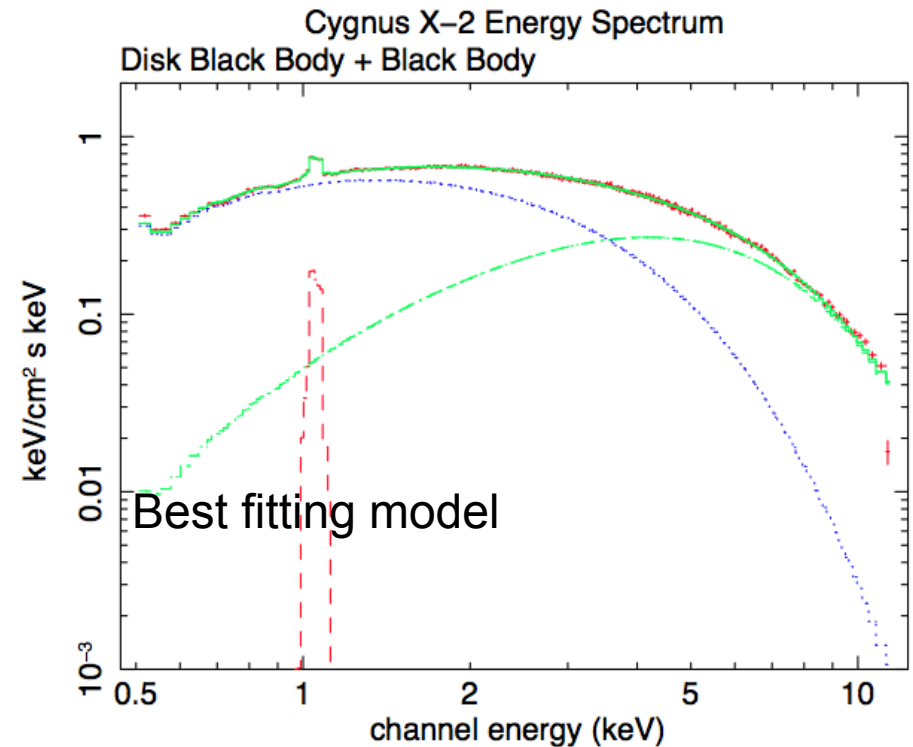
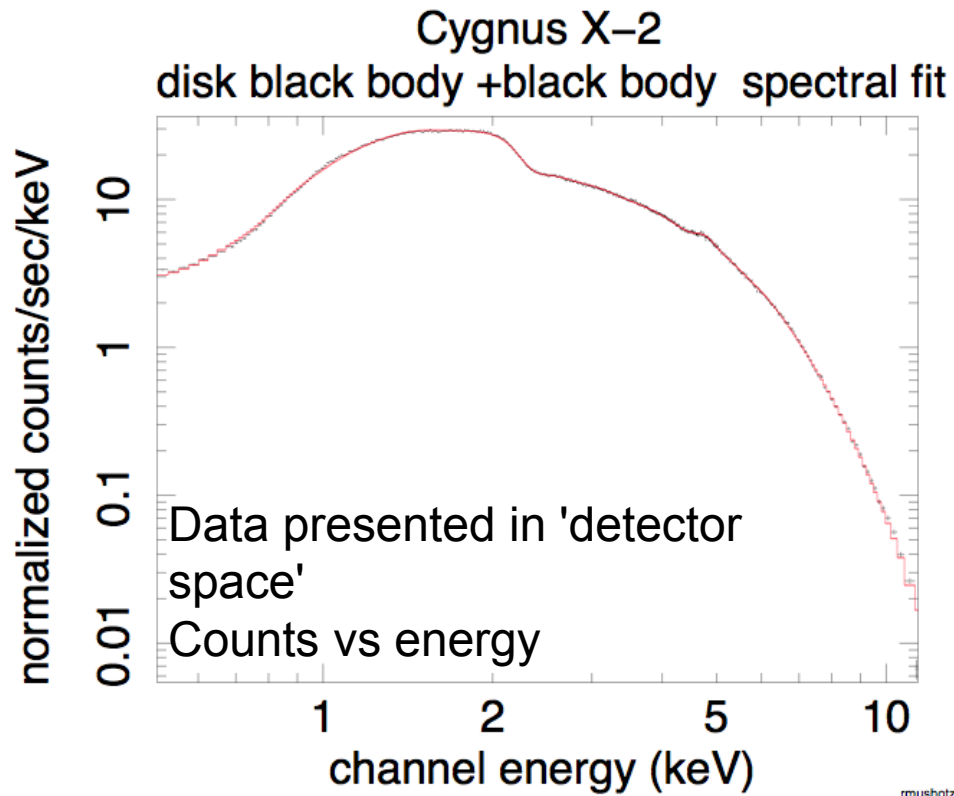
- If each annulus radiates like a black body and the temperature scales as $T \sim r^{-3/4}$
- The emissivity scales over a wide range of energies as $I(\nu) \sim \nu^{-1/3}$
- At lower frequencies the spectrum has a Rayleigh-Jeans ν^2 shape and at higher energies has an exponential cutoff corresponding to the maximum temperature
($e^{-h\nu/kT_{\text{inner}}}$)
- Thus the spectrum from a disc is a sum of blackbody components, with increasing temperature and luminosity emitted from a decreasing area as the radius decreases.

Do They Really Look Like That



- X-ray spectrum of accreting neutron star at various intensity levels
- Right panel is $T(r_{in})$ vs flux - follows the T^4 law

Fit to Real Data



The data is of very high signal to noise
Simple spectral form fits well over a factor of 20 in energy
Emitted energy peaks over broad range from 2-6 keV

Actual Neutron Star X-ray Spectra

- Low Mass x-ray binaries (NS with a 'weak' magnetic field) have a 2 component spectrum
 - The low energy component is well described by a multi-color disk black body spectrum
 - And the hotter temperature black body is related to the boundary layer
 - However the observed temperatures disagree with simple theory due to three effects
 - General relativity
 - The 'non-black body' nature of the radiation
 - Reprocessing of the radiation of the central regions by the outer regions and then re-emission

