Lecture 3: Cosmology of the Scientific Revolution

- More Tycho
- Kepler
- Galileo
- Newton

Jupiter and its 4 major (Galilean) moons

Large quadrant at Uraniborg ~2.6m in diameter - example of the advanced instrumentation Tycho used
**Tycho’s cosmological model**

- Tycho used parallax observations to explore heliocentric model:
  - If Earth moves, then parallax of stars *should* be observable
  - Tycho could not detect any significant parallax; he concluded Earth is stationary
  - In fact, stellar parallax is 100× too small for naked-eye observation to measure; largest values are about 1 arcsecond=(1/3600)°
- Tycho settled on combined geo/helio-centric model
  - Sun orbits Earth; all other planets orbit Sun

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**Stellar Parallax**

<table>
<thead>
<tr>
<th>As seen on the sky in</th>
<th>Star distances are measured in units of the distance from the Sun to the Earth, the Astronomical Unit. The nearer the star, the larger is the angle (called the parallax) between the January and July observations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>January</td>
</tr>
</tbody>
</table>

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**Parallax**

- There are two sources of an observer’s motion with respect to the distant stars
  - rotation of the earth (diurnal motion)
  - rotation of the earth around the sun (annual motion)

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How did Tycho try to estimate how far away the stars are
More on Parallax

Johannes Kepler (1571-1630)

- Born in Germany; originally planned to be ordained as Lutheran minister
- Convinced God made the Universe according to a mathematical plan; saw his Christian duty as understanding works God had created
- Was hired as Tycho Brahe’s assistant in Prague; his job was to make sense of Brahe’s extremely accurate observations of Mars
- Let to the publication of three laws of planetary motion (1601, 1609, 1619)

http://imagine.gsfc.nasa.gov/docs/features/movies/kepler.html
Kepler’s first law

Planets move around the Sun in ellipses, with the Sun at one focus.

\[ d_1 + d_2 = \text{constant} \]

http://www.youtube.com/watch?v=NG18fObqMV4&feature=related
Kepler’s first law

Drawing an ellipse is easy: use two tacks for the focii and a string.

Ellipses

\[(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1\]

- where \(a\) is the semimajor and \(b\) the semiminor axis.
Solar system orbits

Note the low eccentricities!

Eccentricities

<table>
<thead>
<tr>
<th>Planet</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.21</td>
</tr>
<tr>
<td>Venus</td>
<td>0.01</td>
</tr>
<tr>
<td>Earth</td>
<td>0.02</td>
</tr>
<tr>
<td>Mars</td>
<td>0.09</td>
</tr>
<tr>
<td>Jupiter</td>
<td>0.05</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.06</td>
</tr>
<tr>
<td>Uranus</td>
<td>0.05</td>
</tr>
<tr>
<td>Neptune</td>
<td>0.01</td>
</tr>
<tr>
<td>Pluto</td>
<td>0.25</td>
</tr>
<tr>
<td>Haumea</td>
<td>0.19</td>
</tr>
<tr>
<td>Makemake</td>
<td>0.16</td>
</tr>
<tr>
<td>Eris</td>
<td>0.44</td>
</tr>
</tbody>
</table>

‘minor planets’

Kepler’s second law

- The line connecting the Sun and a given planet sweeps out equal areas in equal times.
- Therefore, planets move faster when they are nearer the Sun.
- Consequence of angular momentum conservation.


http://home.cvc.org/science/kepler.gif
Kepler’s second law

http://imagine.gsfc.nasa.gov/Videos/education/kepler2a.avi

The line connecting the Sun and a given planet sweeps out equal areas (blue color) in equal times
Newton showed that for any central force, the area swept out per time will be a constant. (A central force is a force that is always pointed to a center, as the force of gravity on the earth is always pointed to the sun.)

Why?
- For a central force there is no sideways term (torque) and thus the angular momentum will be constant.
- Angular momentum is defined as $\mathbf{L} = \mathbf{r} \times \mathbf{mv}$ (symbols in bold are vectors and $\times$ is the cross product).
- Or $L = rmv$; $r$ is the radius of the orbit, $m$ is the mass of the body, and $v$ is the velocity (for a circular orbit the velocity is always perpendicular to the radius).

Area swept out in time $t$ is $1/2r(vt)$; $v$ is velocity, $t$ time (area of triangles).

Put the two equations together $A = 1/2L(L/m)t$ or in Kepler's language area per unit time $A/t = L/2m$ and since $(L/2m)$ is constant so is $(A/t)$.

This will make more sense after we discuss Newton's Laws.
Time-lapse movies of orbits
Stars around the Galactic center’s black hole

www.mpe.mpg.de/ir/GC/index.php

Kepler’s third law- Law of Periods

- The square of the period $P$ of the orbit is proportional to the cube of the semi-major axis $R$
- Period ($P$) = time it takes for planet to complete one orbit
- Semi-major axis ($R$) = half of the length of the “long” (i.e. major) axis of the ellipse.

$$P^2 = \text{constant} \times R^3$$

Newton determined the constant from his theory of gravity
constant=$4\pi^2/G(M+m)$
For the Earth, we know that:

- \( P = 1 \) year = \( 3.15 \times 10^7 \) seconds
- \( R = 149.6 \) million km (1 Astronomical Unit, A.U.)
- Kepler’s 3rd law says that, for other planets,

\[
\left( \frac{P}{\text{yr}} \right)^2 = \left( \frac{R}{\text{AU}} \right)^3
\]

- Let’s check that \((P/\text{yr})^2 = (R/\text{AU})^3\)
- Fitted line is Kepler’s law
- Boxes are real planet distances and periods

Kepler’s 3rd Law
Kepler's Law for the new planets discovered by Kepler satellite

More Kepler Satellite Discovered Planets - Over 1000 Now Known - see http://kepler.nasa.gov/Mission/discoveries/
How Kepler Satellite Finds Planets

- [http://kepler.nasa.gov/Mission/discoveries/kepler424b/](http://kepler.nasa.gov/Mission/discoveries/kepler424b/)

An imprecise version of Kepler’s laws

- Orbits are not circular
- A planet’s speed changes during its orbit
- There is a definite relationship between orbital period and the distance from the star
Kepler in perspective

- Based on Tycho Brahe’s accurate observations, Kepler calculated and thought his way to a major breakthrough in cosmology.

- Kepler’s three laws of planetary motion
  - Represented a very simple (and correct!) model of the solar system and can be generalized to other similar systems.
  - Swept away thousands of years of prejudice - and his own previous pet theory!
  - Were driven fundamentally by the data, including Tycho’s error estimates.

- Unlike previous models which quantified only what was observed already, Kepler’s Laws had predictive power, consistent with modern idea of a meaningful scientific theory (in fact it was the deviation of Mercury from Kepler’s law that was one of the observational tests of General Relativity).

A Discursion

- Kepler’s laws hold only if the force law is exactly $1/r^2$ - because the sun is not a perfect sphere and the other planets contribute a small amount to the total force the ‘apsides’ (the two points of closest and furthest distance of the orbit) precess.

- However General Relativity does not have an exact $1/r^2$ force law and thus the apsides precess even if one has a perfect sphere and no other planets.
Galileo Galilei
(1564-1642)

- Born in Pisa; worked as professor of mathematics
- Built one of the first telescopes in 1609
- Published “The Starry Messenger” with first telescopic discoveries in 1610
- Telescopic observations: the objects in the sky were not perfect
  - Saw craters and mountains on the Moon
  - Realized sunspots were on surface, not foreground and rotated with Sun
  - Identified four satellites of Jupiter (“Galilean moons”)
  - Saw rings of Saturn
  - Resolved the diffuse Milky Way into many faint stars
  - Observed phases of Venus including gibbous (between full and half) and full

Moon is not smooth, but covered by mountains and craters.

http://www.astromax.org
Galileo observed motion of the sunspots indicating that the Sun was rotating on an axis. The spots showed that, doctrine of an unchanging perfect substance in the heavens and the rotation of the Sun made it less strange that the Earth might rotate on an axis.

Both were new facts made possible by the telescope and thus were unknown to Aristotle and Ptolemy.

Galileo's telescope was so famous it was used for the international year of astronomy in 2009 the 400th anniversary of the astronomical telescope.

http://www.telescope1609.com/Galileo.htm
Letter from Galileo reporting the discovery of Jupiter’s moons...

Galilean Moons

demonstrated that a planet could have moons circling it
Galilean moons
(from Galileo spacecraft!)

Impact of Galileo’s observations

- Chipping away at Aristotelian point of view:
  - Features on Sun, Moon, Saturn indicated they are not perfect orbs
  - Faint stars resolved in Milky Way indicates stars at many distances -- not just single sphere
  - Moons of Jupiter showed that Earth was not sole center of motion
- Crucial experiment ruling out Ptolemaic model:
  - Possible phases of Venus in Ptolemaic model are only crescent or new -- but Galileo observed full phase
  - Observation supported Copernican (or Tycho’s) model (Venus on far side of Sun when full)
- As a result of his observations, Galileo became ardent supporter of Copernican viewpoint
- In 1632, published *Dialogue Concerning the Two Chief Systems of the World - Ptolemaic and Copernican*; the Inquisition banned the book; Galileo was found guilty of heresy in supporting Copernican view, and sentenced to house arrest
Phases of Venus: a test of the Heliocentric system

In the heliocentric model Venus can show certain phases impossible if the sun goes around the earth.

http://www.telescope1609.com/Galileo.htm

1/31/15
Galilean physics

- After 1633 trial, Galileo returned to work on physics of mechanics
- Published *Discourses and mathematical demonstrations concerning the two new sciences* (1642)
- Made experiments with inclined planes; concluded that distance \( d \) traveled under uniform acceleration \( a \) is \( d = at^2 \)
- Used “thought experiments” to conclude that all bodies, regardless of mass, fall at the same rate in a vacuum -- contrary to Aristotle
  
  - Now known as “equivalence principle”
- Realized full principle of inertia:
  - body at rest remains at rest;
  - body in motion remains in motion (force *not* required)
- Realized principle of relative motion (“Galilean invariance”):
  - If everything is moving together at constant velocity, there can be no apparent difference from case when everything is at rest.
  - Ball dropped from top of moving ship’s mast hits near bottom of mast, not behind on deck.

Isaac Newton (1643-1727)

- Attended Cambridge University, originally intending to study law, but reading Kepler, Galileo, Descartes
- Began to study mathematics in 1663
- While Cambridge was closed due to plague (1665-1667), Newton went home and
  - began to work out foundations of calculus
  - realized (contrary to Aristotle) that white light is not a single entity, but composed of many colors
  - began to formulate laws of motion and law of gravity
- Became professor of mathematics starting in 1669 (age 27!)
- Worked in optics, publishing “Opticks” (1704)
  - invented reflecting telescope
  - showed color spectrum from prism recombines into white light with a second prism
  - analyzed diffraction phenomenon

Father of modern physics and cosmology
In 1687, published *Philosophiae naturalis principia mathematica*, or “Principia”.

- Publication was prompted (and paid for) by Halley (of comet fame).
- Partly in response to claim by Hooke that he could prove gravity obeyed inverse-square law.
- Included proof that inverse square law produces ellipses.
- Generalized Sun’s gravity law to universe law of gravitation: all matter attracts all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Many other applications, including tides, precession, etc.
- Showed that Kepler’s laws follow from more fundamental laws.

The *Principia* is recognized as the greatest scientific book ever written!

- Retired from research in 1693, becoming active in politics and government.