Lecture 4: Newton's Laws

- Laws of motion
- Reference frames
- Law of Gravity
- Momentum and its conservation

This week: continue reading Chapter 3 of text

Newton's Laws & Galilean Relativity

- Newton's profound perspective
- Newton's Laws of Motion… 3 ways
- Newton's Law of Gravitation

Newton's profound perspective

- Newton formulated a **universal** theory of motion and gravity
  - Same laws of physics operate anywhere and anytime in the Universe-

    what we measure here on earth is applicable everywhere else in the universe
  - Tears down the wall that Aristotle built between Earthly laws and Heavenly laws

http://physics.learnhub.com/lesson/16618-newton-laws-of-motion

- *Newtons 3 Laws In The Original Latin*
- ♠ Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.
- [An object at rest will remain at rest unless acted upon by an external and unbalanced force . An object in motion will remain in motion unless acted upon by an external and unbalanced force]
- ♠ Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.
- [The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the same direction]
- ♠ Lex III: Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.
- [For every action there is an equal and opposite reaction]

I : Newton's laws of motion

- **Newton's first law (**The Law of Inertia*) **: If a body is not acted upon by any forces, then its velocity remains constant**
- Notes
  - Remember that velocity is a vector quantity (it has direction as well as magnitude)
  - This law sweeps away the idea that "being at rest" is a natural state… this was a major change of thinking
  - Newton's first law is a restatement of what Galileo had already described and Newton gave credit to Galileo.

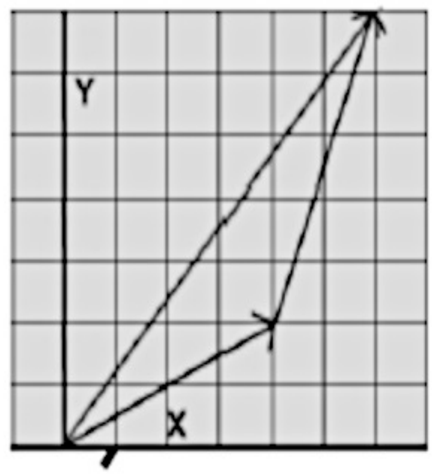
* Inertia is the resistance of any physical object to a change in its state of motion or rest, or the tendency of an object to resist any change in its motion

1st Law Continued

"Velocity" is a vector
  - Thus Newton's "constant velocity" implies both
    - constant speed and constant direction (and also includes the case of zero speed, or no motion).
- Galileo : based on his concept of inertia, it is impossible to tell the difference between a moving object and a stationary one without some outside reference to compare it against.
  - This ultimately led to Einstein's development of the theory of Special Relativity.

Adding Vectors

- Just a reminder that vectors have direction and length and adding them is not just adding the length



Newton's second law-

Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

- If a body of mass M is acted upon by a force F, then its acceleration a is given by F=Ma
  - Remember that both F and a are vectors, but mass is not
  - This law defines the "inertial mass" as the degree to which a body resists being accelerated by a force

F and a are vectors

- Newton's second law requires modification if the effects of special relativity are to be taken into account, because at high speeds the approximation that momentum is the product of rest mass and velocity is not accurate.

2nd Law

- *This is the most powerful of Newton's three Laws, - it allows quantitative calculations of dynamics: how do velocities change when forces are applied.*
- *Notice the fundamental difference between Newton's 2nd Law and the dynamics of Aristotle: according to Newton, a force causes only a change in velocity (an acceleration); it does not maintain the velocity as Aristotle held.*
- *Aristotle's view seems to be more in accord with common sense, but that is because of a failure to appreciate the role played by frictional forces.*

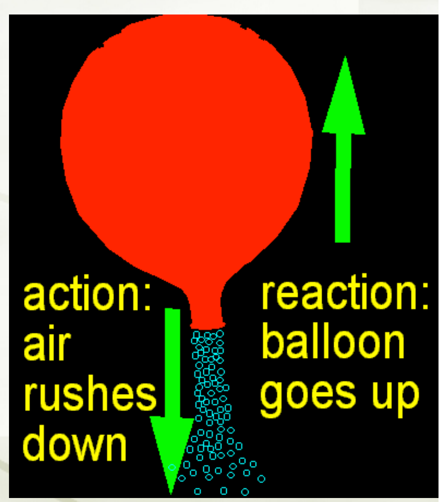
- *Once account is taken of **all** forces acting in a given situation it is the dynamics of Galileo and Newton, not of Aristotle, that are found to be in accord with the observations.*

see
http://en.wikipedia.org/wiki/File:Secondlaw.ogg
for a nice video on the 2nd law

- *http://csep10.phys.utk.edu/astr161/lect/history/newton3laws.html*

Third Law

- **If a body A exerts a force F on body B, then body B exerts a force -F on body A**
- Notes
  - This is the law of "equal and opposite reaction"
  - We will see later that this law is closely tied to conservation of momentum



Review of Goddard's pioneering work on rockets

- "Professor Goddard does not know the relation between action and reaction and the needs to have something better than a vacuum against, which to react. He seems to lack the basic knowledge ladled out daily in high schools."…

  -1921 New York Times editorial

Blast-Off!

Rockets push against ejecta not air

II : Momentum

- Definition : If an object of mass m is moving with velocity $\underline{V}$, its <u>momentum</u> $\underline{p}$ is given by $\underline{p}=m\underline{V}$
- The <u>total momentum</u> $\underline{p}_{tot}$ of a number of objects with masses m_1, m_2, … and velocities $\underline{V}_1$, $\underline{V}_2$, … is just the (vector) sum of the objects' separate momenta

$$\begin{aligned} \underline{p}_{\rm tot} &= m_1\underline{V}_1 + m_2\underline{V}_2 + m_3\underline{V}_3 + \ldots \\ &= \sum_{i=1}^{N} m_i \underline{V}_i \end{aligned}$$

Conservation of momentum

- **: The total momentum of a system of particles is constant if no external forces act on the system**

- For the mathematically inclined
- Proof for a two particle system…
  - Consider two particles with masses m_1 and m_2
  - They exert forces on each other, but there is no force being applied to the pair as a whole
  - At some instant in time, they have velocities $\underline{V}_1$ and $\underline{V}_2$
  - So momentum is $\underline{p}=m_1\underline{V}_1+m_2\underline{V}_2$

Proof… continued

- Consider some instant in time Δt later… individual velocities will have changed due to forces that particles exerted on each other… let new velocities be $\underline{V}_1$' and $\underline{V}_2$'
- Difference between new and old momentum is

$$
\begin{aligned}
\Delta \underline{p} &= \underline{p}_{\text{new}} - \underline{p}_{\text{old}} \\
&= m_1(\underline{V}_1' - \underline{V}_1) + m_2(\underline{V}_2' - \underline{V}_2) \\
&= m_1 \Delta \underline{V}_1 + m_2 \Delta \underline{V}_2 \\
&= \left(m_1 \frac{\Delta \underline{V}_1}{\Delta t} + m_2 \frac{\Delta \underline{V}_2}{\Delta t} \right) \Delta t \\
&= (\underline{F}_1 + \underline{F}_2) \Delta t \\
&= (\underline{F}_1 - \underline{F}_1) \Delta t \\
&= 0
\end{aligned}
$$

Newton's third law used here!

Conservation of Momentum

- Conservation of momentum *is more fundamental* than Newton's Law of Inertia
- Conservation of momentum is critical to ideas like special and general relativity as well as a fundamental principle in quantum mechanics*
- It is equivalent to the concept that physical laws do not depend on position and is a mathematical consequence of the homogeneity (shift symmetry) of space

*Uncertainty in position in space is related to the uncertainty in momentum by the Heisenberg uncertainty principled ΔxΔp~h, where h is Plancks constant.

In quantum mechanics, the Heisenberg uncertainty principle defines limits on how accurately the momentum and position of a single observable system can be known at once. In quantum mechanics, position and momentum are strongly connected in a fundamental way.

Δ is 'a change in'

Newton's second law- Again

- another way of saying the 2nd law is that force = rate of change of momentum
- *p*=momentum=m*v*
- *F*= d*p*/dt=m(d*v*/dt)=m*a*
- derivative dp/dt ~ change in momentum/ unit change in time or "Δp/Δt"

Newton's Laws in terms of Momentum

- We now see that Newton's laws can be rephrased entirely in terms of momentum…
  - **Second law**… the rate of change of momentum of a body is equal to the force applied to that body
  - **First law** is special case of the Second law… the momentum of a body is unchanged if there are no forces acting on body
  - **Third law**… the momentum of an isolated system of objects is conserved

III : Symmetries and frames of reference

- The idea of symmetry is very important in modern advanced physics! Let's have a glimpse of symmetry in action…
- Consider…
  - Two **equal**, connected masses M at rest.
  - At some time, they are suddenly pushed apart by a spring
  - They must fly apart with the same speed in opposite directions (*what else could possibly happen… why would one mass "decide" to move faster*?)

An illustration of Newton's laws

- We can see that aspects of Newton's laws arise from more fundamental considerations.
- Consider two equal masses M at rest. Initial momentum is p = 0. Masses are suddenly pushed apart by a spring… and will move apart with the same speed *V in opposite directions* (by symmetry of space!). Total momentum is p = MV-MV = 0. <u>Total</u> momentum is unchanged.

Before: $v_A = v_B = 0 \Rightarrow p_{tot} = 0$

After: $v_A = -V$, $v_B = V \Rightarrow$
$p_{tot} = Mv_A + M v_B = -MV + MV = 0$

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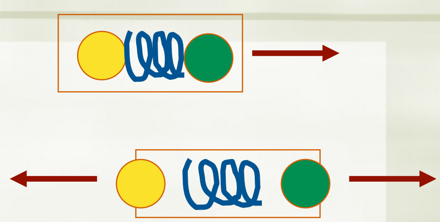
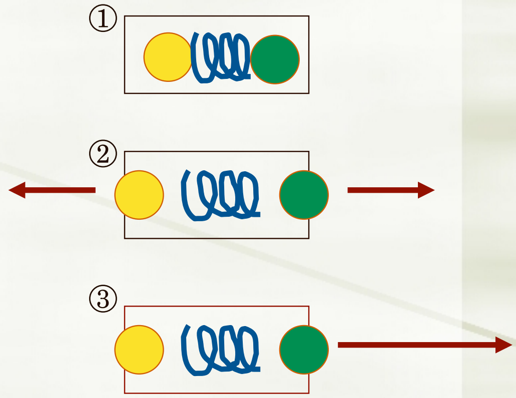
- Same situation, but masses are now both initially moving at velocity *V*. Initial momentum is $p_{tot} = 2MV$.
- Can turn into the previous situation by "moving along with them at velocity V".



1. Change of perspective [subtract V from all velocities] brings masses to rest…
2. Do same problem as before…
3. Change back to original perspective [add V to all velocities] …
4. Final velocity of one ball is *2V*; final velocity of other ball is *0*. Final total momentum is $p_{tot} = 2MV$. No change in total momentum.

① ② ③



- To re-state this , the two connected masses are initially moving at velocity V. Let's turn this into the above situation by "moving along with the masses at velocity V"
  - Change perspective to bring masses to rest…
  - Do same problem as before… find that momentum before = momentum after
  - Change back to the original perspective…
- You have "changed your frame of reference".
  - The "velocity addition" rule is called a **Galilean transformation.**
  - We assume that, after changing our reference frame and using a Galilean transformation, the laws of physics are the same. This is called **Galilean Relativity**.

Galilean Relativity

- Consider two **frames of reference** that differ by some uniform velocity difference (**not** considering accelerated frames of reference)
- The simple "velocity addition rule" is known as a **Galilean transformation**
- The statement that the laws of physics are the same in these two frames of reference (related by a Galilean transformation) is called the **Principle of Galilean Relativity.**

- How do Newton's laws fit into **this** picture?
  - N1 comes directly from Galilean Relativity (there is no difference between a state of rest and a state of motion)
  - N2 and N3 are exactly what's needed to make sure that momentum is conserved and so is related to the symmetry of space
  - ***So… Newton's laws are related to the symmetry of space and the way that different frames of reference relate to each other***.

Force and acceleration

- Forces between two bodies are equal in magnitude, but the observed reaction --the acceleration -- depends on mass ($F = ma$)
- If a bowling ball and a ping-pong ball are pushed apart by a spring, the bowling ball will move very little and the ping-pong ball will move a lot

- Forces in a collision are equal in magnitude, too

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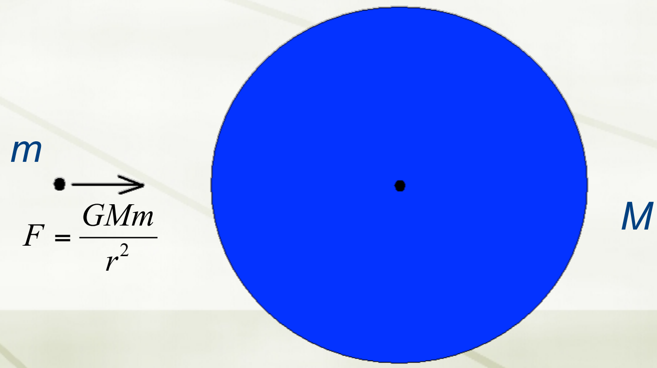
IV: NEWTON'S LAW OF UNIVERSAL GRAVITATION

<u>Newton's law of Gravitation:</u> A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by

$$F = \frac{Gm_1m_2}{r^2}$$

- "G" is called the Gravitational constant (G = 6.67×10^{-11} N m^2 kg^{-2} in mks units)- <u>*concept of a Universal Constant*</u>
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings.

- Newton's Law of Gravitation defines the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass *M* (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass *M* at the sphere's center.



Inertial and gravitational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a$$ m_I=inertial mass

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2}$$ m_G=gravitational mass

So, acceleration due to gravity is:

$$a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) varies, the rate at which objects fall in a gravitational field will vary…

At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Equivalence of inertial and gravitational mass

- *Experimentally*, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)
- So, m_I/m_G must be the *same* for all bodies
- And we can choose the constant "G" such that $m_I = m_G$, and $a = GM/r^2$

This is the **weak equivalence principle**: gravity is equivalent to (indistinguishable from) any other acceleration.

Next time…

- More Newton, including gravity & orbits
- Age of the Earth
- Reference frames & fictitious forces

- **HW #1 due on Thursday!**
- ***Finish Chapter 3 of text***