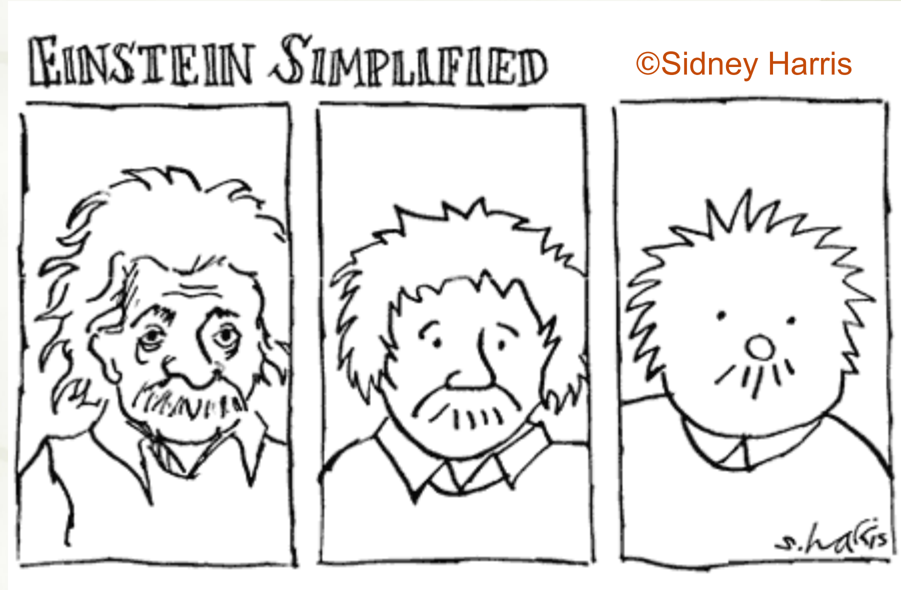
Lecture 7: Special Relativity I

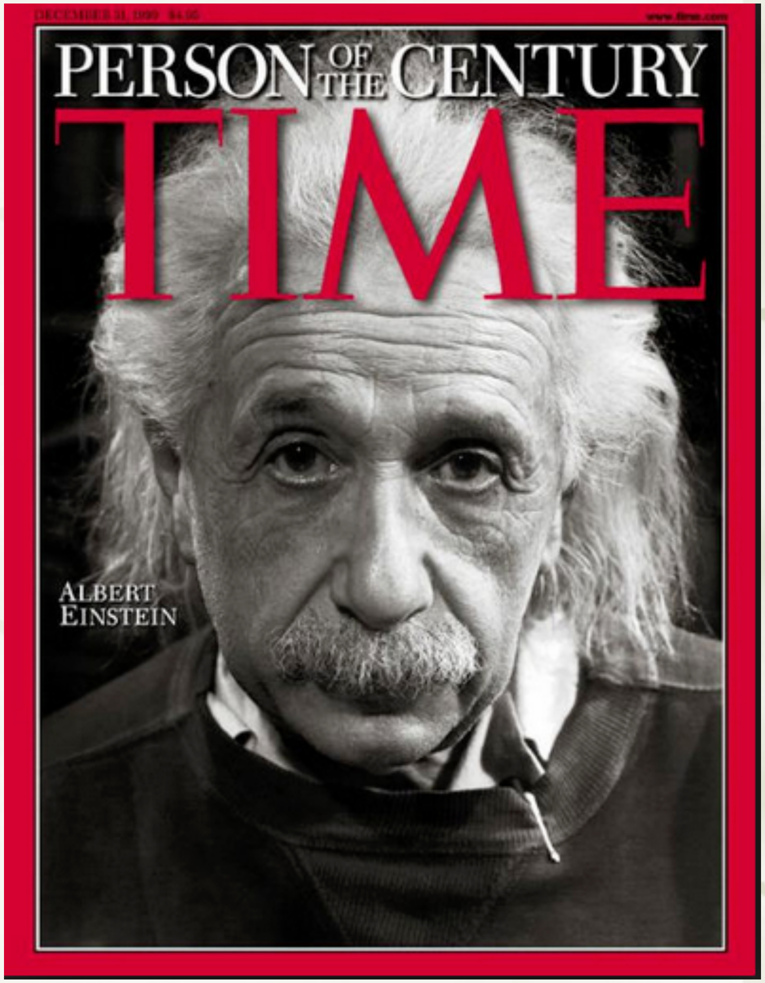
- Einstein's postulates
- Time dilation
- Length contraction
- New velocity addition law



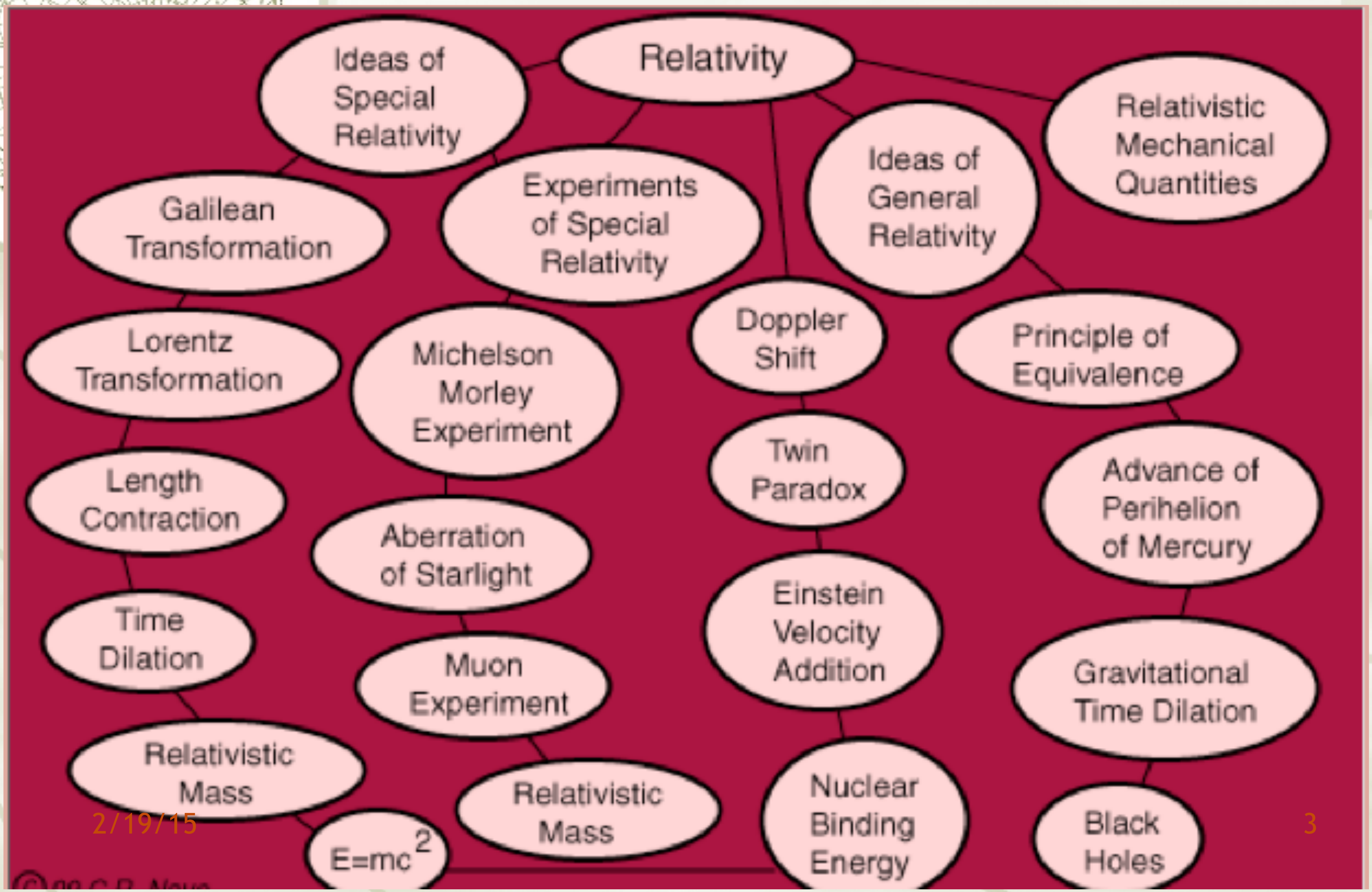
Please read Chapter 7 of the text

Albert Einstein

- Over a 15 year period (1905-1920) came the most explosive ideas of the century. They were catalysts that set in motion a reappraisal of every premise and postulate of modern natural science, a physical revolution whose end is far from sight.
- His ideas, like Newton's and Darwin's, reverberated beyond science, influencing modern culture from painting to poetry.
- Read more: http://www.time.com/time/magazine/article/0,9171,993017,00.html#ixzz1YQbS5kcK



Relativity- Big Picture



No Aether

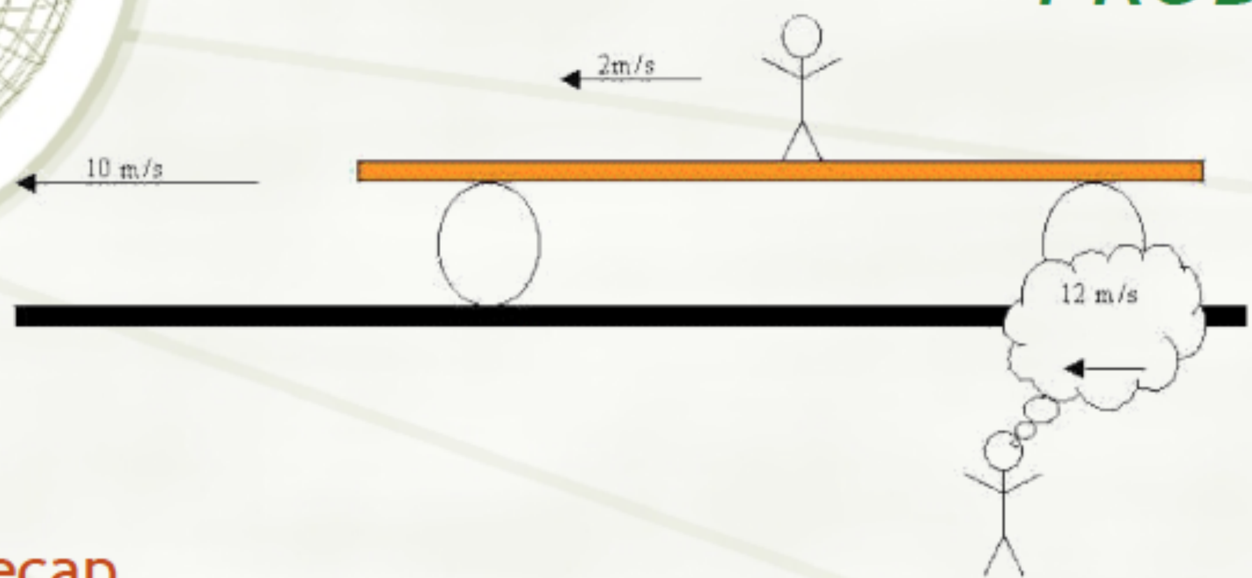
- Ernst Mach in 1883 argued that absolute time and space are meaningless and only relative motion is a useful concept

Einstein enters the picture…

- Albert Einstein (1879-1955)
  - Three papers in 1905: Brownian Motion, Photoelectric Effect (showing that light is quantized in energy), Special Theory of Relativity.
  - Didn’t 'like' idea of a luminiferous ether
  - Knew that Maxwell’s equations were invariant under “*Lorentz* transformation” of space and time a transform is the formula for the conversion of coordinates and times of events in different frames.
- But Newton's Laws are invariant under a *Galilean* transform
- Problem: if Maxwell's laws are 'correct' Newton's are not ?

its a pretty complex argument see http://en.wikipedia.org/wiki Luminiferous_aether#Einstein.27s_views_on_the_aether

I: RECAP-THE SPEED OF LIGHT PROBLEM



- Recap
  - "Relativity" tells us how to relate measurements in different frames of reference
  - Galilean relativity
    - Simple velocity addition law : $v_{total}=v_{run}+v_{train}$
    - Does not seem to work for Maxwell's equations

Problems with the Aether

- Any aether would need to be

  massless, incompressible, entirely transparent, continuous, devoid of viscosity and nearly infinitely rigid and thus not like any known substance.
- In 1905 Albert Einstein realized that Maxwell's equations did not require an aether.
- On the basis of Maxwell's equations he showed that the Lorentz Transformation was sufficient to explain that length contraction occurs and clocks appear to go slow provided that the old Galilean concept of how velocities add together was abandoned.

Einstein's remarkable achievement

- He was the first to propose that Galilean relativity might only be an approximation to reality.
- Using the Lorentz Transformation instead Einstein found that these equations only contain relationships between space and time without any references to the properties of an aether.
- The possibility that phenomena such as length contraction could be due to the physical effects of spacetime geometry rather than the increase or decrease of forces between objects was as unexpected for physicists in 1908 as it is for the modern school student

paraphrase of http://en.wikibooks.org/wiki/Special_Relativity/Introduction

Invariant Under a Transform???

- Invariant :Example: the area of a triangle under a Galilean transform

if $x'=x+\Delta x, y'=y+\Delta y$:the area of the triangle remains the same

- Not invariant: a sum of numbers when you add 1 to each of them; e.g the transform $x'=x+1$ changes the total sum BUT the ordering (cardinal number e.g. first, second, third) is the invariant
- Angles and ratios of distances are invariant under scalings,rotations, translations and reflections. These transformations produce similar shapes . All circles can be transformed into each other and the ratio of the circumference to the diameter is invariant and equal to π

Einstein enters the picture…

- Albert Einstein (1879-1955)
  - How to resolve conflict between mechanics and electromagnetism?
    - 'Throw away' the idea of Galilean Relativity for mechanics!
    - Galilean transformation between frames does not hold: velocities do not simply add/subtract (although the effects are small when the speeds are much less than the speed of light).
  - Came up with the two "Postulates of Relativity"

Special Relativity is a theory of exceptional elegance, crafted from simple postulates about the constancy of physical laws and of the speed of light

Its fundamental - the laws of physics and the constancy of the speed of light are now understood in terms of the most basic symmetries in space and time.

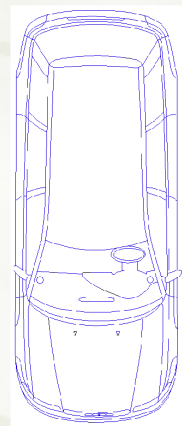
I: EINSTEIN'S POSTULATES OF RELATIVITY

- **Postulate 1** – The laws of nature are the same in all *inertial* frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all *inertial* frames of reference.
  - The second postulate is necessary to allow Maxwell's equations to follow from Postulate 1

- Let's start to think about the consequences of these postulates.
- We will perform "thought experiments" (Gedankenexperimenten) to think of what observers moving at different speeds will think
- For now, we will ignore effect of gravity – we suppose we are performing these experiments in the middle of deep space (or in free fall)

What if the speed of light weren't the same in all inertial frames?

Collision or not? If the speed of light were not the same in all inertial frames, you would see one car reach the collision point earlier than the other. But there either is or isn't a collision!



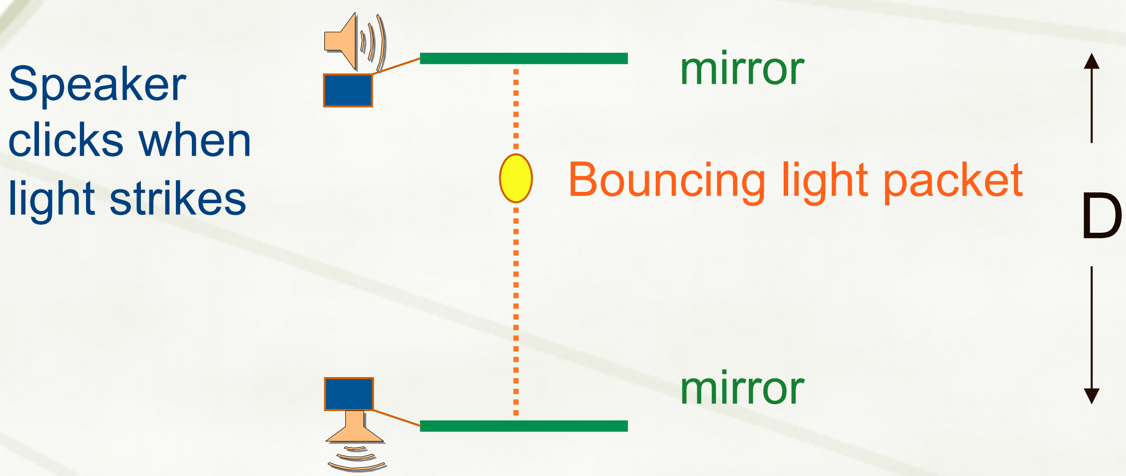
$$\Delta\text{time} = \frac{\text{distance}}{\text{speed}}$$

c

100 km/hr

II: TIME DILATION

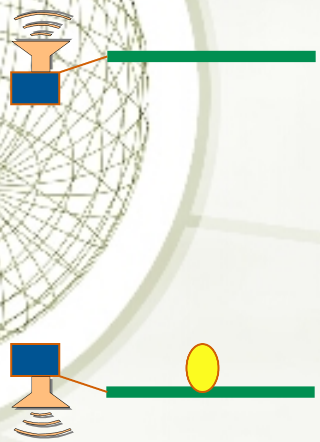
- Imagine building a clock using mirrors and a light beam.



- One "tick" of the clock is the time it takes for light to travel from one mirror to the other mirror.

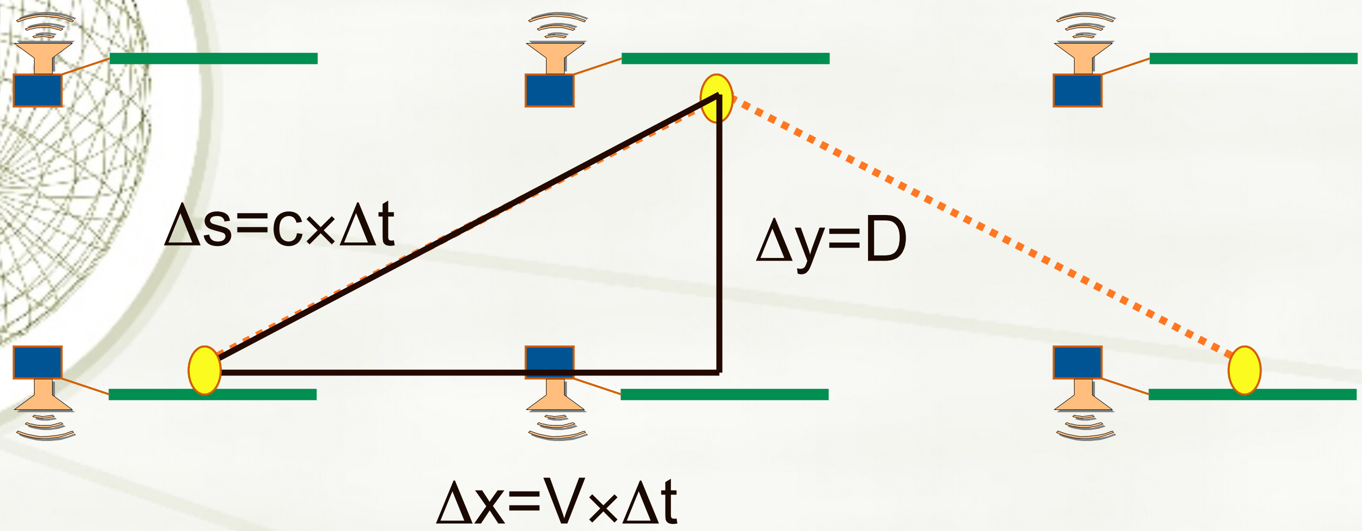
$$\Delta T = \frac{D}{c}$$

Moving clock



- Now suppose we put the same “clock” on a spaceship that is cruising (at constant velocity, V) past us.
- How long will it take the clock to “tick” when we observe it in the moving spacecraft? Use Einstein’s postulates…
- Total distance travelled by light beam is $\Delta s = c \times \Delta t$
- Therefore time $\Delta t = \Delta s / c$
- By Pythagorean theorem, $\Delta s = c\Delta t = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V\Delta t)^2 + D^2}$
- Can solve to obtain $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- Clock appears to run more slowly!!

Moving clock



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- Can solve to obtain $\Delta t = [\,(D/c)/(1-V^2/c^2)^{1/2}\,] > D/c$
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Now change the point of view…

- For ground-based observer, clock on spaceship takes longer to “tick” than it would if it were on the ground
- But, suppose there’s an astronaut in the spacecraft
  - the inside of the spacecraft is also an inertial frame of reference – Einstein’s postulates apply…
  - So, the astronaut will measure a “tick” that lasts

$$\Delta T = \frac{D}{c}$$

  - This is just the same time as the “ground” observers measured for the clock their own rest frame
- So, different observers see the clock going at different speeds!

So time is not absolute!!

It depends on your point of view…

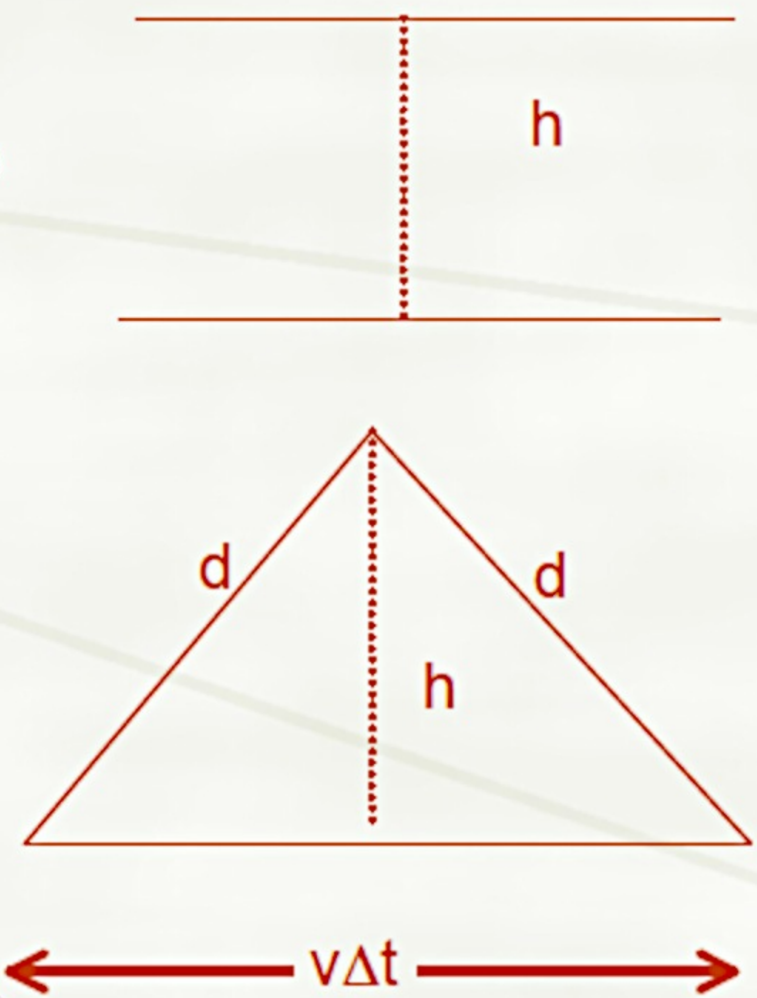
- Again following the text- lets do a different experiment and bounce light off the top of the train car.
- The passenger in the car sees the light go to the ceiling (a distance h) and bounce back- so the total time is t=2h/c
- to the stationary observer because the train is moving the light is observed to travel on an angled path which is longer (d) - since the speed of light is invariant it must take longer to follow this path ! - this is called time dilation
- A bit of math- Pythagorean theorm

$d^2=h^2+(1/2v\Delta t)^2$

in stationary observers frame $\Delta t_o=2d/c$

in train frame $\Delta t_t=2h/c$- replace d with $1/2c\Delta t_o$ and h with $1/2\ c\Delta t_t$ to get

$\Delta t_o=\Delta t_t/\mathrm{sqrt}(1-v^2/c^2)$- since v <<c Δt_o is longer than Δt_t



Time dilation

- This effect called **Time Dilation.**
- Clock always ticks most rapidly when measured by observer in its *own* rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

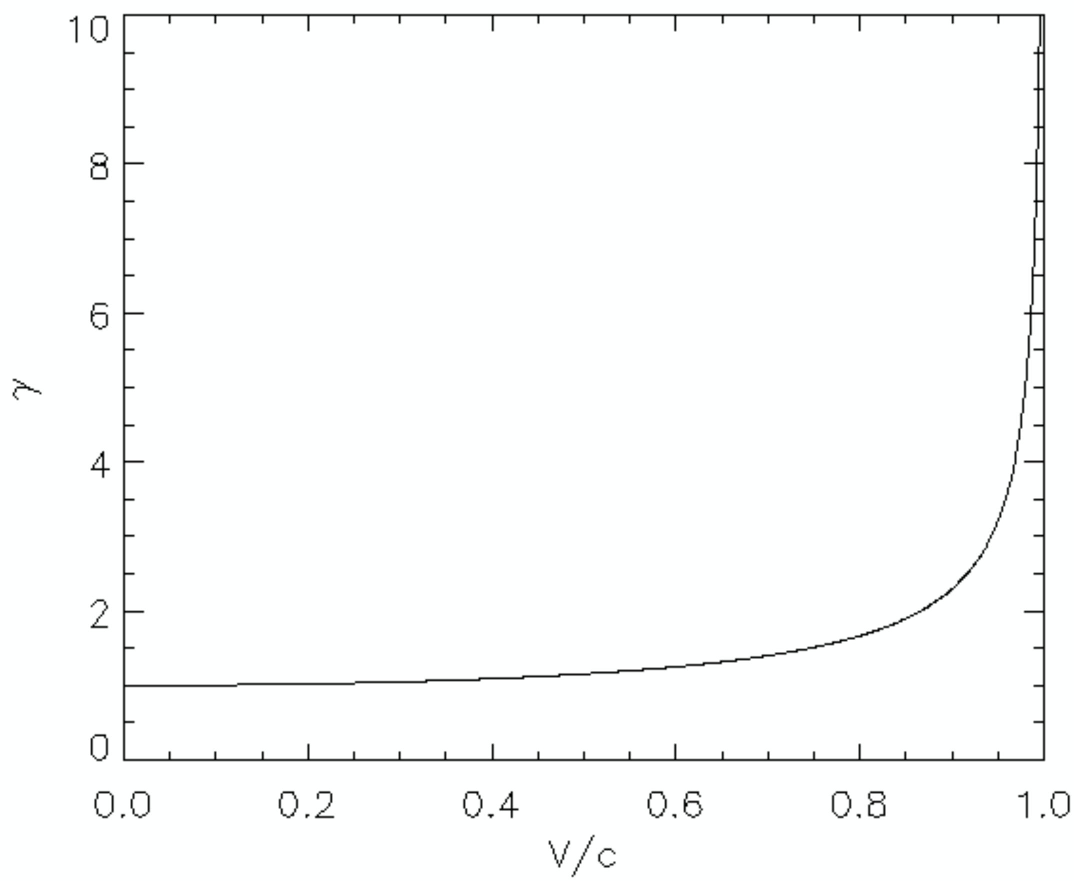
- This slowing factor is called the **Lorentz factor,** γ

$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

Clocks and time

- Does this "time dilation" effect come about because we used a funny clock?
- No, any device that measures time would give the same effect!
- The time interval of an event as measured in its own rest frame is called the ***proper time***
- Note that if the astronaut observed the same "light clock" (or any clock) that was at rest on Earth, it would appear to run slow by the same factor γ , because the dilation factor depends on *relative speed*
- This is called the ***principle of reciprocity***

Lorentz factor



A 1% effect at v = 0.14 *c*, or about 42,000,000 m/s

Lorentz factor goes to infinity when V→c! oops

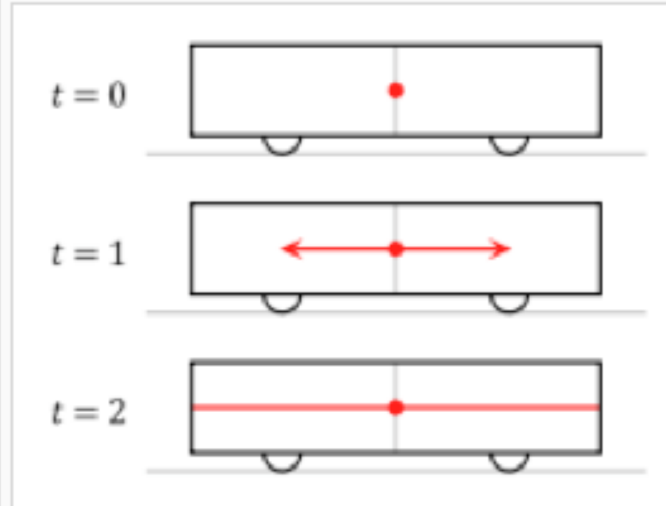
But it is very close to 1 for V/c small- which is why we do not notice this in everyday life

Why don't we ordinarily notice time dilation?

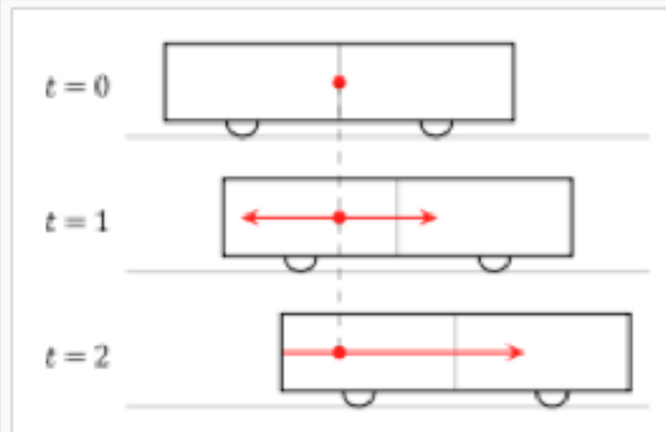
Some examples of speeds in m/s

- 0.0055m/s world record speed of the fastest snail in the Congham,UK
- 0.080 m/s the top speed of a sloth (= 8.0 cm/s)
- 1 m/s a typical human walking speed
- 28 m/s a car travelling at 60 miles per hour (mi/h or mph) or 100 kilometres per hour (km/h); also the speed a cheetah can maintain
- 341 m/s the current land speed record, which was was set by ThrustSSC in 1997.
- 343 m/s the approximate speed of sound under standard conditions, which varies according to air temperature
- 464 m/s Earth's rotation at the equator.
- 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- 1000 m/s the speed of a typical rifle bullet
- 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- 299,792,458 m/s the speed of light (about 300,000 km/s)

- In the moving train analogy in the text(pg185)- we shine a light to the front and the back of the train car-the speed of the train is NOT added to the speed of light
- light ALWAYS moves at the same
- speed.!
- Since the train is moving the stationary observer will
- observe the light to strike the rear of the train before it hits the front
- For the passenger the light strikes the front and the
- back at the same time, for the stationary observer they occur at different times!
- **Time itself is different for the two observers !**



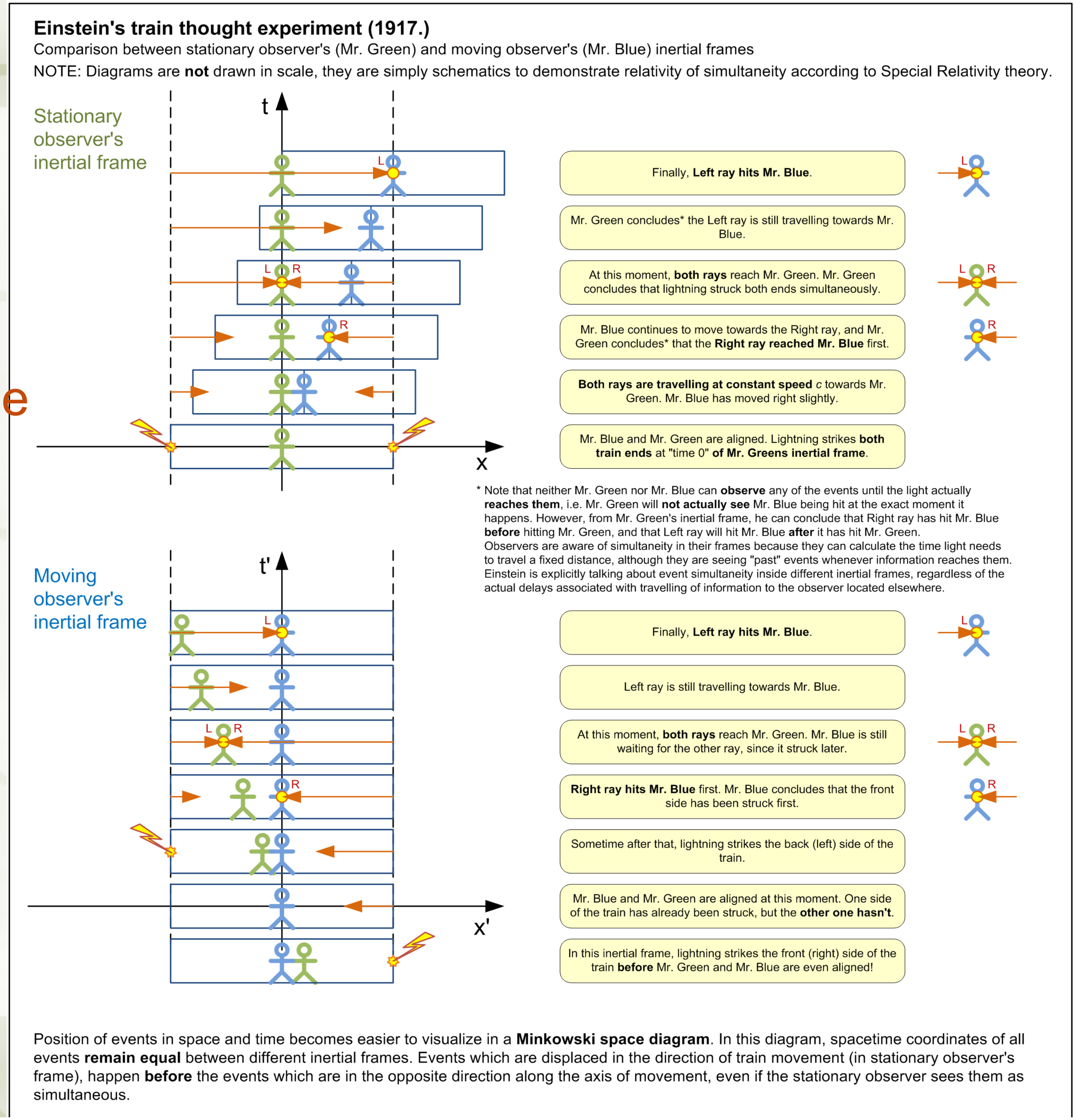
The train-and-platform experiment from the reference frame of an observer onboard the train



Reference frame of an observer standing on the platform (length contraction not depicted)

two lightning bolts striking both ends of the moving train simultaneously (as perceived in the stationary observer's inertial frame)

from http://en.wikipedia.org/wiki/File:Einstein_train_relativity_of_simultaneity.png.



Examples of time dilation

- The Muon Experiment
  - Muons are created in upper atmosphere from cosmic ray hits
  - Typical muon travel speeds are $0.99995\times c$, giving $\gamma=100$
  - Half-life of muons in their own rest frame (measured in lab) is t_h= 2 microseconds =0.000002s
  - Traveling at $0.99995\times c$ for t_h=0.000002s, the muons would go only 600 m
  - But traveling for $\gamma\times t_h$= 0.0002s, the muons can go 60 km
  - *They easily reach the Earth's surface, and are detected!*
  - *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with* $\gamma\times t_h$

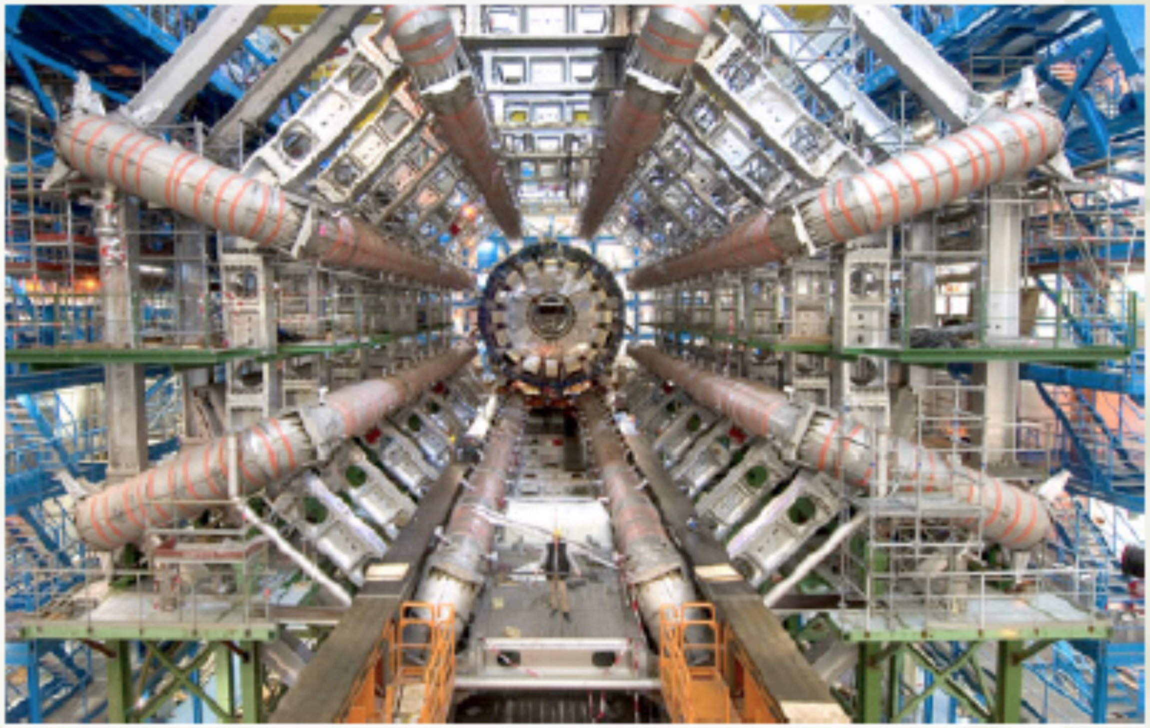
- Why muons?- have comparatively long decay life time (the second longest known for sub-atomic particles) and are relatively weakly interacting so they can penetrate the atmosphere

LHC

- When the power of this machine is discussed, the energy of each proton is often mentioned: The protons each have an energy of 7 TeV. What does that mean?
- With E = 7 TeV: $E=\gamma mc^2$

the Lorentz factor has a value of about 7460 corresponding to v = 0.999999991 times the speed of light

- time passes 7460 times more slowly for the particles than it does for us observers.



A clock traveling at that speed from Earth to Proxima Centauri would measure a journey time of under 5 hours, while an observer who would remain on Earth would have aged over 4 years (Proxima Centauri is about 4.243 light-years away from us).

III: LENGTH CONTRACTION

- Consider two "markers" in space.
- Suppose spacecraft flies between two markers at velocity V.
- A flash goes off when front of spacecraft passes each marker, so that anyone can record it
- Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft…
- Observer at rest w.r.t. markers says:
  - Time interval is t_R; distance is $L_R=V\times t_R$
- Observer in spacecraft says:
  - Time interval is t_S; distance is $L_S=V\times t_S$
- We know from before that $t_R = t_S\,\gamma$
- Therefore, $L_S=V\times t_S= V\times t_R\times(t_S/t_R)=L_R/\gamma$
- *The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor* γ

- So, moving observers see that objects contract *along the direction of motion*.
- **Length contraction**… also called
  - Lorentz contraction
  - FitzGerald contraction
- Note that there is no contraction of lengths that are perpendicular to the direction of motion
  - Recall M-M experiment: results consistent with *one* arm contracting

Muon experiment, again

- Consider atmospheric muons again, this time from point of view of the muons
  - i.e. think in frame of reference in which muon is at rest
  - Decay time in this frame is 2 μs (2/1,000,000 s)
  - How do they get from top of the atmosphere to sea level before decaying?
- From point of view of muon, the atmosphere's height *contracts by factor of γ*
  - Muons can then travel reduced distance (at almost speed of light) before decaying.

IV: NEW VELOCITY ADDTION LAW

- Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simple adding/subtracting frame velocity) gives incorrect results for electromagnetism
- Once we've taken into account the way that time and distances change in Einstein's theory, there is a **new law for adding velocities**
- For a particle measured to have velocity V_p by an observer in a spaceship moving at velocity V_s with respect to Earth, the particle's velocity as measured by observer on Earth is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

IV: NEW VELOCITY ADDTION LAW

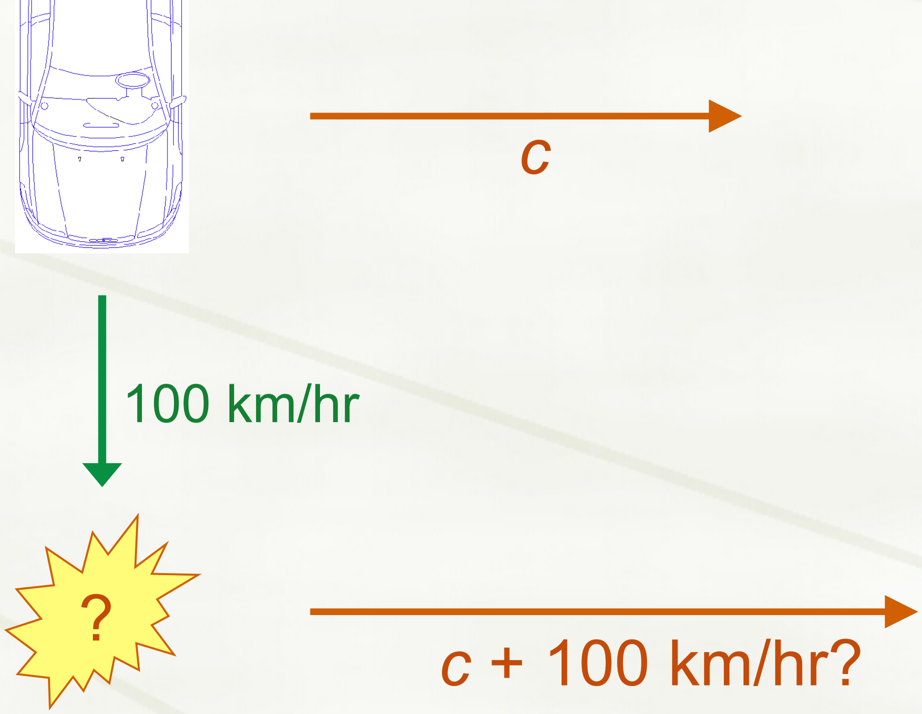
$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

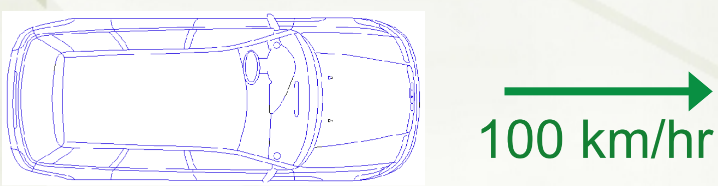
- Notice that if V_p and V_s are much less than c, the extra term in the denominator <<0 and therefore $V << V_p + V_s$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed of light
- This is consistent with everyday experience
- Also notice that if the particle has $V_p = c$ in the spaceship frame, then it has $V_p = c$ in the Earth frame. The speed of light is frame-independent!

What if the speed of light weren't the same in all inertial frames?

Collision or not? If the speed of light were not the same in all inertial frames, you would see one car reach the collision point earlier than the other. But there either is or isn't a collision!

$$\Delta t = \frac{\Delta D}{c}$$





Next time…

- *Special Relativity II:*
  - Simultaneity and causality
  - Space-time diagrams
  - Reciprocity and the twins paradox