Lecture 8 :
Special Theory of Relativity II

- More about time dilation…
  - Twin's paradox
  - The Muon Experiment
- Length (Fitzgerald) contraction
- Simultaneity and causality
- Space time diagrams

NEW VELOCITY ADDTION LAW

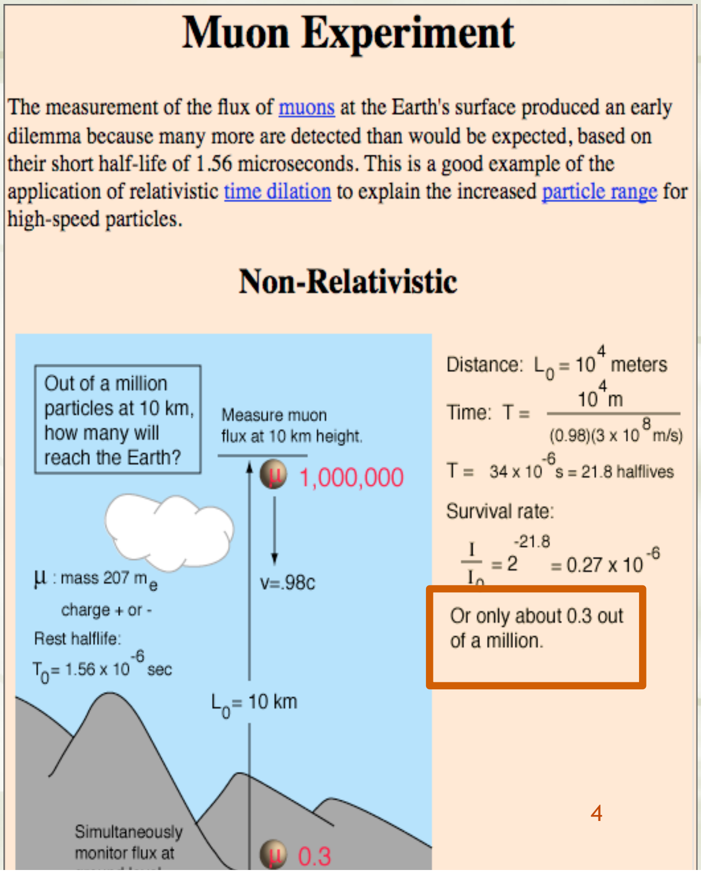
$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator <<0 and therefore $V << V_p + V_s$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed of light
- This is consistent with everyday experience
- Also notice that if the particle has $V_p = c$ in the spaceship frame, then it has $V_p=c$ in the Earth frame. The speed of light is frame-independent!

Rossi & Hall Muon Experiment

- Classic experiment verifying time dilation was performed by Rossi & Hall in 1941…
  - Muons are “electron-like” particles… when at rest, they decay with a half-life of about 2μs
  - Muons are produced when cosmic rays slam into upper atmosphere, then rain down to Earth
  - Rossi & Hall measured the number of muons detected at the top of a 2000m mountain, and compared it to the number at sea-level…
    - Found 560 muons/hour at top of mountain
    - Even at v=c, will take 6.5μs for muon to travel 2000m
    - More than 4 half lives… less than 1/16th of particles should be left by the time they reach the bottom
    - BUT, they measured 422 muons/hour at bottom
    - It seems like only 0.64μs have passed in the muon’s frame of reference… so they are moving with γ≈10

What would we observe if special relativity was not true



Muon Experiment

The measurement of the flux of muons at the Earth's surface produced an early dilemma because many more are detected than would be expected, based on their short half-life of 1.56 microseconds. This is a good example of the application of relativistic time dilation to explain the increased particle range for high-speed particles.

Non-Relativistic

Out of a million particles at 10 km, how many will reach the Earth?

Distance: $L_0 = 10^4$ meters

Time: $T = \frac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

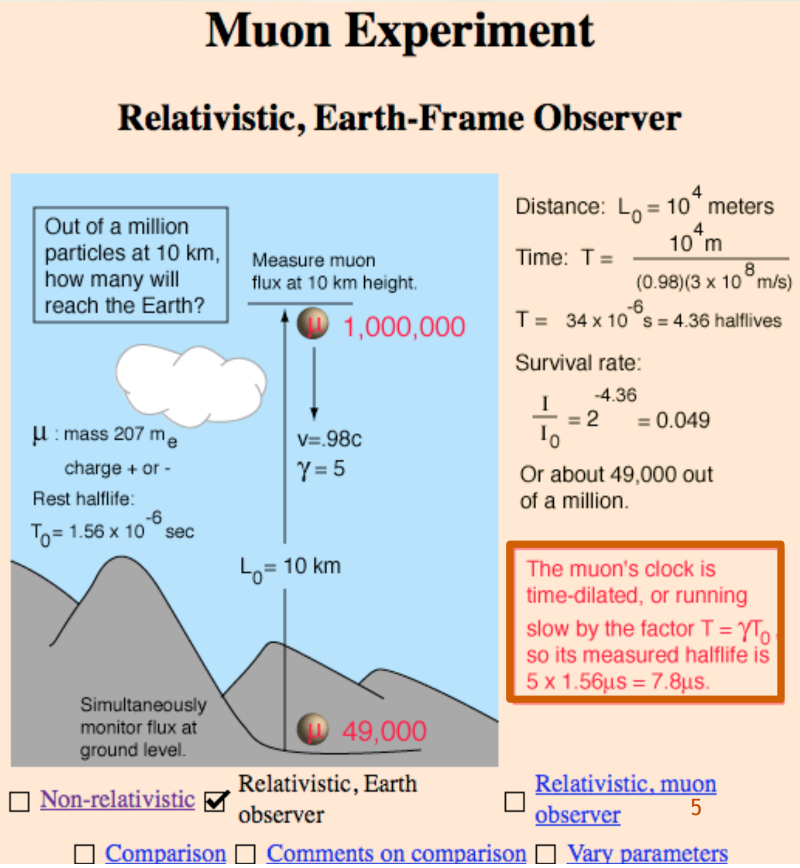
$T = 34 \times 10^{-6}$ s = 21.8 halflives

Survival rate:

$\frac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$

Or only about 0.3 out of a million.

To the observer on the earth the muon's clock is time dilated



Muon Experiment

Relativistic, Earth-Frame Observer

Out of a million particles at 10 km, how many will reach the Earth?

Measure muon flux at 10 km height.

μ : mass 207 m_e

charge + or -

Rest halflife:

$T_0 = 1.56 \times 10^{-6}$ sec

Simultaneously monitor flux at ground level.

Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 34 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$\dfrac{I}{I_0} = 2^{-4.36} = 0.049$

Or about 49,000 out of a million.

The muon's clock is time-dilated, or running slow by the factor $T = \gamma T_0$ so its measured halflife is 5 x 1.56μs = 7.8μs.

☐ Non-relativistic ☑ Relativistic, Earth observer ☐ Relativistic, muon observer

☐ Comparison ☐ Comments on comparison ☐ Vary parameters

Other Experimental Tests of Time Dilation

- Hafele and Keating, in 1971, flew caesium atomic clocks east and west around the Earth in commercial airliners, to compare the elapsed time against that of a clock that remained at the US Naval Observatory. Results were within 4% of the predictions of relativity.
- A reenactment by the National Physical Laboratory took place in 1996 on the 25th anniversary of the original experiment, using more precise atomic clocks during a flight from London to Washington, D.C. and back again. A time gain of 39 ± 2 ns was observed, compared to a relativistic prediction of 39.8 ns.[5] In June 2010, the National Physical Laboratory again repeated the experiment, this time around the globe (London - Los Angeles - Auckland - Hongkong - London). The predicted value was 246 ± 3 ns, the measured value 230 ± 20 ns.[6]
- In 2010 time dilation was observed (Chou et al) at speeds of less than 10 meters per second using optical atomic clocks connected by 75 meters of optical fiber.
- More than 20 more experiments with decaying particles (pion,kaon,muons) in accelerators

If you want more info…

- A good youtube to watch is

http://www.youtube.com/watch?v=xWST2gpbnvw- "Physics-X" lecture series, 15 minutes long

- taught at Michigan Technological University by Dr. Robert Nemiroff

- Relativity In 5 Minutes
- http://www.youtube.com/watch?v=KYWM2oZgi4E

More about time dilation…

the Twin's paradox

one of two twins travels at near the speed of light to a distant star and returns to the earth. Relativity dictates that when he comes back, he is younger than his identical twin brother.

BUT..."Why is the traveling brother younger?"-
relativity says that there is no absolute motion, wouldn't the brother traveling to the star also see his brother's clock on the earth move more slowly? If this were the case, wouldn't they both be the same age?

http://feegics.blogspot.com/2009/12/how-does-relativitytheory-resolve-twin.html

Twin Paradox

The Earth and the ship are not in a symmetrical relationship: the ship has a *turnaround - it undergoes non-inertial motion*, while the Earth has no such turnaround.

- Special relativity does not claim that *all* observers are equivalent, only that all observers at rest in inertial reference frames are equivalent
  - Since there is no symmetry, it is not paradoxical if one twin is younger than the other.

Experimentally confirmed by Bailey et al. (1977), who measured the lifetime of positive and negative muons in the CERN Muon storage ring, muons were sent around a loop, so this experiment also confirms the twin paradox- agrees with Special relativity to accuracy of 2x10^{-3}

Twin Paradox

Its rather involved to do the math and it requires a particular type of diagram (Minkowski space time diagrams- which we will do a bit later) **please see the extra slides at the end of this lecture** (text pg 203-205)

or
http://www.einsteins-theory-of-relativity-4engineers.com/twin-paradox-2.html
or
http://www.oberlin.edu/physics/dstyer/Einstein/SRBook.pdf
for a detailed solution

II : Length (Fitzgerald) contraction

Think again about the muon experiment… but now from a muon's perspective!

- Fitzgerald contraction…
  - A moving object **contracts** by a factor γ (the same Lorentz factor) in the direction of motion

This is really *a* *contraction of space itself*… the object does not experience forces or stresses that make it contract

Again, everything is relative… if someone watches you travel past them at high speed, you will appear to be contracted in the direction of motion

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II : Length (Fitzgerald) contraction

Consider two "markers" in space.

- Suppose spacecraft flies between two markers at velocity V.

A flash goes off when front of spacecraft passes each marker, so that anyone can record it

- Compare what would be seen by observer at rest with respect to (w.r.t.) the markers, and an astronaut in the spacecraft…

Observer at rest w.r.t. markers says:

Time interval is t_R; distance is $L_R=Vxt_R$

Observer in spacecraft says:

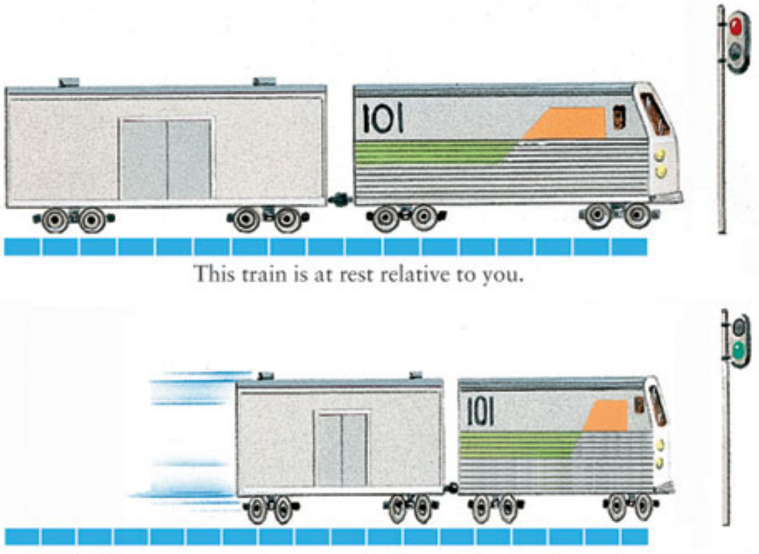
Time interval is $t_{S;}$ distance is $L_S=Vxt_S$

We know from before that $t_R = t_S\,\gamma$

Therefore, $L_S=Vxt_S= Vxt_{Rx}(t_S/t_R)=L_R/\gamma$

The length of any object is contracted in any frame moving with respect to the rest frame of that object, by a factor γ

In addition to time, length depends on your frame of reference !



This train is at rest relative to you.

The same train is now moving relative to you.

(a) Length contraction

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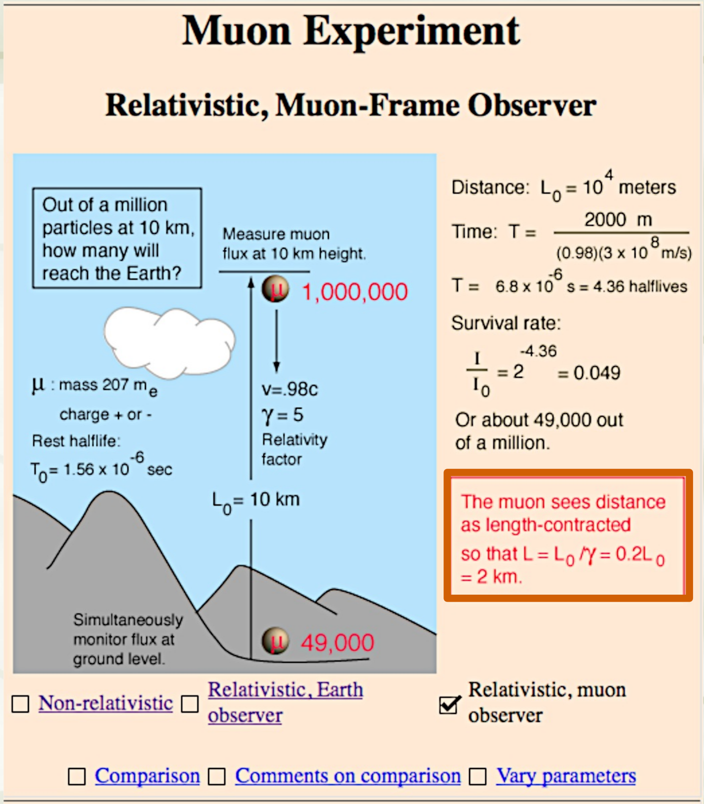
to the muon- the distance from the place it is created to the earth is length contracted

http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/muon.html#c3



Muon Experiment

Relativistic, Muon-Frame Observer

Out of a million particles at 10 km, how many will reach the Earth?

Measure muon flux at 10 km height.

μ 1,000,000

μ : mass 207 m_e
charge + or -
Rest halflife:
$T_0 = 1.56 \times 10^{-6}$ sec

v=.98c
$\gamma = 5$
Relativity factor

$L_0 = 10$ km

Simultaneously monitor flux at ground level.

μ 49,000

Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{2000\ \text{m}}{(0.98)(3 \times 10^8\ \text{m/s})}$

$T = 6.8 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$\dfrac{I}{I_0} = 2^{-4.36} = 0.049$

Or about 49,000 out of a million.

The muon sees distance as length-contracted so that $L = L_0/\gamma = 0.2L_0$ = 2 km.

☐ Non-relativistic ☐ Relativistic, Earth observer ☑ Relativistic, muon observer

☐ Comparison ☐ Comments on comparison ☐ Vary parameters

Muons ... again!

- Consider atmospheric muons again, this time from point of view of the muons i.e. think in frame of reference in which muon is at rest
- Decay time in this frame is 2 μs (2/1,000,000 s)

How do they get from top of the atmosphere to sea level before decaying?

- From point of view of muon, the atmosphere's height *contracts by factor of* γ
- Muons can then travel the reduced distance (at almost speed of light) before decaying.

Length contraction

- So, moving observers see that objects contract *along the direction of motion.*
- **Length contraction...** also called

Lorentz contraction

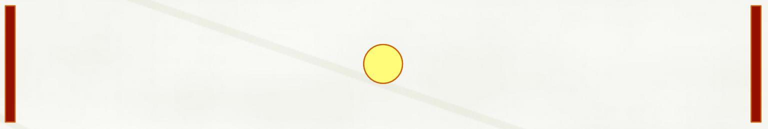
FitzGerald contraction

Note that there is *no contraction of lengths* that are **perpendicular** to the direction of motion

- *Recall M-M experiment: results consistent with one arm contracting*

III: RELATIVITY OF SIMULTANEITY

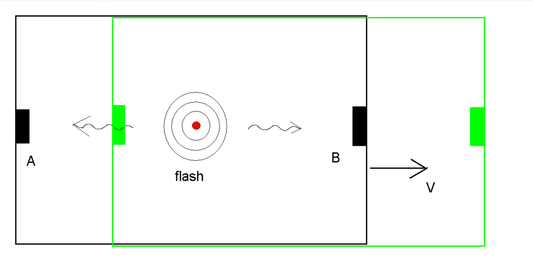
- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



- **Since speed of light is constant**, light rays will hit opposite walls at precisely the same time. Call these events A and B.

Change frames…

- Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.



- But astronaut in spacecraft thinks events are simultaneous.
- Concept of "events being simultaneous" (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

Change frames…

- Imagine performing same experiment aboard a moving spacecraft, and observing it from the ground.
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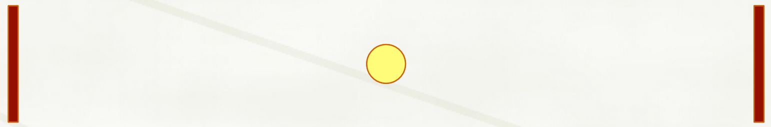
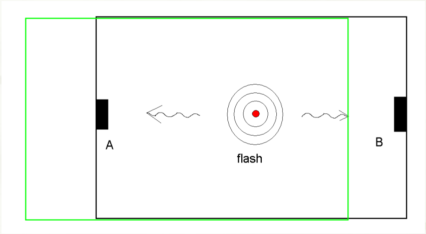


- But astronaut in spacecraft thinks events are simultaneous.
- Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity**).

Change frames again!

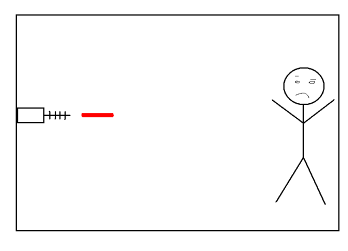
- What about perception of a 3rd observer who is moving faster than spacecraft?



- 3rd observer sees event B before event A
- So, order in which events happen can depend on the frame of reference.

The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.



- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., *can the victim get hit **before** the gun fires?*

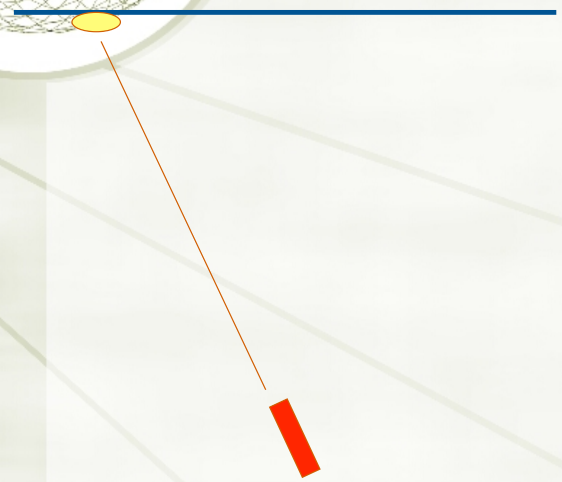
- This is a question of **causality.**
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- **<u>It is not possible</u>** to change the order of these events by changing frames, according to Special Relativity theory.

- This is true provided that
  - The laser blast does not travel faster than the speed of light
  - We do not change to a frame of reference that is going *faster* than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information.**

Causality

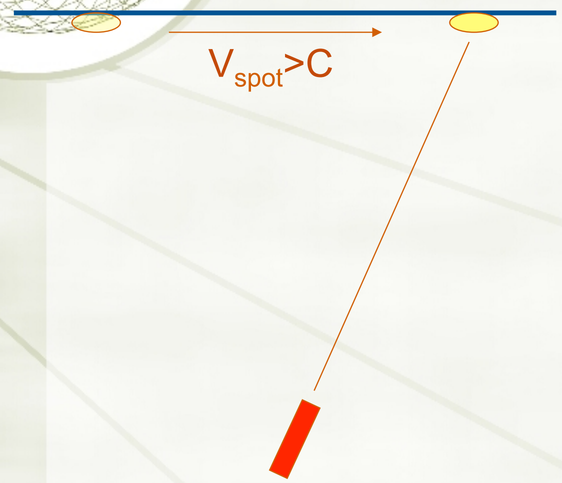
- Can causality be proved?
  - No, it is an axiom of physics
- What if causality doesn't hold?
  - Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - This would be a true "end of science."
- So we will *insist* on causality as we continue to explore relativity.

Some things move faster than light



- But they transmit no information
- E.g., light spot on a distant screen

Some things move faster than light



- But they transmit no information
- E.g., light spot on a distant screen

Distances in *time* and space

- Two events A and B separated by distance Δs in space (x, y, z):

  $\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$

  (Thanks, Pythagoras!)

  where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

Distances in *time* and space

- Two events A and B separated by distance Δs in time (Δt):

  $\Delta s = [(c\Delta t)^2]^{1/2}$

  where $\Delta t = t_A - t_B$, and we've multiplied by c to make the units of Δs come out as a distance

- Two events A and B separated in x and t:

  $\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$

  one space dimension+time

Space-time intervals

- Two events A and B in space-time are separated by an **invariant interval**, given by

$$\Delta s=[(c\Delta t)^2 -(\Delta x)^2 -(\Delta y)^2 -(\Delta z)^2]^{1/2}$$

where $\Delta t = t_A - t_B$, $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$,

- The formula is analogous to Pythagorean equation, but modified to account for the difference between space (x) and time (c*t*)
- The invariant space-time interval is an important quantity because it is independent of the frame in which it is measured; *all* observers agree on it!
  - This is true even though the Δt, Δx , *etc individually* are different for different observers (due to time dilation, space contraction)
  - The invariant interval is equal in value to (proper time of event) × c

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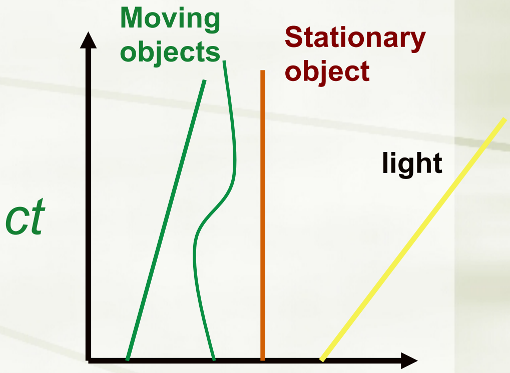
Proper Length and Time

- Proper length is the length of an object as measured in its own frame
  - Proper length is the largest possible length to any observer

- Proper time is the time as measured by a clock at rest with respect to the observers inertial frame
  - Proper time is the fastest rate for any observer

Space-time diagrams

- Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- This is simplest to do for events that take place along a line in space (one-dimensional space)
  - Plot as a 2D graph
  - use two coordinates: x and ct *(has units of distance (velocityxtime)*
- Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image): x, y and ct
- a straight line represents an object moving at constant velocity (slope represents its velocity)- a wiggly line an object that is accelerating.



World lines of events

world line of an object is the unique path of that object as it travels through 4-dimensional spacetime

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World Lines

- Space-time interval is zero for any two points on light ray *world line*
- Proper time between two events connected by a curved world line is computed by adding up results for small straight intervals along curve
  - Even if two curved world lines start and end at the same place, they may result in different proper time intervals

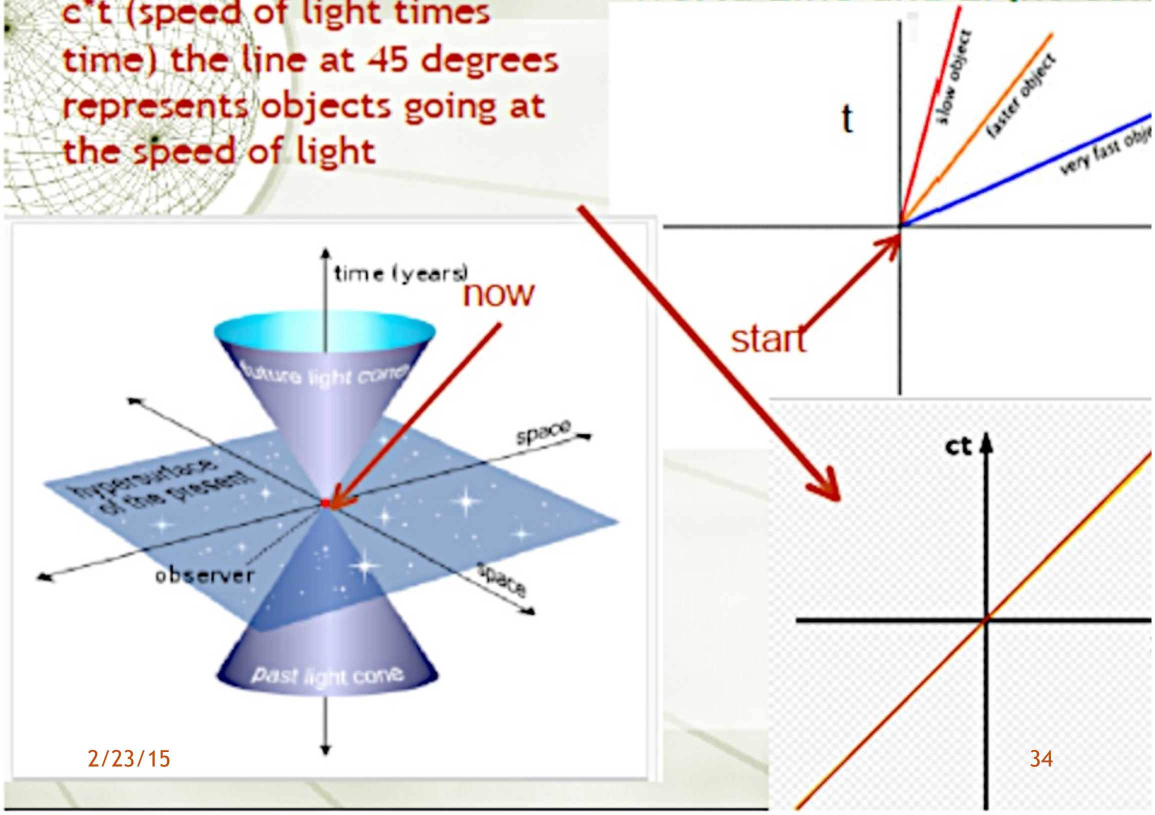
World Line and Light Cone

- In the short story Life-Line, Robert A. Heinlein describes the world line of a person
- He stepped up to one of the reporters. "Suppose we take you as an example. Your name is Rogers, is it not? Very well, Rogers, you are a spacetime event having duration four ways. You are not quite six feet tall, you are about twenty inches wide and perhaps ten inches thick. In time, there stretches behind you more of this space-time event, reaching to perhaps nineteen-sixteen, of which we see a cross-section here at right angles to the time axis, and as thick as the present. At the far end is a baby, smelling of sour milk and drooling its breakfast on its bib. At the other end lies, perhaps, an old man someplace in the nineteen-eighties.

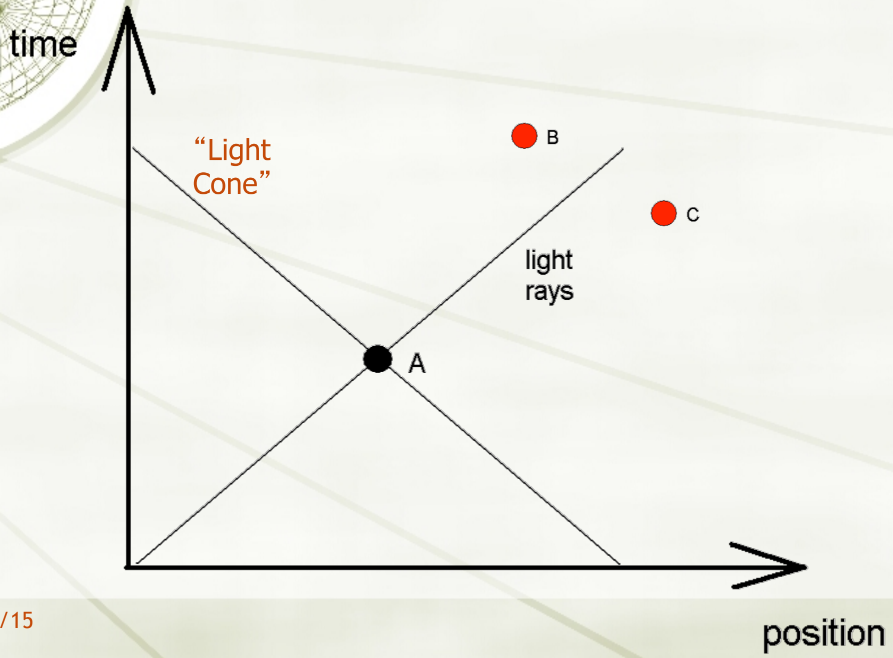
"Imagine this space-time event that we call Rogers as a long pink worm, continuous through the years, one end in his mother's womb, and the other at the grave..."

World Line and Light Con

- If we make the vertical axis c*t (speed of light times time) the line at 45 degrees represents objects going at the speed of light



Light cone for event "A"

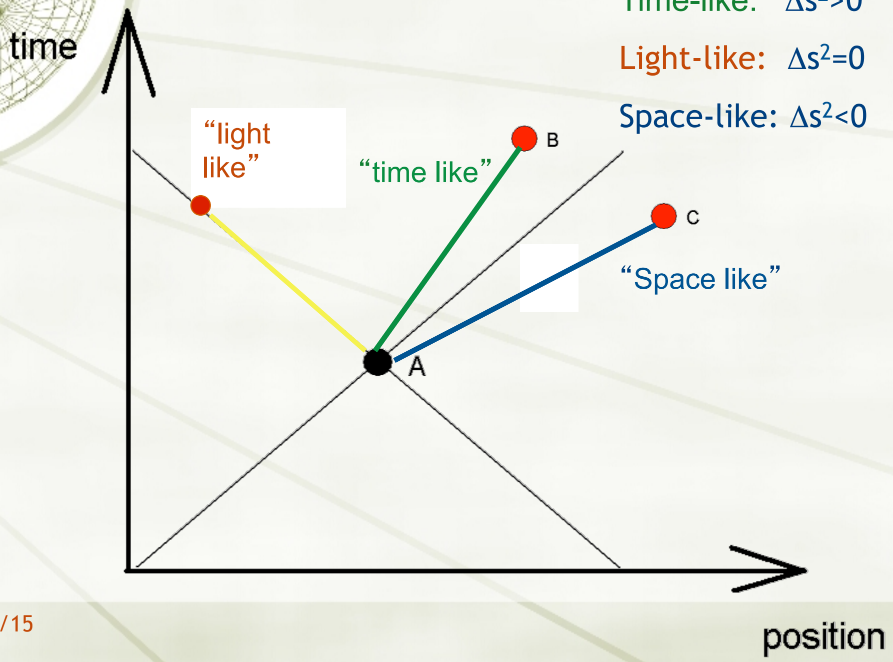


Different kinds of space-time intervals

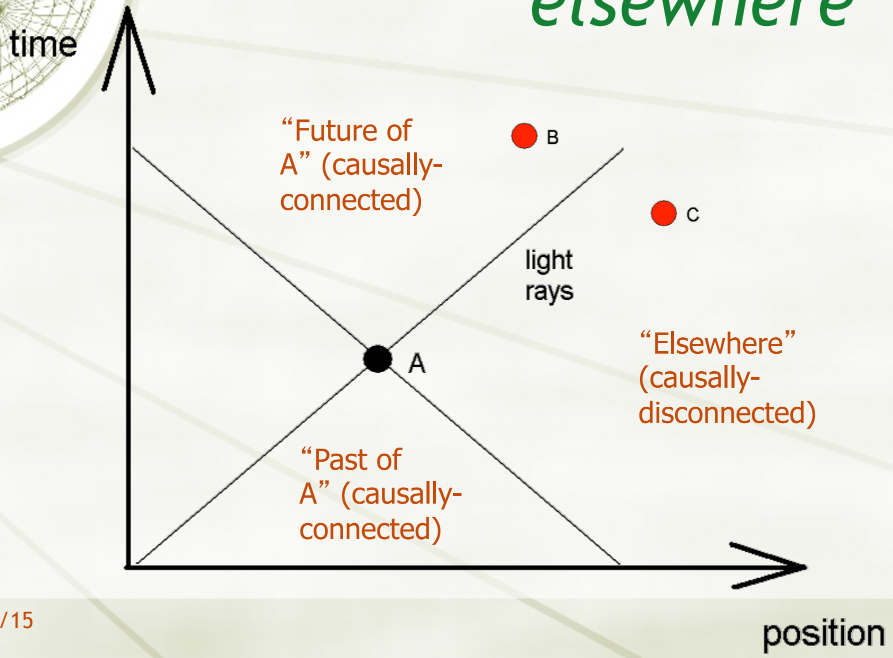
Time-like: $\Delta s^2 > 0$

Light-like: $\Delta s^2 = 0$

Space-like: $\Delta s^2 < 0$



Past, future and "elsewhere".



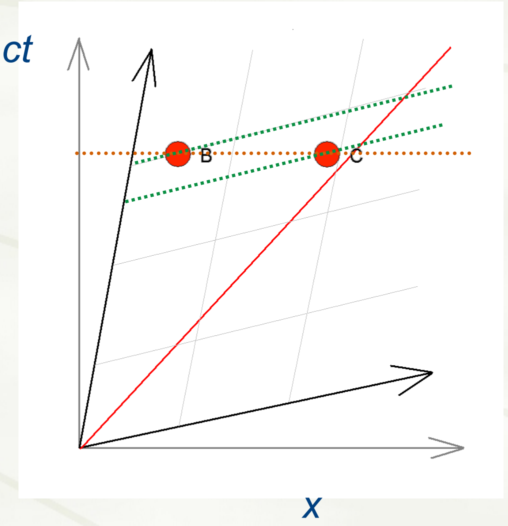
"Future of A" (causally-connected)

"Elsewhere" (causally-disconnected)

"Past of A" (causally-connected)

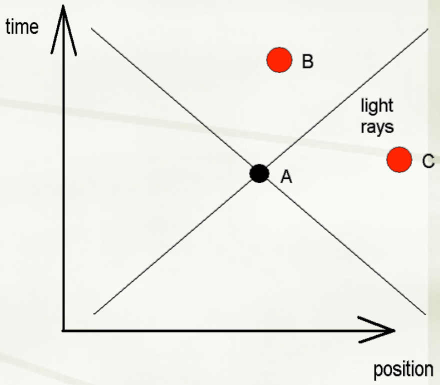
Spacetime diagrams in different frames

- Changing from one reference frame to another…
  - Affects time coordinate (time dilation)
  - Affects space coordinate (length contraction)
  - Leads to a distortion of the space-time diagram as shown in figure.
- Events that are simultaneous in one frame are not simultaneous in another frame



Causality

- Events A and B…
  - Cannot change order of **A** and **B** by changing frames of reference.
  - A can also communicate information to **B** by sending a signal at, or less than, the speed of light.
  - This means that **A** and **B** are causally-connected.
- Events A and C…
  - Can change the order of **A** and **C** by changing frame of reference.
  - If there were any communication between **A** and **C**, it would have to happen at a speed faster than the speed of light.
- If idea of **cause and effect** is to have any meaning, we must conclude that *no communication can occur at a speed faster than the speed of light*.



- For a <u>light ray</u> since $\Delta x= c\Delta t$

  $\Delta s^2=sqrt((c\Delta t)^2-(\Delta x)^2))=0$

- Not like Euclidean space

if $(\Delta x)^2 > (c\Delta t)^2$ the events are separated by a 'spacelike' interval - can't get from here to there or more formally

not enough time passes between their occurrence for there to exist a causal relationship crossing the spatial distance between the two events at the speed of light or slower

Other Experimental Tests of Time/Length Dilation

- frequency of wave (sound) due to motion towards or away from the observer
- If ν_{obs} is the frequency seen by the observer and ν_{emit} is the wavelength emitted by the object moving at velocity ,v, then
- The Doppler shift is $\nu_{obs}=\nu_{emit}(1+v/c)$ when v<<c

when viewed from in front the

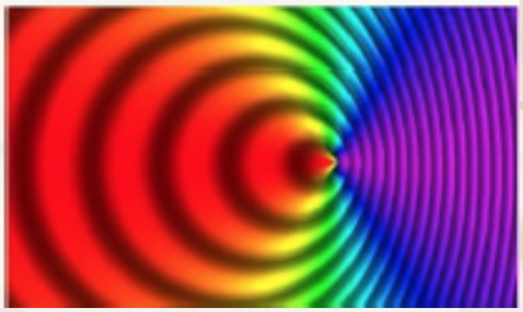
pitch, (frequency of sound), gets higher)

- (see page 100-101 in text)

$\nu_{obs}=\nu_{emit}$*sqrt((1+v/c)/(1-v/c)) for the observer in front of the source and

ν_{obs}=vemit*sqrt((1-v/c)/(1+v/c)) for the observer in back

result is due to contraction of length (change in wavelength) or time dilation (change in frequency)



Observer in front see contraction one in back expansion

Doppler Shift- the Full story

- Assume the observer and the source are moving away from each other with a relative velocity v
- Let us consider the problem in the reference frame of the source.
- Suppose one wavefront arrives at the observer. The next wavefront is then at a λ=c/*f*s away from him (where λ is the wavelength, *f*s is the frequency of the wave the source emitted, and c, is the speed of light).
- the wavefront moves with velocity c, and the observer escapes with velocity v, the time (as measured in the reference frame of the source) between crest of the wave arrivals at the observer is
- t= λ/(c-v)=1/(1-v/c)*f*s
- due to relativisic time dilation the observer will be t_0=t/γ
- γ=1/sqrt(1-v^2/c^2) is the Lorentz factor
- the corresponding frequency *f*o=1/t 0= γ(1-v/c)*f*s=$f_s$sqrt((1-v/c)/(1+v/c)
- the ratio *f*s/*f*o=sqrt((1+(v/c)).(1-(v/c)) is the Doppler factor

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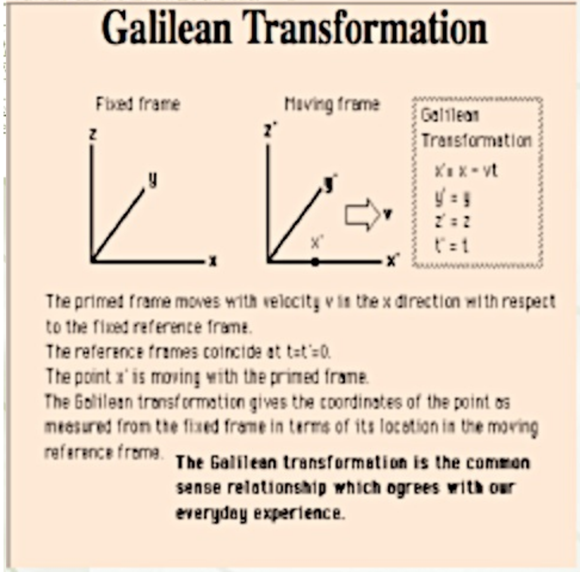
Summary

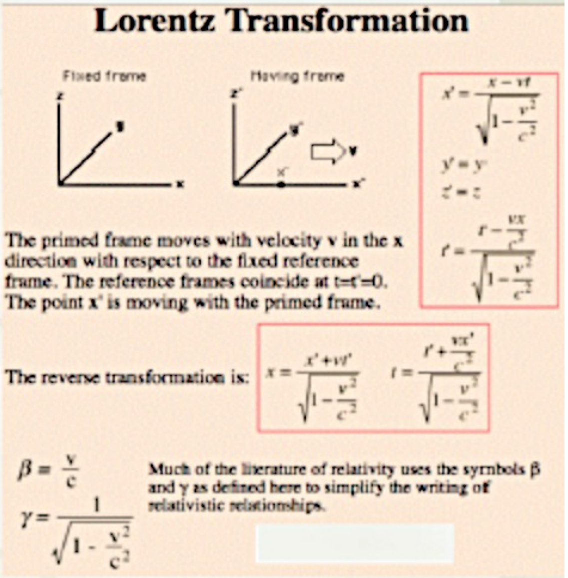


Galilean Transformation

Galilean Transformation
x' = x - vt
y' = y
z' = z
t' = t

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame.
The reference frames coincide at t=t'=0.
The point x' is moving with the primed frame.
The Galilean transformation gives the coordinates of the point as measured from the fixed frame in terms of its location in the moving reference frame.

The Galilean transformation is the common sense relationship which agrees with our everyday experience.



Lorentz Transformation

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

The reverse transformation is: $x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$ $t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Much of the literature of relativity uses the symbols β and γ as defined here to simplify the writing of relativistic relationships.

http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/relcon.html#c1