## *Lecture 10: General Relativity I*

- Einstein Tower Experiment
- Gravitational redshifting
- Strong Equivalence Principal
- Read Chapter 8
- Due to snow and confusion the mid-term is delayed to Thursday March 26
- Homework 3 will be due Tuesday March 24



## *0: RECAP OF SPECIAL RELATIVITY*

- Einstein's postulates
  - Laws of physics look the same in any inertial frame of reference.
  - The speed of light is the same in any inertial frame of reference
- Strange consequences
  - Time dilation and length contraction
  - Relativity of simultaneity and ordering of events
  - Equivalence and conversion of mass and energy
- **Why have we been so carefully avoiding gravity until now?**

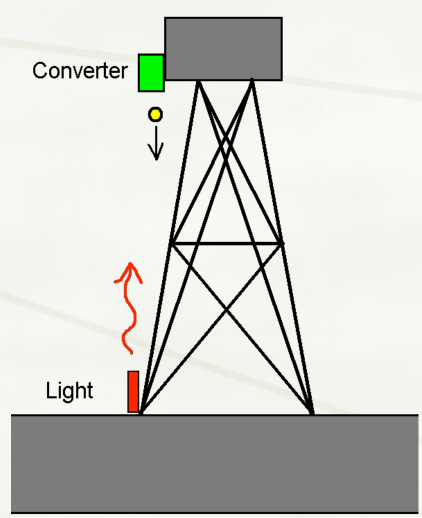
## *First Newtonian mechanics (special relativity), now his law of gravity (general relativity)*

- As we have just learned we have to understand
  - In whose frame do we measure ?
  - Does the force depend based on your reference frame?
  - Can gravity information travel (communicate) faster than *c*? If not, shouldn't there be some reference to *c* in the expression above?



## *Motivation for General Relativity: Einstein's tower*

- So far,we have ignored the effects of gravity. Is this really okay??
- Consider another thought experiment to test whether **light** can be **affected by gravity.**
- Consider a tower on Earth
  - Shine a light ray from bottom to top
  - When light gets to top, turn its energy into mass.
  - then drop this mass to bottom of tower in Earth's gravity field
  - then turn it back into energy- BUT now have more mass (the sum of the original mass+ the energy gained by 'falling' from the height of the tower)



# Perpetual motion?

- If we could do this, then we could get energy from nothing!
  - Original energy in light beam=$E_{start}$
  - Turn photon into mass at top created mass is $m=E_{start}/c^2$
  - then drop mass…at bottom of tower it has picked up speed (and energy) due to the effects of gravitational field *.
  - When we turn it back into energy,we have $E_{end}=E_{start}+E_{grav}$
  - But, we started off with only $E_{start}$-we have made energy! We're rich!

* a little physics.. the energy due to falling in a gravitational field is $E_{fall}=mgh$ (h=height you fall, m is the mass falling, and g is the local acceleration due to gravity)



- details
  - Suppose original photon energy E
  - *By assumption*, it is not affected by gravity so it has energy **E** once it reaches top
  - Thus, mass created at top is $\mathbf{m=E/c^2}$
  - Then drop mass… at bottom of tower, it has picked up energy due to the conversion of gravitational potential energy ($\mathbf{E_{grav}=mgh}$)
  - When we convert it back into energy, we made energy! We're rich!!!!

$$E_{new} = E + mgh = E\left(1 + \frac{gh}{c^2}\right)$$

Physics formula

the new energy of the photon= the old energy+the energy gained by falling

- *Clearly something is wrong with our assumptions…*
  - Only way we can conserve energy is to suppose that light **is** affected by gravity…
  - ***the photon has to lose energy as it climbs upwards***… at top of tower, to balance things

$$E_{top} = E\left(1+\frac{gh}{c^2}\right)^{-1}$$

  - This is known as ***gravitational redshift***- for light the energy of a photon is related to its frequency, $\nu$, (alternatively its wavlength, $\lambda$)

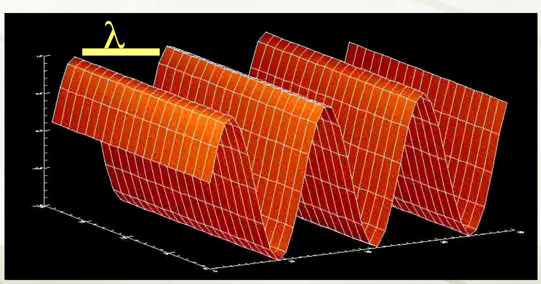
**$E=h^*\nu=hc/\lambda \rightarrow$ lower energy, longer wavelength- redder**

- The profound nature of gravitational redshift…
  - Imagine a clock based on the frequency of light
  - Place the clock at the base of the tower… observe it from the top.
  - Photons lose energy… so they decrease frequency
  - Thus, we see the clock running slowly!
  - **Time passes at a slower rate in a gravitational field! (units of $\nu$ is cycles per sec)**

***h= Planck's constant** (6.62606957 × $10^{-34}$ $m^2$ kg/s)

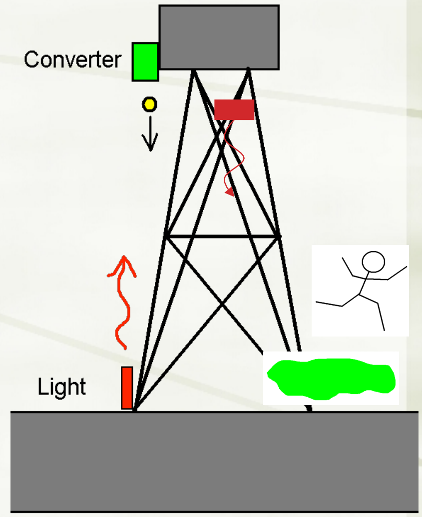
## *Properties of Waves*

- **Recall properties of waves:**
- Waves characterized by **(refer to Ch. 4 for review):**
  - Wavelength ($\lambda$) = distance between crests- (units length)
  - Frequency ($f$ or $\nu$) = number of crests passing a given point per second (units cycles/time)
- Speed of a crest; $s_c = \lambda\nu$ (this is generic to waves, for light $s_c$=c)
- **Energy** of a wave is proportional to frequency $\nu$, E = h$\nu$.



# Resolving the tower problem

- Now consider light ray aimed from top to bottom of tower
- Free-falling(FF) observer sees light ray travel **unaffected** by gravity, since freefall is an inertial frame
- From "Earth's" frame...
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity
  - If FF observer is **supposed to** see a constant frequency light beam, then light must get relatively *blueshifted* as it falls in a gravitational field, to compensate

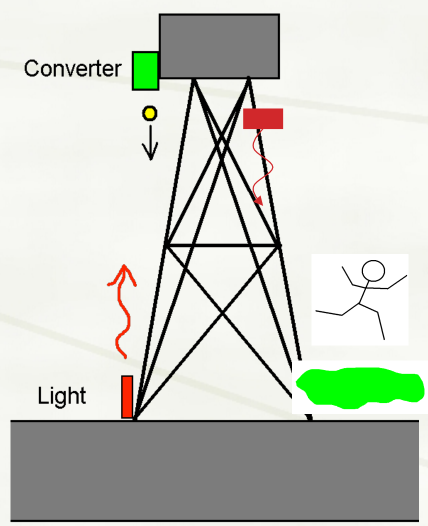





# Resolving the tower problem

- Light beam aimed upward must conversely be increasingly *redshifted* with height
- ***Gravitational redshifting removes just the right amount of energy to solve the tower paradox!***

***(remember that the energy of a photon is $E=h\nu=hc/\lambda$)***



# *Maxwell and gravity*

- Clearly,our assumption must be wrong…
  - light **must** be affected by gravity.
  - But gravity does not appear in Maxwell's equations,which govern light
  - Thus, Maxwell's equations are not complete* and are not exactly valid in the reference frame of Earth's surface,where there **is** gravity.
  - the Earth's surface must not be an inertial frame of reference!
  - * Einstein–Maxwell equations



# *RECAP*

- Einstein's postulates for Special Relativity
  - Laws of physics look the same in any inertial frame of reference.
  - The speed of light is the same in any inertial frame of reference
- Strange consequences of Special Relativity
  - Time dilation and length contraction
  - Relativity of simultaneity and ordering of events
  - Equivalence and conversion of mass and energy
- Behavior of light in gravity field ("tower" experiment)
  - Energy of light must vary in a gravitational field to ensure that free-fall is an inertial frame

## *What makes free-fall an inertial frame? Think back to the astronauts…*

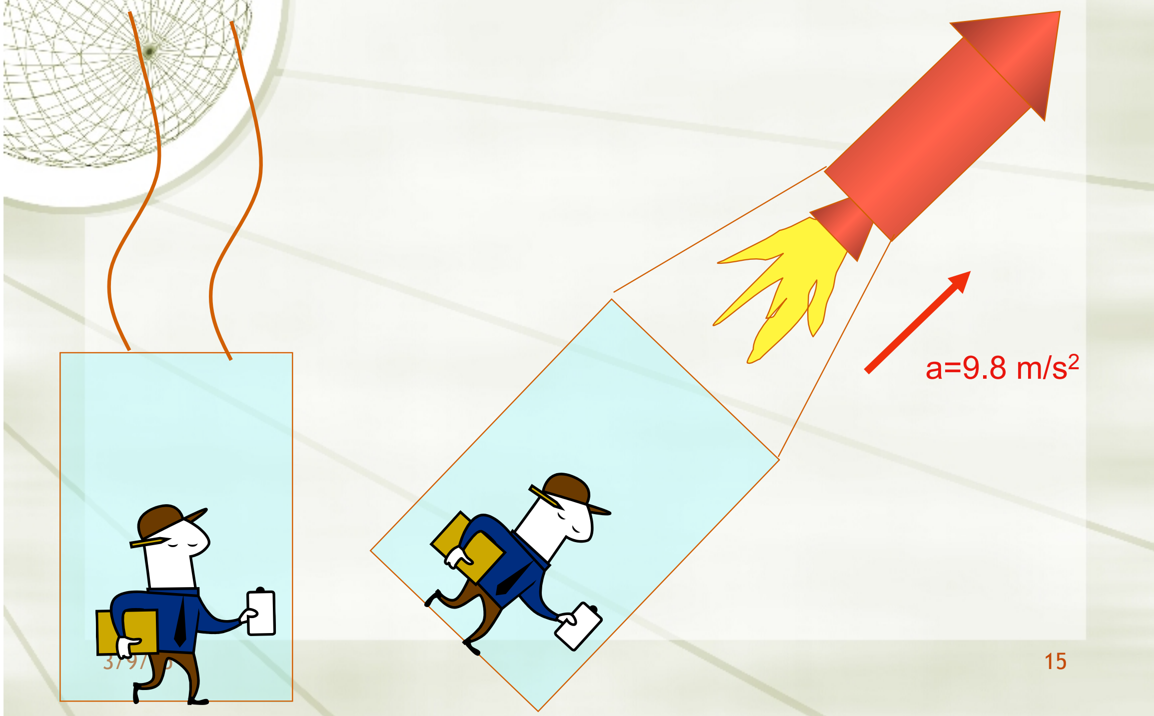
Apollo 10: Lots of gravity, but everything is falling together



## *EQUIVALENCE PRINCIPLES*

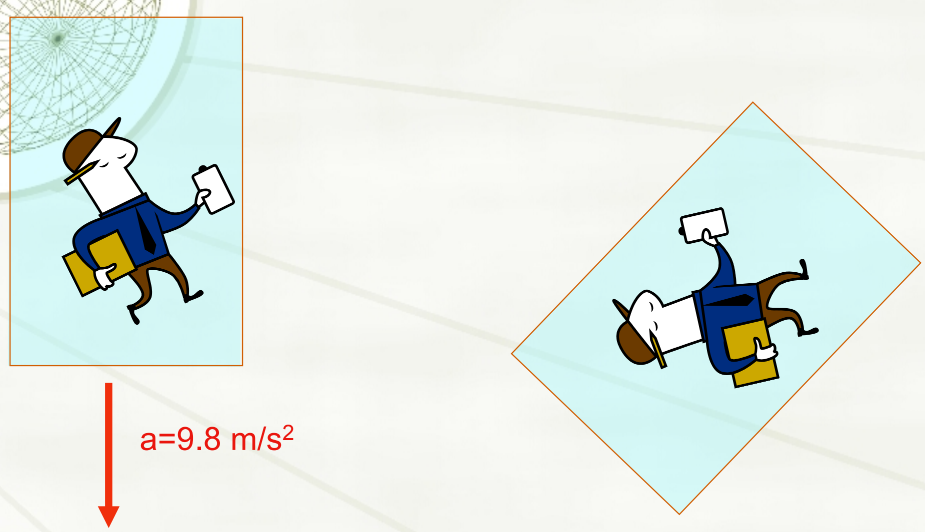
- Recall the "**weak" equivalence principle:**
  - All objects are observed to accelerate at the same rate in a given gravitational field (Galileo tower of Pisa experiment).
  - Therefore, the inertial and gravitational masses must be the same for any object.
  - This has been verified experimentally, with fractional difference in masses $<10^{-11}$
- As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot, locally, be distinguished

## *Elevator at rest on Earth equivalent to elevator being pulled by accelerating rocket in deep space*






## *Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space*

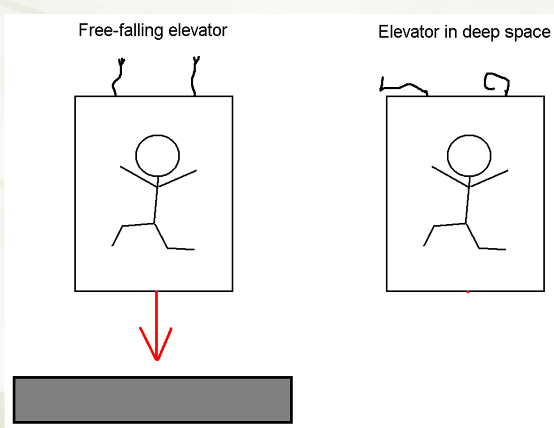


# II: STRONG EQUIVALENCE PRINCIPLE

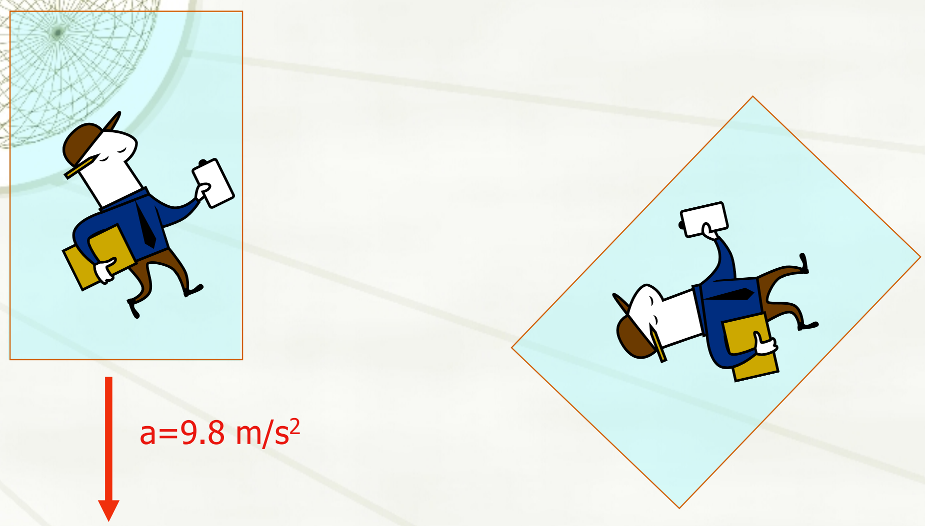
- Recap of the **weak equivalence principle**
  - All objects accelerate at the same rate in a given gravitational field.
  - In other words, inertial and gravitational masses are the same for any object.
- Einstein introduced the **strong equivalence principle** – when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- What does this mean???

# Free Fall vs 'No' Gravity

- There is **no** way of telling the difference between a free-falling frame in a gravitational field and an inertial frame in no gravitational field… the two are equivalent.

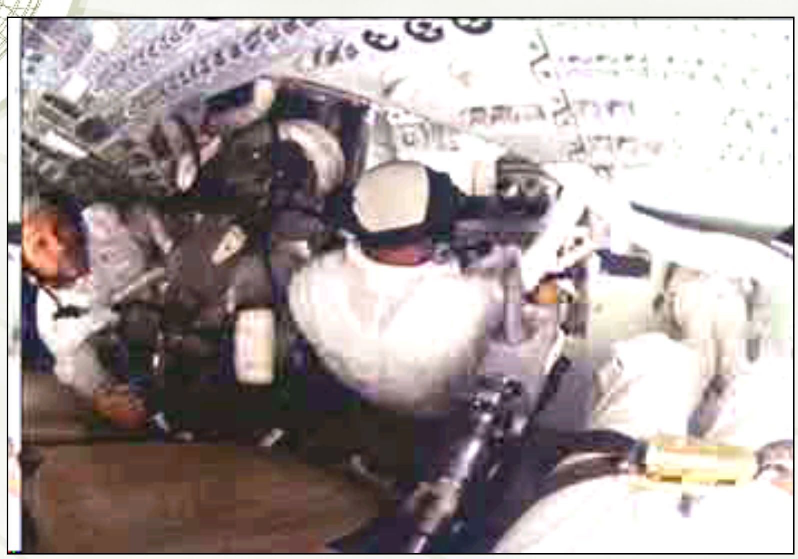


*Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space*





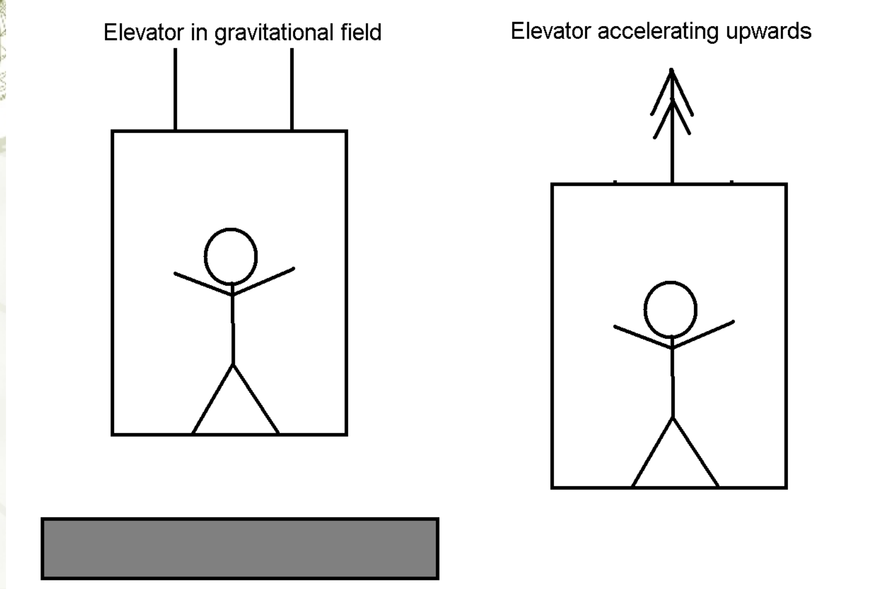

# *Back to the Astronauts…*



## What about gravity?

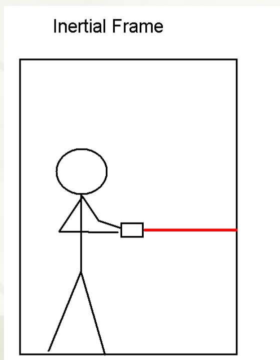
- Suppose that you decide that your frame of reference is not inertial…
  - Freely moving bodies change velocity
  - Is it because of gravity or is the frame accelerating?
  - Einstein says: *you cannot tell the difference!*
  - **Is Gravity a "fictitious force"??** – i.e., a force which appears to exist because we are living in a non-inertial frame of reference.

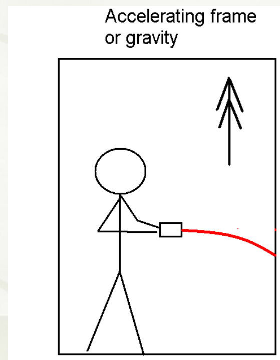
## Accleration vs Gravity



# *What about light? It "falls", too!*

- Astronaut in inertial frame with flashlight
  - Inertial frame, so light goes in straight lines
  - It doesn't matter whether this is free fall or far from masses
- What if we now put flashlight in a gravitational field (accelerated frame)?
  - Light beam will bend: it must accelerate at the same rate and direction as the elevator
  - Strong equivalence principle ⇨ frame with gravity acts the same
  - Important conclusion – light "falls" due to gravity!- **how can we test this idea?**





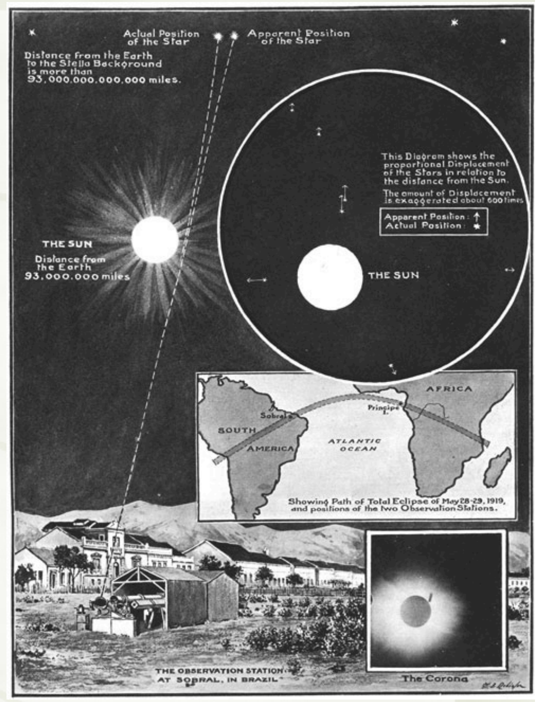

# *The Eddington Test*

- 1919 – the first "accessible" total Solar eclipse since Einstein postulated the Strong Equivalence Principle
- Arthur Eddington
  - Famous British Astronomer
  - Lead expedition to South Africa to observe eclipse
  - Was looking for effects of gravitational light bending by searching for shifts in positions of stars just next to the Sun*.
  - The shifts were exactly as Einstein calculated! (1.75 seconds )

    Extremely important

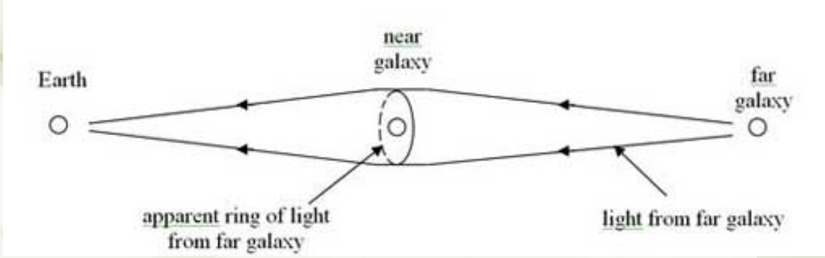
*Einstein predicted that the magnitude of the shift depended on how large the angle was between the sun and the background star and the mass of the sun

- The total eclipse of 29th May 1919 gave scientists the chance to test the theory for the first time. Eddington travelled to Príncipe to observe the eclipse and measure the apparent locations of stars near the Sun. Heavy clouds parted minutes before the eclipse and, with the Sun almost directly in front of them, the stars appeared to be shifted from their positions - direct evidence that our nearest star shapes the space around it.
- 'Without Eddington's clever experiment and the fortuitous timing of this total eclipse it might have taken ages before Einstein's theory of gravity, first proposed in 1915, was proven to be correct,' (Pedro Ferreira Oxford University)

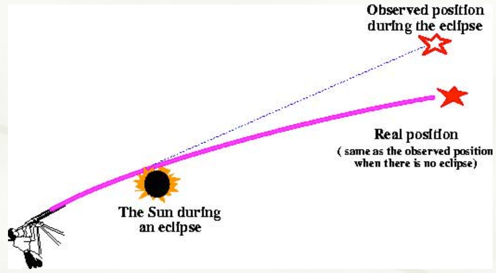



- Prior to this, the GR was untested – just an interesting, intelligent, idea.
- After Eddington, Einstein (and his science) was finally be taken seriously, and even the non-scientists of the world somehow found themselves standing in awe of the genius of this Swiss-German physicist.


- Can Gravity Bend Light?
- If light, does not have mass, how could it possibly be affected by gravity?
- After all, the force of gravity – – is directly dependent upon the mass of two objects.
- http://isaacmmcphee.suite101.com albert-einstein-and-bending-light-a43865#ixzz1nboyQEaH



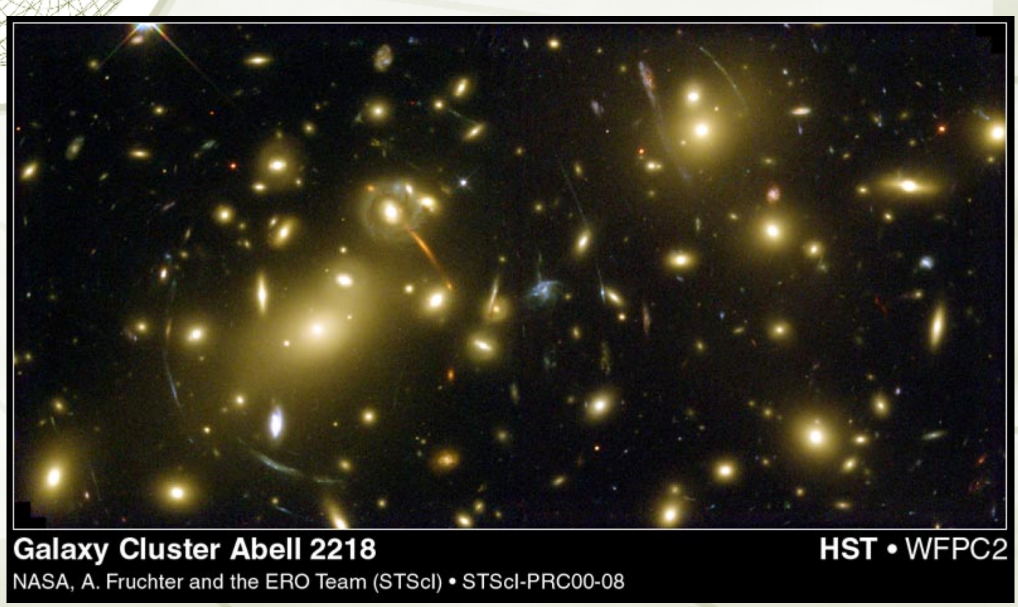

- Extraordinary claims require extraordinary evidence, and it's hard to think of something more extraordinary than the idea that massive objects warp space and time.

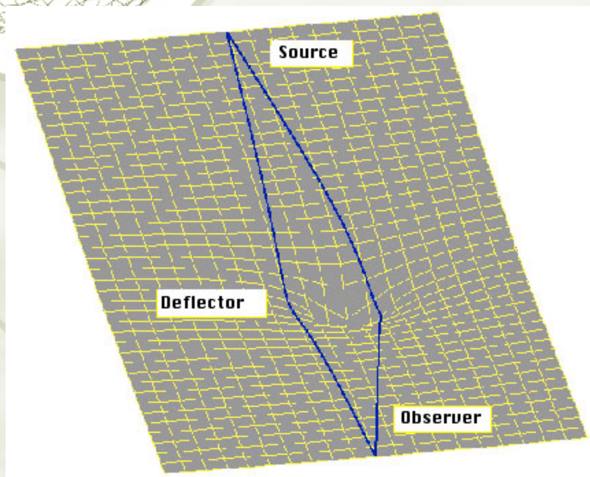



Angle is very small

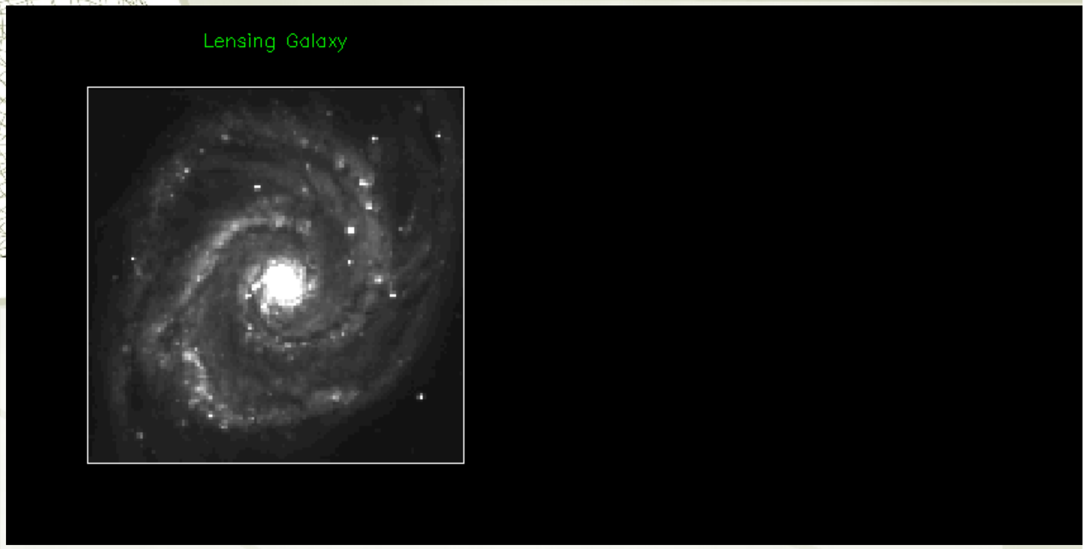
http://sunearthday.nasa.gov/2006/events/einstein.php

## *Also have light bending by distant galaxy clusters: "giant lenses" in the sky*






light 'bending' is a consequence of the bending of space itself

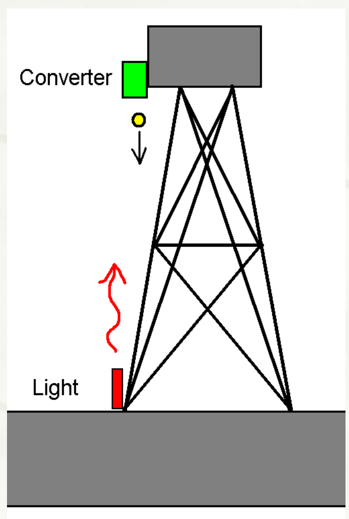



- http://spiff.rit.edu/classes/phys240/lectures/grav_lens/lens_large.gif

## *Remember the tower…*

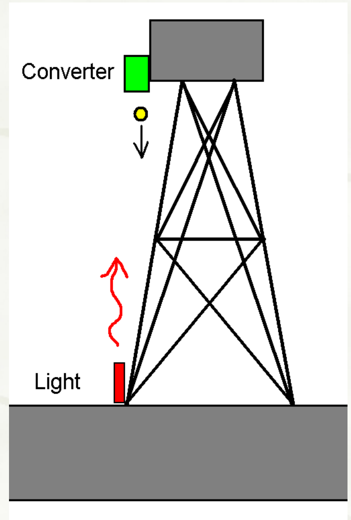
Light beam must lose energy as it climbs up

- So…frequency must decrease
- i.e., light is redshifted.
- Gravitational redshifting



## Remember the tower…

- Imagine a clock based on frequency of laser light…
  - 1 “tick” = time taken for fixed number of crests to pass
  - Gravitational redshifting slows down the clock.
  - **Clocks in gravitational fields must run slowly**

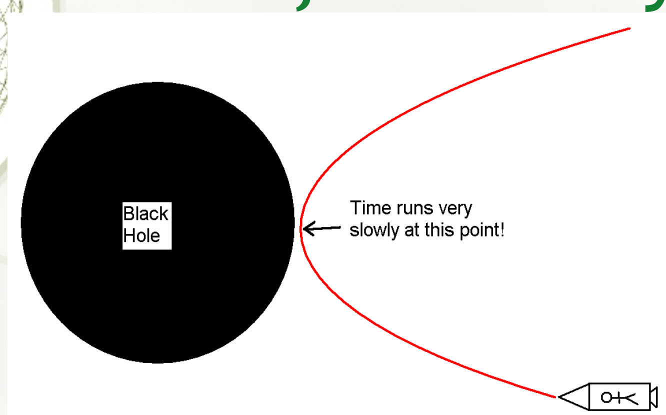



$$t_{grav} \approx \left(1 - \frac{GM}{c^2 r}\right) t_{space}$$

if gravitational field is "weak"



## How to live for a 1000 years!

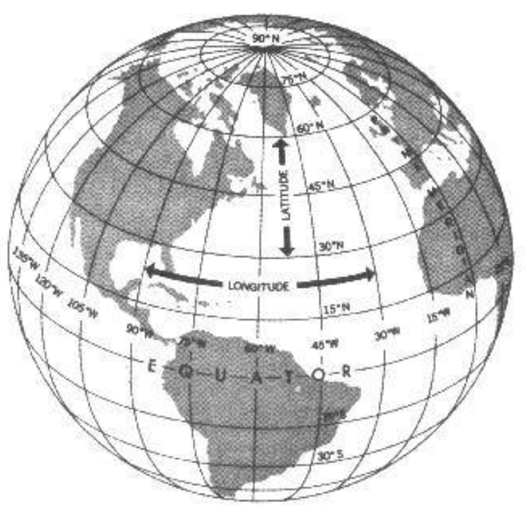



- Go where gravity is very strong!
- Observer on Earth would see astronaut’s clock running **very** slowly when close to black hole – astronaut would age very slowly.
- (In fact, there are other discomforts from of being near a black hole!)

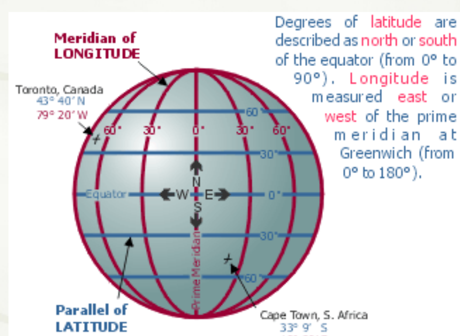
# Gravitational time dilation has practical importance!

- Global Positioning System (GPS)
  - System of satellites that emit timing signals
  - Detector on Earth receives signals
  - Can figure out position on Earth's surface by measuring time delay between signals from different satellite (light travel time gives distance to satellite)
  - Need to measure time of signal from satellite very well!
    - 10m positioning requires ~30ns time accuracy
  - Satellites are at varying heights; clocks run at varying rates
    - Satellite clocks drift by ~38us per day wrt Earth clocks!
- If GR effects were not included, computed GPS positions would drift from true position by **kilometers per day**!



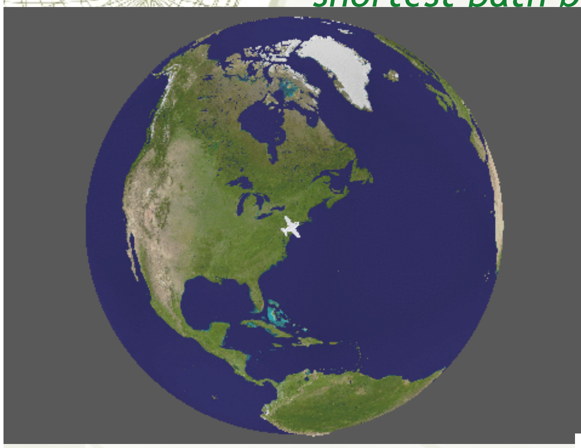



# On Globe…



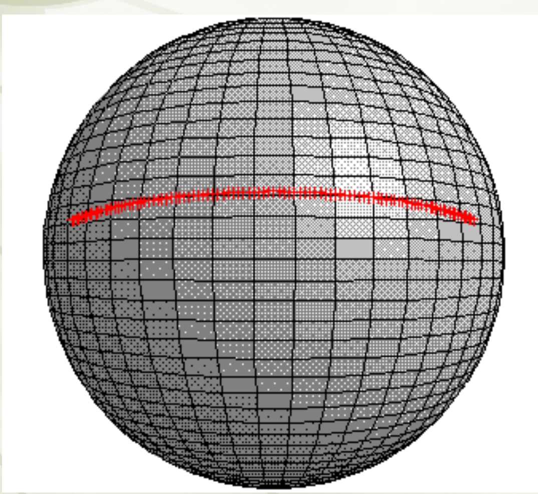

- Constant-longitude lines (meridians) are geodesics
- On the Earth, geodesics are Great Circles, the shortest distance between two points on the surface.
- Constant-latitude lines (parallels) are **not geodesics**

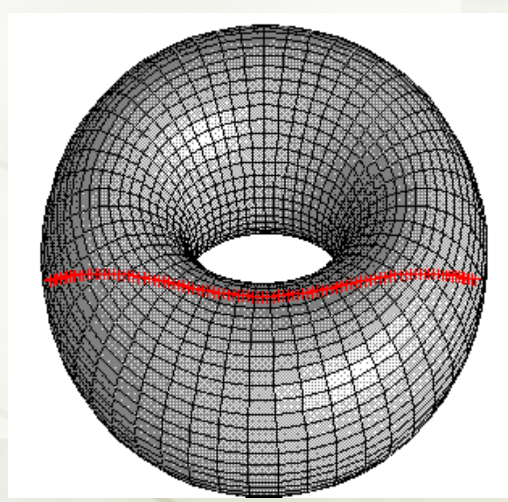
# *Shortest flight paths are geodesics-geodesics are defined to be the shortest path between points in the space.*





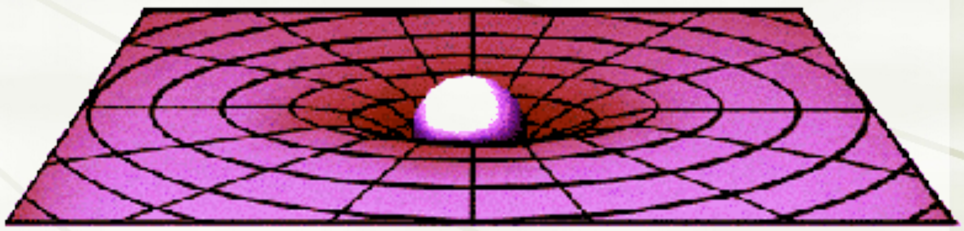
# *Geodesics on sphere and torus*





# How does matter "warp" space?

- Use two-dimensional space as an analogy: think of how rubber sheet is affected by weights
- Any weight causes sheet to sag locally
- Amount that sheet sags depends on how heavy weight is

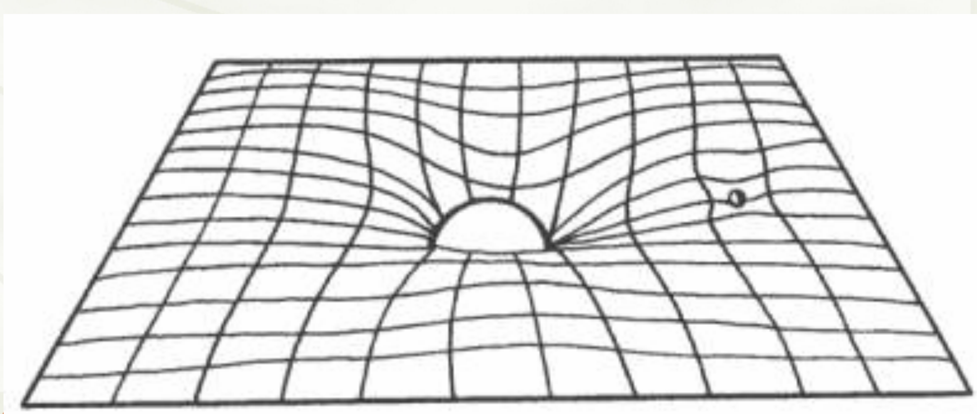


From web site of UCSD

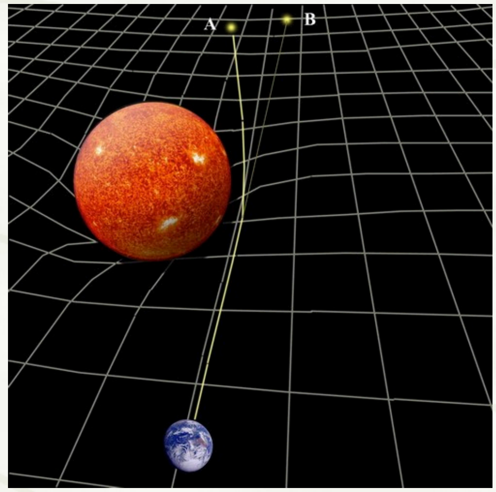


# Effect of matter on coordinates

- Lines that would be straight become curved (*to external observer*) when sheet is "weighted"



## *Is Light or Space 'Bent'?*




## *Next time…*

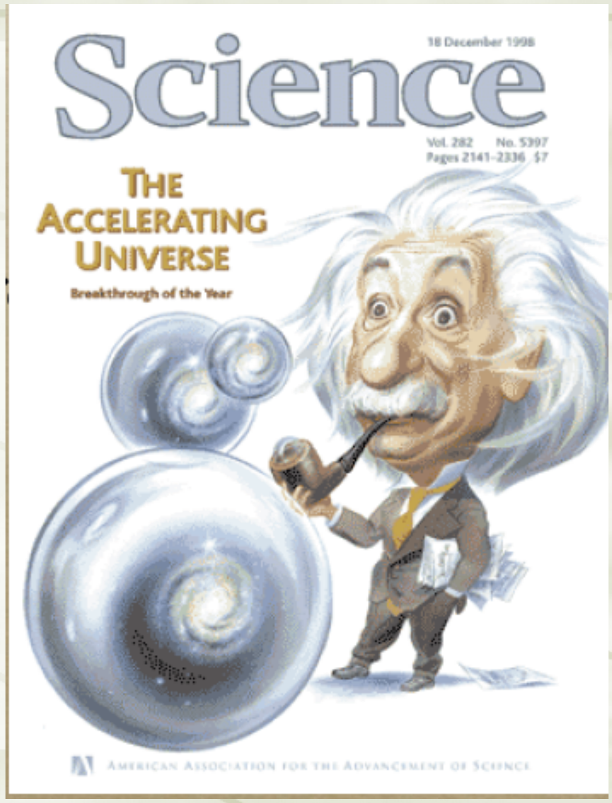
- More on General Relativity
  - Relativity Equations
  - Consequences of GR

Continue to read Chapter 8 of the book

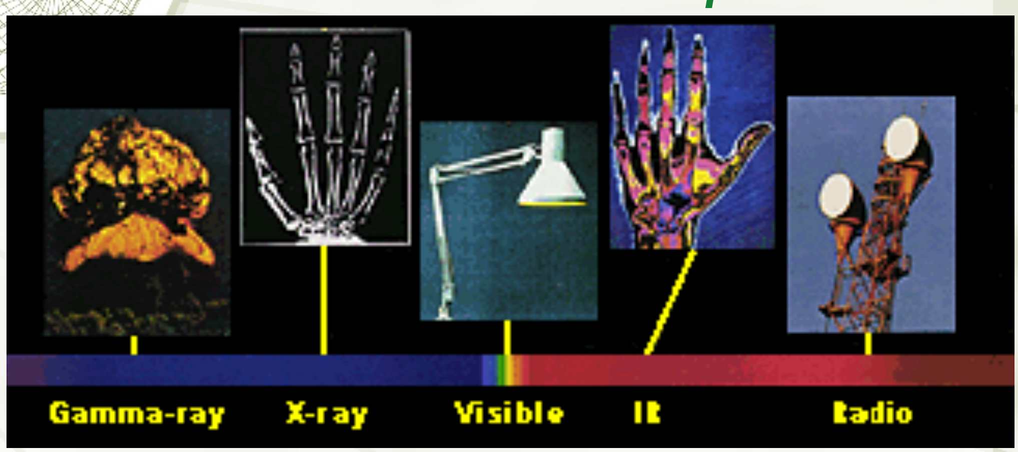
**Midterm on Thursday !**

## Nobel Prize in Physics 2011

Data from distant exploding stars called supernovae indicates, that the universe contains a large-scale repulsive force that is accelerating its expansion




## *The electromagnetic spectrum*




Small wavelength
**High Frequency**
**High energy**

Large wavelength
**Low frequency**
**Low energy**