Lecture 15:
Cosmological Principles

- The basic Cosmological Principles
- The geometry of the Universe
  - The scale factor R and curvature constant k
  - Comoving coordinates
- Einstein's initial solutions

Ch 10-11

I : BASIC COSMOLOGICAL ASSUMPTIONS

- 1915:
  - Einstein just completed theory of GR
  - Explains anomalous orbit of Mercury perfectly
  - Schwarzschild solution for black holes (1 month after publication of Einstein's paper!)
  - Einstein turns his attention to modeling the universe as a whole…apply his theory to the structure of the universe, he was dismayed to find that it predicted either an expanding or contracting universe--something entirely incompatible with the prevailing notion of a static universe.
- How to proceed… it's a horribly complex problem

- In his gravitational field equations, Einstein provided a compact mathematical tool that could describe the **general configuration of matter and space of the universe as a whole.**
- The existence of the curvature of space predicted in the equations was quickly checked (e.g bending of light by Sun)
- By the early 1920s most leading scientists agreed that Einstein's field equations could make a foundation for **cosmology.**
- The only problem was that finding a solution to these simple equations — that is, producing a model of the universe — was a *mathematical nightmare*.

http://www.aip.org/history/cosmology/ideas/expanding.htm

The First Solutions (other than the black Hole)

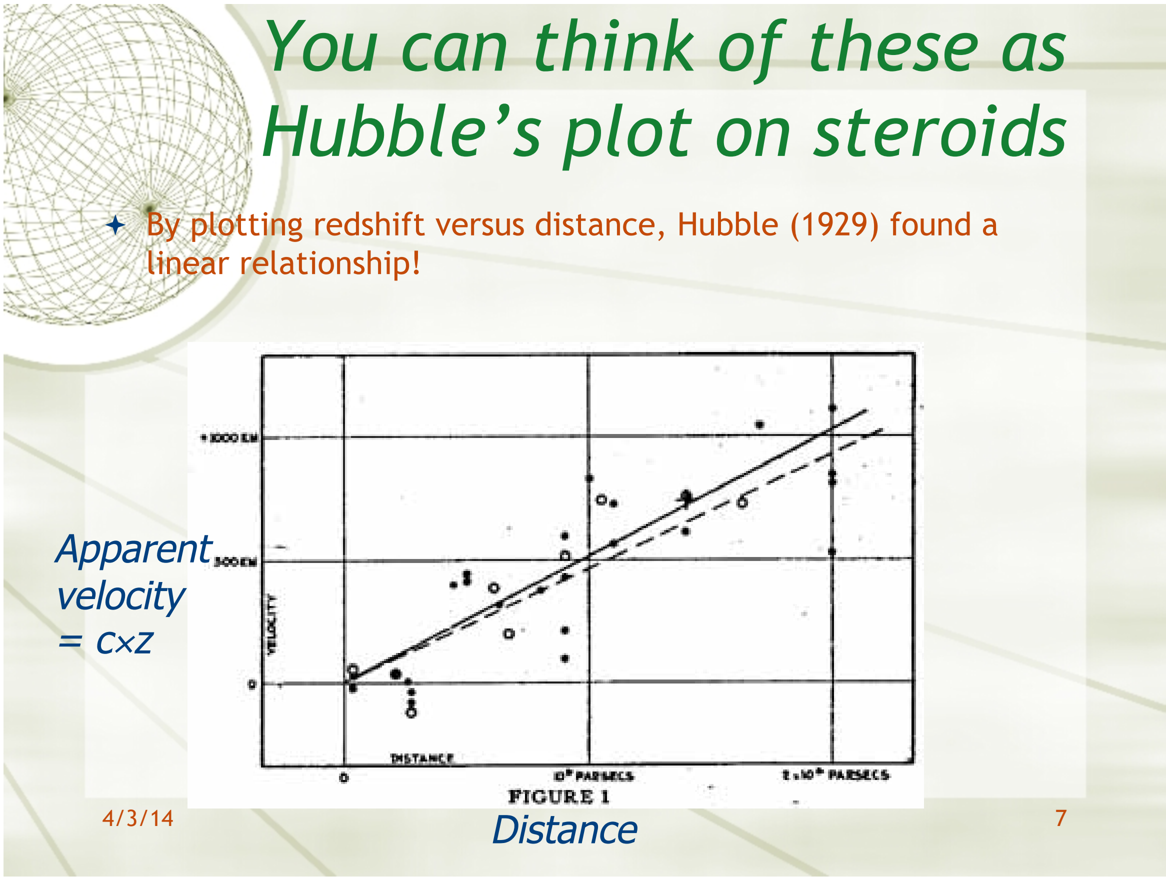
- Due to Einstein and deSitter (1917)
- De Sitter: Odd results:
  - Model was stable only if it contained no matter. Perhaps it could describe the real universe, if the density of matter was close enough to "zero".
  - Also an odd effect on light — the farther one went from the mathematical center (the origin of coordinates), the slower the frequency of light vibrations. That meant that the farther away an object was, the more the light coming from it would seem to have a reduced frequency (redshift -before Hubble !!!)
- Einstein's model
  - Likewise could not contain matter and be stable. The equations showed that if the universe was static at the outset, the gravitational attraction of the matter would make it all collapse in upon itself. That seemed ridiculous, for there was no reason to suppose that space was so unstable.

How to make progress…

- Proceed by ignoring details…
    - Imagine that all matter in universe is "smoothed" out
    - i.e., ignore details like stars and galaxies, but deal with a smooth distribution of matter
- Then make the following assumptions
    - Universe is **homogeneous** – every place in the universe has the same conditions as every other place, on average.
    - Universe is **isotropic** – there is no preferred direction in the universe, on average.

Observational evidence for homogeneity and isotropy

- Let's look into space… see how matter is distributed on large scales.
- "Redshift surveys"…
    - Make 3-d map of galaxy positions
    - Use redshift and Hubble's law to determine distance
    - look at class 6 (repeated here)



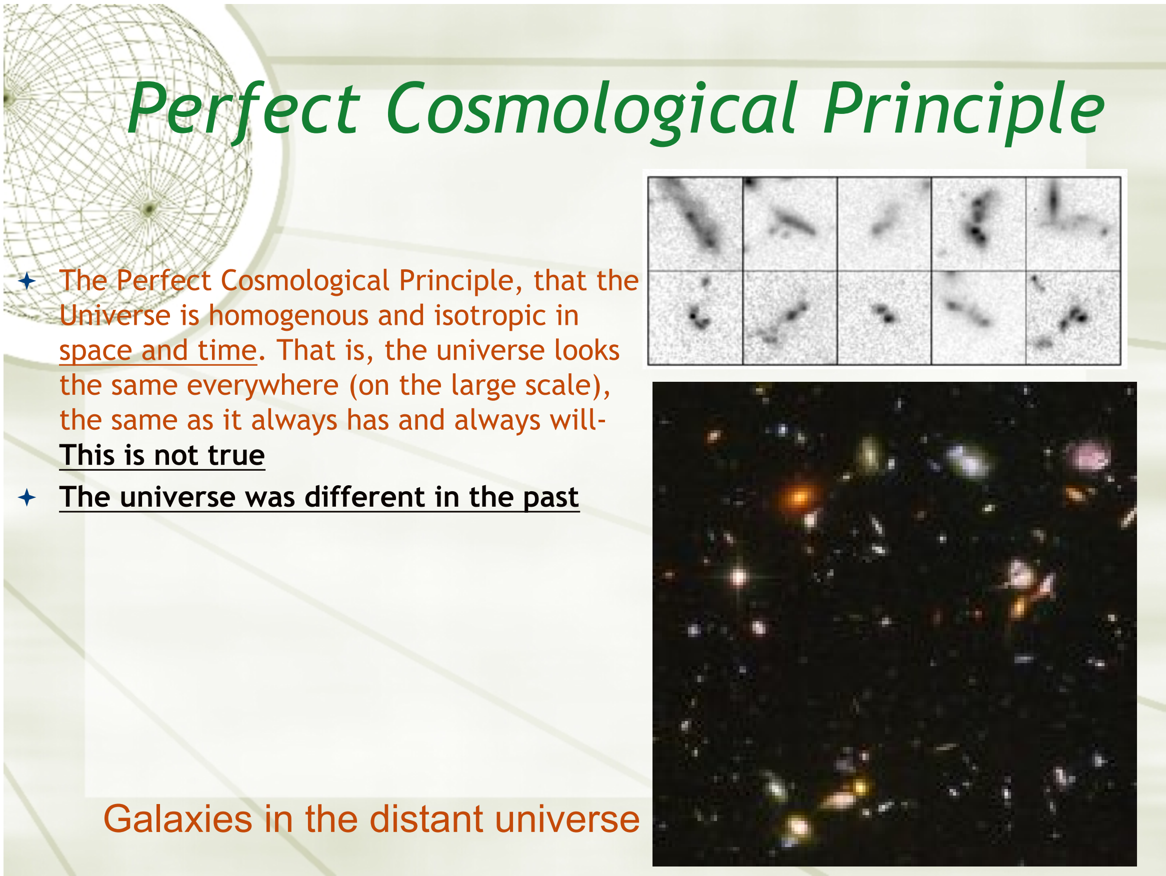
You can think of these as Hubble’s plot on steroids

- By plotting redshift versus distance, Hubble (1929) found a linear relationship!

FIGURE 1

Apparent velocity = c×z

Distance



Perfect Cosmological Principle

- The Perfect Cosmological Principle, that the Universe is homogenous and isotropic in <u>space and time</u>. That is, the universe looks the same everywhere (on the large scale), the same as it always has and always will- **<u>This is not true</u>**
- **<u>The universe was different in the past</u>**

Galaxies in the distant universe

How to Test Isotropy and (Homogenity)

- (i) the large scale spatial distribution of galaxies, which form a *randomly* tangled web of clusters and voids up to around 400 megaparsecs in width.
- ii) the distribution of radio galaxies, which are *randomly* distributed across the entire sky.
- (iii) the cosmic microwave background radiation, the relic radiation produced by the expansion and cooling of the early universe, constant temperature in **all** directions to one part in 10^5
- (iv) spatial distribution of gamma-ray bursts, objects at cosmological distances

Homogenity is very difficult to test since the universe is evolving- use consistency relations between distances and expansion rates: a bit messy to show

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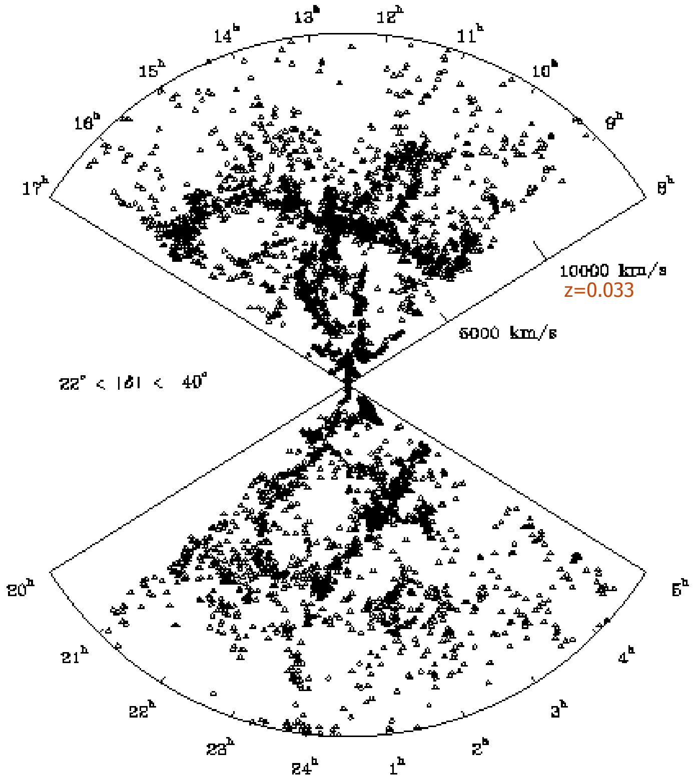
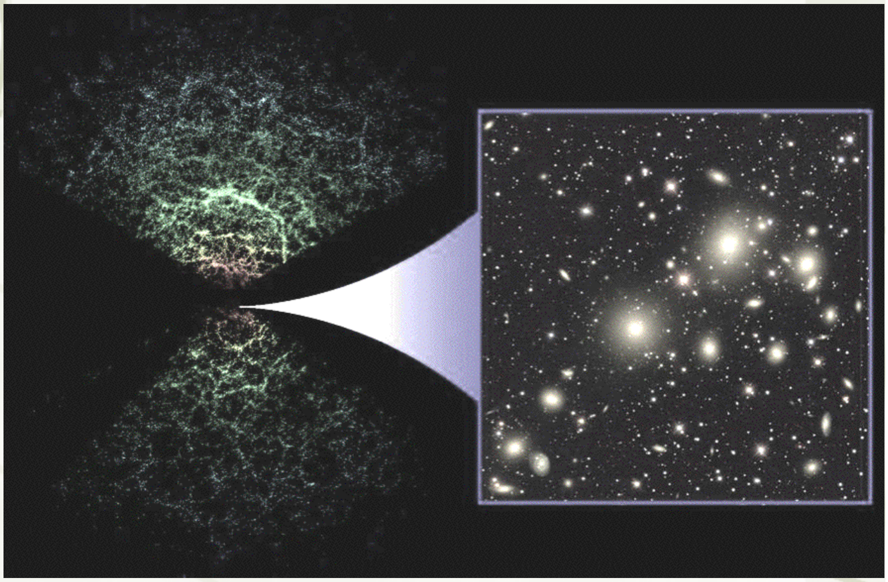


Each point is a bright galaxy

CfA redshift survey

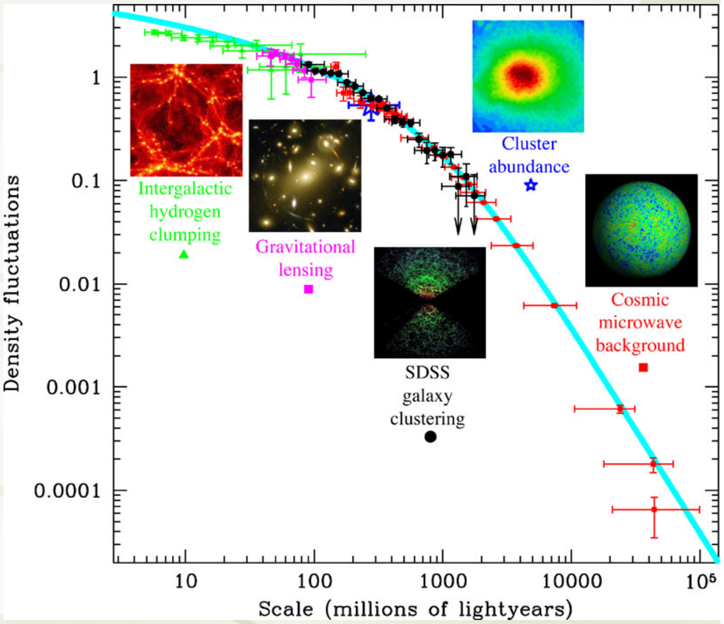
Large Scale distribution of normal galaxies

- On scales $<10^8$pc the universe is 'lumpy'- e.g. non-homogenous
- On larger scales it is homogenous- and isotropic



Sloan Digital Sky Survey- http://skyserver.sdss3.org/dr8/en/

- As one goes to larger scales the universe gets less lumpy (on average)



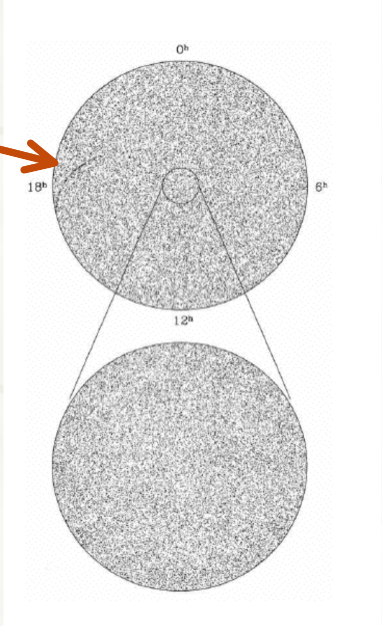
Lumpiness of universe

Tegmark 2004

size of box

Distribution of Radio Galaxies

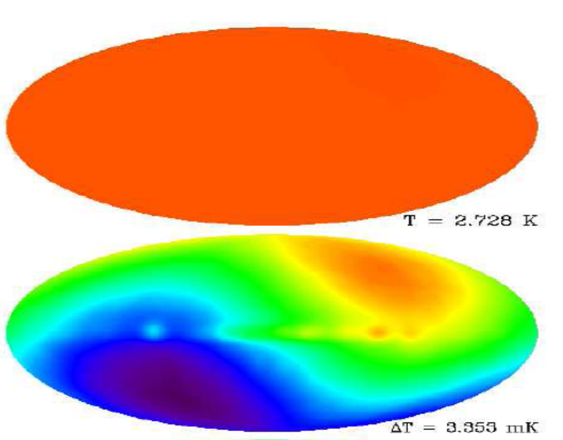
- Position of 40,000 brightest radio sources in northern sky
- 40,000 brightest sources near celestial north pole
- Mean distance between sources is ~10^7 pc- probe very large distances up to when universe was 2 billion years old



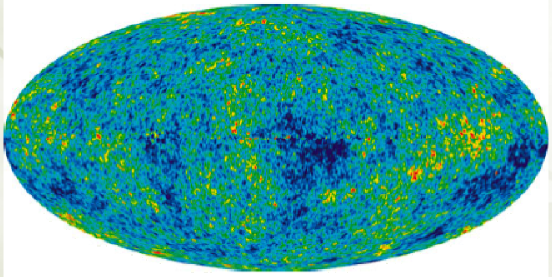
Condon 1999 PNAS97,4756

Microwave Background- relic of the big bang

- CMB probes whole universe at 300,000 yrs after Big Bang
  - isotropic to $\Delta T/T \sim 10^{-5}$



Dipole term- our motion with respect to CMB frame



CMB temperature map with dipole removed from WMAP showing anisotropies at level of 10^{-5}

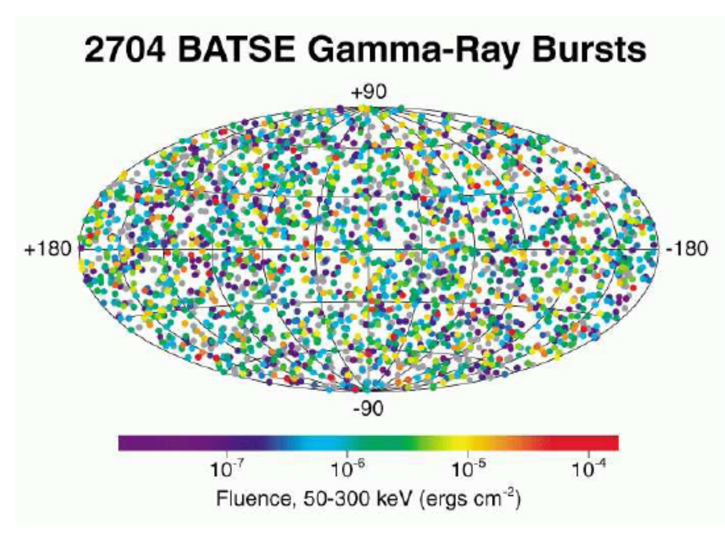
Dipole and Doppler Shift

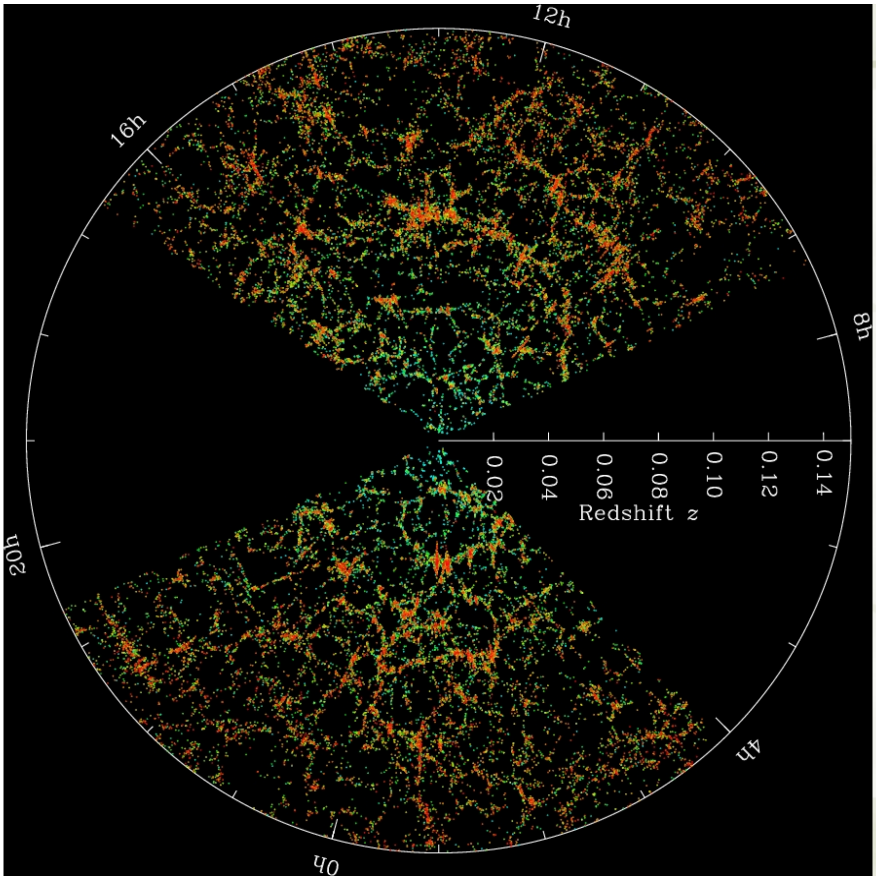
- The CMB (which we will discuss in detail later) defines a frame of reference
- We are moving at some speed and direction with respect to that frame
- This changes the apparent brightness and temperature $T(\theta)$ of the CMB with a particular pattern- a dipole
- $\beta=v/c$, θ=angle our velocity makes with respect to the CMB frame

$$T(\theta) = T_0(1-\beta^2)^{1/2}/(1-\beta\cos\theta)$$

Gamma-Ray Bursts

- Gamma-ray bursts come from very far away, on average
- The mean redshift is z~1 (half the 'age' of the universe)- very isotropic

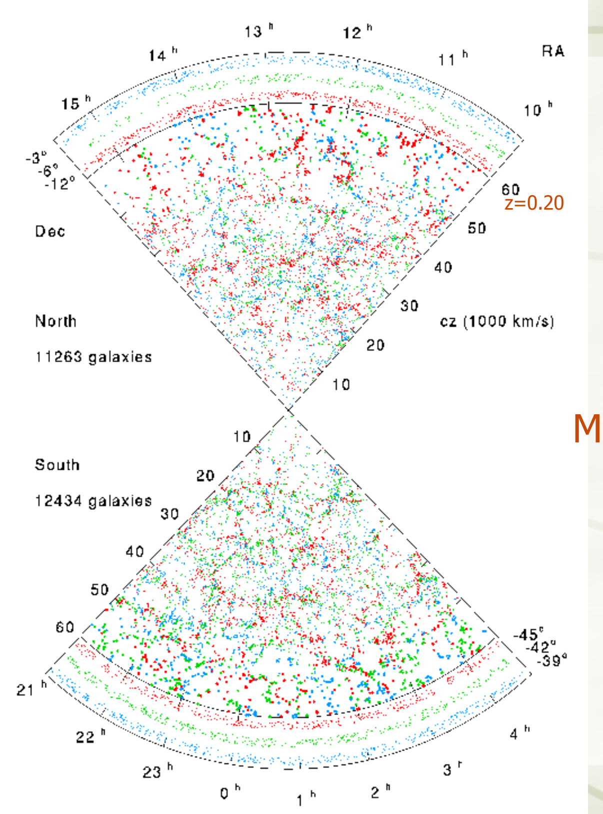




Sloan Digital Sky Survey

Galaxies color coded by the age of their stars

http://www.sdss.org



z=0.20

Las Campanas Redshift survey

Max distance ~ 10^9 pc !

Homogeneous?

- There is clearly large-scale structure
  - Filaments, clumps
  - Voids and bubbles
- But, homogeneous on **very** large-scales.
- So, we have the…
- **The Generalized Copernican Principle…** there are no special points in space within the Universe. The Universe has no special place (like a center)!
- These ideas are collectively called the **Cosmological Principles.**

II : POSSIBLE GEOMETRIES FOR THE UNIVERSE

- The Cosmological Principles constrain the possible geometries for the space-time that describes Universe on large scales.
- The problem at hand - to find curved 4-d space-times which are both homogeneous and isotropic…
- Early in 1930, de Sitter admitted that neither his nor Einstein's solution to the field equations could represent the observed universe.
- Eddington next raised "one puzzling question." Why should there be only these two solutions? Answering his own question, Eddington supposed that the trouble was that people had only looked for static solutions.
- Solution to this mathematical problem is the Friedmann-LeMaitre-Robertson-Walker (FLRW) metric.

See http://www.aip.org/history/cosmology/ideas/expanding.htm) for the interesting history

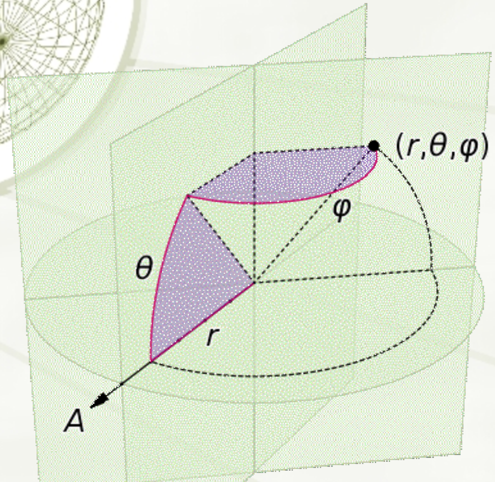
FLRW Solution

Exact solution of Einstein's field equations of general relativity; it describes a homogeneous, isotropic expanding or contracting universe

The general form of the solution follows from
homogeneity and isotropy

Einstein's field equations are only needed to derive the scale factor of the universe as a function of time (R(t)).

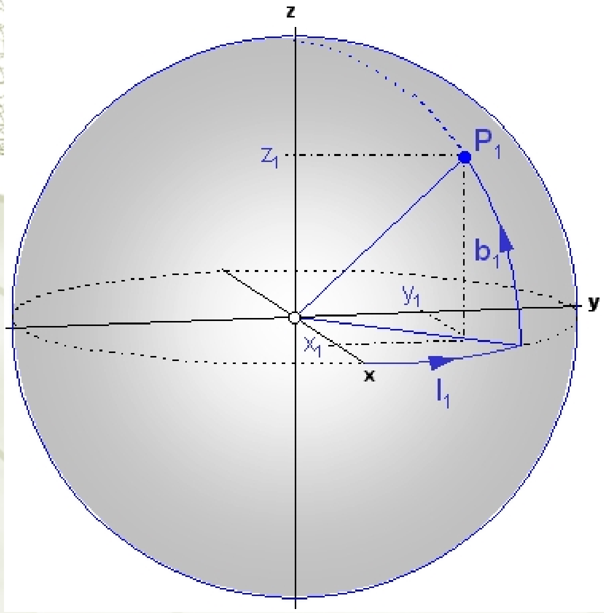
Coordinates



Think of a sphere of size r; θ,ϕ are the 'latitude and longitude'

- Coordinates are just recipes to get from here (the origin) to there.
- Spherical coordinates tell you how to get there using one distance and two angles.
- The vector **r** tells how 'far' (the scale of the system)

Coordinates



- Coordinates are just recipes to get from here (the origin) to there.
- Cartesian (rectangular) coordinates use three distances to do the same.

Friedmann-Robertson-Walker metric

- **A “metric” describes how the space-time intervals relate to local changes in the coordinates**
- We are already familiar (lecture 8 and 10) with the formula for the space-time interval in flat space (now generalized for arbitrary space coordinate scale factor *R*):

$$\Delta s^2 = (c\Delta t)^2 - R^2\left(\Delta x^2 + \Delta y^2 + \Delta z^2\right)$$

- In spherical coordinates (radius and angles) instead of x,y,z, this is written (text eq 10.6):

$$\Delta s^2 = (c\Delta t)^2 - R^2\left(\Delta r^2 + \Delta\theta^2 + \sin^2\theta(\Delta\varphi)^2\right)$$

- General solution for isotropic, homogeneous *curved* space is (k is related to the type of curvature (next slide)):

$$\Delta s^2 = (c\Delta t)^2 - R^2\left(\frac{\Delta r^2}{1-\underline{k}r^2} + \Delta\theta^2 + \sin^2\theta(\Delta\varphi)^2\right)$$

- In general the scale factor is a function of time, i.e. *R(t)- remember the universe is expanding!*

- The spacetime being modeled by this equation can be neatly separated into time and space, so we can talk of this spacetime as representing the evolution of space in time.

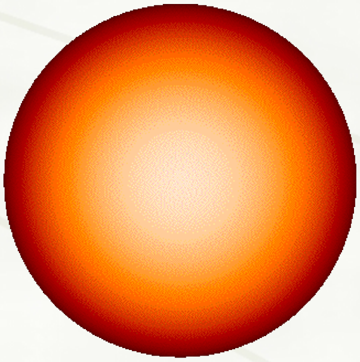
- The space part of this spacetime is homogeneous (looks the same at any point in a given direction) and isotropic (looks the same in any direction from a given point). This is an abstract ideal approximation to the Universe, but it's one that has worked extremely well from an observational point of view

- R(t) indicates the characteristic curvature of space-notice that for the universe as a whole R does not depend on x,y,z.

Curvature in the FRW metric

- Three possible cases…k is a constant representing the curvature of the space.

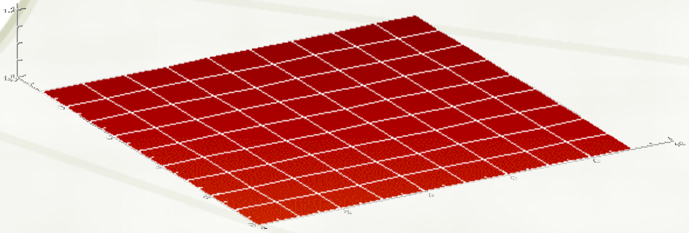
Spherical spaces (closed) k=1 (closed)



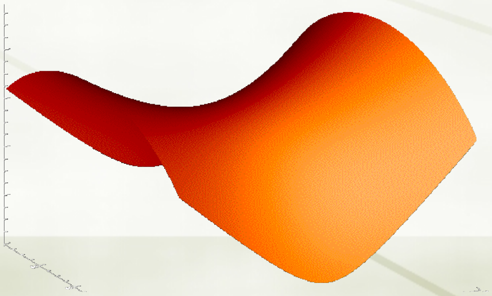
http://www.superstringtheory.com/cosmo/cosmo2a.html

Curvature in the FRW metric

Flat spaces (open)k=0 (flat)

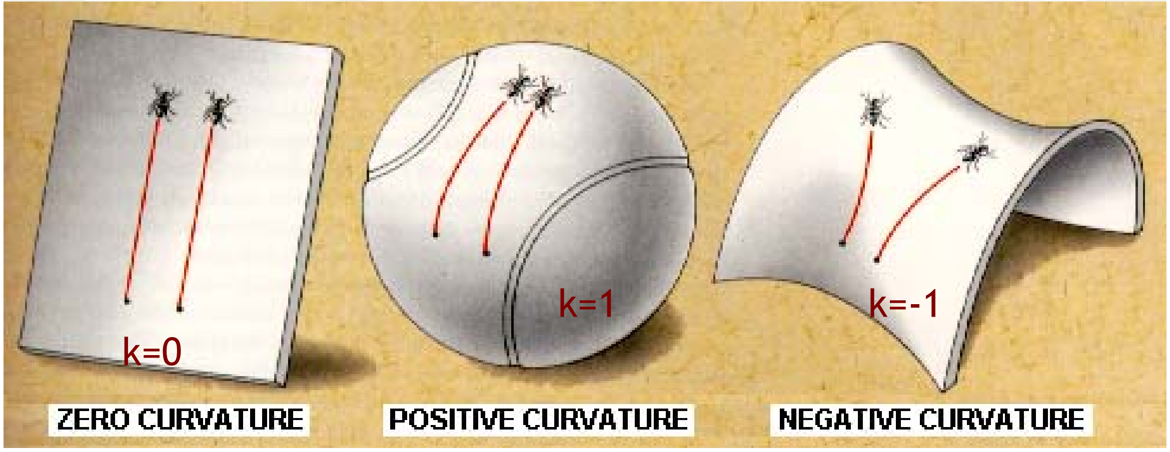


Hyperbolic spaces (k=-1 open)



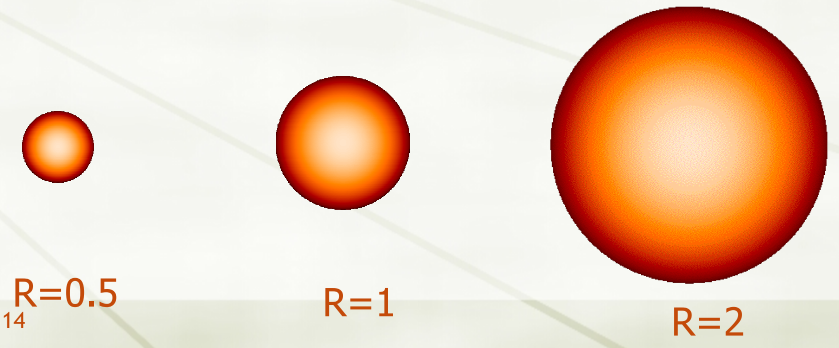
Curvature of Universe

- 3 types of general shapes: flat surface at the left :zero curvature, the spherical surface : positive curvature, and the saddle-shaped surface : negative curvature.
- GR tells us that each of these possibilities is tied to the amount of mass (and thus to the total strength of gravitation) in the universe, and each implies a different past and future for the universe.



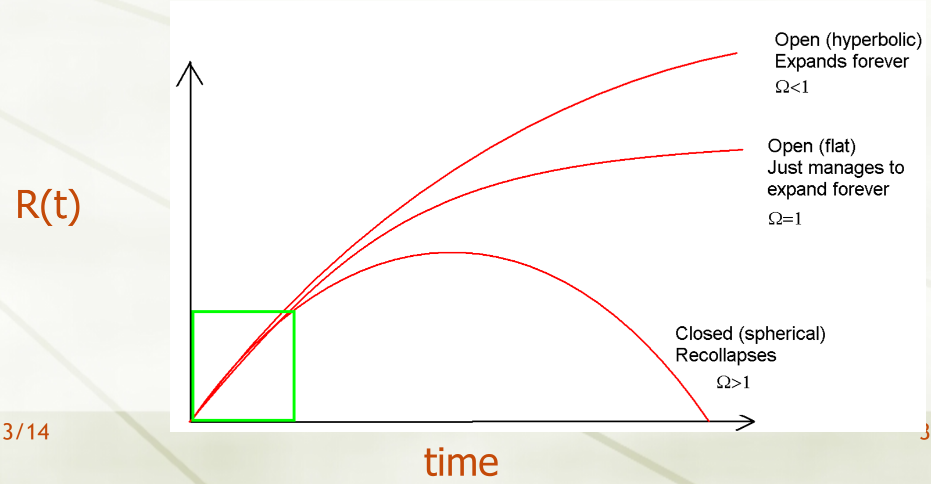
Meaning of the scale factor, R(t)

- Scale factor, *R(t)*, is a central concept!
  - *R(t)* tells you how "big" the space is…
  - Allows you to talk about changing the size of the space (expansion and contraction of the Universe - even if the Universe is infinite).
- Simplest example is spherical case
  - Scale factor is just the radius of the sphere



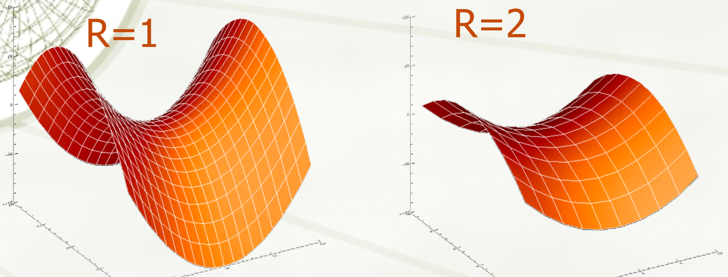
Scale Factor

- Whether case with *k=-1,0, or 1* applies depends on the ratio of the actual density to the "critical" density, Ω
- Properties of standard model solutions:
  - k=-1, Ω<1 expands forever
  - k=0, Ω=1 "just barely" expands forever
  - k=+1, Ω>1 expands to a maximum radius and then recollapses

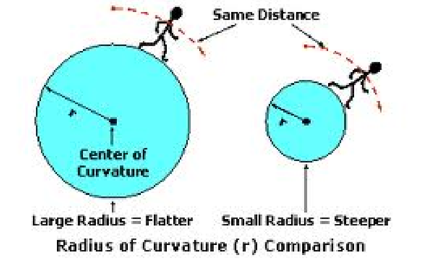


The scale factor R

- What about k=-1 (hyperbolic) universe?
  - Scale factor gives “radius of curvature”- how highly curved the saddle is



- For k=0 universe, there is no curvature… shape is unchanged as universe changes its scale (stretching a flat rubber sheet)

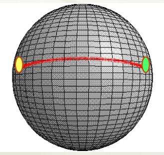


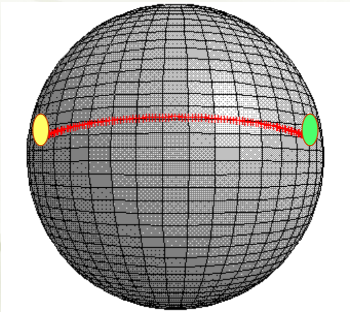
Radius of Curvature (r) Comparison

Smaller circles bend more sharply, and hence have higher curvature.

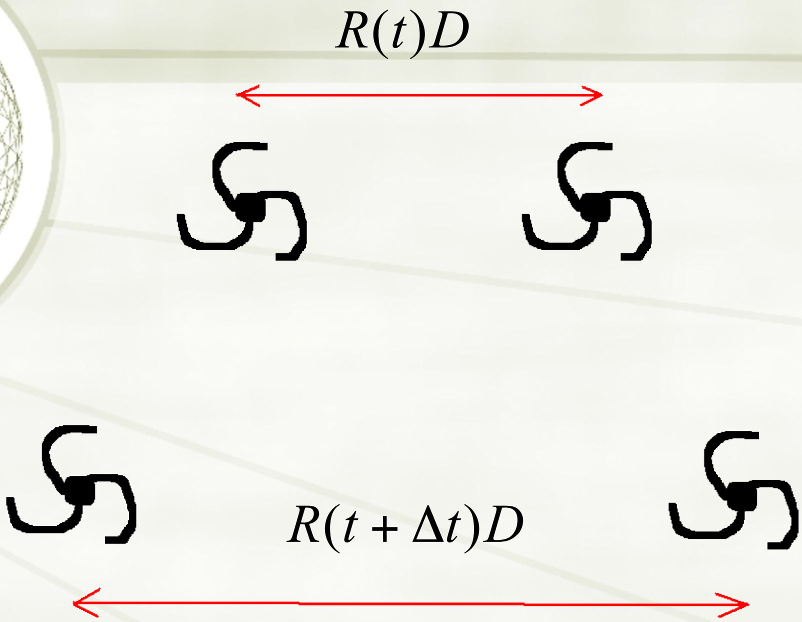
Co-moving coordinates.

- What do the coordinates x,y,z or r,θ,φ represent?
- They are positions of a body (e.g. a galaxy) in the space that describes the Universe
- Thus, Δx can represent the separation between two galaxies
- But what if *the size of the space itself changes*?
- e.g. suppose space is sphere, and has a grid of coordinates on surface, with two points at a given latitudes and longitudes θ_1,φ_1 and θ_2,φ_2
- If the sphere expands, the two points would have the same latitudes and longitudes as before, but *distance between them would increase*
- *Coordinates defined this way are called* *comoving coordinates*





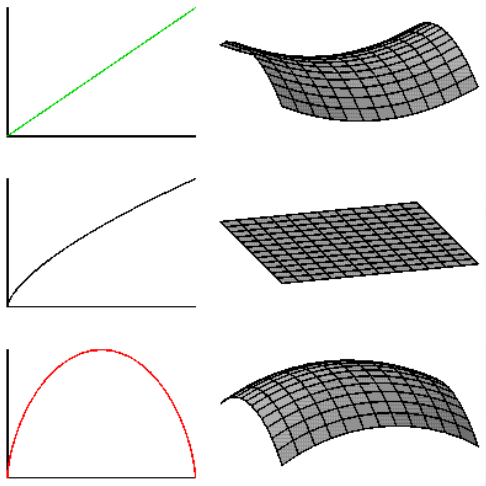
- If a galaxy remains at rest relative to the overall space (i.e. with respect to the average positions of everything else in space) then it has fixed co-moving coordinates.
- Consider two galaxies that have fixed co-moving coordinates.
  - Let's define a "co-moving" distance *D*
  - Then, the real (proper) distance between the galaxies is $d = R(t) \times D$; *where R(t) is describing the expansion of space*



R(t) is the (time-dependent) factor that relates the proper distance (which can change over time, unlike the comoving distance which is constant) for a pair of objects moving with the Hubble flow in an expanding or contracting FLRW universe

Shape of the Universe and Scale Factor

- open universe- scale factor increases with time
- flat universe- rate of change of scale factor slows down with time
- 'closed' universe- scale factor changes sign with time



- applet to solve Friedman eq can be found at
- http://www.astro.virginia.edu/~jh8h/Foundations/Foundations_1/Friedmann.html

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Cosmological redshift, z and Hubble Constant H

- As galaxies move apart, z describes a Doppler shift from the expansion velocity
- More fundamentally, it comes from the change in metric scaling, *R*(*t*)
- It's more like the gravitational redshift than a Doppler shift (nothing is truly moving)
- Since it's relativistic, it affects time as well as length
- Hubble's law v=Hd (v is velocity and d is distance);

H has the units of Km/s/Mpc (or in standard units Km/s/Km or 1/s- *inverse time*)

- 1/H is a time (Hubble time)-estimate of the age of the universe and c/H is length (size of universe). If H=50 $t_H=2x10^{10}$ yrs and c/H=20x10^9 ly

III : THE DYNAMICS OF THE UNIVERSE – EINSTEIN'S MODEL

- Back to Einstein's equations of GR

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

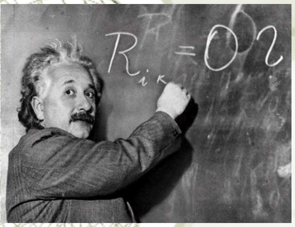
"G" describes the space-time curvature (including its dependence with time) of Universe… here's where we plug in the FRW geometries.

"T" describes the matter content of the Universe. Here's where we tell the equations that the Universe is homogeneous and isotropic.

- Einstein plugged the three homogeneous/isotropic cases of the FRW metric formula into his equations of GR to see what would happen…
- Einstein found…
  - That, for a static universe (*R(t)=constant*), only the **spherical** case worked as a solution to his equations
  - If the sphere started off static, it would rapidly start collapsing (since gravity attracts)
  - The only way to prevent collapse was for the universe to start off expanding… **there would then be a phase of expansion followed by a phase of collapse**

A bit of scientific sociology

- So… Einstein could have used this to **predict** that the universe must be either expanding or contracting!
- … but this was **before** Hubble discovered expanding universe (more soon!)– everybody thought that universe was static (neither expanding nor contracting).
- So instead, Einstein modified his GR equations!
  - Essentially added a *repulsive* component of gravity
  - New term called **"Cosmological Constant,"** Λ
  - Could make his spherical universe remain static
  - BUT, it was unstable… a fine balance of opposing forces. Slightest push could make it expand violently or collapse horribly.



A stroke of genius?

- Soon after, Hubble discovered that the universe **was** expanding!
- Einstein called the Cosmological Constant "Greatest Blunder of My Life"!
- ….but very recent work suggests that he may have been right (more later!)

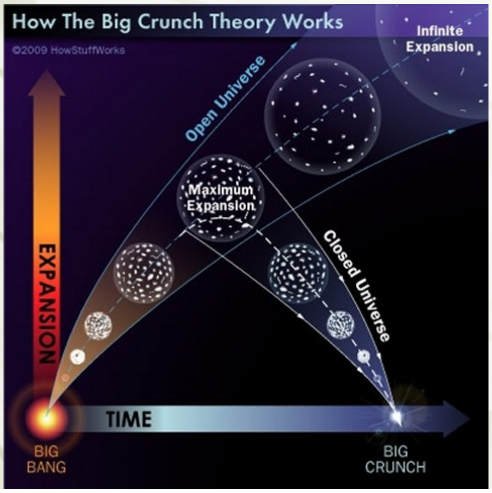
- What determines whether a Universe is open or closed ?(In a closed universe gravity eventually stops the expansion of the universe, after which it starts to contract)
- For a closed Universe, the total energy density (mass+energy)* in the Universe has to be greater than the value that gives a flat Universe, called the critical density

* excludes vacuum energy from cosmological constant (http://www.superstringtheory.com/cosmo/cosmo21.html)

- The 3 Possibilities in a $\Lambda=0$ Universe- Relationship of total mass to curvature
- If space has negative curvature, there is insufficient mass to cause the expansion of the universe to stop-the universe has no bounds, and will expand forever- an open universe (k=-1).
- If space is flat, there is <u>exactly</u> enough mass to cause the expansion to stop, but only after an infinite amount of time-the universe has no bounds and will also expand forever, but the rate of expansion will gradually approach zero after an infinite amount of time. This "flat universe" is called a Euclidian universe (high school geometry)(k=0)

Historical solutions without a cosmological constant

- If space has positive curvature, there is enough mass to stop the expansion of the universe. The universe in this case is not infinite, but it has no end (just as the area on the surface of a sphere is not infinite but there is no point on the sphere that could be called the "end"). The expansion will eventually stop and turn into a contraction.
    - Thus, at some point in the future the galaxies will stop receding from each other and begin approaching each other as the universe collapses on itself. This is called a closed universe(k=+1)



http://www.physicsoftheuniverse.com/topics_bigbang_bigcrunch.html

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