Class 2 : Newton's Laws (in three acts)

ASTR350 Black Holes (Spring 2020)
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This class

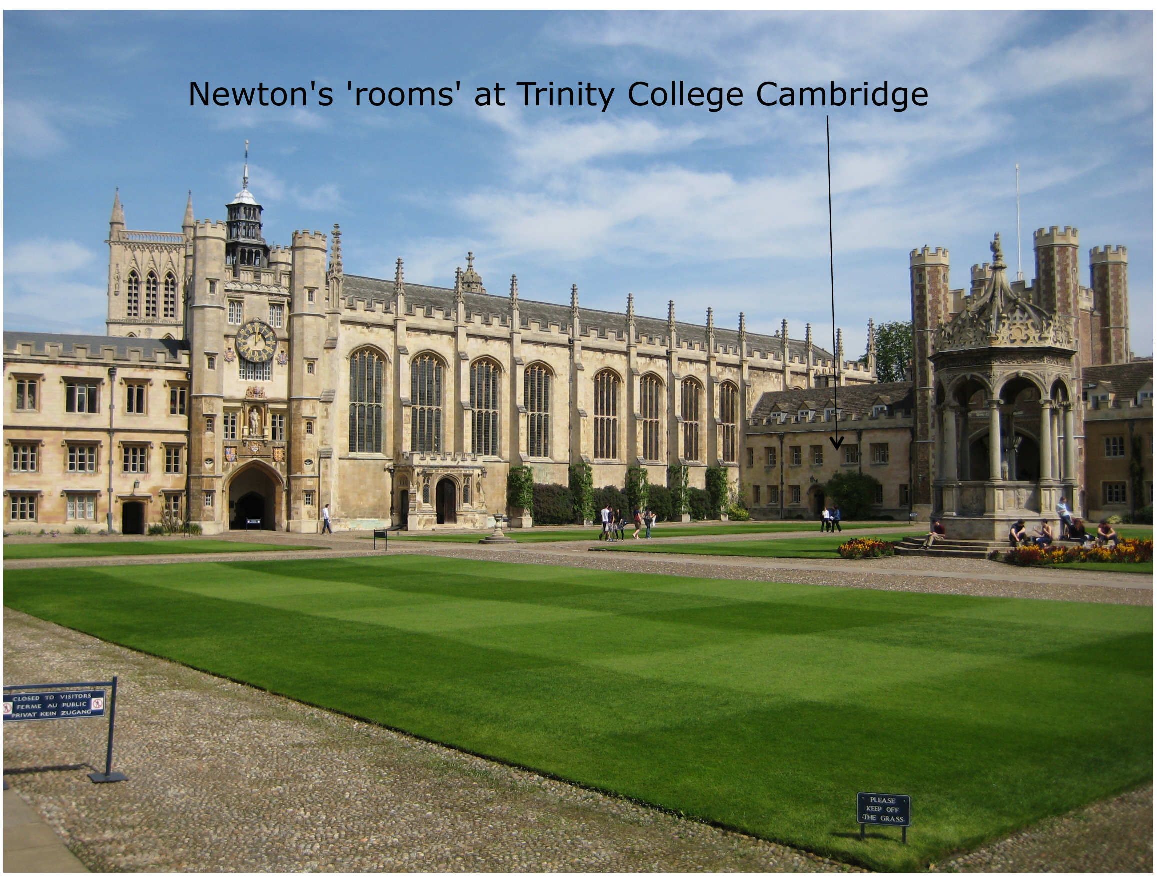
- A bit about Newton
- Act I : Newton's Laws, the "Classic" version
- Act II : Newton's Laws in terms of momentum
- Act III : Newton's Laws from symmetry

Isaac Newton in 1689, by Sir Godfrey Kneller.

Father of modern physics and cosmology

Isaac Newton (1643-1727)

- Attended Cambridge University, originally intending to study law, but reading Kepler, Galileo, Descartes
- Began to study mathematics in 1663
- While Cambridge was closed due to plague (1665-1667), Newton went home and
  - began to work out foundations of calculus
  - realized (contrary to Aristotle) that white light is not a single entity, but composed of many colors
  - began to formulate laws of motion and law of gravity
- Became professor of mathematics starting in 1669 (age 27!)
- Worked in optics, publishing “Opticks” (1704)
  - invented reflecting telescope
  - showed color spectrum from prism recombines into white light with a second prism
  - analyzed diffraction phenomenon

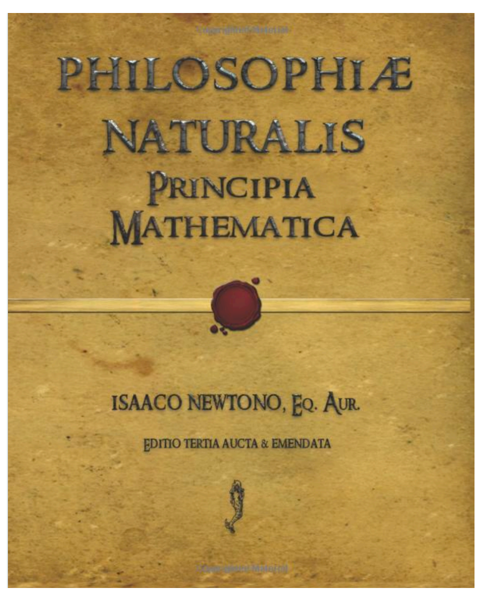


Newton's history, cont.

- In 1687, published *Philosophiae naturalis principia mathematica, or "Principia"*
  - publication was prompted (and paid for) by Halley (Comet fame)
  - partly in response to claim by Hooke that he could prove gravity obeyed inverse-square law
  - included proof that inverse square law predicts planetary orbits are ellipses
  - generalized Sun's gravity law to universe law of gravitation: ***all matter attracts all other matter with a force proportional to the product of their masses and inversely proportional to the square of the distance between them***
  - many other applications, including tides, precession, etc.
  - laid out general physics of mechanics -- laws of motion
  - showed that Kepler's laws follow from more fundamental laws

Newton's history, cont.

- The *Principia* is recognized as the greatest scientific book ever written! (available from Amazon !)
- Retired from research in 1693, becoming active in politics and government and was recognized in his lifetime (first scientist to be knighted)
  - (see https://www.britannica.com/biography/Isaac-Newton/Career)



Newton’s profound perspective

- Newton formulated a **universal** theory of motion and gravity
  - ***Same laws of physics operate anywhere and anytime in the Universe***
- Helped spur the Age of Enlightenment

ACT I – “CLASSIC” DISCUSSION

- *Newtons 3 Laws In The Original Latin*
- ♠ Lex I: Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.
  - [An object at rest will remain at rest unless acted upon by an external and unbalanced force . An object in motion will remain in motion unless acted upon by an external and unbalanced force]
- ♠ Lex II: Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.
  - [The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the same direction]
- ♠ Lex III: Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi.
  - [For every action there is an equal and opposite reaction]

I : Newton’s laws of motion

- **<u>Newton’s first law(</u>**The Law of Inertia*) **: If a body is not acted upon by any forces, then its velocity remains constant**
- Notes
  - Remember that velocity is a <u>vector</u> quantity (it has direction as well as magnitude)
  - This law sweeps away the idea that “being at rest” is a natural state… this was a major change of thinking (originated with Galileo)!

*Inertia is the resistance of any physical object to a change in its state of motion or rest, or the tendency of an object to resist any change in its motion

I : Newton's laws of motion

- "Velocity" is a vector
  Thus Newton's "constant velocity" implies both constant speed and constant direction (and also includes the case of zero speed, or no motion).
- Galileo : based on his concept of inertia, it is impossible to tell the difference between a moving object and a stationary one without some outside reference to compare it against.
- This ultimately led to Einstein's development of the theory of Special Relativity.

- **Newton's second law : If a body of mass M is acted upon by a force F, then its acceleration a is given by F=Ma**
- Notes
  - *Remember that both **F** and **a** are vectors* but mass is not
  - This law defines the "inertial mass" as the degree to which a body resists being accelerated by a force

Define acceleration =change of velocity/time

2^{nd} Law

- This is the most powerful of Newton's three Laws, - it allows quantitative calculations of dynamics: how do velocities change when forces are applied.
- According to Newton, a force causes only a change in velocity (an acceleration).

https://www.wisc-online.com/learn/general-education/technical-physics/tp1917/newtons-second-law-of-motion---video

Newton's second law- Again

- another way of saying the 2nd law is that force = rate of change of momentum
- p=momentum=mv
- F= dp/dt=m(dv/dt)=ma
- derivative dp/dt ~ change in momentum/unit change in time or "Δp/Δt"

What is Mass?

1) **Mass** is a measure of the amount of matter in an object

2) **Mass** is a measure of the amount of energy in an object (Einstein)

3) The degree to which a body resists being accelerated by a force

Weight is NOT the same as mass, weight depends on the gravitational field the mass is in.

An object on the Moon weighs less than it does on Earth because of the lower gravity, but has the same mass. This is because weight is a force, while mass is the property that (along with gravity) determines the strength of this force.

Tesla model-S
"0-60mph in 2.8s"

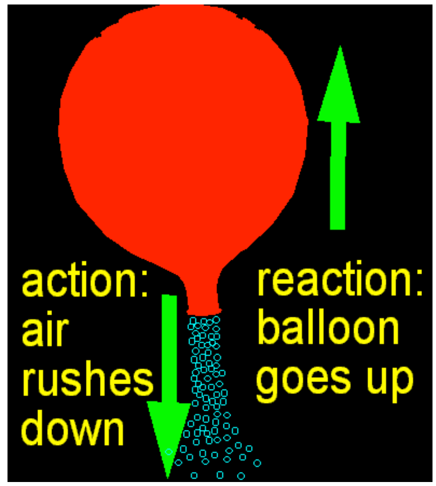
caranddriver.com

2^{nd} Law

- **Newton's third law - If a body A exerts a force $\underline{F}$ on body B, then body B exerts a force $-\underline{F}$ on body A**
- Notes
  - This is the law of "equal and opposite reaction"
  - We will see later that this law is closely tied to conservation of momentum

Third Law

- **If a body A exerts a force F on body B, then body B exerts a force -F on body A**
- Notes
  - This is the law of "equal and opposite reaction"
  - We will see later that this law is closely tied to conservation of momentum

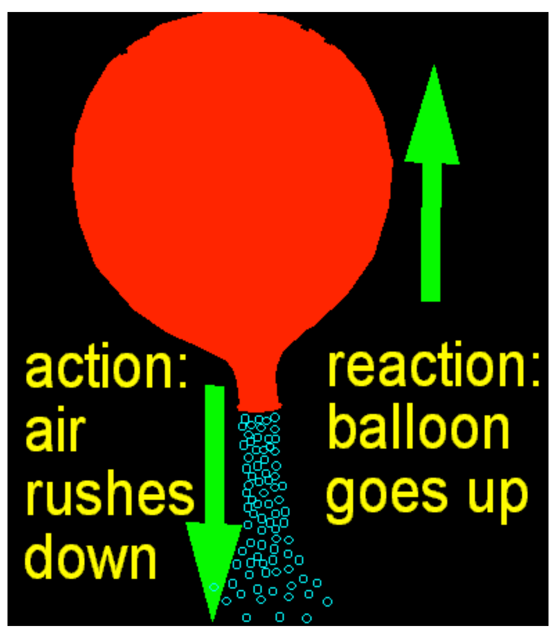


Shuttle take off!

https://www.youtube.com/watch?v=2aCOyOvOw5c

Third Law

If a body A exerts a force F on body B, then body B exerts a force -F on body A



Notes
This is the law of "equal and opposite reaction"

This law is closely tied to conservation of momentum

Review of Goddard's pioneering work on rockets

- "Professor Goddard does not know the relation between action and reaction and the needs to have something better than a vacuum against, which to react. He seems to lack the basic knowledge ladled out daily in high schools."…

  -1921 New York Times editorial

Goddard's Rockets

- https://airandspace.si.edu/stories/editorial/robert-goddard-and-first-liquid-propellant-rocket

ACT II – INTRODUCING MOMENTUM

Momentum

- **Definition :** If an object of mass m is moving with velocity $\underline{V}$, its momentum $\underline{p}$ is given by
  $\underline{p}=m\underline{V}$

- **Definition :** total momentum $\underline{p}_{tot}$ of a number of objects with masses m_1, m_2, … and velocities $\underline{V}_1$, $\underline{V}_2$, … is just the (vector) sum of the objects' separate momenta

- **Conservation of momentum :** The total momentum of a system of particles is constant *if no external forces act on the system*

- Newton's laws can be rephrased entirely in terms of momentum…
  - **Second law**… the rate of change of momentum of a body is equal to the force applied to that body
  - **First law** is special case of the Second law… the momentum of a body is unchanged if there are no forces acting on body
  - **Third law**… the momentum of an isolated system of objects is conserved

Conservation of Momentum

- Conservation of momentum is more fundamental than Newton's Law of Inertia
- Conservation of momentum is critical to ideas like special and general relativity as well as a fundamental principle in quantum mechanics*

- It is equivalent to the concept that physical laws do not depend on position and is a mathematical consequence of the homogeneity (shift symmetry) of space

ACT III – SYMMETRIES AND FRAMES OF REFERENCE

Symmetries and frames of reference

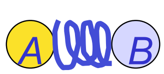
- Can we derive Newton's laws / conservation of momentum from more fundamental considerations? Yes! Use the idea of symmetry…
- The idea of symmetry is very important in modern advanced physics! Let's have a glimpse of symmetry in action… [see discussion on board]

Galilean Relativity

- Consider two **frames of reference** that differ by some uniform velocity difference (so we are not considering accelerated frames of reference)
  - How do we relate velocity in one frame to the other??
- We use a "velocity addition rule" $\mathbf{v_{total}=v_1+v_2}$– this is an example of a **Galilean transformation**
- The idea that the laws of nature are the same for a moving observer as for a stationary observer (two different frames of reference connected by a Galilean transformation) is called the **Principle of Galilean Relativity**

An illustration of Newton's laws

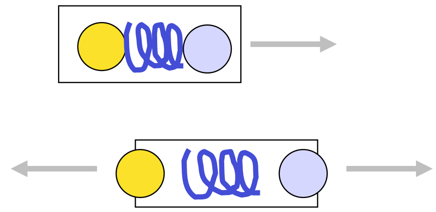
- We can see that aspects of Newton's laws arise from more fundamental considerations.
- Consider two equal masses M at rest. Initial momentum is $p = 0$. Masses are suddenly pushed apart by a spring… and will move apart with the same speed *V in opposite directions* (by symmetry of space!). Total momentum is $p = mv - mv = 0$. **Total momentum is unchanged.**



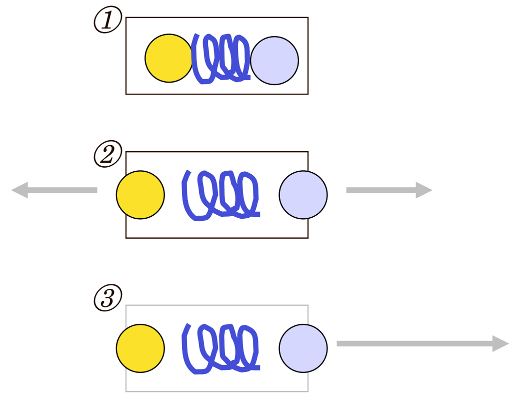
Before: $v_A = v_B = 0 \Rightarrow p_{tot} = 0$

After: $v_A = -V,\ v_B = V \Rightarrow$
$p_{tot} = Mv_A + M v_B = -MV + MV = 0$

- Same situation, but masses are now both initially moving at velocity *V*. Initial momentum is p_{tot}=2*MV*.
- Can turn into the previous situation by "moving along with them at velocity V".



1. *Change of perspective [subtract V from all velocities] brings masses to rest…*
2. *Do same problem as before…*
3. *Change back to original perspective [add V to all velocities] …*
4. *Final velocity of one ball is 2V; final velocity of other ball is 0. Final total momentum is* p_{tot}= *2MV. No change in total momentum.*



- To re-state this , the two connected masses are initially moving at velocity V. Let's turn this into the above situation by "moving along with the masses at velocity V"
  - Change perspective to bring masses to rest…
  - Do same problem as before… find that momentum before = momentum after
  - Change back to the original perspective…
- You have "changed your frame of reference".
  - The "velocity addition" rule is called a **Galilean transformation.**
  - We assume that, after changing our reference frame and using a Galilean transformation, the laws of physics are the same. This is called **Galilean Relativity**.

- Example of Galilean transformation
  - a person on a uniformly moving car has the impression of being at rest -they see a iphone falling vertically downward.
  - from the point of view of a person standing on the side of the road, the motion of falling downwards in the car would be combined with, or added to, the forward motion of the ship.
  - Thus the velocity of the falling body relative to the sidewalk equals the velocity of that body relative to the car plus the velocity of the ship relative to the sidewalk.
  - The two observers infer something different!

Newtons Laws

- How do Newton's laws fit into **this** picture?
  - N1 comes directly from Galilean Relativity (there is no difference between a state of rest and a state of motion)
  - N2 and N3 are exactly what's needed to make sure that momentum is conserved and so is related to the symmetry of space
  - *So… Newton's laws are related to the symmetry of space and the way that different frames of reference relate to each other.*

Symmetry and Physics

- Different symmetries and their conservation laws (Emmy Noether's theorem)
- MOMENTUM - TRANSLATION SYMMETRY OF SPACE
- ANG. MOMENTUM - ROTATIONAL SYMMETRY OF SPACE
- ENERGY - TIME SYMMETRY
- CHARGE - GAUGE SYMMETRY

#quantumchangers

https://www.ias.edu/ideas/2017/emmy-noether%E2%80%99s-paradise

Emmy Noether

Amalie "Emmy" Noether was described by Einstein as the most important woman in the history of mathematics.

Noether discovered that for every symmetry in the universe there is a conservation law - a fundamental property of our universe and a powerful concept for physicists.

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Summary

- **Newton's 1st law :** ***V*** = *constant* if ***F*** = *0*
- **Newton's 2nd law :** ***F*** = *Ma*
- **Newton's 3rd law:** for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference ($\mathbf{v}_{tot}=\mathbf{v}_1+\mathbf{v}_2$)
- **Galilean Relativity** – the idea that the laws of nature are the same for a moving observer as for a stationary observer.

- **v**=velocity, **a**=acceleration, **F**=force, *M*=mass

Next time…

- Newtonian gravity!

- Reminder! HW1 (due next class)