Class 3 : Newtonian Gravity

ASTR350 Black Holes (Spring 2020)

Prof. Richard Mushotzky

Today's class

- Newton's Law of Gravity
- Gravitational and Inertial Mass
- Free fall & the Weak Equivalence Principle
- Escape velocity (and Newtonian Black Holes)

What questions do we have from Jacobson's talk !

HW1 due

RECAP

- **Newton's 1st law** – $\boldsymbol{V}$ = *constant* if $\boldsymbol{F} = 0$
- **Newton's 2nd law** – $\boldsymbol{F} = M\boldsymbol{a}$
- **Newton's 3rd law** – for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference. $\mathbf{v}=\mathbf{v}_1+\mathbf{v}_2$
- **Galilean Relativity** – the idea that the 'laws of nature' are the same for a moving non-accelerated observer as for a stationary observer.
  - If everything is moving together at constant velocity, there can be no apparent difference from case when everything is at rest.
    - Ball dropped from top of moving ship's mast hits near bottom of mast, not behind on deck.

(reminder of terms, **V**= velocity, **F**= force, **a**= acceleration, all are vectors)

Waitlist

- There are 11 students on the waitlist as of this morning.

What Was Known About Gravity Before Newton

NOT MUCH (various ideas)

- But it was known how the planets move (Keplers 3 Laws)
  - **Planets move around the Sun in ellipses, with the Sun at one focus.**
  - **The line connecting the Sun and a given planet sweeps out equal areas in equal times.**
    - Therefore, planets move faster when they are nearer the Sun
  - **The square of the period *P* of the orbit is proportional to the cube of the semi-major axis *R*-** *P^2=constant $\times$ R^3; constant determined by observations*

Kepler in perspective

- Based on accurate observations, Kepler calculated and thought his way to a major breakthrough
- Kepler's three laws of planetary motion
  - Represented a very simple (and correct!) model of the solar system
  - Swept away previous ideas thousands of years old
  - Were driven fundamentally by the **data**, including Tycho's error estimates
- Unlike previous models which quantified only what was observed already, Kepler's Laws had predictive power, consistent with modern idea of a meaningful scientific theory (in fact it was the deviation of Mercury from Kepler's law that was one of the observational tests of General Relativity)

I: Newton's Law of Universal Gravitation

Newton's law of Gravitation: A particle with mass m_1 will attract another particle with mass m_2 and distance r with a force F given by

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

- "G" is called the Gravitational constant ($G = 6.67\times10^{-11}$ N m^2 kg^{-2} in SI units)
- This is a **universal** attraction. Every particle in the universe attracts every other particle! Gravity often dominates in astronomical settings- but weak compared to other forces (electricity and magnetism, nuclear force)

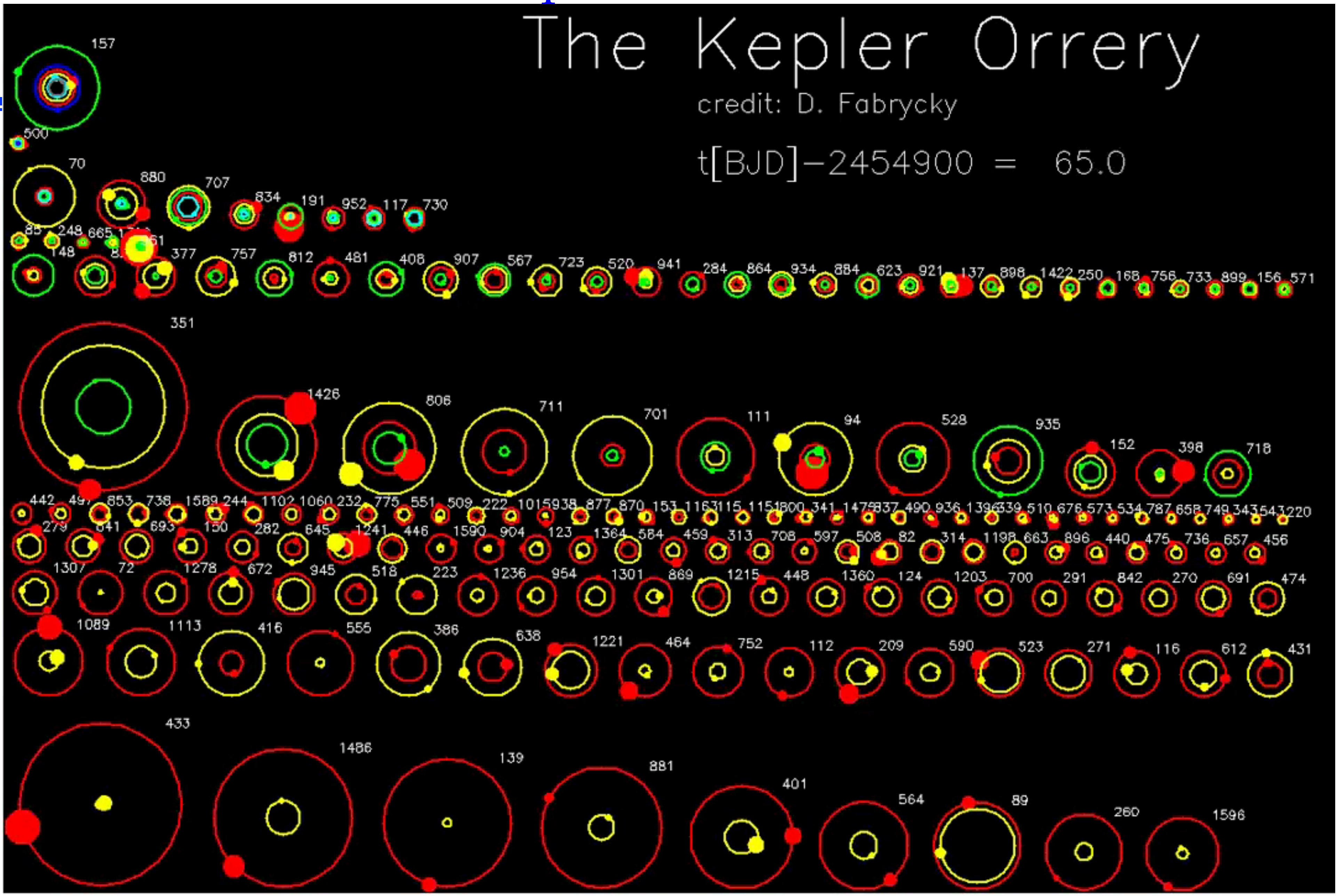
The Apple Story

- *After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees…he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself…"*

https://thonyc.files.wordpress.com/2016/05/newton.jpg?w=500

https://www.newscientist.com/article/2170052-newtons-apple-the-real-story/#ixzz6Ck4e8hFN

How Universal- Orbits of Planets Determined by Kepler Satellite

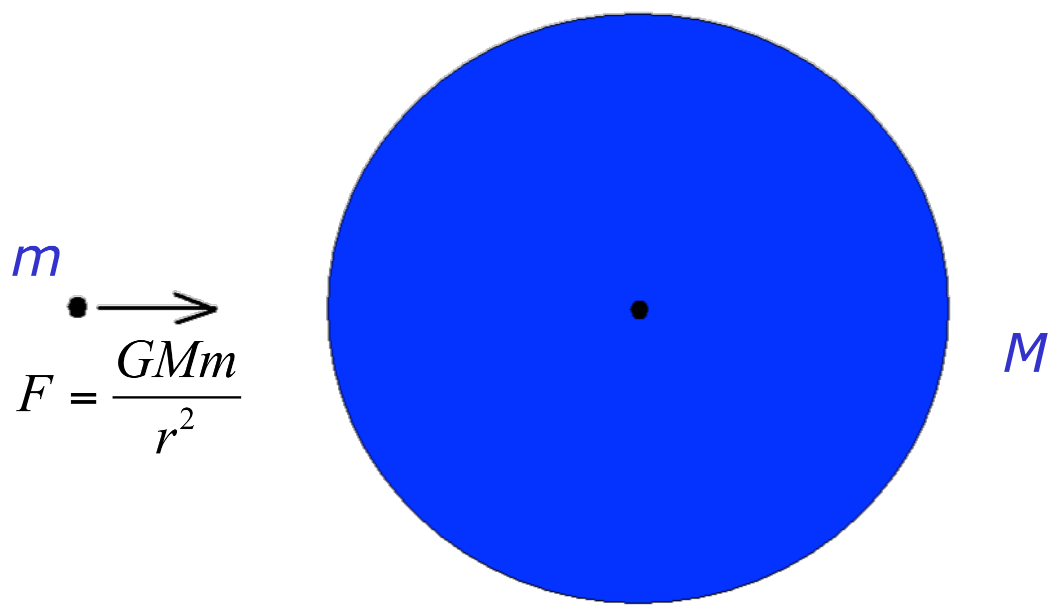


How Weak is Gravity

- Gravity is the weakest force (much weaker than electromagnetic force)- both of which depend on $1/R^2$
- **BUT it controls the motion of all the planets, stars and galaxies and is dominant in the universe on large scales**

Gravitational Mass

- Newton's Law of Gravitation defines the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass *M* (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass *M* at the sphere's center.



$$F = \frac{GMm}{r^2}$$

How Does it work?

- Newton's law did not identify any mediator of gravitational interaction. His theory assumed that gravitation acts instantaneously, regardless of distance. But Newton recognized that and said

It is inconceivable that inanimate Matter should, without the Mediation of something else, which is not material, operate upon, and affect other matter without mutual Contact.... Gravity must be caused by an Agent acting constantly according to certain laws; but whether this Agent be material or immaterial, *I have left to the Consideration of my readers.*

Newton's theory described how objects attracted each other, it didn't explain why.

II : Inertial and gravitational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a$$

m_I=inertial mass

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2}$$

m_G=gravitational mass

So, acceleration due to gravity is:

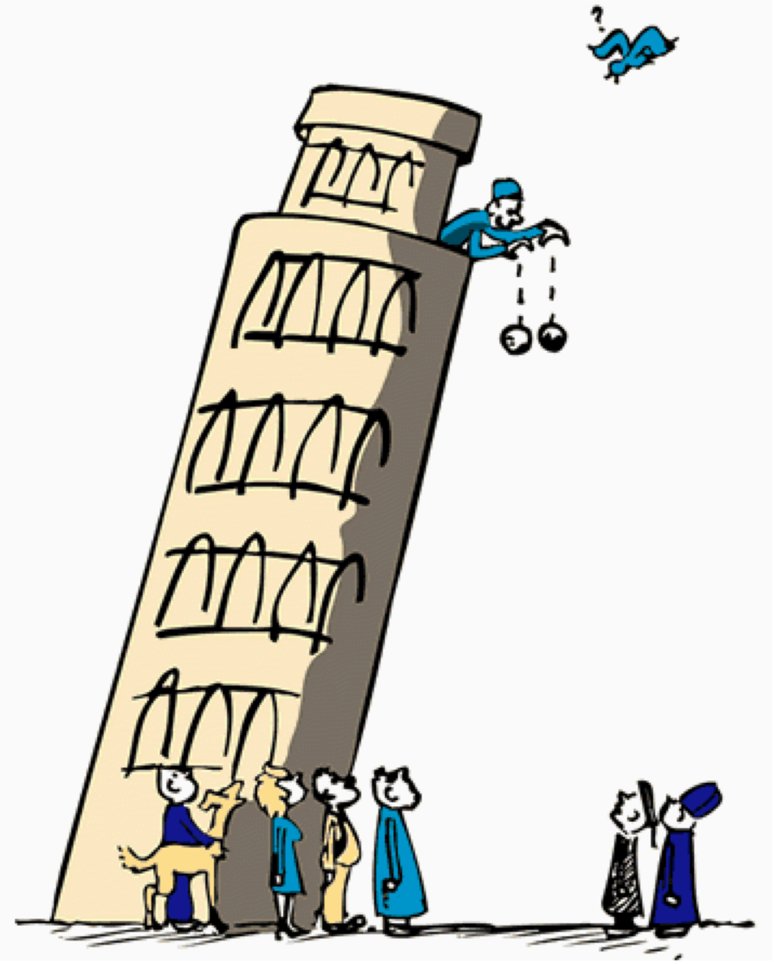
$$\boxed{a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}}$$

So, if the ratio (m_G/m_I) *varies*, the rate at which objects fall in a gravitational field will vary…

Galileo Tower Experiment

- Galileo said 1638 in *Dialogues Concerning Two New Sciences*: "[T]he variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that … I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed."

https://science.sciencemag.org/content/347/6226/1096.full?intcmp=collection-generalrelativity



At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Video https://www.youtube.com/watch?v=E43-CfukEgs

Equivalence Principle in other words

- Inertial mass determines how much an object resists moving when pushed by a force—as when you shove a car.
- Gravitational mass determines how strongly gravity pulls on the object.
- The equivalence principle says: the two masses are one and the same, regardless of how heavy a thing is or what it's made of.
- That explains Galileo's experiment: If the two types of mass are identical, then for all objects the pull of gravity varies in strict proportion to the resistance to motion, ensuring that all things fall at the same rate

https://science.sciencemag.org/content/347/6226/1096.full?intcmp=collection-generalrelativity

Equivalence of inertial and gravitational mass

- *Experimentally*, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo in the famous tower experiment**)
- So, m_I / m_G must be the *same* for all bodies
- And we can choose the constant "G" such that $m_I = m_G$, and $\boldsymbol{a} = GM/r^2$
- Gravity is equivalent to (indistinguishable from) any other acceleration(!)- this is the **weak equivalence principle**

Equivalence of inertial and gravitational mass

- *Experimentally*, if all forces apart from gravity can be ignored, all objects fall at the same rate (first demonstrated by **Galileo**)

- So, m_I/m_G must be the *same* for all bodies-this has now been experimentally verified (Eotvos parameter)*

- And we can choose the constant "G" such that $m_I = m_G$, and $a = GM/r^2$

- This is a consequence of the **weak equivalence principle**

Equivalence Principle- Test*

- Last year the 'Microscope' (**Micro-S**atellite à traînée **C**ompensée pour l'**O**bservation du **P**rincipe d'**E**quivalence) satellite derived a limit on the ratio of the interial to gravitational mass of **1 part in 10^{-15}** (https://microscope.cnes.fr/en/MICROSCOPE/index.htm) by having two free-floating weights of different materials and testing whether one experiencs a stronger tug from Earth's gravity than the other.

* why bother since we know Einstein is 'right' – well ...the violation of the Equivalence Principle would open the way to the demonstration of a new force, the existence of which is predicted by many quantum theories of gravity.

How Many 'Types' of Mass are There?

- Intertial mass
- Gravitational mass
- 'Einsteinian mass' (eg $E=mc^2$)
- How much curvature of spacetime does a given mass cause (General relativity)
- In quantum mechanics objects have both a wavelike and particle behavior- in this description mass has a wavelength λ

  ($\lambda=h/mc$); h=Plancks constant

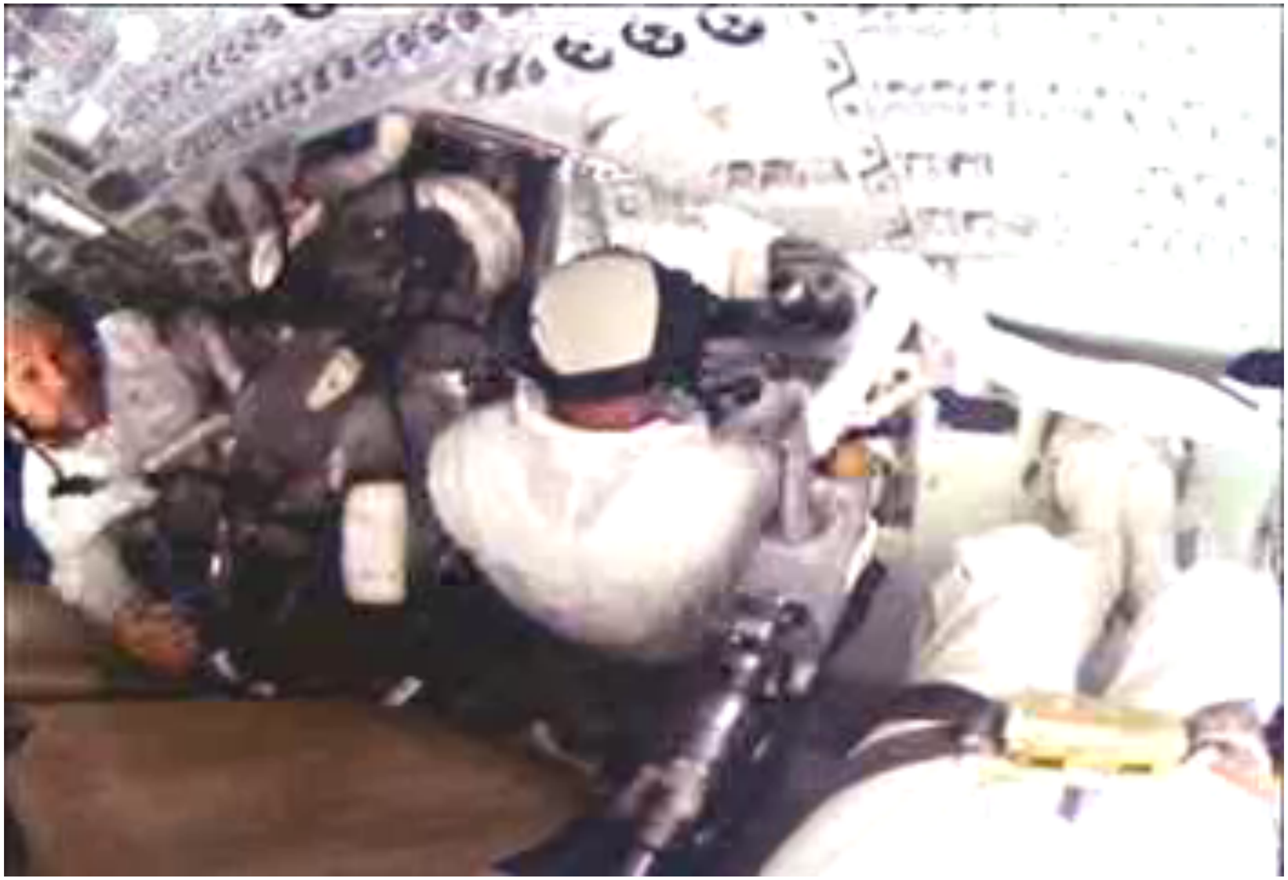
ALL are the same

Equivalence Principle

- **The Weak Principle of Equivalence** states all the laws of motion for freely falling particles are the same as in an unaccelerated reference frame.
  - the Weak Equivalence Principle is a restatement of the equality of gravitational and inertial mass.
  - **There is no way of distinguishing between the effects on an observer of a uniform gravitational field and of constant acceleration-** the equality of gravitational and inertial mass. The foundation of the General Theory of Relativity.

Einstein said that this was 'the happiest thought of his life' (http://physicsbuzz.physicscentral.com/2015/03/the-happiest-thought-of-my-life-100.html)

“Weightlessness”



Apollo 10, in orbit (May 18-26, 1969)

“Weightlessness”

Quiz :Why do objects and astronauts in spacecraft appear weightless?

1. There is no gravity in space and they do not weigh anything.

2. Space is a vacuum and there is no gravity in a vacuum.

3. The astronauts are too far away from Earth’s surface to be subject to its gravitational pull.

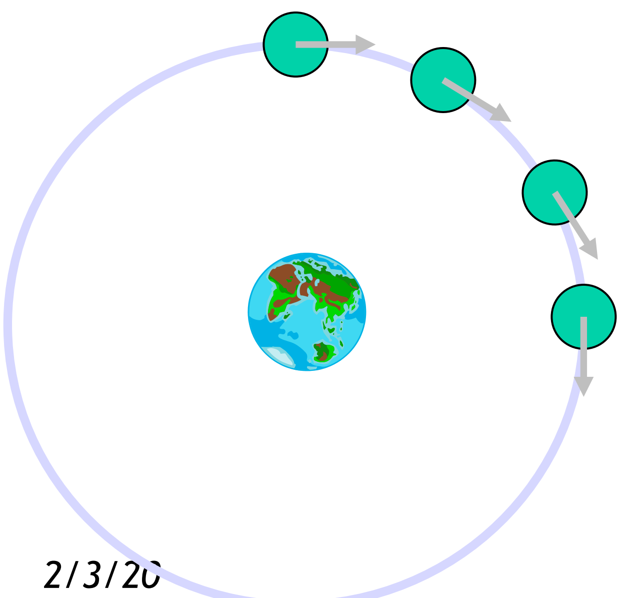
4. The astronauts, the ISS itself and other objects in Earth orbit aren’t floating, they are actually falling.

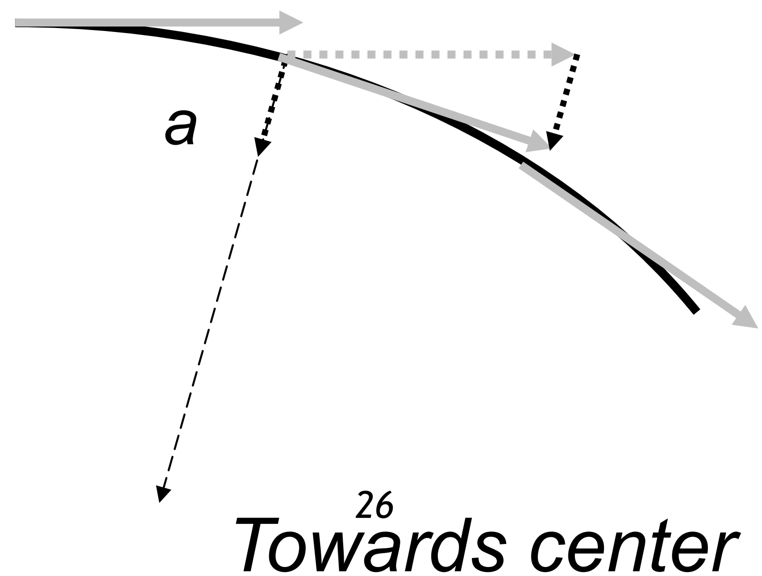
Weightless, or free-fall?

- Space Station orbits about 500km above Earth's surface. Radius of Earth is 6300km.
- Newton's inverse square law:
  - Gravitational acceleration at location of space station is 86% of what it is on the Earth's surface! (compare $(6300+500)^2$ to 6300^2)
- So, why do the astronauts feel weightless?
  - The astronauts "fall" toward Earth at the same rate as the space station – another example of the equivalence principle.

“Falling” in a circular orbit

- Newton arrived at the theory of gravity by imagining that the same force causes an apple to fall towards the Earth as the Moon to orbit Earth (continually “falling”), with a decrease in the magnitude of the force with distance
- In what sense is a body in orbit *“falling”?*
- Direction of acceleration (= rate of change of vector velocity) is always *directly towards center of orbit*
- *Acceleration* must be toward center because *gravitational force* is toward center, and **F=ma** is a vector equation

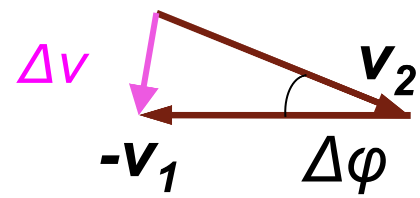


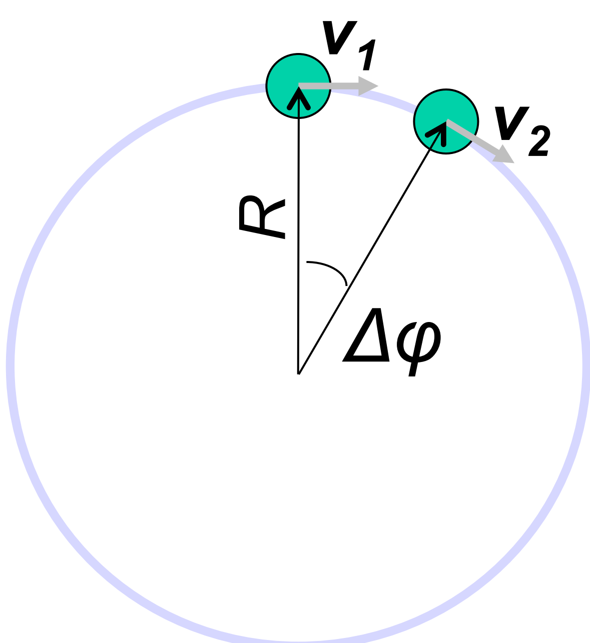


Acceleration in a circular trajectory

http://galileo.phys.virginia.edu/classes/109N/lectures/newtongl.html

$\Delta \mathbf{v} = \mathbf{v}_2 + (-\mathbf{v}_1)$





For small φ, $\Delta v \sim v\,\Delta\varphi$

For constant speed v, the time it takes to go around is
T = perimeter/v = $2\pi R/v$

$\Delta\varphi = \Delta t\, 2\pi/T = \Delta t\, v/R$

So $\Delta v/\Delta t = v\,\Delta\varphi/\Delta t = v\,(v/R)$

$$\boxed{a = \Delta v/\Delta t = v^2/R}$$

Weightless, or free-fall?

- Earth-**orbiting** astronauts are **weightless** for the same reasons riders of a free-falling amusement park ride or a free-falling elevator are **weightless**.
- They are **weightless** because there is no external contact force pushing or pulling upon their body (free-fall)

Orbital speed and period

- An object of mass m going in a circle of radius r with speed v requires a force F given by

$$\boxed{F = \frac{mv^2}{r}}$$

to keep it in orbit

- For an object mass m (satellite, space station, Moon) in a circular orbit about a large body of mass M (such as the Earth), this force is provided by gravity. So equate the two to get...

$$\boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}}$$

Orbital speed and period

$$\boxed{\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r}}$$

The mass of the smaller object cancels (the weak equivalence principle that we've just seen)

Take square roots and get

$$v = \sqrt{\frac{GM}{r}}$$

- Consequences of this…
  - The mass of the smaller object cancels (the weak equivalence principle that we’ve just seen)
  - The speed is given by

$$v = \sqrt{\frac{GM}{r}}$$

  - The period T (time taken for one orbit) is given by

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Kepler's third law
Period^2 α r^3

The period (time taken for one orbit) is given by

$2\pi r$= circumference of circle/velocity of body

substitute above formula for v and do a little algebra

$$\boxed{T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}}$$

This as one of Kepler's laws

$T^2 \sim r^3$ Newton determined the constant from his theory of gravity

constant=$4\pi^2/GM$

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make predictions
  - these laws can be applied to stars in galaxies, galaxies in clusters, etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the typical orbital period and cube of the orbital distance.
  - Its the *deviation* of the observations from our understanding of how much mass objects have that has led to the idea of dark matter.

- As Newton's physics came to be widely known, there was a huge cultural impact. With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc. *Key to shift in thought known as the Enlightenment*. *The universe is a giant machine! (?)*

III : Escape Velocity

- What goes up must come down.   Arh, actually…

- Suppose we are on the surface of a planet with mass M and radius R. Then there is a critical speed above which we can leave the surface and *never fall back*. This is the **escape velocity**.
- Using Newton's Laws and a little calculus, we find that

$$V_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$

- Some numbers…
  - $G=6.67\times10^{-11}\ Nm^2kg^{-2}$
- The Earth
  - $M_{Earth}=6.0\times10^{24}kg$
  - $R_{Earth}=6.4\times10^{6}m$

$V_{esc}=11.2km/s$ *(25,020 mph)*

7 miles per second

- The Sun
  - $M_{Sun}=2.0\times10^{30}kg$
  - $R_{Sun}=7.0\times10^{5}km$

$V_{esc}=617km/s$

Escape Speed = Speed of Light

- Work the equation backward and calculate what mass and size is required
- Lets take the sun- what radius is required such that the escape speed is the speed of light e.g. $R=2GM/c^2$
- As we will see later the speed of light is special in Einsteinian relativity.

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make new predictions about orbits of solar system bodies
  - Halley argued that several comet appearances separated by 76 years were actually the same comet, and predicted its recurrence in 1758 (last return in 1910,1986)
  - Planets have near-elliptical orbits, but they are not exact ellipses due to gravity of *other* planets and thus show small deviations from Kepler's laws
- Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet.
  - In 1845, John Adams and Urbain Leverrier, predicted the existence of an unseen planet, to account for the fact that Uranus was being pulled slightly out of position in its orbit by the gravitational effect of an unknown body, and calculated its position and motion in the sky. Observations confirmed this leading to the discovery of Neptune in 1846

Early indications of black holes

- ``The gates of hell are open night and day;
  Smooth the descent, and easy is the way:
  But to return, and view the cheerful skies,
  In this the task and mighty labor lies.''
  - Virgil, The Aeneid, Book VI [John Dryden translation]

Rev. John Mitchell (1784)

- If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, **since their light could not arrive at us**; or if there should exist any other bodies of a somewhat smaller size, which are not naturally luminous; of the existence of bodies under either of these circumstances, **we could have no information from sight**; yet, if any other luminous bodies should happen to revolve about them we might still perhaps from the motions of these revolving bodies infer the existence of the central ones with some degree of probability, as this might afford a clue to some of the apparent irregularities of the revolving bodies, which would not be easily explicable on any other hypothesis; but as the consequences of such a supposition are very obvious, and the consideration of them somewhat beside my present purpose, I shall not prosecute them any further.

Laplace (1798)

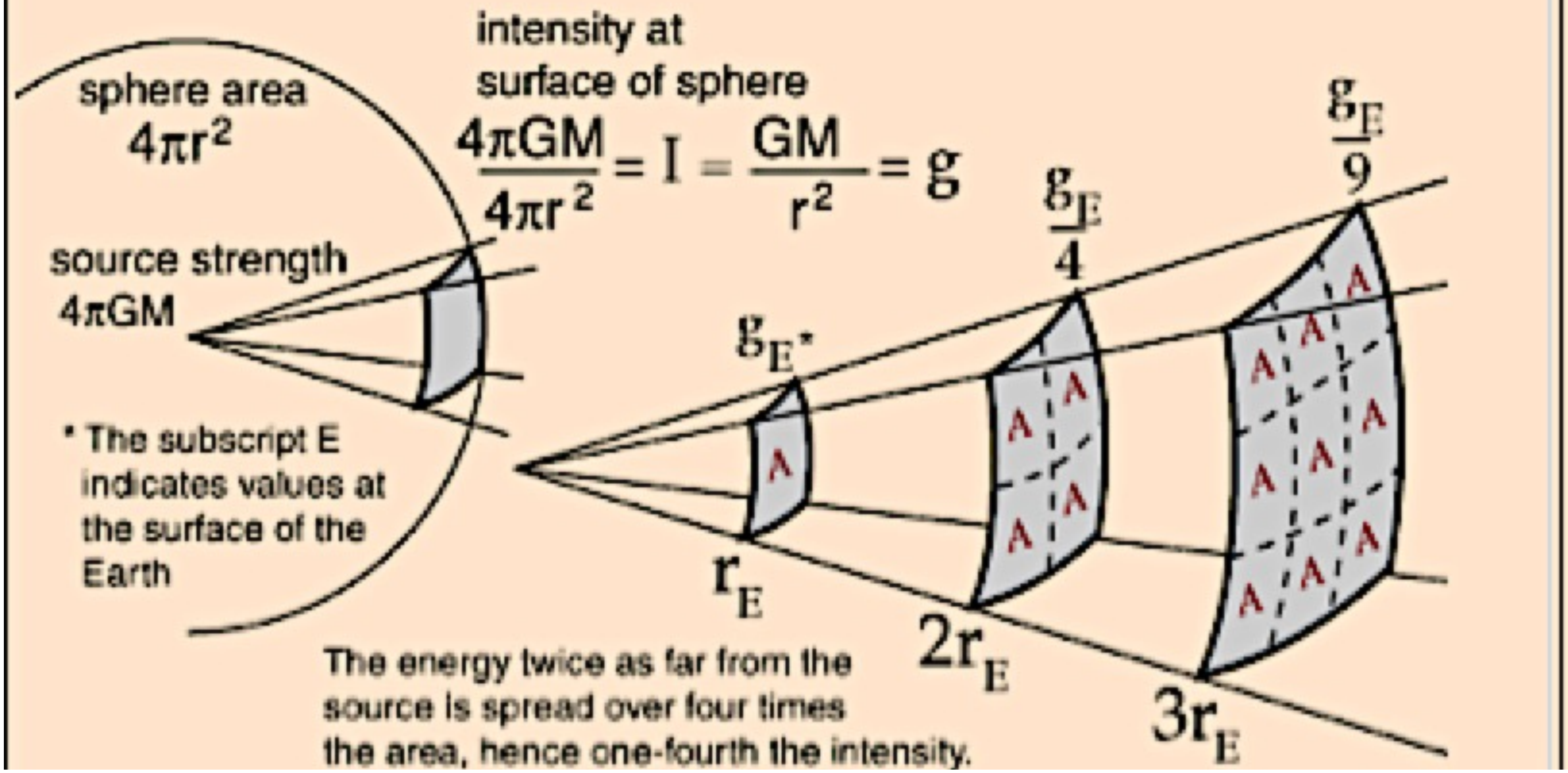
- *A luminous star, of the same density as the Earth, and whose diameter should be two hundred and fifty times larger than that of the Sun, would not, in consequence of its attraction, allow any of its rays to arrive at us; it is therefore possible that the largest luminous bodies in the universe may, through this cause, be invisible*.

Next Lecture- Special Relativity

- Speed of light problem
- Einstein’s postulates
- Time dilation – time is not absolute!!!

Inverse Square Law , Gravity

As one of the fields which obey the general inverse square law, the gravity field can be put in the form shown below, showing that the acceleration of gravity, g, is an expression of the intensity of the gravity field.



http://hyperphysics.phy-astr.gsu.edu/hbase/forces/isq.html