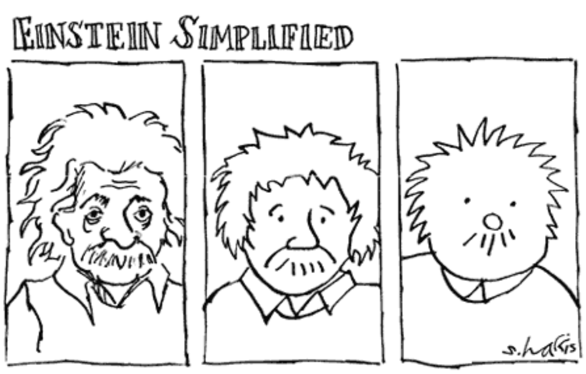
Class 4 : Special Relativity

ASTR350 Black Holes (Spring 2020)
Prof. Richard Mushotzky

Today's class

- Speed of light problem
- Einstein's postulates
- Time – time is not absolute!!!
- **HW2 due Feb 13 in class or via ELMS**
- **grades for HW 1 on ELMS**



RECAP

- Newton's Laws of Motion
  - Connection with notion of frame of reference
- Newtonian Gravity
  - Action at a distance ?
  - Predicts notion and value of escape velocity, v_{esc}
- First ideas for black holes in late 1700s
  - Hypothetical objects with v_{esc}>speed of light
  - Rev John Mitchell and Laplace

Galilean Transformations

•*In Galilean relativity* ***length intervals*** *and* ***time intervals are the same for all*** ***inertial observers*** ***of the same events***

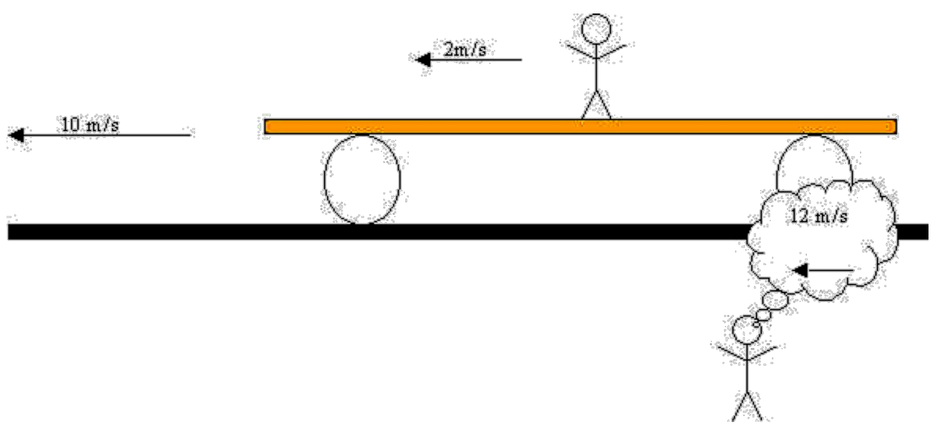
- *if* ***meter sticks*** *are of the* ***same length*** *when measured and compared* ***at rest****, they are of the* ***same length*** *when compared in* ***relative motion to one another****;*
- *if* ***clocks*** *are* ***calibrated*** *and* ***synchronized*** *when at rest, their* ***readings*** *and* ***rates will agree*** *even if they are in* ***relative motion*** *to one another.*

Even Great Scientists Make Mistakes- A. Michelson 1894

- ...it seems probable that most of the grand underlying principles (of physics) have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice..... An eminent physicist remarked that the future truths of physical science are to be looked for in the sixth place of decimals.

- Right before relativity and quantum mechanics!

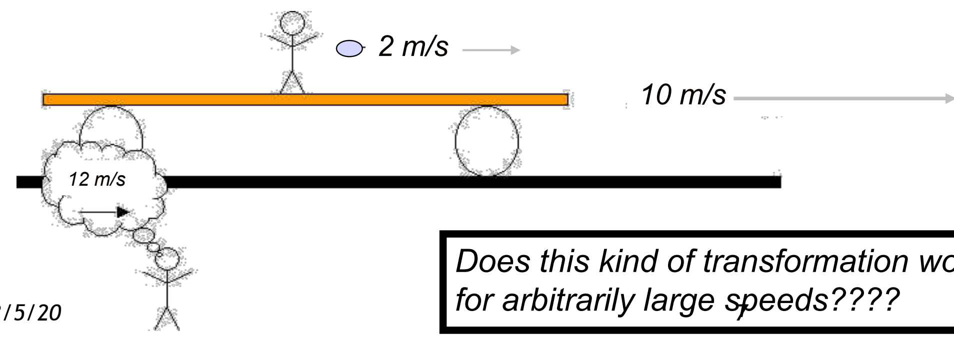
I: THE SPEED OF LIGHT PROBLEM



- Recap
  - "Relativity" tells us how to relate measurements in different frames of reference
  - Galilean relativity
    - Simple velocity addition law : $v_{total}=v_{run}+v_{train}$

Relativity

- "Relativity" refers in general to the way physical measurements made in a given inertial frame are related to measurements in another frame.
- An inertial observer is one whose rest frame is inertial
- A quantity is invariant if all inertial observers obtain the same value
- Under Galilean relativity, measurements are transformed simply by adding or subtracting the velocity difference between frames:
- v_{ball}(measured on ground)=v_{train} (measured on ground)+v_{ball}(measured on train)
  12 m/s = 10m/s + 2 m/s
- V_{ball}(measured on train)=v_{ground}(measured on train)+ v_{ball}(measured on ground)
  2 m/s = -10m/s + 12 m/s



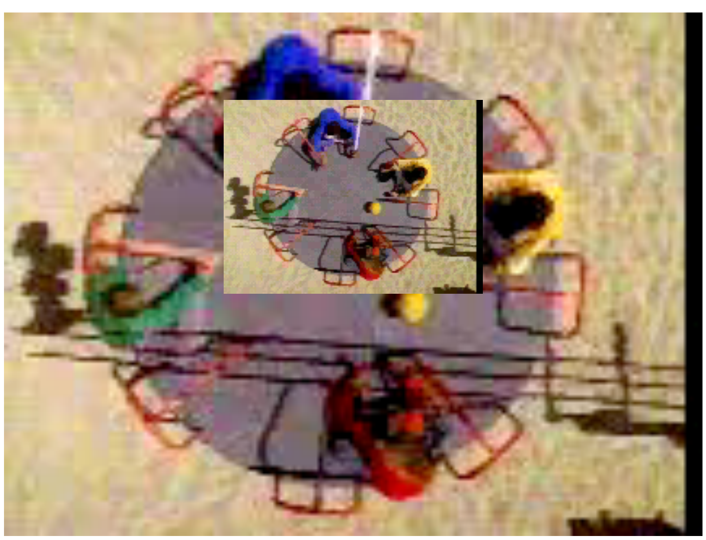
Does this kind of transformation work for arbitrarily large speeds????

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

- Newton's laws were clearly powerful. But they also led to some puzzles, particularly relating to **reference frames.**
- We have already come across idea of **frames of reference that move with constant velocity.** In such frames, Newton's laws (esp. N1) hold. These are called **inertial frames of reference.**
- Suppose you are in an accelerating car looking at a freely moving object (i.e., one with no forces acting on it). You will measure its velocity changing because you are accelerating! *In accelerating frames of reference, N1 doesn't hold* – this is a **non-inertial frame of reference.**

Real and fictitious forces

- In *non-inertial frames* you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces point opposite to the direction of acceleration
- Fictitious forces are always proportional to the inertial mass of the body.



Coriolis force: a fictitious force

Inertial frame

Noninertial frame

Fictitious Forces- Examples

- In a passenger vehicle that is *accelerating* in the forward direction – the passengers perceive that they are acted upon by a force in the rearward direction pushing them back into their seats.
- An example in a *rotating reference frame* is the force that appears to push objects outwards towards the rim of a centrifuge. These apparent forces are examples of fictitious forces.

…does this seem familiar?

Recall that from weak equivalence principle, inertial mass=gravitational mass ⇒ *gravitational force is proportional to inertial mass.*

Maybe gravity is a fictitious force…

… and we live in an accelerating frame of reference???

Physics after Newton and Before Einstein: Electromagnetic waves

- The physics of electricity and magnetism:
  - In the late 1700's to mid 1800's scientists like Coulomb, Oerstead, Ampere, Faraday, Gauss developed ideas of how electricity and magnetism 'worked'-like Kepler and Brahe before Newton
- These quantitative theories of electricity and magnetism were formulated in terms of *forces acting at a distance*, analogous to Newton's law of gravitation.
- The fields of electromagnetism and understanding of light (optics) *were* unrelated and treated as separate branches having no connection.

Electromagnetic waves

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations).

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / \boxed{c} + (1/\boxed{c})\partial \mathbf{E} / \partial t$$

Equations are just written out for fun… *<u>you do not need to know them!</u>*

Notice that a new constant "c" appears which is a velocity

B= magnetic field, **E**= electric field, **J** is the current, ρ *is charge density*, $\nabla \cdot$ is the divergence*,* $\nabla \times$ *is the curl* and ∂ is the *partial derivative*

Solution to Maxwell's Equations

- Maxwell said "*.. the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*[
- Maxwell's Equations show that Electric and Magnetic Fields in "Free Space" - a region without charges or currents like air - can travel with any shape, and will propagate at a single speed - c.

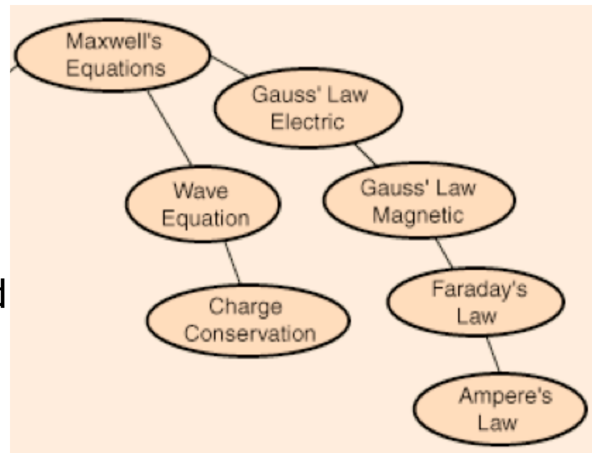
(there is a nice derivation for the mathematically inclined at http://maxwells-equations.com/equations/wave.php)

A little bit more detail?

- Maxwell equations showed the waves speed is equal to the speed of light **from a purely theoretical argument**
- Thus light is an electromagnetic *wave.* (and that there must be other such waves with different wavelengths).
- characterized by a wavelength (λ), a frequency (ν) and a speed (c). It carries energy E.

Maxwell's equations. are core to an understanding of electricity, magnetism and light. but require a level of mathematical sophistication to understand

The speed of light is related to 2 constants that come out of the laws describing electric and magnetic forces

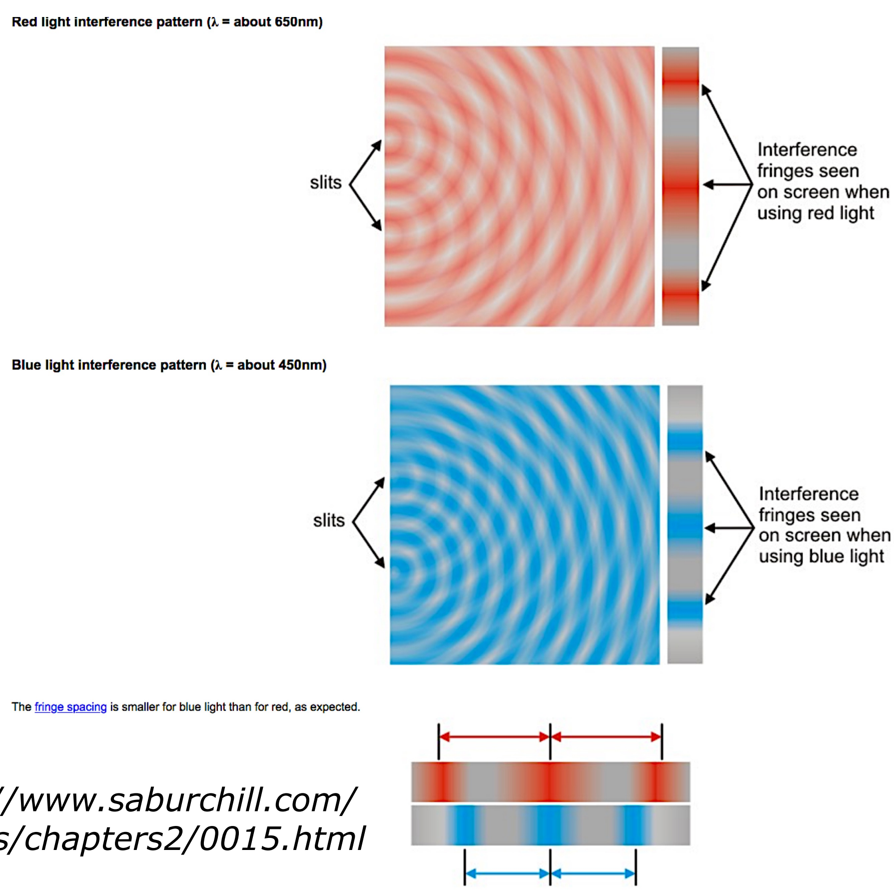


- Maxwell's equations:
  - Basic laws of physics governing electricity and magnetism
  - Solutions are waves of electromagnetic energy – quickly realized that these were light waves!
  - *The speed of light "c" appears as a fundamental constant in the equations* same in all frames (But what about Galilean transform ??)
    - c=299,792,458 km/s (1.0 feet/nanosecond).

- **BUT, what frame of reference is this speed measured relative to?**
- **If Light is a Wave, What is Waving?**

Basic Problem of How does Light Propagate

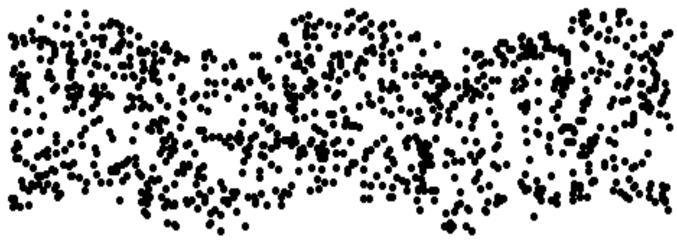
- According to Maxwell's equations <u>light is a wave</u>
- In the 19th century it was known that waves need a medium to move in (sound waves, ocean waves, earth quake waves, etc.)
  - sound waves are compressional waves in air.
- If light is a wave, just what is waving? It clearly isn't air, because light reaches us from the sun, and stars, and we know the air doesn't stretch that far 



https://www.saburchill.com/physics/chapters2/0015.html

- Before Maxwell all waves (sound, pressure waves in earthquakes etc.) were known to be disturbances in the substance (water, air) through which they traveled
- sound waves propagate via air molecules bumping into their neighbors

Waves



http://www.kettering.edu/physics/drussell/Demos/waves/wavemotion.h

- Luminiferous Aether* (19th century)
  - Hypothetical substance that fills space – provides "medium" through which light travels.
  - Was presumed that "c" should be measured with respect to the rest frame of the Ether.
  - This would explain why the speed of wave propagation "c" is a constant in the equations
  - Albert Michelson & Edward Morley attempted to measure motion of Earth through aether…Is the speed of light the same in all directions in a moving reference frame?

serious problem with aether- invisible and infinite material with no interaction with physical objects.

*"luminiferous", means "light-bearing"- invoked to explain the ability of the light wave to propagate through empty space, something that waves should not be able to do

How to Measure How Motion with Respect to the Aether

- As the Earth moved in its orbit around the sun, the flow of the ether should produce a detectable "ether wind".
- The speed of a beam of light emitted from a source on Earth would depend on the magnitude of the ether wind and on the direction of the beam with respect to it.

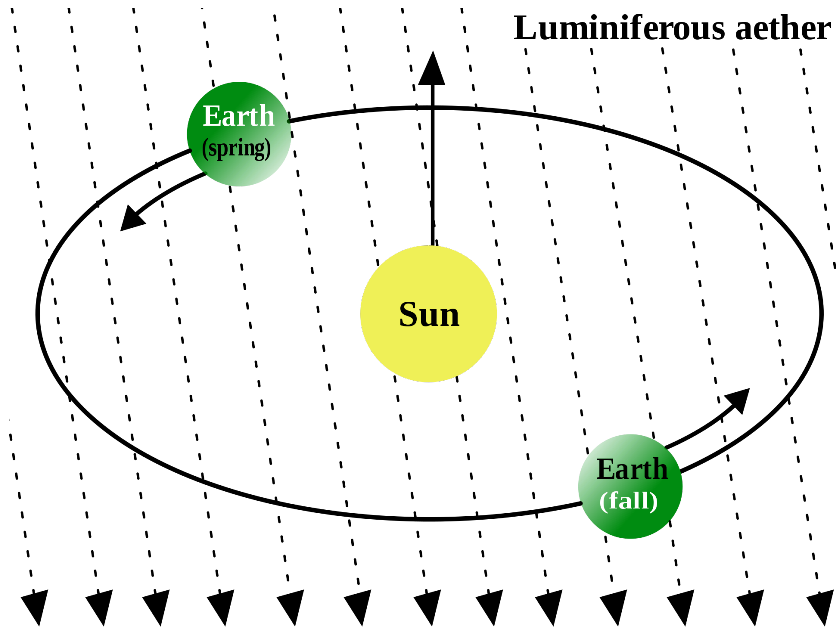
The experiment was designed to measure the speed of light in different directions in order to measure the speed of the ether relative to Earth, thus establishing its existence.

https://www.aps.org/programs/outreach/history/historicsites/michelson-morley.cfm

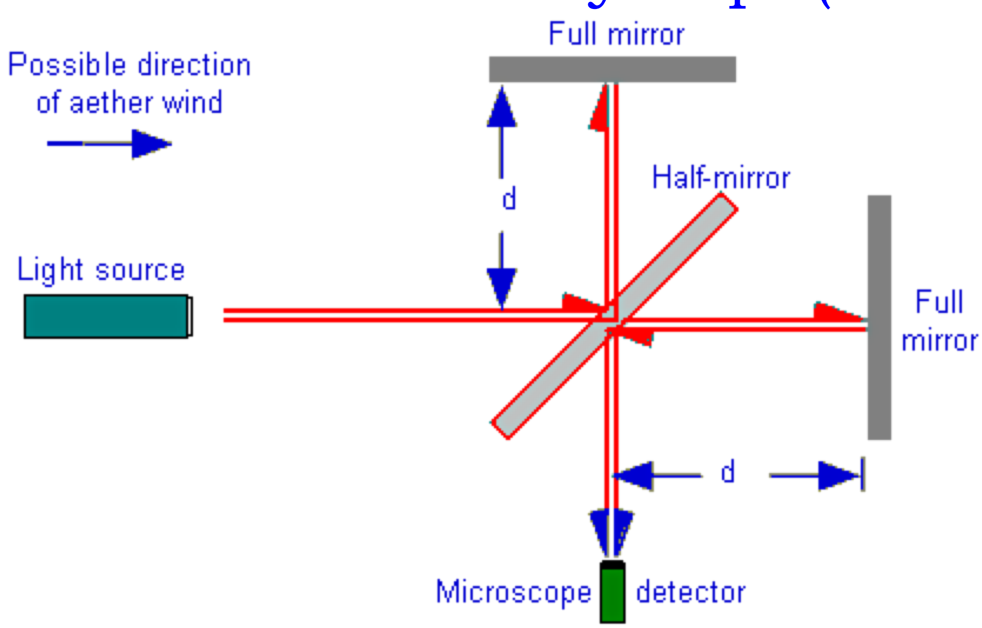
How Big is the Effect

- If we do the Galilean transform ($v_{total}=v_{light}+v_{earth}$) and compare the velocity of light ($3x10^5$km/sec) and the velocity of the earth around the sun (30 km/sec) the effect is on the order of v_{earth}/v_{light} $\sim10^{-4}$
- Need sensitive experiment !

Light Waves Coming From Distant source



Michelson-Morley Expt (1887)



See http://galileoandeinstein.physics.virginia.edu/lectures/michelson.html for a clear detailed description

Do the M-M experiment for yourself !!

- http://galileoandeinstein.phys.virginia.edu/more_stuff/Applets/MichelsonMorley/michelsonmorley.html

Interferometer

- Sends a light beam through a half-silvered mirror which splits it into two beams traveling at right angles to one another.
- the beams travel out to the ends of long arms where they were reflected back to the middle by small mirrors.
- They then recombined on the far side of the splitter producing a pattern of constructive and destructive interference based on the length of the arms.
- Any slight change in the amount of time the beams spent in transit would then be observed as a shift in the positions of the interference fringes.

Michelson-Morley Experiment

• *Light leaves source, and is partly reflected 45°/partly transmitted at half-glazed mirror*

•*Light returning from both paths is collected at detector*

•*Path length of light along either "arm" of apparatus is the same*

•If one arm is along Earth's motion through ether, and the other arm is perpendicular to motion through ether, then light travel time was expected to be shorter for perpendicular arm

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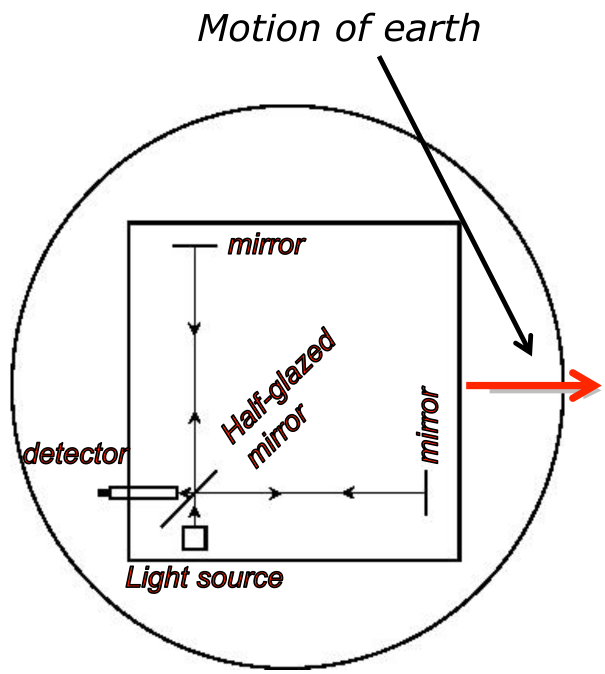
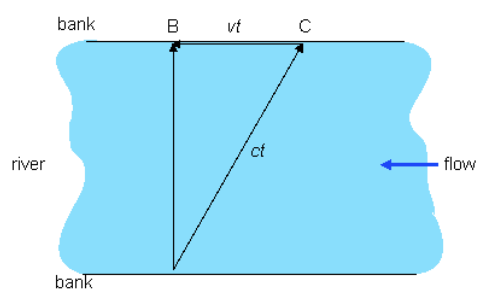


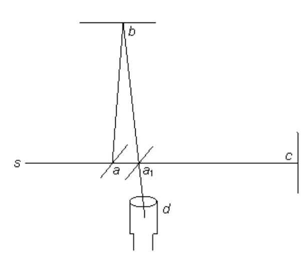
- Michelson's critical idea was to construct an a race for pulses of light, with the aether wind playing the part of the river.
- **expected difference in time is ~ 2(pathlength/c)x($v^2/2c^2$)- where *v is the velocity of the aether***

Michelson-Morley



In time *t*, the swimmer has moved *ct* relative to the water, and been carried downstream a distance *vt*.

Tried to measure annual effect effect to due rotation of earth and at different altitudes-nothing seem



This is also from the original paper, and shows the expected path of light relative to the aether with an aether wind blowing.

http://galileoandeinstein.physics.virginia.edu/home.html

M-M mounted their interferometer on a massive stone slab for stability and floated it in mercury for smooth rotation.

Michelson-Morley results

- Travel time difference would be measured using interference fringes of light from two paths
- Apparatus could be rotated to make sure no effects from set-up
- Repeated at different times of year, when Earth's motion differs; Earth's speed around the Sun is ~30 km/s
- Experiment performed in 1887
- Results
  - M-M showed that speed of light was same in any direction to within 5 km/s
  - Modern versions of the experiment show constancy to better than 1 micron/s
- So, what's going on??

Attempts to deal with M-M results

- Maybe the ether "sticks" to the Earth?
  - ?
- Maybe the ether squeezes the arms of the M-M experiment and distorts the result? "Fitzgerald contraction" (1889)?
  - A contraction (in the direction parallel to motion through ether) would change the light travel time to compensate for the difference expected due to different speed of light

$$L = L_0\sqrt{1 - V^2 / c^2}$$

Impact of the M-M results

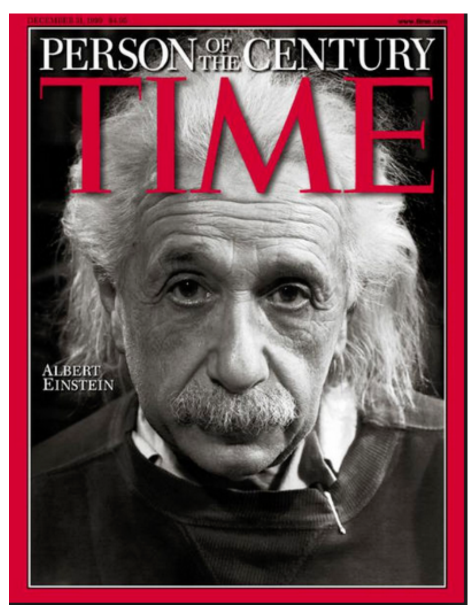
- Major mystery ("crisis") in 19th century physics – two highly successful theories seemed incompatible!
  - Mechanics – Galilean Relativity and Newton's laws
  - Electromagnetism – Maxwell's equations

Einstein enters the picture…

- Albert Einstein (1879-1955)
  - Knew that Maxwell's equations were invariant under "*Lorentz* transformation" of space and time
    a transform is the formula for the conversion of coordinates and times of events in different frames.
- But Newton's Laws are invariant under a *Galilean* transform
- Problem: if Maxwell's laws are 'correct' Newton's are not ?

Albert Einstein

- Over a 15 year period (1905-1920) came the most explosive ideas of the century. They were catalysts that set in motion a reappraisal of every premise and postulate of modern natural science, a physical revolution whose end is far from sight.
- His ideas, like Newton's and Darwin's, reverberated beyond science, influencing modern culture from painting to poetry.
- Read more: http://www.time.com/time/magazine/article/0,9171,993017,00.html#ixzz1YQbS5kcK



II : Einstein's Postulates of Special Relativity

- Albert Einstein
  - Didn't "like" idea of Aether
  - Any aether would need to be massless, incompressible, entirely transparent, continuous, devoid of viscosity and nearly infinitely rigid and thus not like any known substance.

How to resolve conflict between mechanics and electromagnetism?

- 'Throw away' the idea of Galilean Relativity for mechanics!
  - Galilean transformation between frames does not hold: velocities do not simply add/subtract (although the effects are small when the speeds are much less than the speed of light).

Special Relativity is a theory of exceptional elegance, crafted from simple postulates about the constancy of physical laws and of the speed of light

Its fundamental - the laws of physics and the constancy of the speed of light are now understood in terms of the most basic symmetries in space and time.

II : Einstein's Postulates of Special Relativity

- **Postulate 1** – The laws of nature are the same in all inertial* frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all inertial frames of reference.

- The second postulate is necessary to allow Maxwell's equations to follow from Postulate 1

* inertial frame of reference- no acceleration (no forces on object, no rotation)

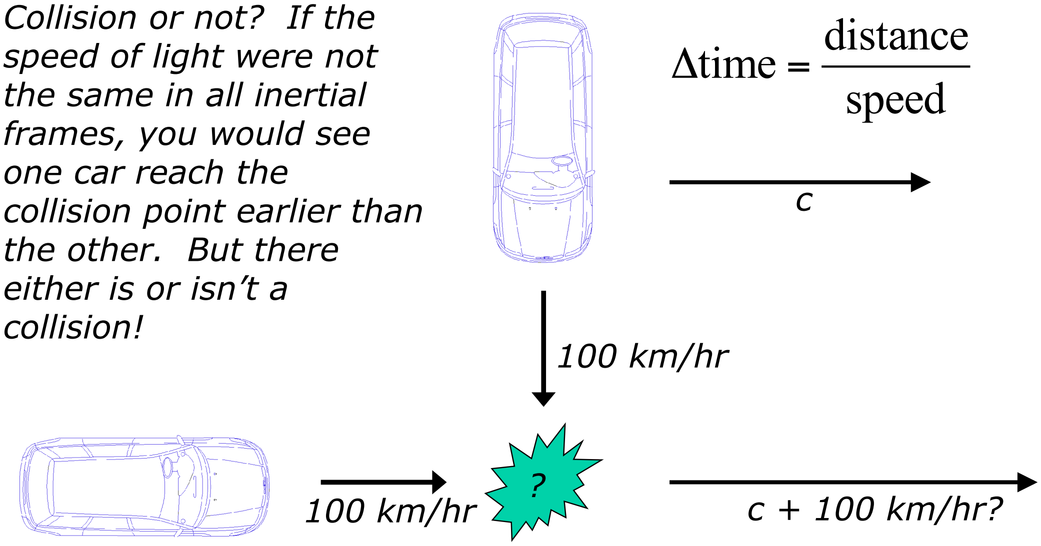
Consequences of EINSTEIN'S POSTULATES OF RELATIVITY

- Let's start to think about the consequences of these postulates.
- We will perform "thought experiments" (Gedankenexperimenten) to think of what observers moving at different speeds will think
- For now, we will ignore effect of gravity – we suppose we are performing these experiments in the middle of deep space (or in free fall)

What if the speed of light weren't the same in all inertial frames?

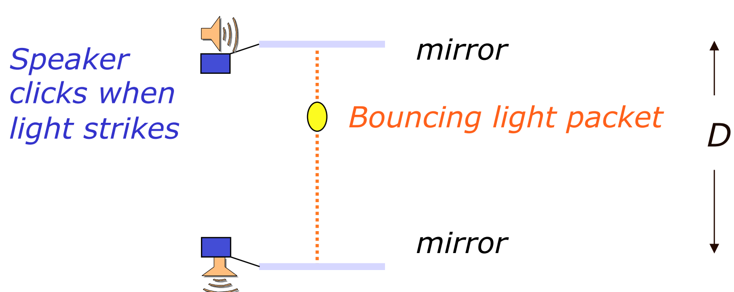
Collision or not? If the speed of light were not the same in all inertial frames, you would see one car reach the collision point earlier than the other. But there either is or isn't a collision!

$$\Delta\text{time} = \frac{\text{distance}}{\text{speed}}$$



III: TIME

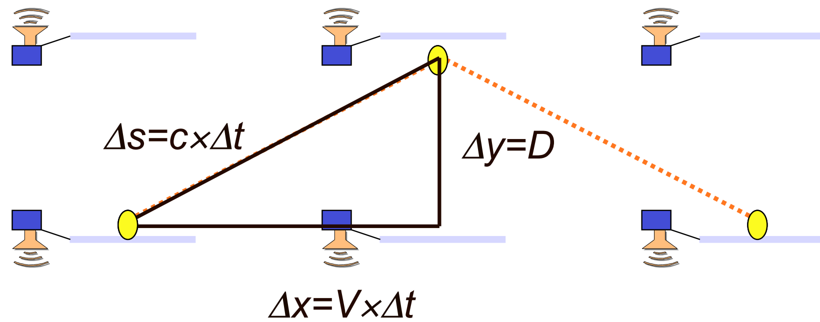
- Imagine building a clock using mirrors and a light beam.



- One "tick" of the clock is the time it takes for light to travel from one mirror to the other mirror.

$$\Delta T = \frac{D}{c}$$

Moving clock



- Now suppose we put the same "clock" on a spaceship that is cruising (at constant velocity, V) past us.
- How long will it take the clock to "tick" when we observe it in the moving spacecraft? Use Einstein's postulates…
- Total distance travelled by light beam is $\Delta s = c \times \Delta t$
- Therefore time $\Delta t = \Delta s / c$
- By Pythagorean theorem, $\Delta s = c\Delta t = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V\Delta t)^2 + D^2}$
- Can solve to obtain $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- Clock appears to run more slowly!!

Now change the point of view…

- For ground-based observer, clock on spaceship takes longer to "tick" than it would if it were on the ground
- But, suppose there's an astronaut in the spacecraft
  - the inside of the spacecraft is also an inertial frame of reference – Einstein's postulates apply…
  - So, the astronaut will measure a "tick" that lasts

$$\Delta T = \frac{D}{c}$$

  - This is just the same time as the "ground" observers measured for the clock their own rest frame
- So, different observers see the clock going at different speeds!

So time is not absolute!!

It depends on your point of view…

Time dilation

- This effect called **Time Dilation**.
- Clock always ticks most rapidly when measured by observer in its *own* rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

- This slowing factor is called the **Lorentz factor,** γ

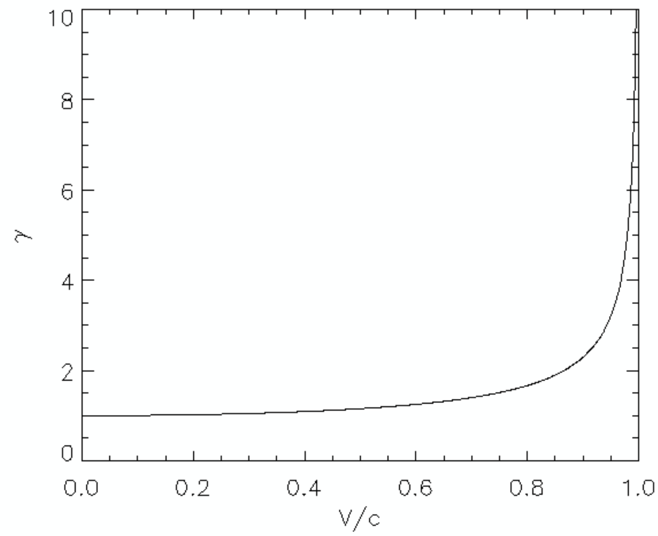
$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

- The moving clock ticks more slowly by a factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- This effect is called **time-dilation**, and the quantity “γ” is the **Lorentz factor**.
- There is nothing “wrong” with the moving clock! Time really is flowing more slowly in the moving frame.

Lorentz factor



A 1% effect at v = 0.14 c, or 42,000 km/s

Lorentz factor goes to infinity when V→c!

But it is very close to 1 for V/c small

Why don't we ordinarily notice time dilation?

Some examples of speeds in m/s

- 0.0055m/s world record speed of the fastest snail in the Congham,UK
- 0.080 m/s the top speed of a sloth (= 8.0 cm/s)
- 1 m/s a typical human walking speed
- 28 m/s a car travelling at 60 miles per hour (mi/h or mph) or 100 kilometres per hour (km/h); also the speed a cheetah can maintain
- 341 m/s the current land speed record, which was was set by ThrustSSC in 1997.
- 343 m/s the approximate speed of sound under standard conditions, which varies according to air temperature
- 464 m/s Earth's rotation at the equator.
- 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- 1000 m/s the speed of a typical rifle bullet
- 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- 299,792,458 m/s the speed of light (about 300,000 km/s)

Do Frequent Fliers Age More Slowly?

- https://scienceline.org/2010/10/do-frequent-fliers-age-more-slowly/
- For small velocities at which the relativity factor is very close to 1, then the time dilation can be expanded in a binomial expansion to get the approximate expression:
- $\gamma = (1+v^2/2c^2)$

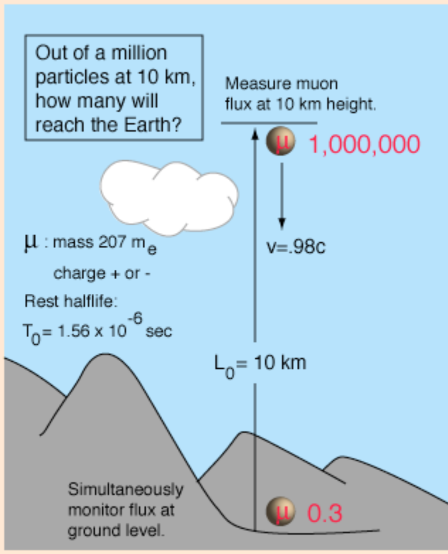
Rossi & Hall Muon Experiment

- Classic experiment verifying time dilation was performed by Rossi & Hall in 1941…
  - Muons are "electron-like" particles… when at rest, they decay with a half-life of about 1.56μs
  - Muons are produced when cosmic rays slam into upper atmosphere, then rain down to Earth
  - Rossi & Hall measured the number of muons detected at the top of a 2000m mountain, and compared it to the number at sea-level…
    - Find 560 muons/hour at top of mountain
    - Even at v=c, will take 6.5μs for muon to travel 2000m
    - More than 4 half lives… less than 1/16th of particles should be left by the time they reach the bottom
    - BUT, they measured 422 muons/hour at bottom
    - It seems like only 0.64μs have passed in the muon's frame of reference… so they are moving with $\gamma \approx 10$

Muon Experiment

The measurement of the flux of muons at the Earth's surface produced an early dilemma because many more are detected than would be expected, based on their short half-life of 1.56 microseconds. This is a good example of the application of relativistic time dilation to explain the increased particle range for high-speed particles.

Non-Relativistic



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4\,\text{m}}{(0.98)(3 \times 10^8\,\text{m/s})}$

$T = 34 \times 10^{-6}$ s = 21.8 halflives

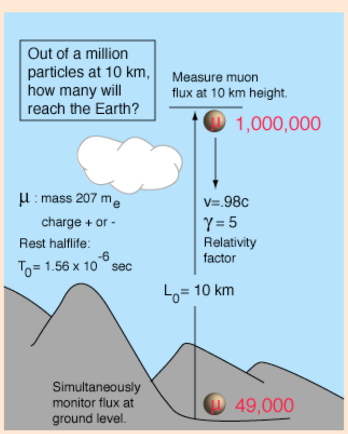
Survival rate:

$\dfrac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$

Or only about 0.3 out of a million.

Muon Experiment

Relativistic, Muon-Frame Observer



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{2000\,\text{m}}{(0.98)(3 \times 10^8\,\text{m/s})}$

$T = 6.8 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$\dfrac{I}{I_0} = 2^{-4.36} = 0.049$

Or about 49,000 out of a million.

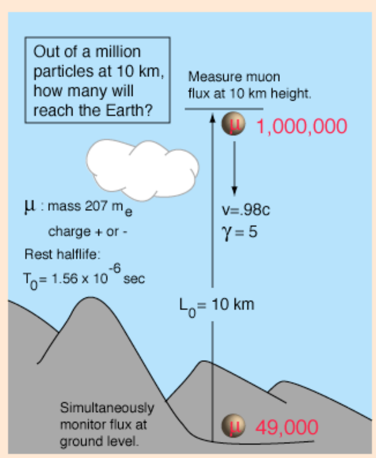
The muon sees distance as length-contracted so that $L = L_0/\gamma = 0.2L_0$ = 2 km.

☐ Non-relativistic ☐ Relativistic, Earth observer ☑ Relativistic, muon observer

☐ Comparison ☐ Comments on comparison ☐ Vary parameters

Muon Experiment

Relativistic, Earth-Frame Observer



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The muon's clock is time-dilated, or running slow by the factor $T = \gamma T_0$, so its measured halflife is 5 x 1.56μs = 7.8μs.

☐ Non-relativistic ☑ Relativistic, Earth observer ☐ Relativistic, muon observer

☐ Comparison ☐ Comments on comparison ☐ Vary parameters

Examples of time dilation

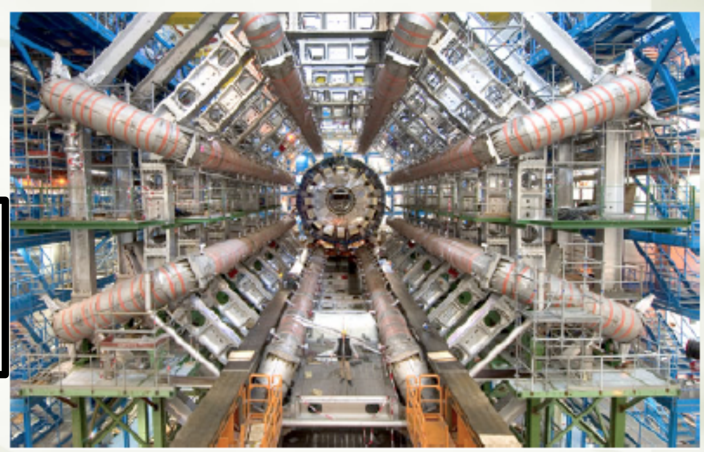
- The Muon Experiment
  - Muons are created in upper atmosphere from cosmic ray hits
  - Typical muon travel speeds are $0.99995\times c$, giving $\gamma=100$
  - Half-life of muons in their own rest frame (measured in lab) is t_h= 2 microseconds =0.000002s
  - **Traveling at $0.99995\times c$ for t_h=0.000002s, the muons would go only 600 m**
  - **But traveling for $\gamma\times t_h$= 0.0002s, the muons can go 60 km**
  - *They easily reach the Earth's surface, and are detected!*
  - *Half-life can be measured by comparing muon flux on a mountain and at sea level; result agrees with $\gamma\times t_h$*
- Why muons?- have comparatively long decay life time (the second longest known for sub-atomic particles) and are relatively weakly interacting so they can penetrate the atmosphere

LHC

- When the power of this machine is discussed, the energy of each proton is often mentioned: The protons each have an energy of 7 TeV. What does that mean?
- With E = 7 TeV: $E=\gamma mc^2$

the Lorentz factor has a value of about 7460 corresponding to v = 0.999999991 times the speed of light

- time passes 7460 times more slowly for the particles than it does for us observers.



A clock traveling at that speed from Earth to Proxima Centauri would measure a journey time of under 5 hours, while an observer who would remain on Earth would have aged over 4 years (Proxima Centauri is about 4.243 light-years away from us).

Simultaneity

- Time can be different for two observers
- whether two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame.
- reference frame that is moving relative to the first will generally assign different times to the two events
- More later on....