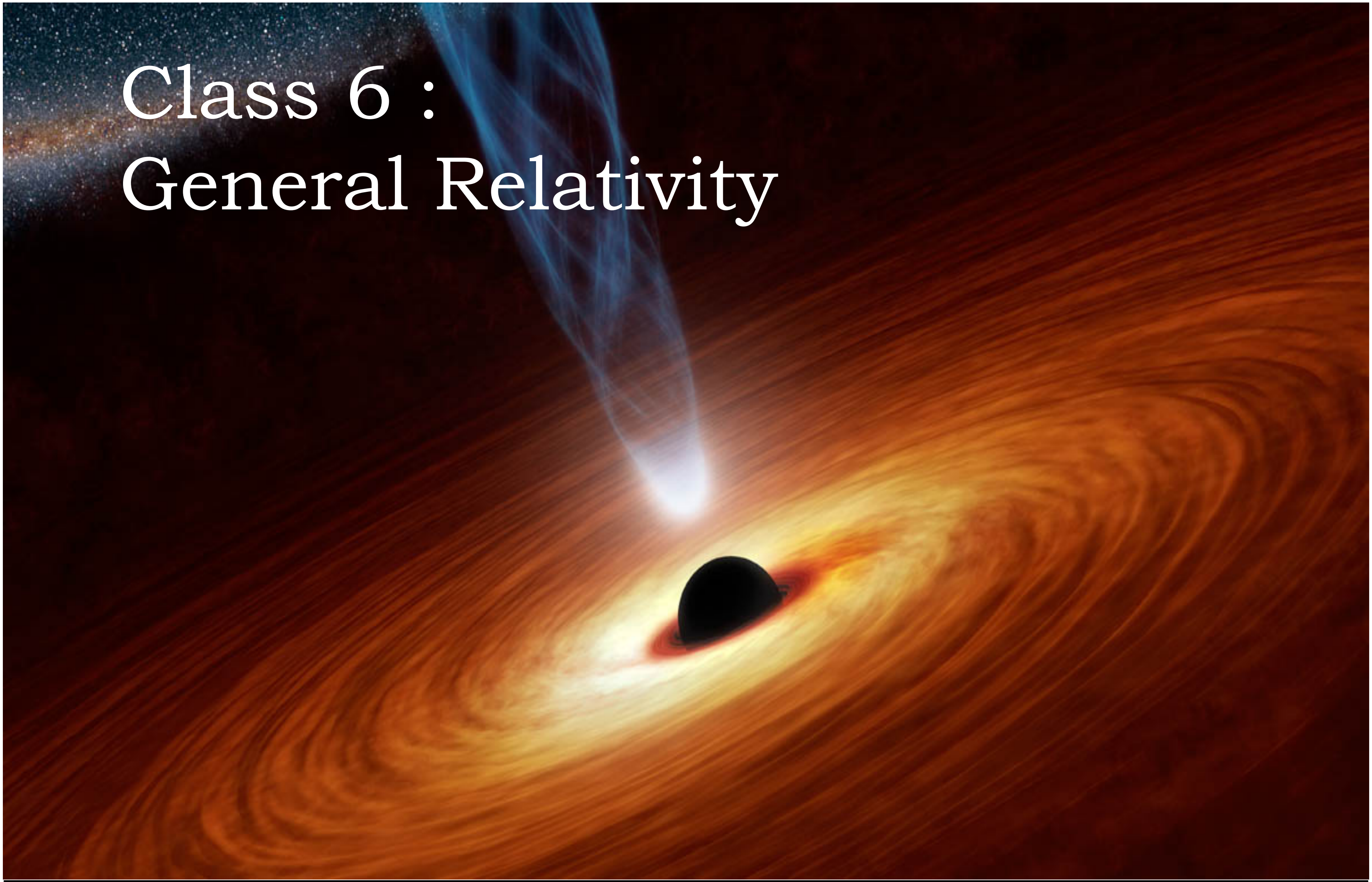
Class 6 : General Relativity



ASTR350 Black Holes (Spring 2020)
Prof. Richard Mushotzky

RECAP

- Einstein's postulates
  - Laws of physics look the same in any inertial frame of reference.
  - The speed of light is the same in any inertial frame of reference
- Strange consequences
  - Time dilation and length contraction
  - Relativity of simultaneity and ordering of events… need to have ultimate speed limit
  - Equivalence and conversion of mass and energy… reason for ultimate speed limit
- **We have been carefully avoiding gravity**

This class

- Finally talk about gravity
- Affect of gravity on light
- Affect of gravity on time
- Strong equivalence principle

- HW 3 due Feb 20
- Alex D's office hours for Monday Feb 17 are 12:00-1:00 – ONLY FOR THIS WEEK

HW 3

- Homework 3 : The Theory of Relativity
- (Due 20h February 2020)

- 1. Read the New York Times article on Einstein https://www.nytimes.com/2015/11/24/science/a-century-ago-einsteins-theory-of-relativity-changed-everything.html
- 2. Read/listen to NPR story on atomic clocks at http://www.npr.org/2014/11/03/361069820/new-clock-may-end-time-as-we-know-it

- For each of these pieces, submit via ELMS one question that came to your mind, and one issue that you found particularly interesting.

The New York Times

WEDNESDAY, DECEMBER 3, 1919

THE NEW

EINSTEIN EXPOUNDS HIS NEW THEORY

It Discards Absolute Time and Space, Recognizing Them Only as Related to Moving Systems.

IMPROVES ON NEWTON

Whose Approximations Hold for Most Motions, but Not Those of the Highest Velocity.

INSPIRED AS NEWTON WAS

But by the Fall of a Man from a Roof Instead of the Fall of an Apple.

Special Cable to THE NEW YORK TIMES.

BERLIN, Dec. 2.—Now that the Royal Society, at its meeting in London on Nov. 6, has put the stamp of its official authority on Dr. Albert Einstein's much-debated new "theory of relativity," man's conception of the universe seems likely to undergo radical changes. Indeed, there are German savants who believe that since the promulgation of Newton's theory of gravitation no discovery of such importance has been made in the world of science.

When THE NEW YORK TIMES correspondent called at his home to gather from his own lips an interpretation of what to laymen must appear the book with the seven seals, Dr. Einstein himself modestly put aside the suggestion that his theory might have the same revolutionary effect on the human mind as Newton's theses. The doctor lives on the top floor of a fashionable apartment house on one of the few elevated spots in Berlin—so to say, close to the stars which he studies, not with a telescope, but rather with the mental eye, and so far only as they come within the range of his mathematical formulae; for he is not an astronomer but a physicist.

It was from his lofty library, in which this conversation took place, that he observed years ago a man dropping from a neighboring roof—luckily on a pile of soft rubbish—and escaping almost without injury. This man told Dr. Einstein that in falling he experienced no sensation commonly considered as the effect of gravity, which, according to Newton's theory, would pull him down violently toward the earth. This incident, followed by further researches along the same line, started in his mind a complicated chain of thoughts leading finally, as he expressed it, "not to a disavowal of Newton's theory of gravitation, but to a sublimation or supplement of it."

When he read in the message from THE TIMES requesting the interview a reference to Dr. Einstein's statement to his publishers on the submission of his last book that not more than twelve persons in all the world could understand

electrodynamics, as well as mechanics, hitherto irreducible by the old formulae—and there are multitudes—were satisfactorily explained.

"Till now it was believed that time and space existed by themselves, even if there was nothing else—no sun, no earth, no stars—while now we know that time and space are not the vessel for the universe, but could not exist at all if there were no contents, namely, no sun, earth, and other celestial bodies.

"This special relativity, forming the first part of my theory, relates to all systems moving with uniform motion; that is, moving in a straight line with equal velocity.

"Gradually I was led to the idea, seeming a very paradox in science, that it might apply equally to all moving systems, even of difform motion, and thus I developed the conception of general relativity which forms the second part of my theory.

"It was during the development of the formulas for difform motions that the incident of the man falling from the roof gave me the idea that gravitation might be explained by difform motion."

"If there is no absolute time or space, supposedly forming the vessel of the universe," the correspondent asked, "what becomes of the ether?"

"There is no ether, as hitherto conceived by science, which is proved by the well known experiment of the celebrated American savant, Michelson, showing that no influence by the motion of the earth on the ether is perceptible through change in velocity of light, such as ought to be produced if the old conception were true."

"Are you yourself absolutely convinced of the correctness of this revolutionary theory of relativity, or are there still any reservations?"

"Yes, I am," Dr. Einstein answered. "My theory is confirmed by the two crucial cases mentioned before. But there is still one test outstanding, namely, the spectroscopic. According to my theory, the lines of the spectra of fixed stars must be slightly shifted through the influence of gravitation exerted by the very stars from which they emanate. So far, however, the results of the examinations have been contradictory; but I have no doubt of final confirmation, even through this test."

Just then an old

in the library chim

reminding Dr. Eins

Large Measure of Home Rule for Ireland Believed in View

LONDON, Wednesday, Dec. 3.—The Irish situation was the principal subject discussed by the Cabinet at its meeting yesterday forenoon.

Ministers attending included Viscount French, the Lord Lieutenant of Ireland; Earl Curzon, the Foreign Secretary; A. J. Balfour, Lord President of the Council; Winston Churchill, Secretary for War, and Edwin S. Montagu, Secretary for India.

There is reason to believe that the Government's original Home Rule proposals are undergoing considerable alteration, says The Daily Mail this morning, particularly with reference to the proposed executive uniting of the two provincial Legislatures. It is also believed that a considerably larger measure of Home Rule is contemplated.

"It is significant," the newspaper adds, "that a strong body of young Unionist Members of Parliament are pressing the Government to adopt a very forward policy with regard to Ireland."

SAYS TEN MILLIONS FACE HUNGER DEATH

Morgenthau Urges America to Try to Save Dying Nations of the Old World.

DESCRIBES SCENES HE SAW

Ex-Ambassador Convinced Nothing Except Miracle Can Prevent Great Horror Winter Holds in Store.

Special to The New York Times.

WASHINGTON, Dec. 2.—In an address here tonight, Henry Morgenthau, former American Ambassador to Turkey, said that "nothing on earth except a miracle from heaven can prevent the death by freezing and starvation of from 5,000,000 to 10,000,000 people in Europe and the Near East this Winter."

Many prominent officials were in the audience that heard Mr. Morgenthau, who recently returned to this country from an official mission to Poland.

"I wish that I could adequately describe a scene which I witnessed in Pinsk last August," said Mr. Morgenthau. "It has haunted me ever since and has served as a complete expression of the misery and injustice which is prevalent over such a large part of the world today. A few months before my arrival a particularly atrocious Jewish massacre had taken place.

"A Polish officer with troops had entered an assembly hall where the leading Jewish residents had gathered, arrested them, and marched them hurriedly to the public square. He took thirty-five men, and, in the dim light of an automobile lamp, placed them against the Cathedral and shot them in cold blood. A somewhat hazy charge had been made that these men were Bolsheviki, but no trial was given them and indeed the charge was subsequently shown to be untrue.

"Returning to the scene of the execution the next morning the troops found that three of their victims were still breathing; these they dispatched with bullets and all the thirty-five corpses were then thrown into a pit in an old Jewish cemetery, without an opportunity for decent burial or religious exercises and with nothing to mark the graves.

"A proceeding to make one shudder, it is true, yet not a particularly horrible event compared with the crimes that have been perpetrated in Central Europe, in the Balkans, and in Asia Minor in the last five years. The lives of only thirty-five Jews were sacrificed, but in a few months nearly a million Armenians were destroyed under conditions that were far more hideous. It was the scene which I witnessed in my visit that I wish particularly to bring to your minds.

"Up to the time I came not a single Jew had been permitted to visit that cemetery. But I was allowed to inspect the scene of this martyrdom; and, when I entered, a great crowd of Jews who had followed me also went in. As soon as they reached the burial place of their relatives they threw themselves as a mass upon the ground, and set up a wailing that still rings in my ears. It was the anguish cry of a terribly persecuted race; to my mind it expressed the misery of centuries and the misery not only of the Jews, but the numerous other peoples that for ages have looked for justice and have not found it.

"That same evening I attended divine service at the synagogue. Inside this building a crowd of more than 5,000 had gathered to express their grief for the loss of their leaders. This large mass of men, women and children screamed until it seemed that the heavens would burst. I had read of such public expressions of agony in the Old Testament but this is the first time that I ever completely realized what the collective

First Newtonian mechanics (special relativity), now his law of gravity (general relativity)

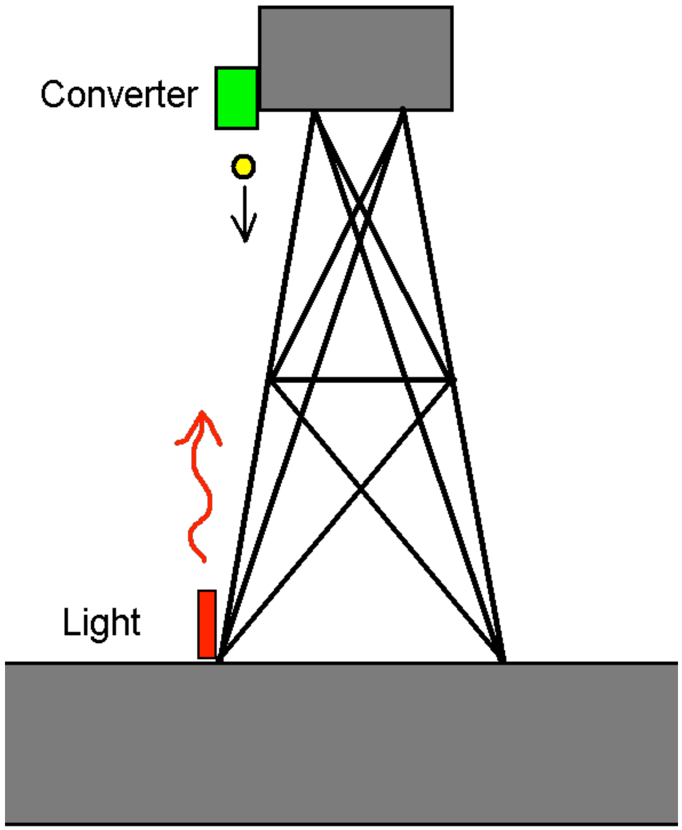
- As we have just learned we have to understand
  - In whose frame do we measure ?
  - Does the force depend based on your reference frame?
  - Can gravity information travel (communicate) faster than *c*?

General Relativity

- Like special relativity, the general theory predicts phenomena which differ significantly from those of classical physics,
  - especially concerning the passage of time,
  - the geometry of space,
  - the motion of bodies in free fall, and the *propagation* of light.
- Examples of such differences include gravitational time dilation, gravitational lensing, the gravitational redshift of light, and gravitational time delay

I: Einstein's Tower

- Another thought experiment… **suppose that light is not affected by gravity.**
- Consider a tower on Earth
  - Shine a light ray from bottom to top
  - When light gets to top, turn its energy into mass.
  - Then drop mass to bottom of tower.
  - Then turn it back into energy



Perpetual motion?

- So…
  - Suppose original photon energy E
  - *By assumption*, photons are is not affected by gravity so it has energy **E** once it reaches top
  - Convert this energy into mass, mass created at top is **m=E/c²**
  - Then drop mass… at bottom of tower, it has picked up speed due to the conversion of gravitational potential energy (**E_{grav}=mgh**)*

$$\boxed{E_{new} = E + mgh = E\left(1 + \frac{gh}{c^2}\right)}$$

  - We have made energy! We're rich!!!! Infinite amount of energy!

* A little physics.. the energy due to falling in a gravitational field is E_{fall}=mgh (h=height you fall, m is the mass falling, and g is the local acceleration due to gravity)

- Clearly something is wrong with our assumptions…
  - Only way we can conserve energy is to suppose that light **is** affected by gravity…
  - We need the photon to lose energy as it climbs upwards… at top of tower, we must have

$$E_{top} = E\left(1+\frac{gh}{c^2}\right)^{-1}$$

  - This is known as **gravitational redshift (?)**

Gravitational Redshift

- <u>for light the energy of a photon is related to its frequency,ν, (alternatively its wavlength,λ)</u>

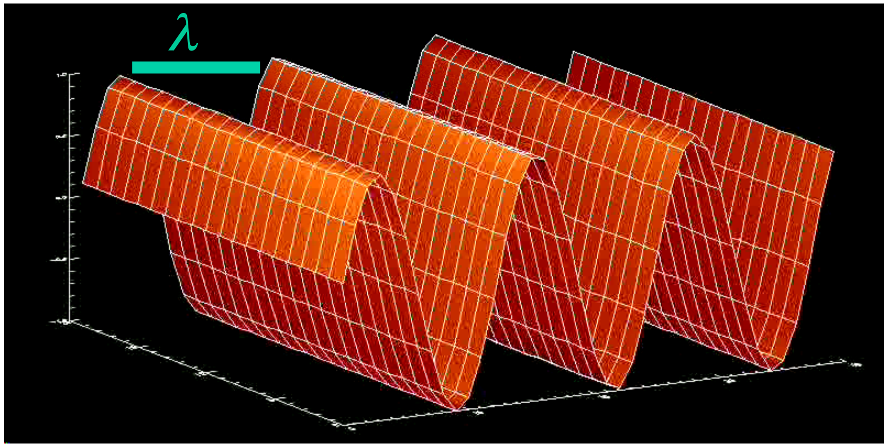
  $E=h^*\nu=hc/\lambda \rightarrow$ **lower energy, longer wavelength– redder**

- The profound nature of gravitational redshift…
  - Imagine a clock based on the frequency of light
  - Place the clock at the base of the tower… observe it from the top.
  - Photons lose energy… so they decrease frequency
  - Thus, we see the clock running slowly!
  - **Time passes at a slower rate in a gravitational field! (units of ν is cycles per sec, λ is wavelength of light)**

***h= Planck's constant** ($6.62606957 \times 10^{-34}$ m^2 kg/s)

Properties of Waves

- **Recall properties of waves:**
- Waves characterized by
  - Wavelength (λ) = distance between crests- (units length)
  - Frequency (f or ν) = number of crests passing a given point per second (units cycles/time)
- Speed of a crest; $s_c = \lambda\nu$ (this is generic to waves, for light s_c=c)
- **Energy** of a wave is proportional to frequency ν, $E = h\nu$.



To repeat....

- The profound nature of gravitational redshift…
  - Imagine a clock **based on the frequency of light**
  - Place the clock at the base of the tower…

$$E_{top} = E\left(1 + \frac{gh}{c^2}\right)^{-1}$$

  - observe it from the top.
  - Photons lose energy… **so their frequency decreases**
  - Thus, we see the clock running slowly!
  - **Time passes at a slower rate in a gravitational field!**

Gravitational Redshift

- To work it out with equations
- For a photon $\mathbf{E=mc^2=h\nu}$
- The gravitational potential energy (the energy gained by falling in a gravitational field) $PE=-GMm/r=(-GMh/rc^2)\nu_0$
- So as the photon rises up in the gravitational field
- $h\nu=h\nu_0\{1-(GM/rc^2)\}$; $\nu=\nu_0\ \{1-(GM/rc^2)\}$; $\Delta\nu/\nu=GM/rc^2$

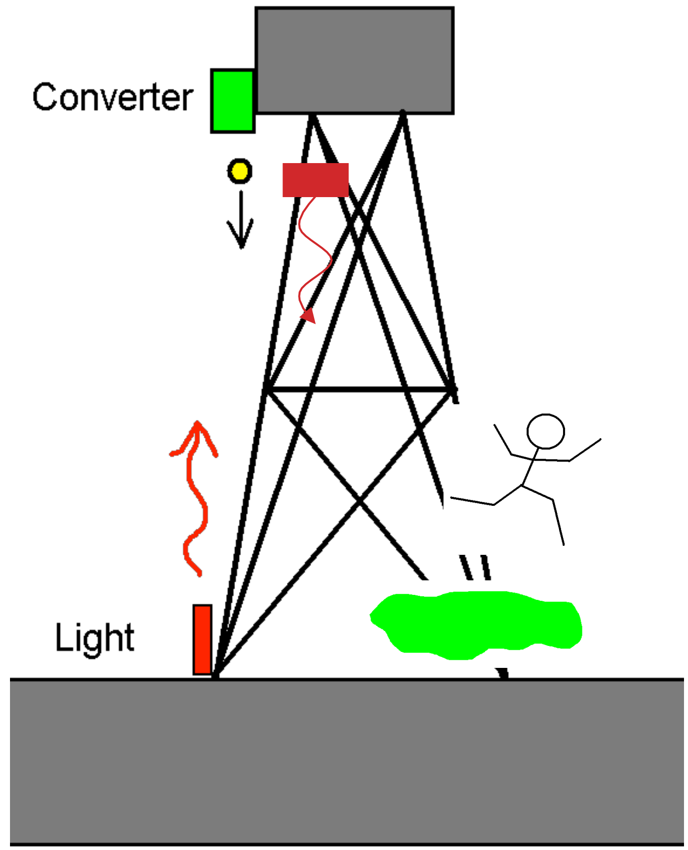
where $\Delta\nu$ is the change in frequency of the photon $\Delta\nu=(\nu-\nu_0)$

Tthe photon is reduced in frequency (and thus lengthened in wavelength) this is called the **gravitational redshift**

Resolving the tower problem

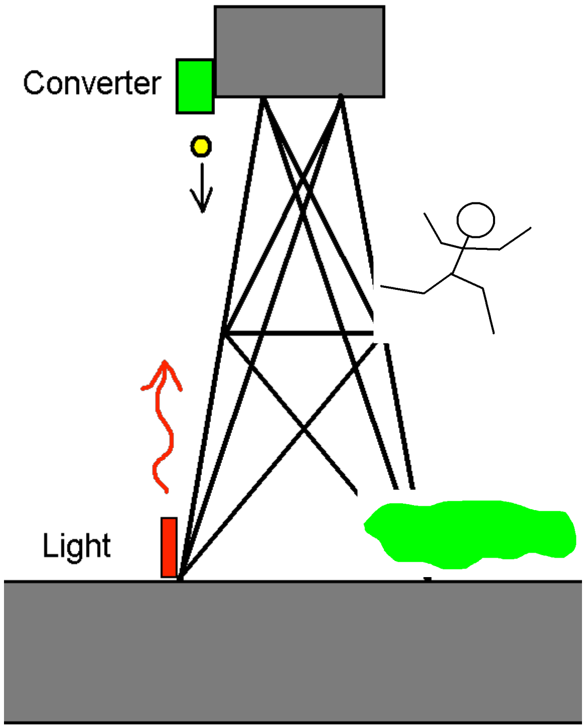
- Now consider light ray aimed from top to bottom of tower
- Free-falling(FF) observer sees light ray travel **unaffected** by gravity,since freefall is an inertial frame

- From"Earth's"frame…
  - Free-falling (FF) observer is traveling faster and faster
  - Falling observer would see an increasing *redshift* of light source according to special relativity (doppler effect)
  - ***If FF observer is supposed to see a constant frequency light beam,then light must get relatively blueshifted as it falls in a gravitational field, to compensate***



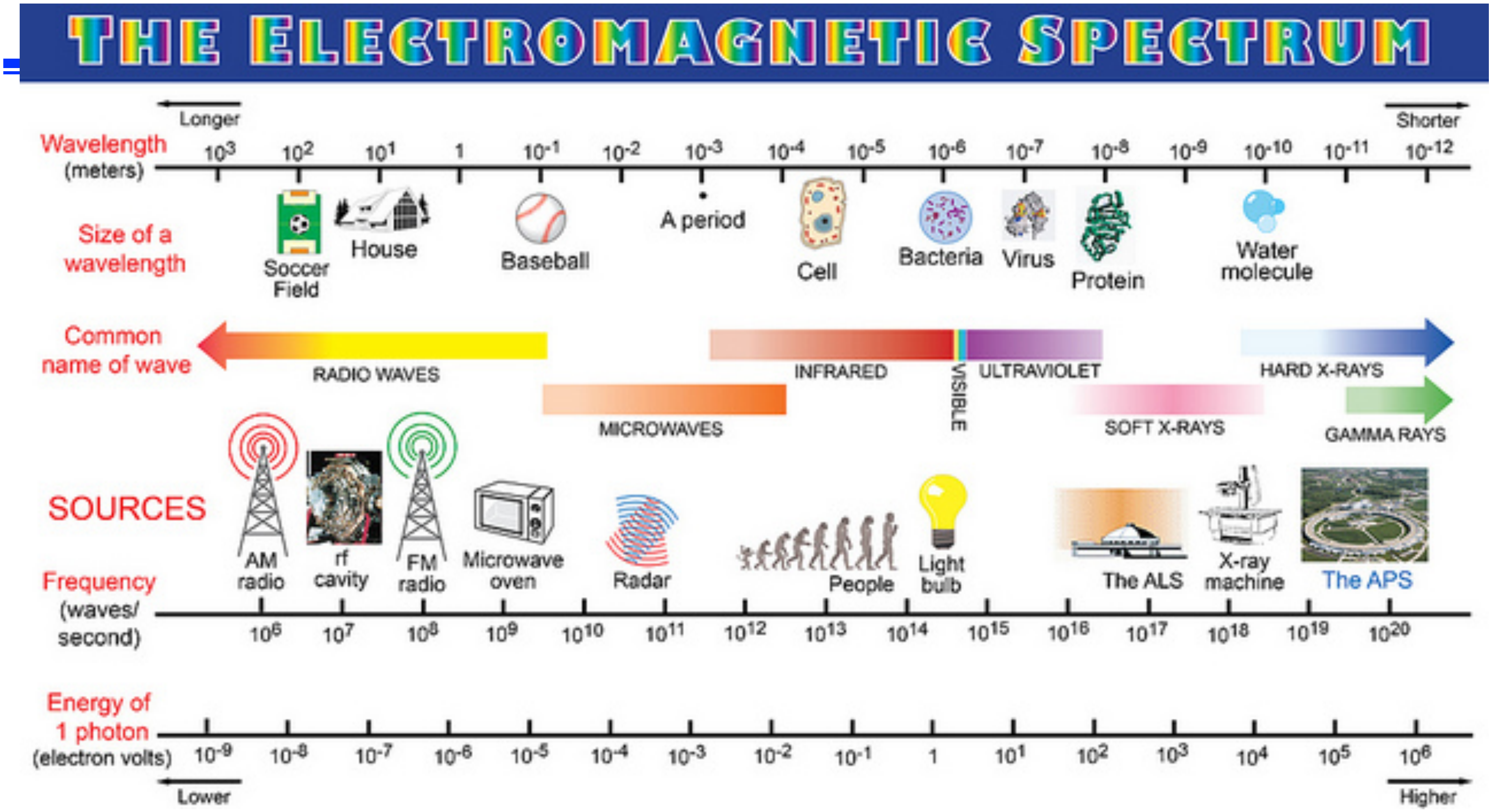
Gravitational Redshift

- Free-falling (FF) observer sees light ray travel unaffected by gravity.
- From "Earth's" frame…
  - Free-falling (FF) observer traveling faster and faster
  - FF observer would an increasing *blueshift*
  - Since FF observer sees an unaffected (I.e. constant frequency) light beam, light must get progressively *redshifted* as it climbs up.
  - **Redshifting removes just the right amount of energy to solve tower paradox.**



Maxwell and gravity

- Clearly,our assumption must be wrong…
  - ***light must be affected by gravity*.**
  - But gravity does not appear in Maxwell's equations,which govern light
  - Thus, Maxwell's equations are not complete* and are not exactly valid in the reference frame of Earth's surface,where there **is** gravity.
  - the Earth's surface must not be an inertial frame of reference!



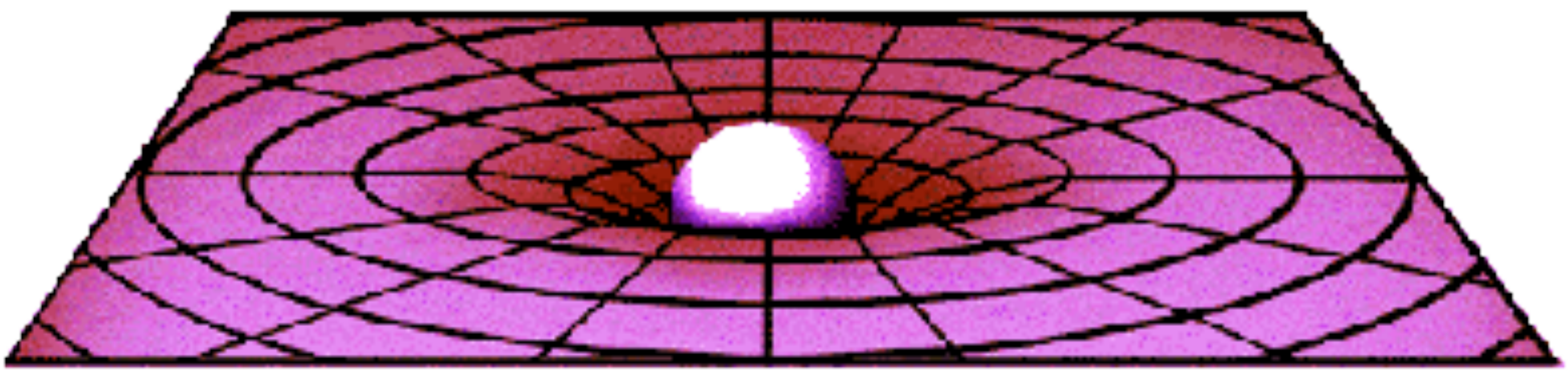
THE ELECTROMAGNETIC SPECTRUM
Longer
Shorter
Wavelength (meters)
10^3 10^2 10^1 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12
Size of a wavelength
Soccer Field
House
Baseball
A period
Cell
Bacteria
Virus
Protein
Water molecule
Common name of wave
RADIO WAVES
MICROWAVES
INFRARED
VISIBLE
ULTRAVIOLET
SOFT X-RAYS
HARD X-RAYS
GAMMA RAYS
SOURCES
AM radio
rf cavity
FM radio
Microwave oven
Radar
People
Light bulb
The ALS
X-ray machine
The APS
Frequency (waves/second)
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20
Energy of 1 photon (electron volts)
10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6
Lower
Higher

Another Effect of Gravity on Light

- Gravitational time delay- time it takes for light to travel to a source is effected by gravity (paths longer in curved space)

Curved Space (more in next lecture)

- Curved space around the Earth looks something like this…



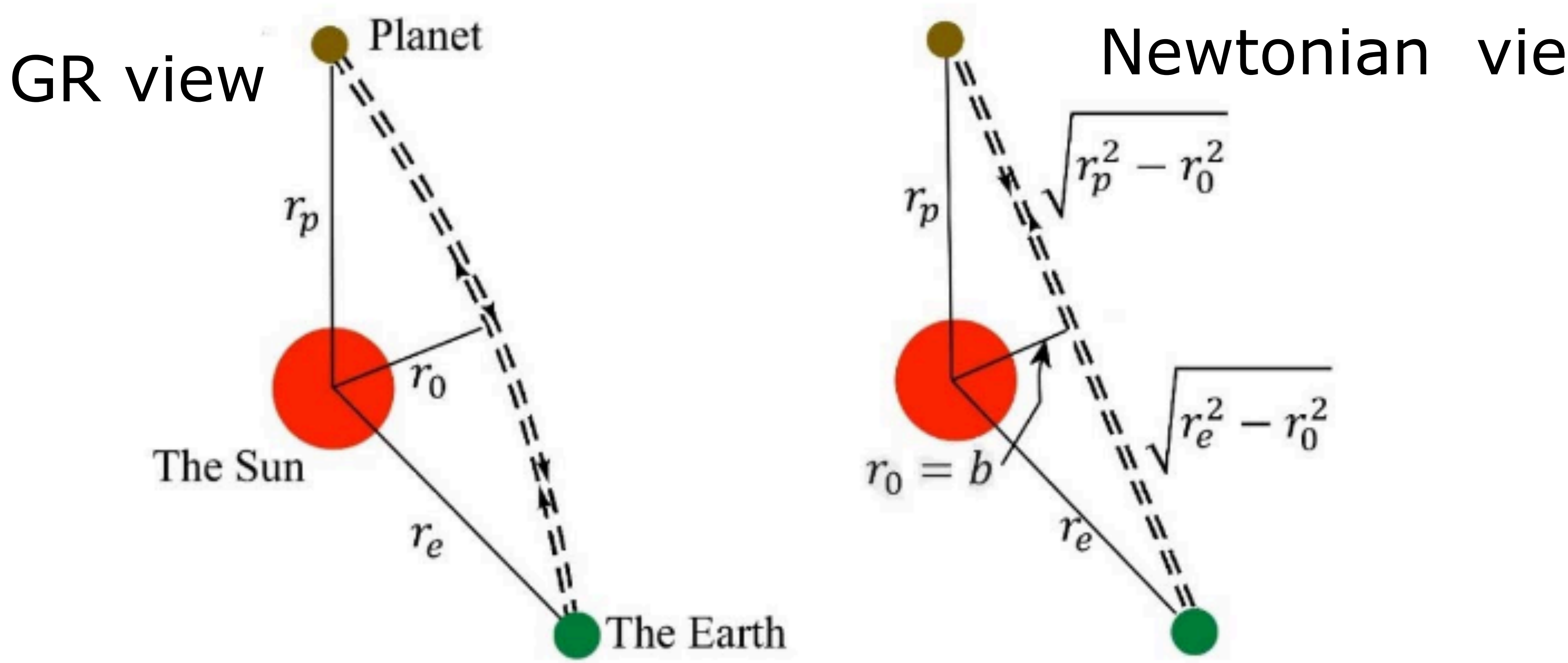
From web site of UCSD

The **Shapiro time delay** effect, or **gravitational time delay**

- the gravitational time delay effect, one of the four classic solar-system tests of general relativity.
- Radar signals passing near a massive object take slightly longer to travel to a target and longer to return than they would if the mass of the object were not present
- bounce radar beams off the surface of Venus and Mercury and measure the round-trip travel time as a function of orbit.
- Results agree with GR to 20 parts per million

Shapiro Delay Geometry

- Estelle Asmodelle (UCLAN Honors



igure 8 The radar reflection of photons from the Earth to a planet and back. The left image is the actual path, exaggerated. The right image is the Euclidean form. Illustration E. Asmodelle.

Hafele–Keating experiment

- Hafele & Keating (1971) flew around world with atomic clocks…
- Clock on plane gained time relative to one on ground by…
  - 273±7ns (Westbound)
  - -59±10 (Eastbound)
  - But TWO terms
    - special relativity
    - general relativity

	nanoseconds gained			
	predicted			measured
	gravitational (general relativity)	kinematic (special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

Two terms are need- special and general relativity

II: Strong Equivalence Principle

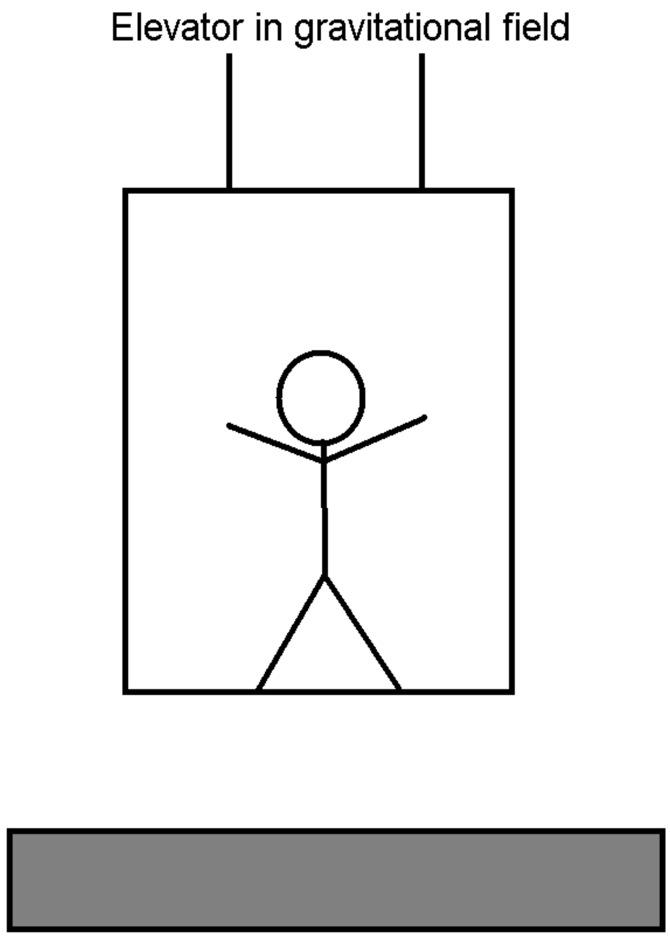
- Recap of the **weak equivalence principle**
  - All objects accelerate at the same rate in a given gravitational field.
  - In other words, inertial and gravitational masses are the same for any object.

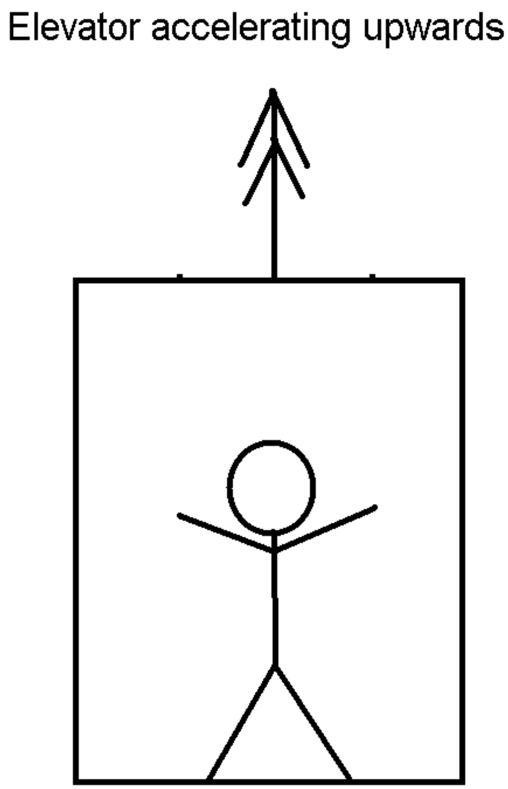
- Einstein introduced the **strong equivalence principle** – when gravity is present, the inertial frames of Special Relativity should be identified with free-falling frames of reference.
- What does this mean???

EQUIVALENCE PRINCIPLES

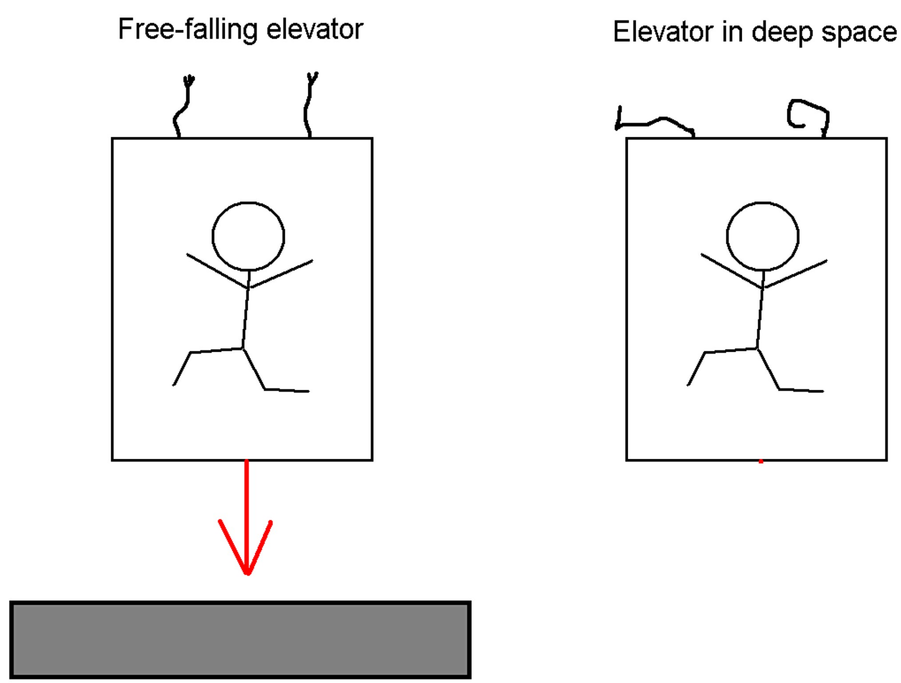
- Recall the "**weak**" **equivalence principle:**
  - All objects are observed to accelerate at the same rate in a given gravitational field (Galileo tower of Pisa experiment).
  - Therefore, the inertial and gravitational masses must be the same for any object.
  - This has been verified experimentally, with fractional difference in masses $<10^{-11}$
- As a consequence, the effects of gravity and of inertial forces (fictitious forces associated with accelerated frames) cannot, locally, be distinguished

Accleration vs Gravity

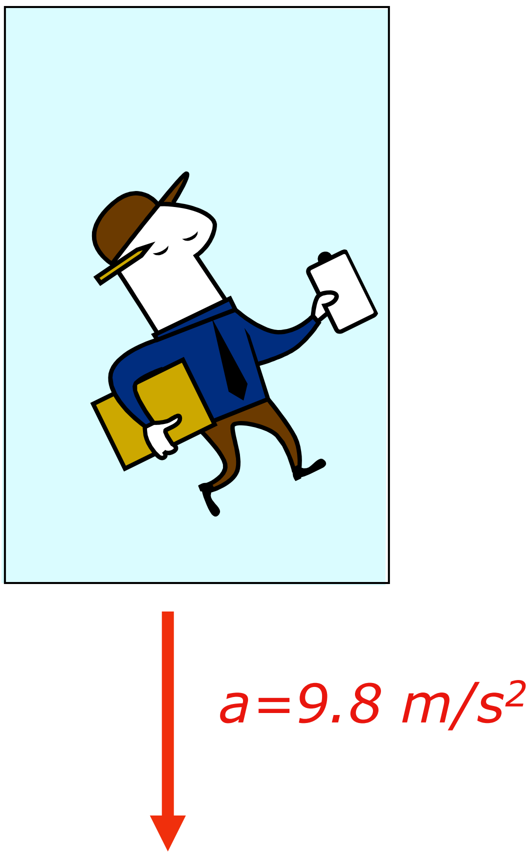


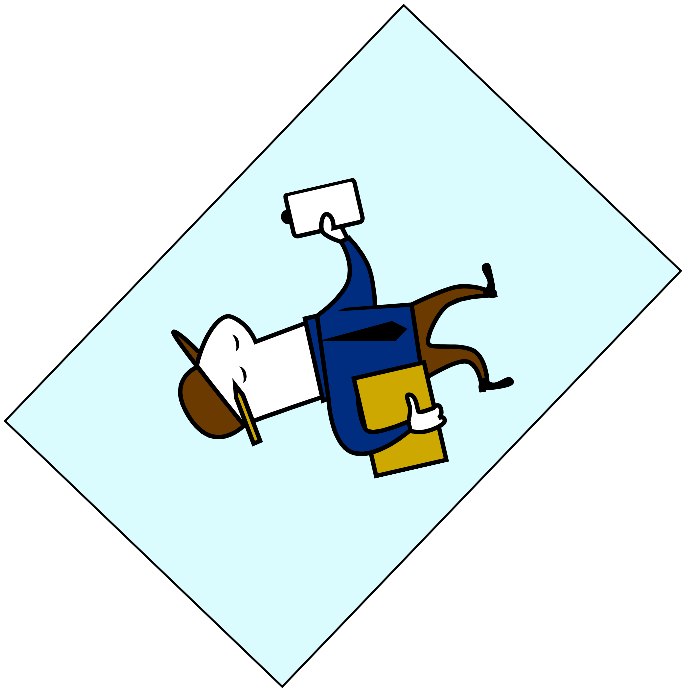


- There is **no** way of telling the difference between a free-falling frame in a gravitational field and an inertial frame in no gravitational field… the two are equivalent.

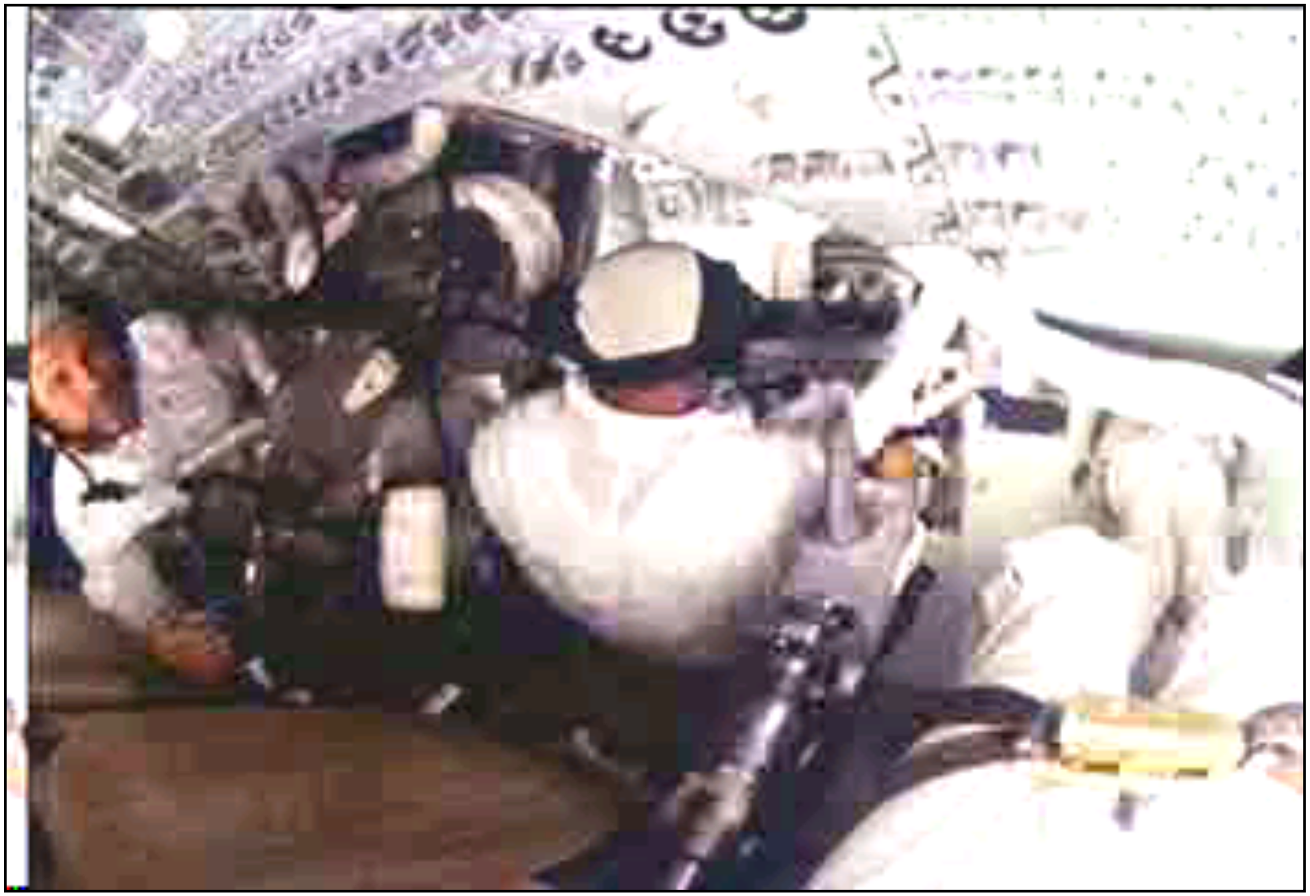


Interior of elevator free-falling on Earth is equivalent to interior of elevator floating freely in deep space





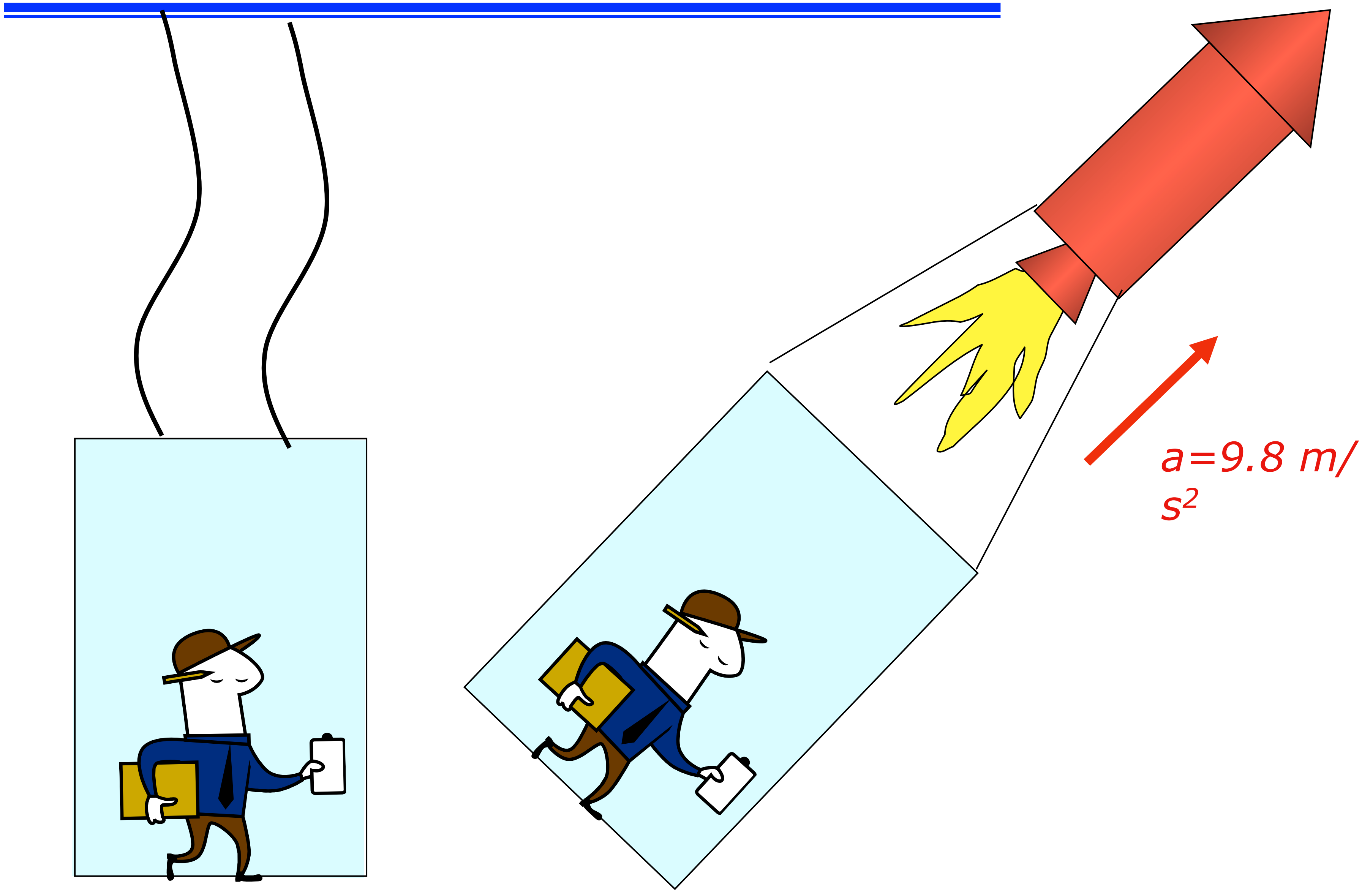
Back to the Astronauts…



What about gravity?

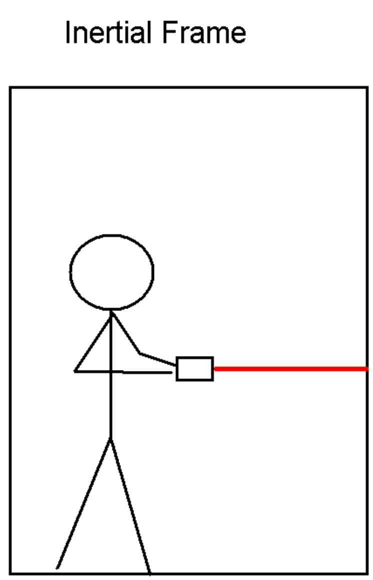
- Suppose that you decide that your frame of reference is not inertial…
  - Freely moving bodies change velocity
  - Is it because of gravity or is the frame accelerating?
  - Einstein says that you cannot tell the difference!
  - **Gravity is a "fictitious force"** – i.e., a force which appears to exist because we are living in a non-inertial frame of reference.

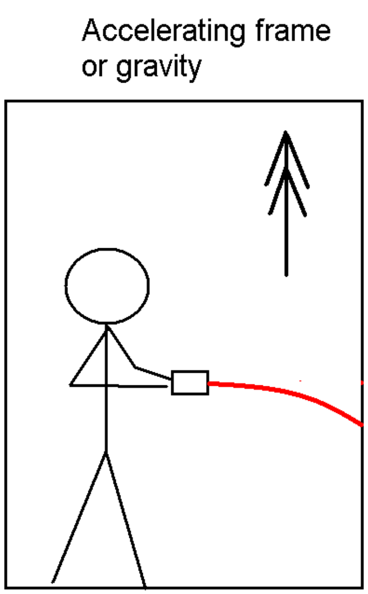
Elevator at rest on Earth equivalent to elevator being pulled by accelerating rocket in deep space



What about light? It “falls”, too!

- Astronaut in inertial frame with flashlight
  - Inertial frame, so light goes in straight lines
  - It doesn’t matter whether this is free fall or far from masses
- What if we now put flashlight in a gravitational field (accelerated frame)?
  - Light beam will bend: it must accelerate at the same rate and direction as the elevator
  - Strong equivalence principle ⇨ frame with gravity acts the same
  - Important conclusion – light “falls” due to gravity!- **how can we test this idea?**





The Eddington Test

- 1919 – the first "accessible" total Solar eclipse since Einstein postulated the Strong Equivalence Principle (SEP)
- Arthur Eddington
  - Famous British Astronomer
  - Lead expedition to South Africa to observe eclipse
  - Was looking for effects of gravitational light bending by searching for shifts in positions of stars just next to the Sun*.
  - The shifts were exactly as Einstein calculated! (1.75 seconds)

    Extremely important

**Einstein predicted that the magnitude of the shift depended on how large the angle was between the sun and the background star and the mass of the sun*

- Prior to this, the GR was untested – just an interesting, intelligent, idea.
- After Eddington, Einstein (and his science) was finally be taken seriously, and even the non-scientists of the world somehow found themselves standing in awe of the genius of this Swiss-German physicist.

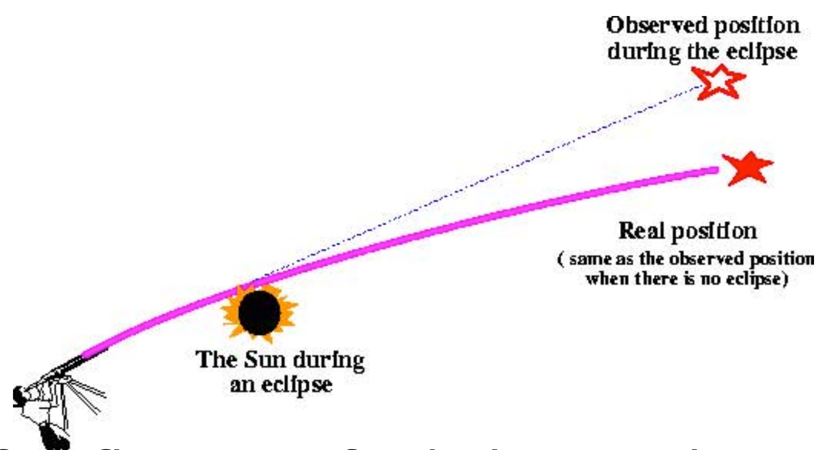
✦*Can Gravity Bend Light?*

✦*If light, does not have mass, how could it possibly be affected by gravity?*

✦*After all, the force of gravity - - is directly dependent upon the mass of two objects.*

✦*http://isaacmmcphee.suite101.com albert-einstein-and-bending-light-a43865#ixzz1nboyQEaH*

- Extraordinary claims require extraordinary evidence, and it's hard to think of something more extraordinary than the idea that massive objects warp space and time.

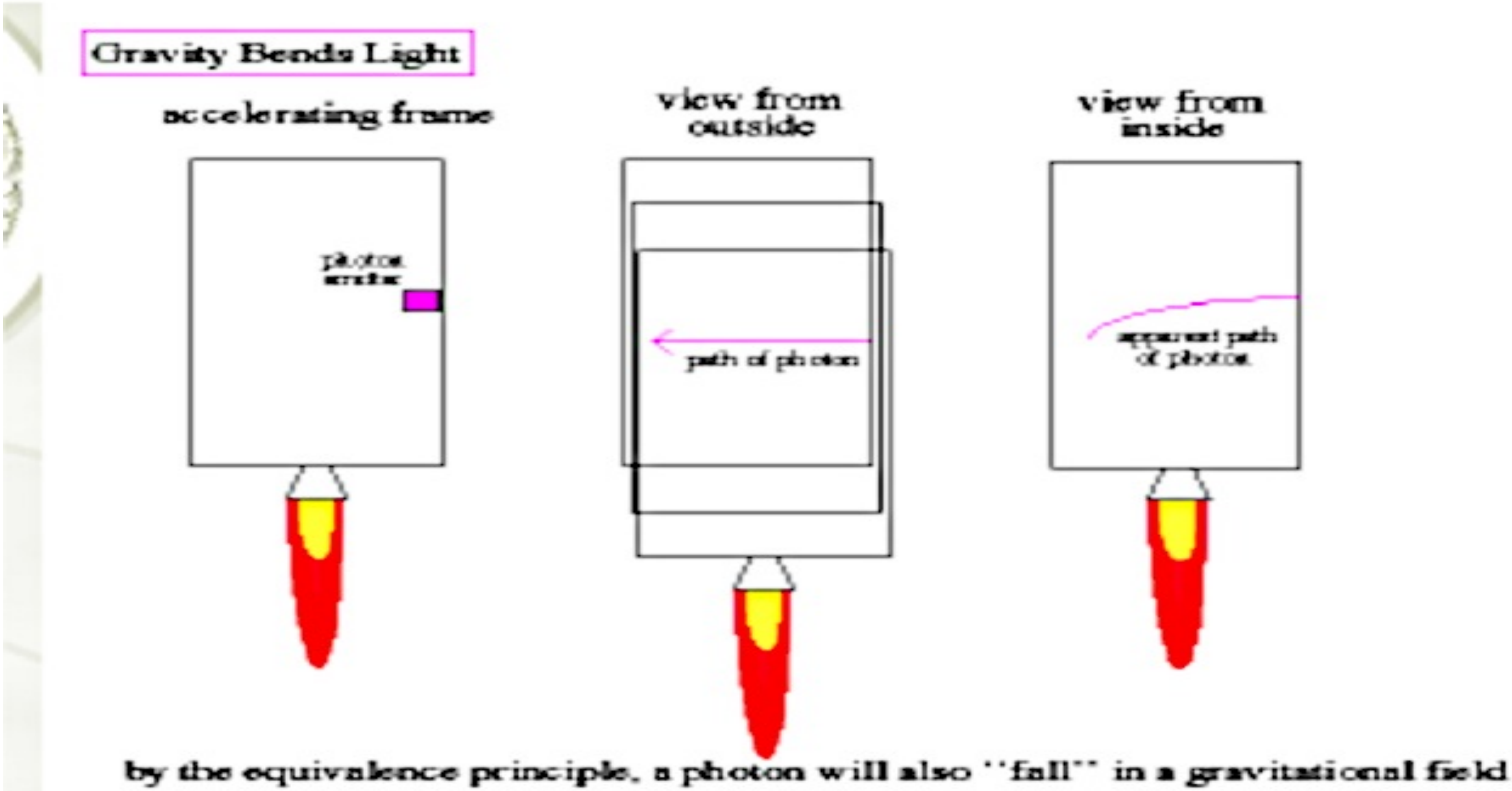


Angle is very small

the angle of deflection of a light ray that just grazes the surface of the Sun should be $\theta_d = 4GM_\odot/c^2R_\odot = 8.5\times10^{-6}$ *radians = 1.75 arcsec*

http://sunearthday.nasa.gov/2006/events/einstein.php

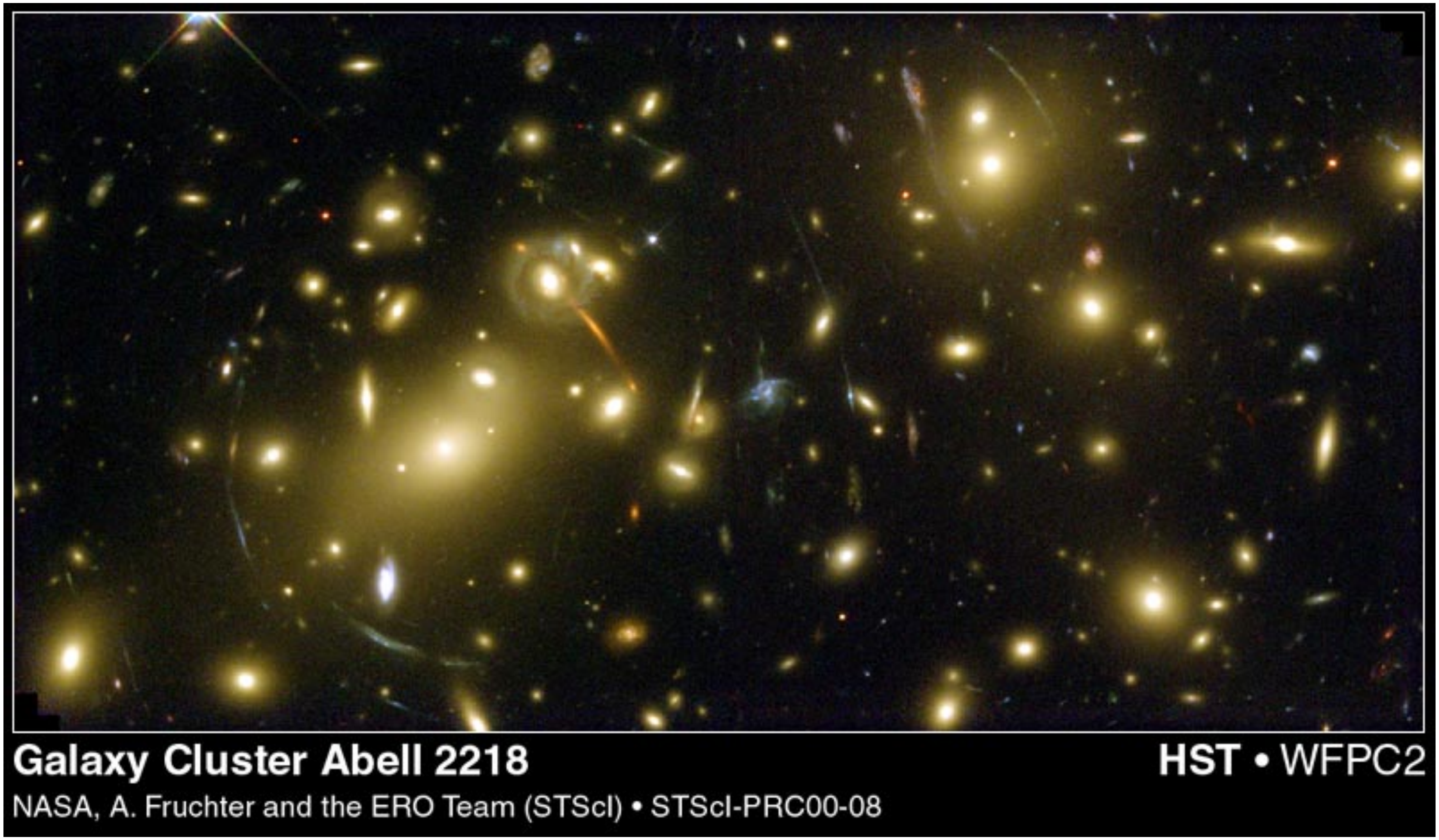
Bending of light by Gravity



The principle of equivalence renders the gravitational field fundamentally different from all other force fields encountered in nature.

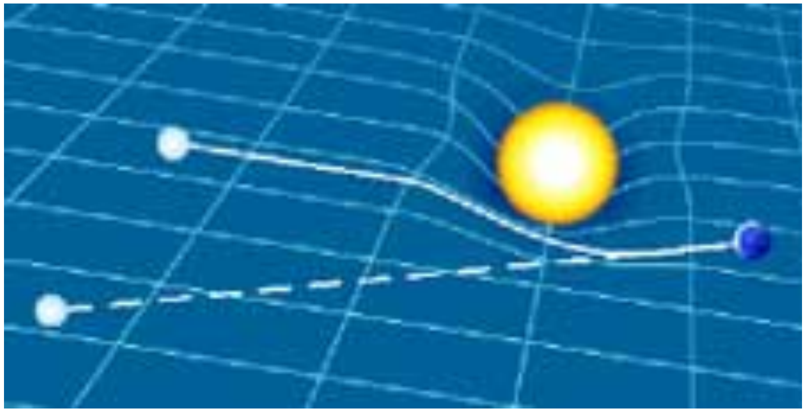
http://abyss.uoregon.edu/~js/cosmo/lectures/lec06.html

Giant lenses in the sky

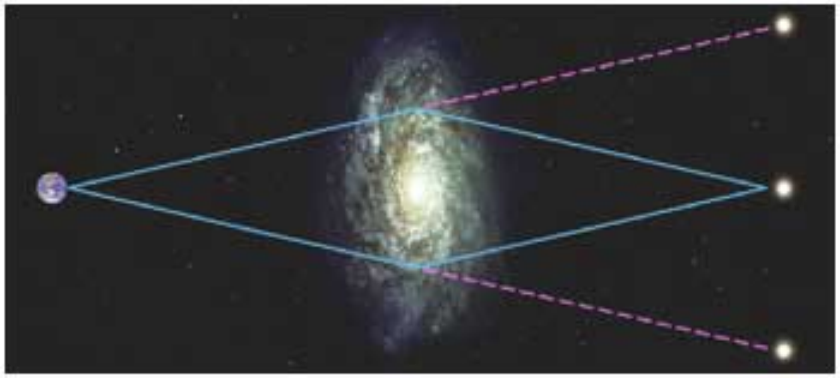


Gravitational Lensing

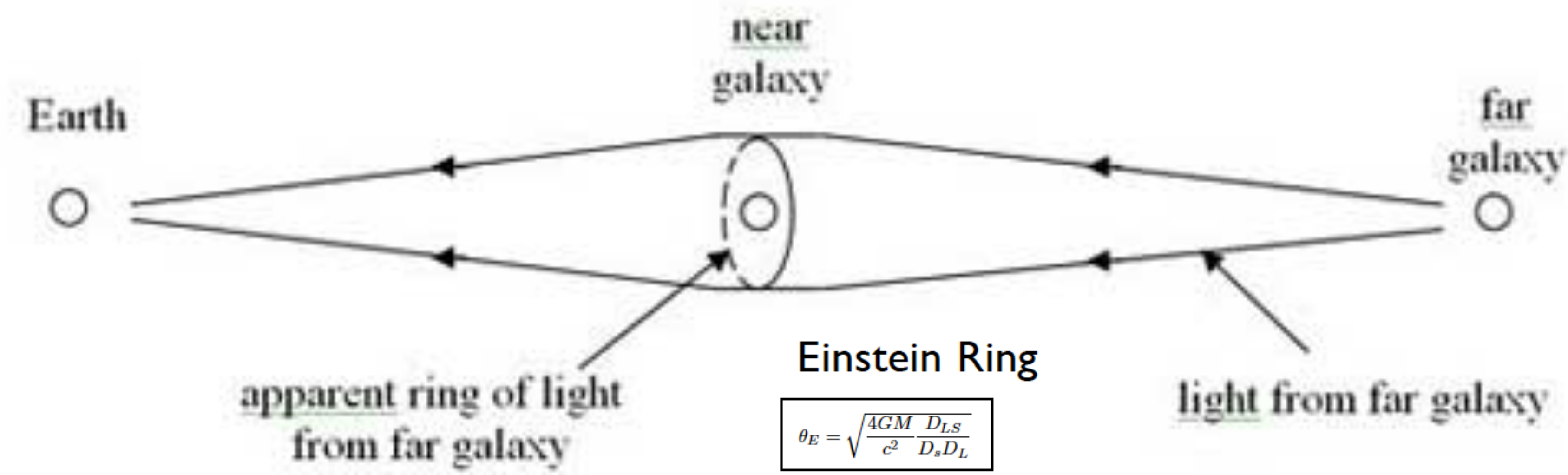
- Einstein's prediction that gravity distorts space-time was verified by observing a 1.7″ deflection of star close to the sun during a solar eclipse in 1919.



- The gravity from a foreground galaxy bends or images the light from a background galaxy or quasar.

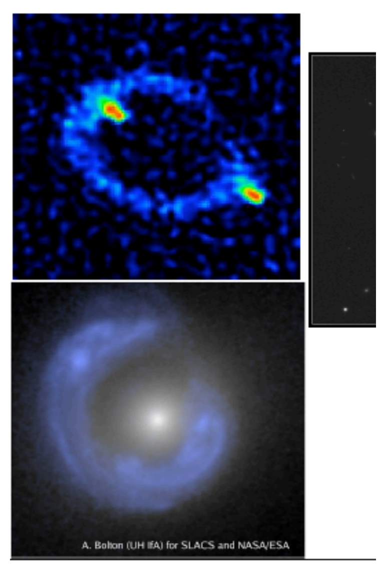


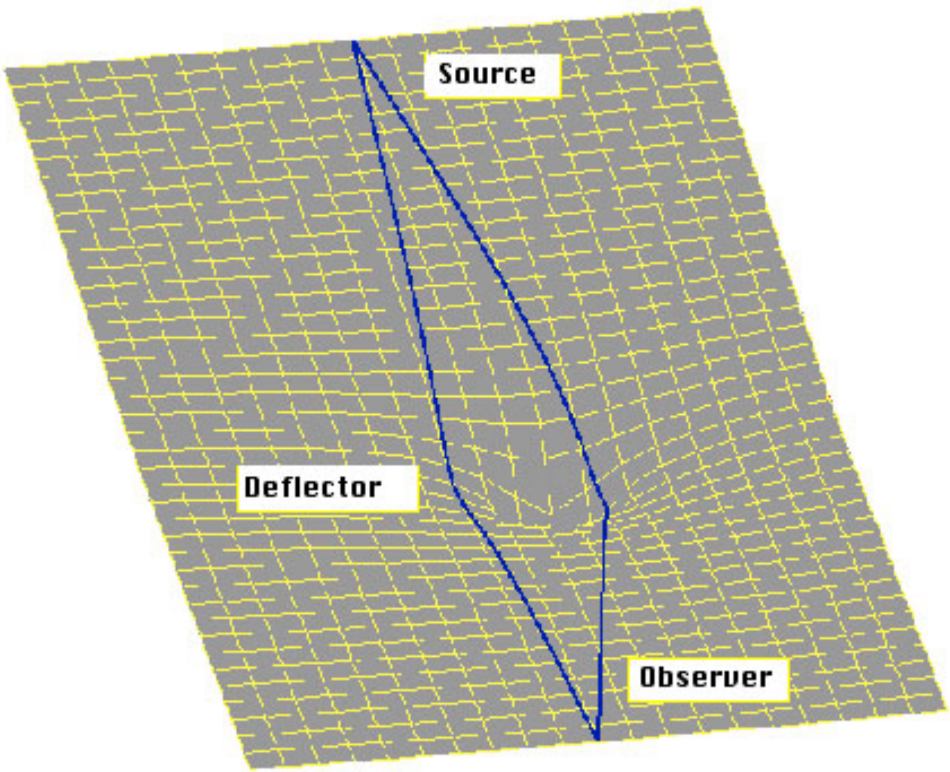
Light Is Bent On Cosmic Scales



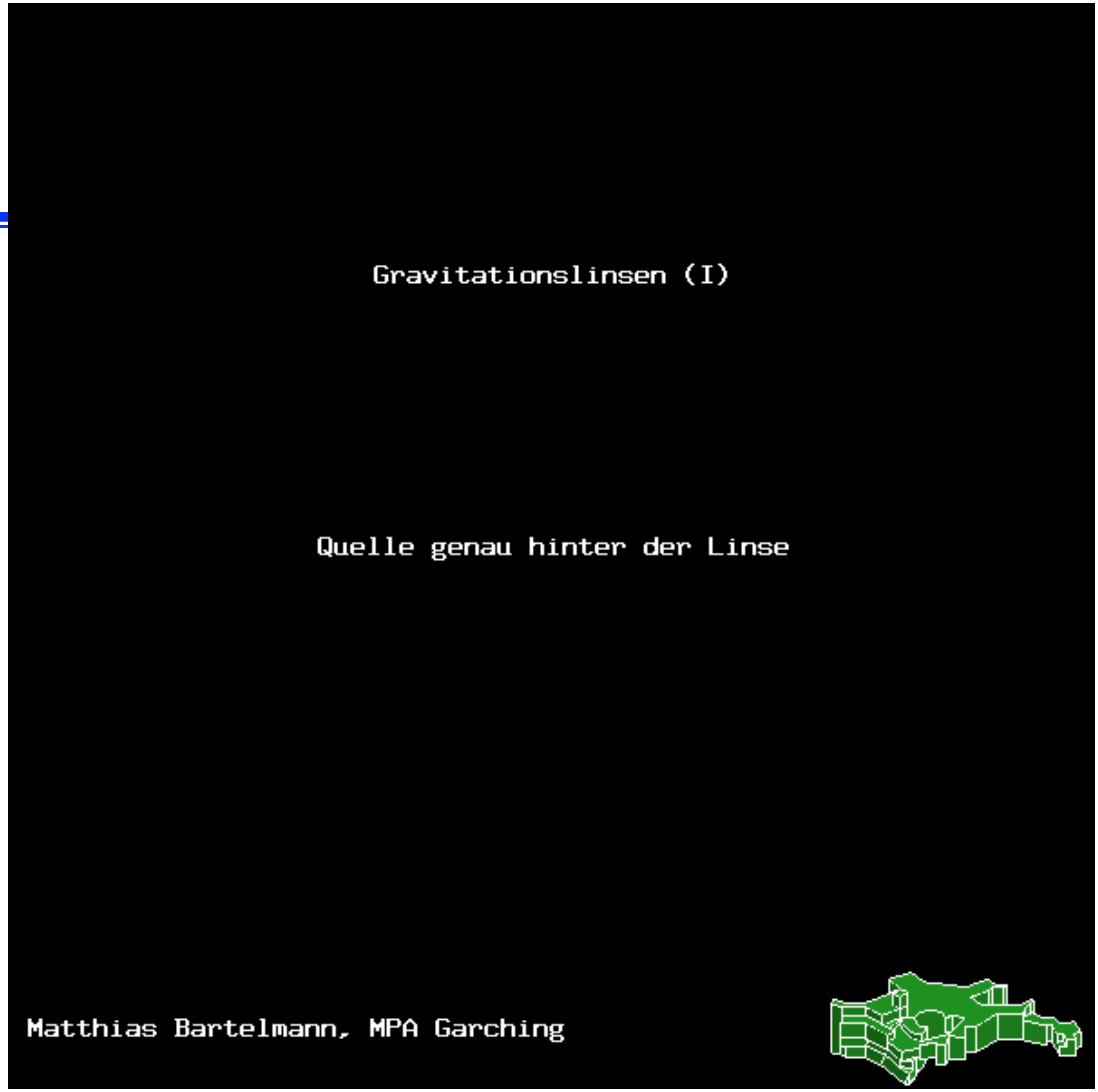
Einstein Ring

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_s D_L}}$$





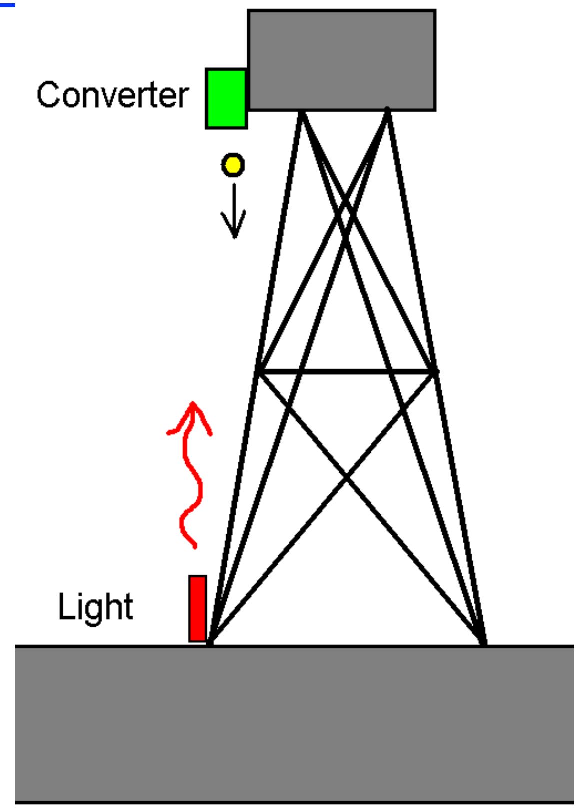
light 'bending' is a consequence of the bending of space itself


Gravitationslinsen (I)
Quelle genau hinter der Linse
Matthias Bartelmann, MPA Garching

Remember the tower…

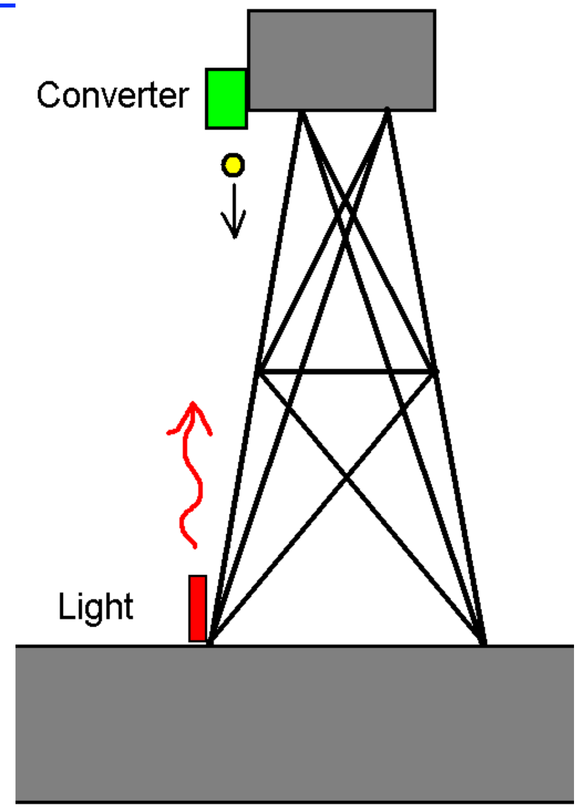
Light beam must lose energy as it climbs up

- So…frequency must decrease-wavekength increase
- i.e., light is redshifted.
- Gravitational redshifting



Remember the tower…

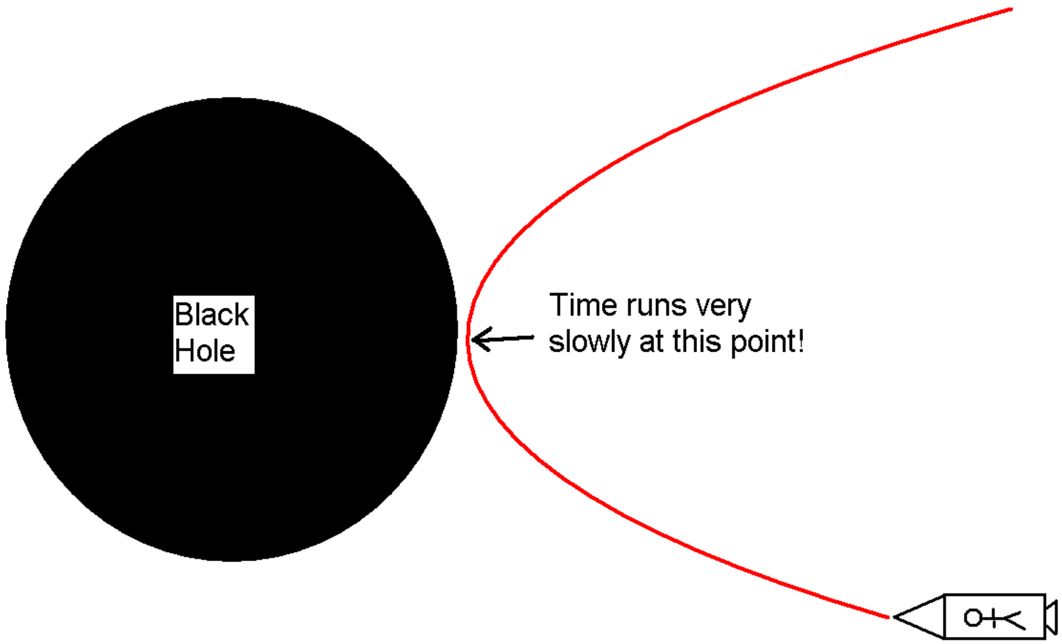
- Imagine a clock based on frequency of laser light…
  - 1 “tick” = time taken for fixed number of crests to pass
  - Gravitational redshifting slows down the clock.
  - **Clocks in gravitational fields must run slowly**



$$t_{grav} \approx \left(1 - \frac{GM}{c^2 r}\right) t_{space}$$

$$\text{if gravitational field is "weak"}$$

How to live for a 1000 years!- Interstellar the movie



- Go where gravity is very strong!
- Observer on Earth would see astronaut's clock running **very** slowly when close to black hole – astronaut would age very slowly.
- (In fact, there are other discomforts from of being near a black hole!)

Gravitational time dilation has practical importance!

- Global Positioning System (GPS)
  - System of satellites that emit timing signals
  - Detector on Earth receives signals
  - Can figure out position on Earth's surface by measuring time delay between signals from different satellite (light travel time gives distance to satellite)
  - Need to measure time of signal from satellite very well!
    - 10m positioning requires ~30ns time accuracy
  - Satellites are at varying heights; clocks run at varying rates
    - Satellite clocks drift by ~38us per day wrt Earth clocks!
- If GR effects were not included, computed GPS positions would drift from true position by **kilometers per day!**

GPS Geometry

Have both special (velocity of satellites) and GR effects (gravity)

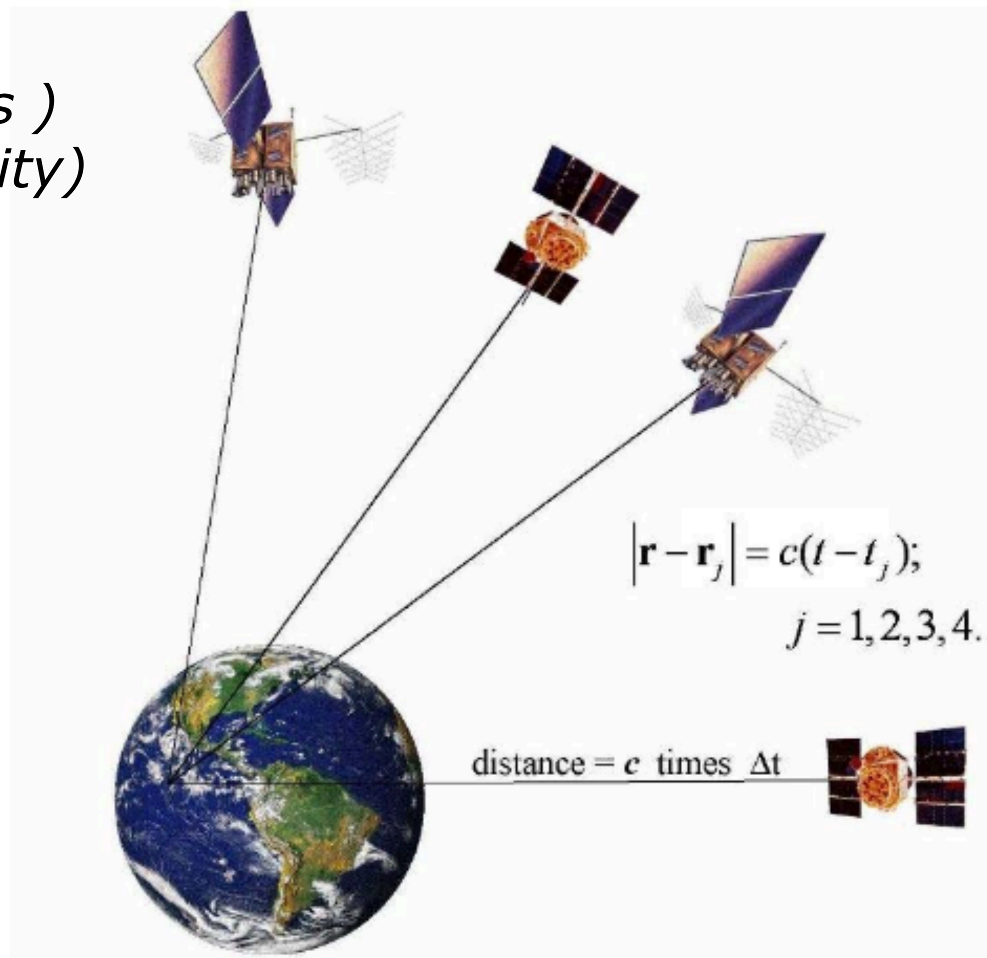


Figure 11: The positions of receivers are shown. Transmitted at t_j, and at positions r_j. The values of GvT is found by solving four simultaneous equations. Courtesy: (Ashby, 2002).

Consequences of Strong Equivalent Principle

Gravity effects space-time itself !

Gravity can bend light

Gravitational redshift

Gravity can slow down time

Gravity can change lengths

see for more info
https://www.einstein-online.info/en/spotlight/equivalence_principle/