ASTR350 Black Holes (Spring 2020)
Prof. Richard Mushotzky

RECAP

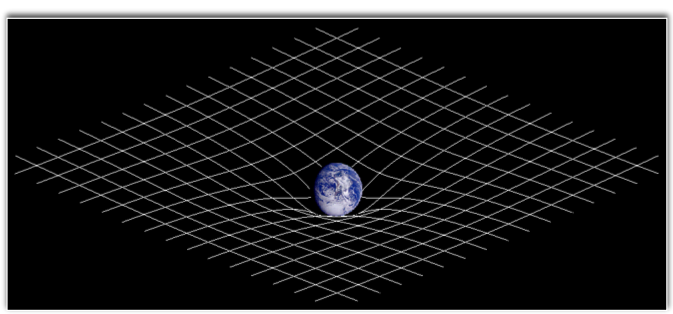
- Special and General Relativity
  - Highlights importance of frames of reference
  - Measurements of time affected by motion
  - Time dilation necessarily implies length contraction. Highlights fact that space and time get "mixed together" when changing reference frame… instead, think of space-time.
  - Idea of light cones, the past/future, and causality
- General Relativity
  - Measurements of time affected by gravity/acceleration
  - Gravity can be made to (locally) vanish by going to a free-falling reference frame

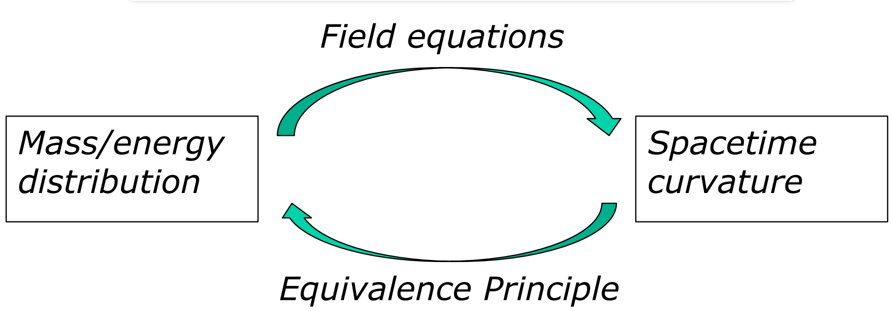
more material online in https://www.einstein-online.info/en/category/elementary/
from Max Planck Institute for Gravitational Physics (Albert-Einstein-Institut)

More Recap

- Laws of physics same in all inertial reference frames
- Special relativity tells us how to relate measurements in different reference frames
- Einstein tower- gravity effects light
- Strong equivalence principle-free-falling frames are inertial frames- in such a frame can make local effects of gravity "disappear"
- GR- free-falling matter (and light) follow geodesics in space-time
- Predictions and effects of GR- bending of light, precession of Mercury...

Recap





Discussion of HW3

1. Questions??
New York Times article on Einstein https://www.nytimes.com/2015/11/24/science/a-century-ago-einsteins-theory-of-relativity-changed-everything.html

2. NPR story on atomic clocks at http://www.npr.org/2014/11/03/361069820/new-clock-may-end-time-as-we-know-it
(update at:https://www.discovermagazine.com/the-sciences/atomic-clocks-so-accurate-they-can-measure-gravity and https://www.newsweek.com/portable-atomic-clocks-can-measure-height-mountain-and-it-could-help-805431)

HW4- Due Feb 27

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see ELMS

THIS CLASS- Black Holes

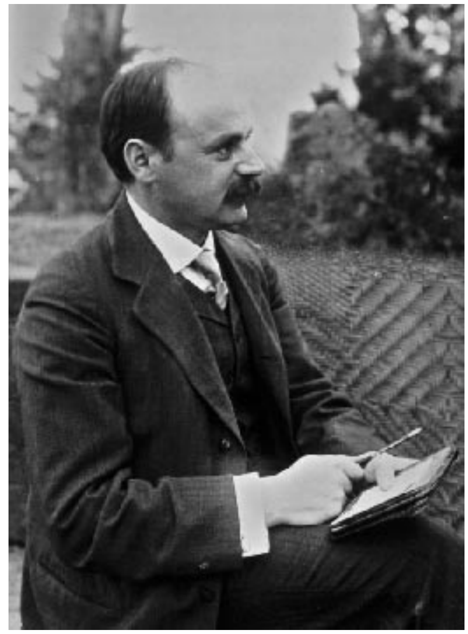
- The General Relativistic view of black holes
- “Schwarzschild” black holes- the simplest kind
  - View of an external observer
  - View of an infalling observer
  - Spaghettification

a source for the special and general relativity material is https://web.stanford.edu/~oas/SI/SRGR/

Einstein DID NOT get the Nobel prize for Relativity

- THE ROYAL SWEDISH ACADEMY OF SCIENCE in November 9, 1922, in accordance with the regulations in the November 27, 1895, will of ALFRED NOBEL, decided to, independently of the value that, after possible confirmation, may be attributed to the relativity and gravitation theory, award the prize that for 1921 is given to the person who within the domain of physics has made the most important discovery or invention, to ALBERT EINSTEIN for his contributions to Theoretical Physics, especially his discovery of the photoelectric effect.

I : Schwarzschild



- Karl Schwarzschild (1873-1916*)
  - Solved the Einstein field equations for the case of a "spherically-symmetric" point mass. (1 year after Einstein's paper !)
  - First exact (non-trivial) solution of Einstein's equations
  - Describes a non-spinning, 'spherical',non-charged black hole… a *Schwarzschild* black hole

*died in WWI

Black Holes Newtonian

Remember escape velocity?

By making M larger and R smaller, V_{esc}^2 increases- in Newtonian physics to arbitrary values

Idea of an object with gravity so strong that light cannot escape first suggested by Rev. John Mitchell in 1783

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Black Holes General Relativity

Karl Schwarzschild (1916) First solution of Einstein's equations of GR

Describes gravitational field in (empty) space around a non-rotating mass Space-time interval in Schwarzschild's solution

Features of Schwarzschild's solution:

Yields Newton's law of gravity, with flat space, at large distances (Large R)

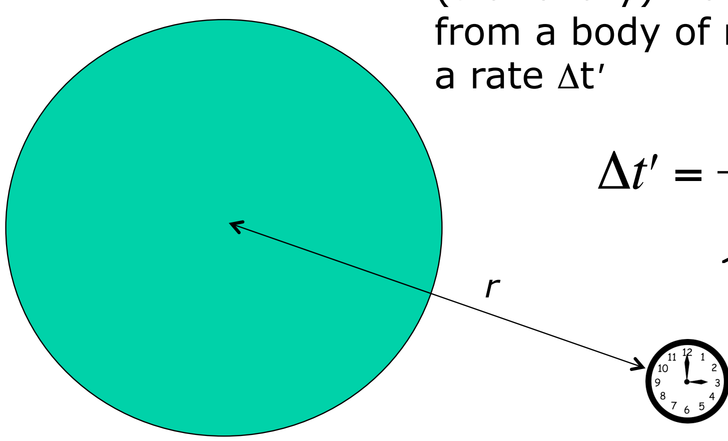
Space-time curvature becomes infinite at center (R=0; this is called a **space-time singularity**)

- Gravitational time-dilation effect becomes infinite on a spherical surface known as the **event horizon**
- Radius of the sphere representing the event horizon is called the **Schwarzschild radius,** $R_s = 2GM/c^2$

The view of a distant observer

Distant observer sees a (stationary) clock at a distance r from a body of mass M ticking at a rate $\Delta t'$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$



Δt is what it tick at in the distant observers frame

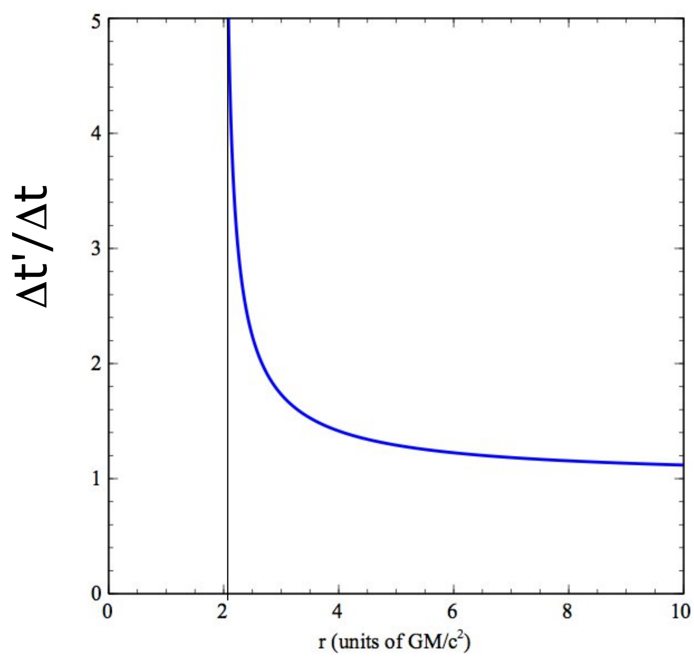
Questions to Class

What happens as r increases to infinity ? ($r \longrightarrow \infty$)

What happens as r gets to $2GM/c^2$?

What happens as r gets to 0 ?

Time 'stops'



Clock appears to tick more slowly as it gets closer to the black hole… seems to stop ticking when it gets to ***$r=2GM/c^2$***.

The **event horizon**… sphere on which the gravitational redshift is **infinite**

Event horizon

Point of no-return… the location where the **escape velocity equals the speed of light**

- The **gravitational redshift becomes infinite** here (as seen by an outside static observer)
- Nothing occurring inside can be seen from outside (or have any causal effect on the external Universe!)
- So… as a *practical matter*, we never need concern ourselves with the Universe interior to the event horizon ?

$$R_{Sch} = \frac{2GM}{c^2} \approx 3\left(\frac{M}{M_\odot}\right)\ \mathrm{km}$$

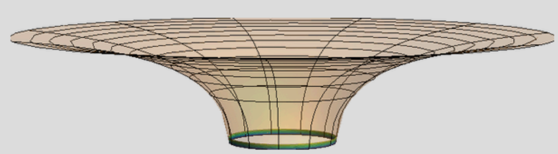
⊙symbol for sun, $M_\odot$ =solar mass

- Radius corresponding to event horizon for a *non-spinning* black hole is known as the **Schwarzschild radius R_{Sch}**

A typical star (the dark disc) creates a gentle curvature of spacetime

in general relativity we can make a black hole by throwing a lot of matter together into a region.

But we can never 'unmake' the hole, since nothing that went in the hole can ever come out.



Allowing more curvature eventually leads to a black hole. The exterior of the hole, depicted above, ends in a horizon (the green circle); anything falling past the horizon cannot come back out.

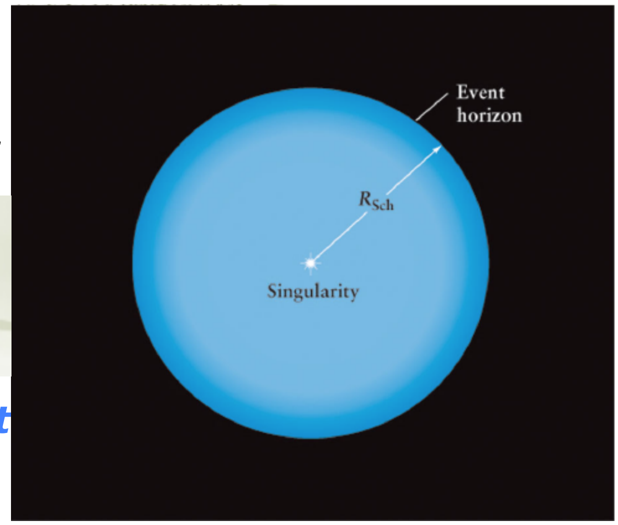
https://www.asc.ohio-state.edu/mathur.16/tutorial1sep17/bhtemplate3.html

GR black holes

Gravitational redshift outside of a *spherical object with mass M is*

$$\nu_{obs} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \nu_{emit}$$

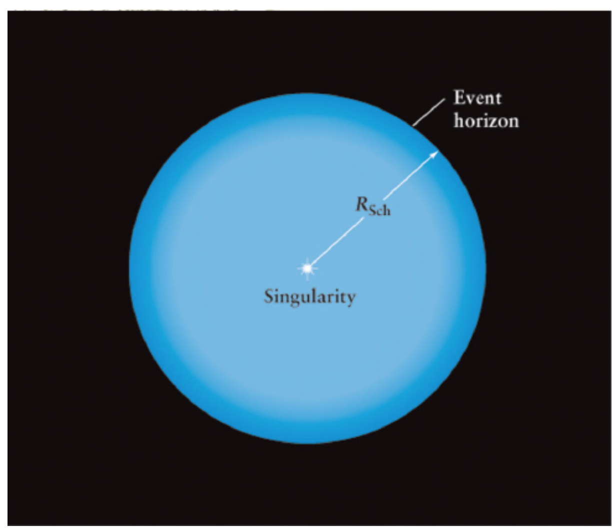
ν is frequency of the light



As r → R_s
ν_{obs} goes to zero
wavelength of emitted radiation goes to ∞
infinite redshift !!!

- *Radius corresponding to event horizon for a non-spinning black hole is known as the* ***Schwarzschild radius R_S***

GR black holes



Gravitational length contraction
$L' = L\ (1 - R_s/R)^{1/2}$

Gravitational time dilation
$t_0 = t_f (1 - R_s/R)^{1/2}$; *$t_0$ is the time that the observer near the BH measures*
t_f is the time that a distant observer measures – ***as you get closer to the black hole time slows down***

$R_s = 2GM/c^2$

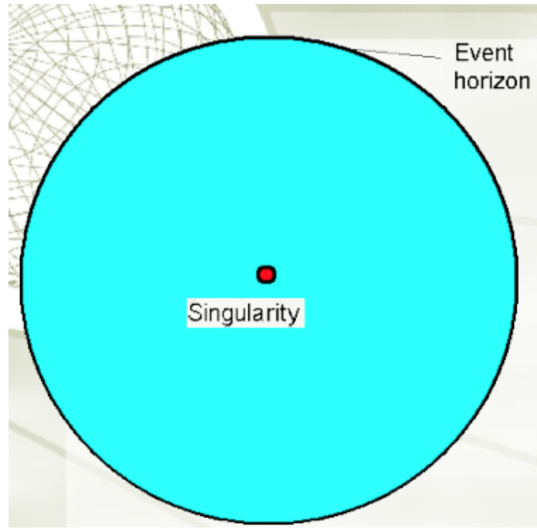
As R → R_s time goes to ∞
length goes to zero

- *Radius corresponding to event horizon for a non-spinning black hole is known as the* ***Schwarzschild radius R_S***

GR black holes

For a body of the Sun's mass,
Schwarzschild radius
$R_S = 2GM/c^2 \rightarrow 3$km

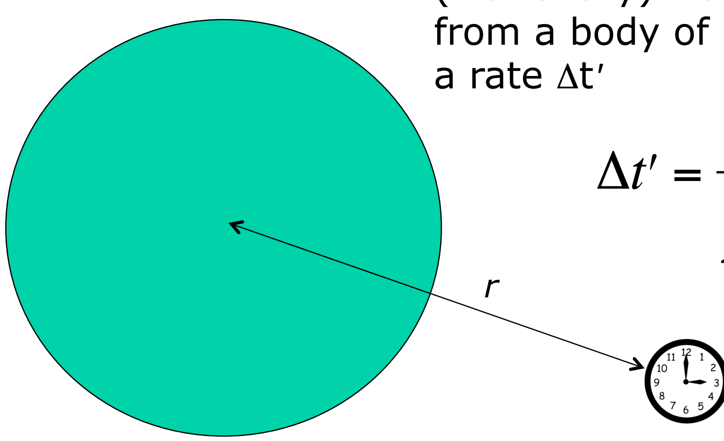
- ***Singularity*** *– spacetime curvature is infinite. Everything destroyed. Laws of GR break down.*
- ***Event horizon (R_s)*** *– gravitational time-dilation is infinite as observed from large distance.*
- *Any light emitted at R_s would be infinitely redshifted - hence could not be observed from outside*



Schwarschild radius is NOT the singularity

At the Schwarzschild radius gravitational time dilation goes to infinity and lengths are contracted to zero

The view of a distant observer-repeat



Distant observer sees a (stationary) clock at a distance r from a body of mass M ticking at a rate $\Delta t'$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

Δt is what it tick at in the distant observers frame

View of an External Observer

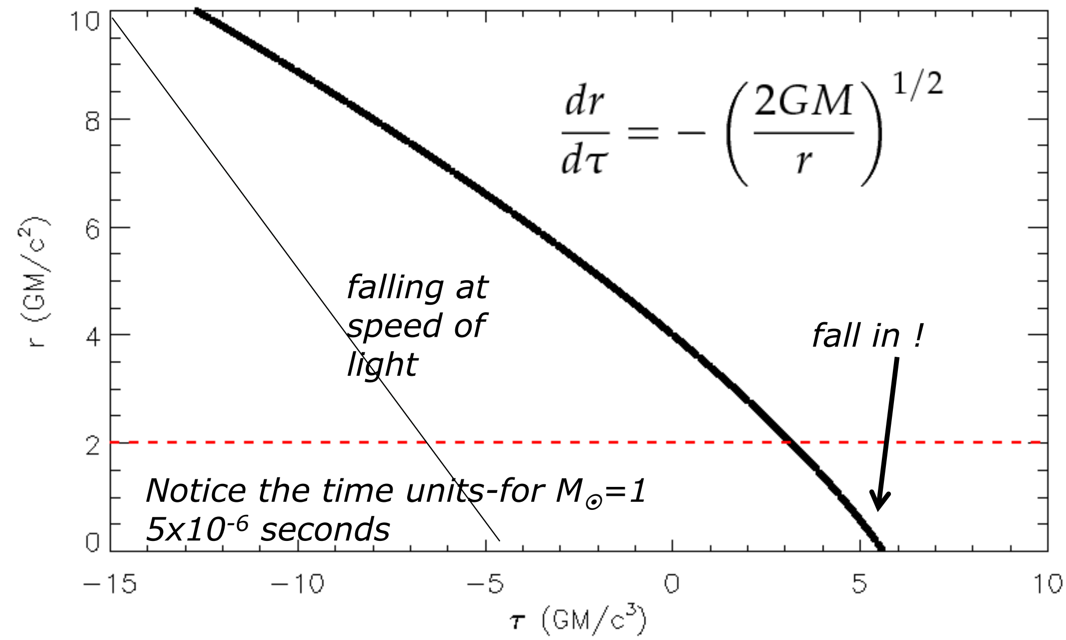
- The event horizon
  - Surface of infinite gravitational redshift
  - *From point of view of distant observer*, infalling objects will appear to freeze at event horizon
  - Old name for black holes was "Frozen Star" (referring to the star that collapsed to create the black hole)
  - Infalling object will also appear to fade away as it freezes (why?)

The view of an infalling observer

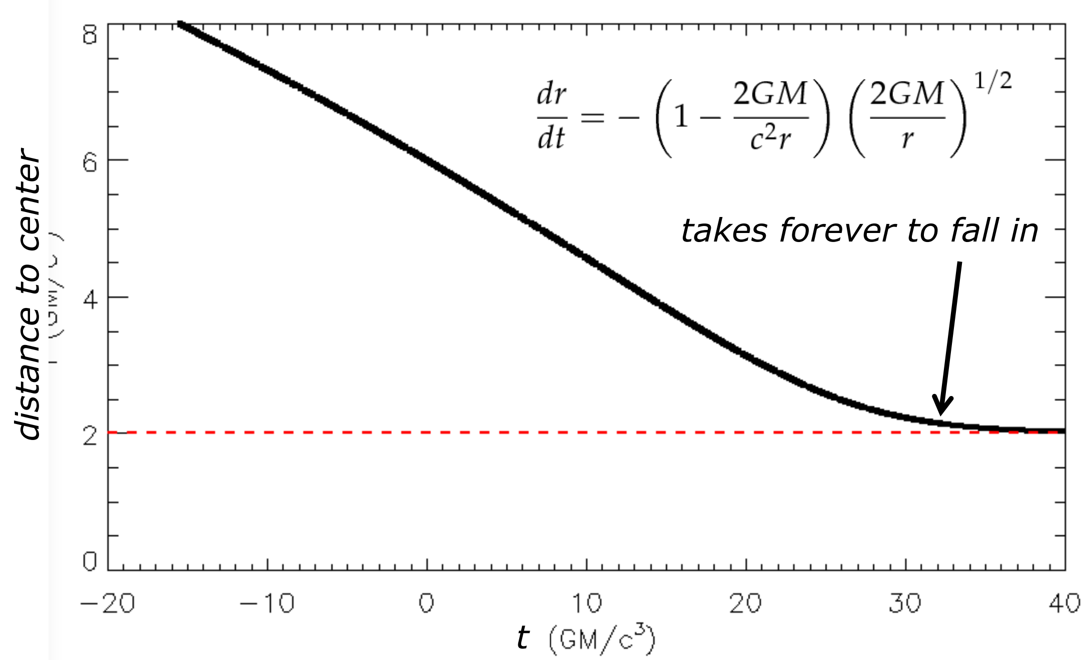
- Very different view by infalling observer
  - Pass through the event horizon without "fuss" – see later
  - Eventually reach the center (r=0)

- What happens at the center?
  - Equations of General Relativity break down (predict infinite space-time curvature, corresponding to the infinite density of matter that has been crushed there).
  - Called a **spacetime singularity**
  - Means that GR is invalid and some other, deeper, laws of physics are needed to describe this location (Quantum Gravity)

Falling radially into a black hole – victim's view



Falling radially into a black hole – **external** view



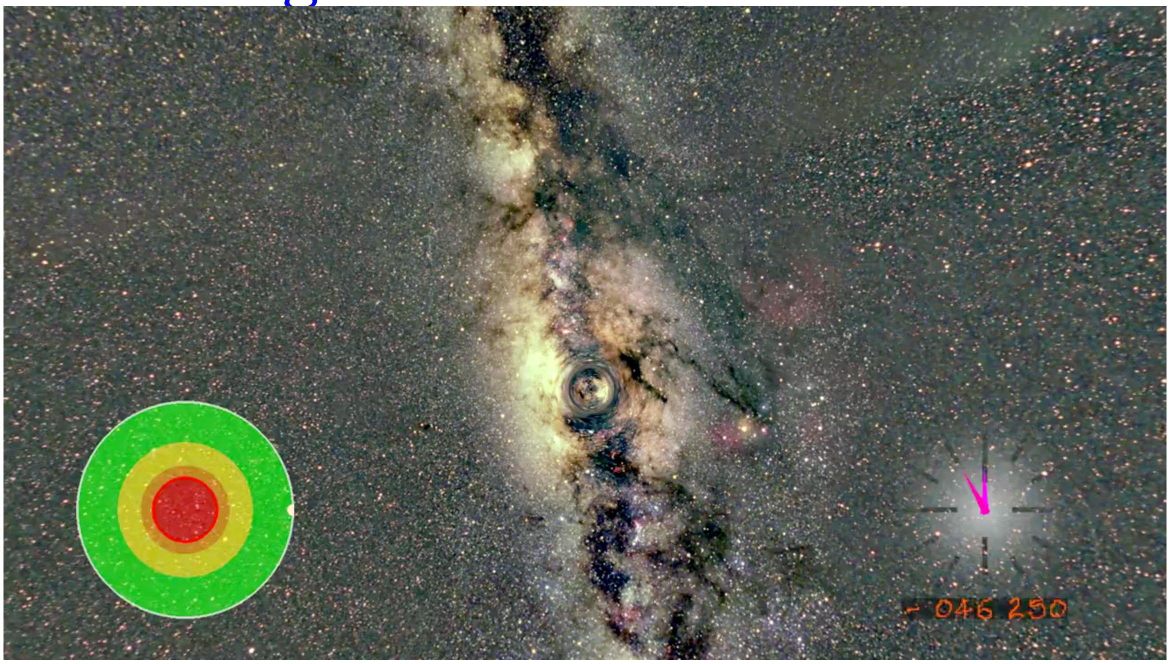
- The Schwarzschild metric is singular at the horizon ($r=2GM/c^2$) but this is only a coordinate artifact.
  - *A free falling observer feels no drama going through the horizon.*

    It takes the observer a finite amount of proper time but infinite coordinate time.

  We can remove the singularity by a proper coordinate transformation.
  - In contrast, the **origin r=0 is a singularity**. The scalar curvature is infinite and the general relativity is no longer valid at this point.

- How do we reconcile the two views (infalling observer and distant observer)?
  - The distant observer only sees *part* of the timeline of the infalling observer… they never see the part of the timeline inside of the event horizon!
  - This is because the distant observer's measure of time freezes at the event horizon… but the infalling observer has a perfectly well behaved measure of time as they pass across the horizon
  - We say that the event horizon is a "coordinate singularity", not a "physical singularity" (similar to the latitude/longitude situation at the north/south pole)

Falling Into a Black Hole



Explaining the Movie

The inset to the bottom right of the movie is a clock, the time left until you hit the central singularity.
The clock records your "proper" time, the time that your watch shows it is in seconds if the black hole has a mass of 5 millions suns and it takes 16 seconds to fall from the horizon to the singularity.

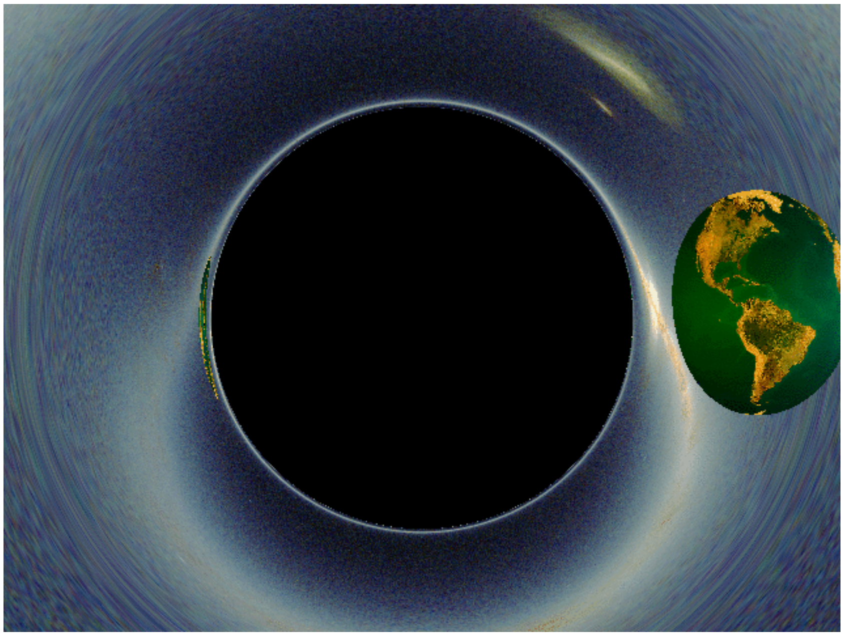
*In the lower left is your path into the BH- **green** is where there are stable orbits, **orange** no orbits, **red** inside event horizon*

The BH bends light around it, which appears to "repel" the image radially, which in turn stretches the image transversely. Parts of the image nearer the black hole are repelled more, so the image appears compressed radially.

*The horizon is painted **dark red***

3 Schwarzschild radii marks the radius of the innermost stable orbit. Outside this radius circular orbits are stable, whereas within it circular orbits are unstable.

Orbiting Object as Seen by External Observer



Another Trip to A Black Hole

https://www.einstein-online.info/en/spotlight/descent_bh/

A transit of Einstein

More features of **Schwarzschild** black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)-

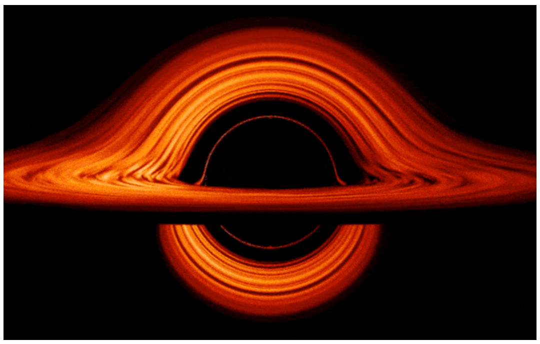
- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)
- Observer who enters event horizon would only "feel" “strange” gravitational effects if the black hole mass is small.

- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady
- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
  - Thus black hole would produce “shadow” on sky (EHT picture)

Black Hole Shadow

Real data

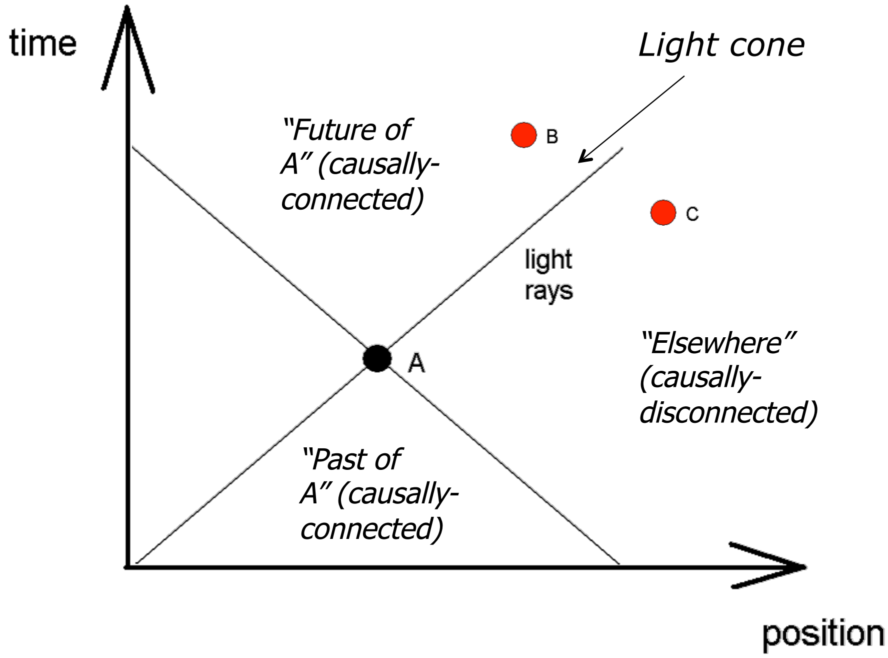
Simulation



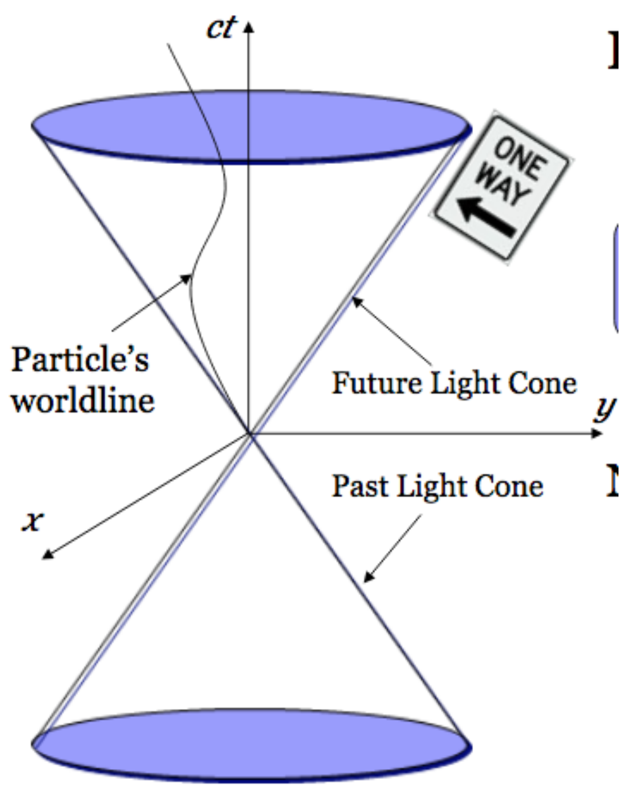
The nature of the event horizon

- So, we have learned important things about the nature of the ***event horizon***…
- The Event Horizon is…
  - The infinite redshift surface
  - The place where infalling objects appear to "freeze" according to external observers
  - NOT a real singularity, since infalling observers pass through it unharmed
  - The boundary of the causally disconnected region (we didn't prove this!)
- *The real singularity is at r=0*

Remember space-time diagrams?



Remember Light Cones??



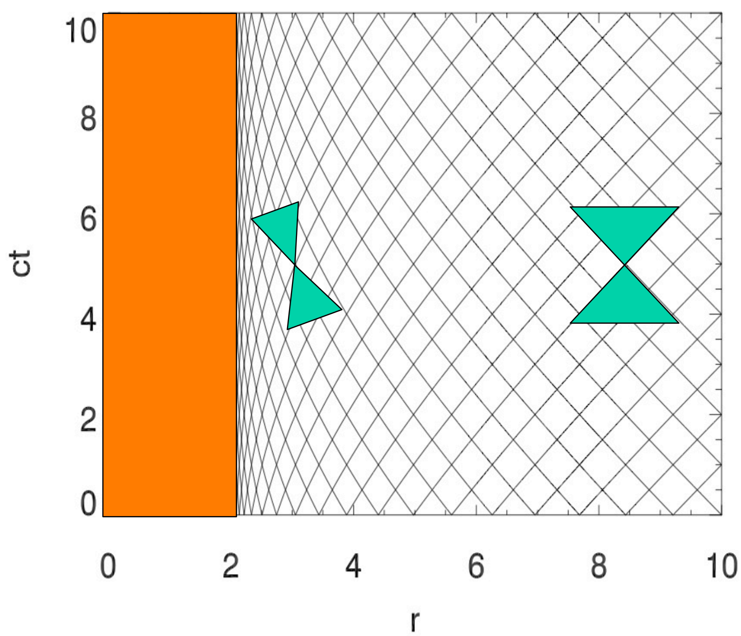
C. Middleton

Spacetime diagram... path of ingoing/outgoing light rays as seen by distant observer

r is in units of $2GM/c^2$
(ct no units)

No signal can reach a distant from observer from within $r=2GM/c^2$

this region is 'causally' disconnected



Meaning of 'grid'

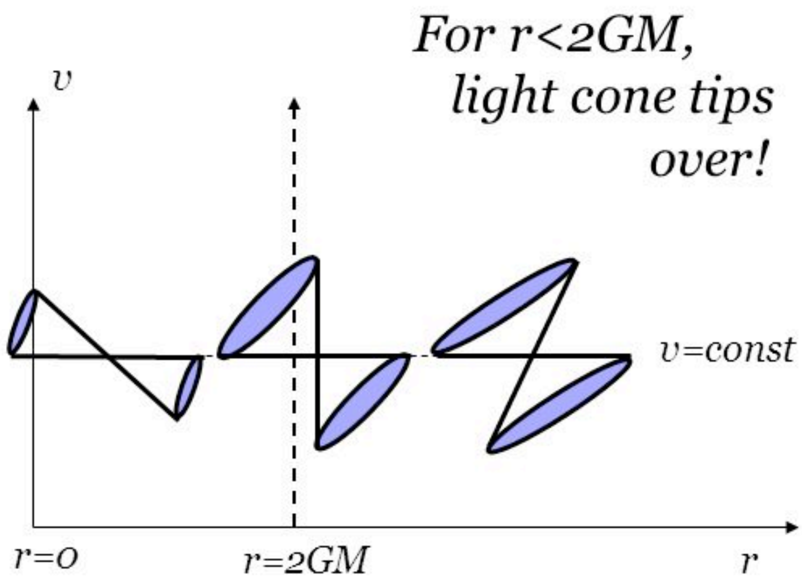
the lines are the paths that photons take

the photons are emitted at ct=0 in space-time coordinates at different values of r
- one line represents photons 'shot' outward the other photons 'shot' inward.

The lines are curved due to curvature of space time near the BH
The cones are the 'usual' notice the shape of the cones change as one gets close to the BH

Spacetime diagram-Falling in

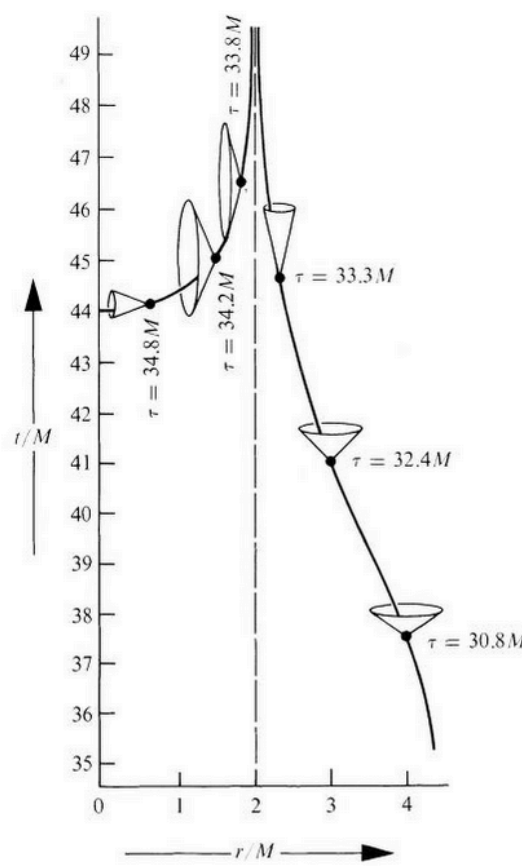
This coordinate system shows that the event horizon is NOT a singularity- But light still heading into horizon- what happens?



What Happens to light cones after crossing event horizon

Notice that the cones do not point 'out' e.g the time direction is bent more and more inward as you get closer to the black hole.
As the bending is strong enough to prevent any object from moving in any direction but inwards, you cross the horizon.

Misner,Wheeler Thorne



Fate of the photons

A photon starting on the outside of the black hole can travel through the Schwarzschild Radius.

A photon that starts on the inside of the Schwarzschild Radius, never has a path that leads outside of the black hole. It is trapped.

If we had a dense collection of mass with a physical surface boundary less than its Schwarzschild Radius, light from the surface could not escape from it to the outside .

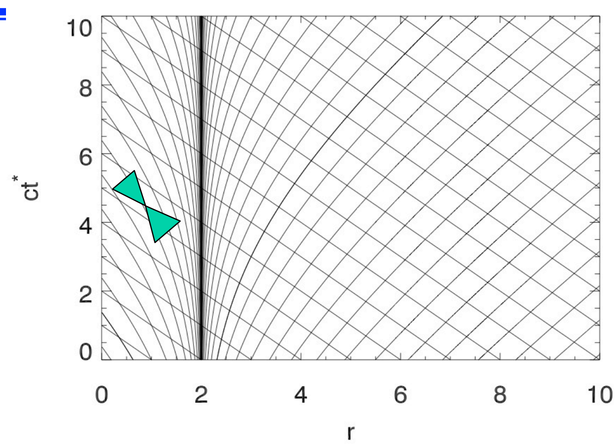
An outside observer could detect the object's existence through its gravitational field but that observer would not be able to see it. This is the reason why such objects came to be known as black holes.

The sphere defined by $r=2GM/c^2$ shuts off the outside world from observing what's inside.

For this reason the region is called the Event Horizon.

Spacetime diagram inside event horizon-all future light cones point inwards!

The increase in the time coordinate to infinity as one approaches the event horizon is why information cannot be received back from any probe that is sent through an event horizon.



This is despite the fact that the probe itself can nonetheless travel past the horizon.
It is also why the space-time metric of the black hole, <u>when expressed in Schwarzschild coordinates,</u> becomes singular at the horizon.

River Model of a Black Hole

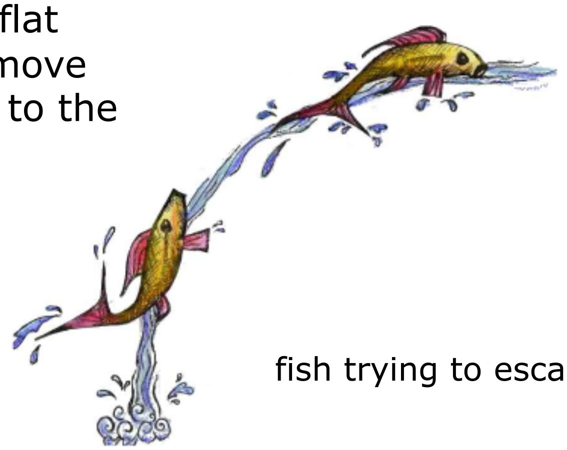
https://arxiv.org/abs/gr-qc/0411060

River Model of a Black Hole

https://arxiv.org/abs/gr-qc/0411060

In the river model, space itself flows like a river through a flat background, while objects move through the river according to the rules of special relativity

Inside the horizon, the river flows inward faster than light, carrying everything with it.



fish trying to escape

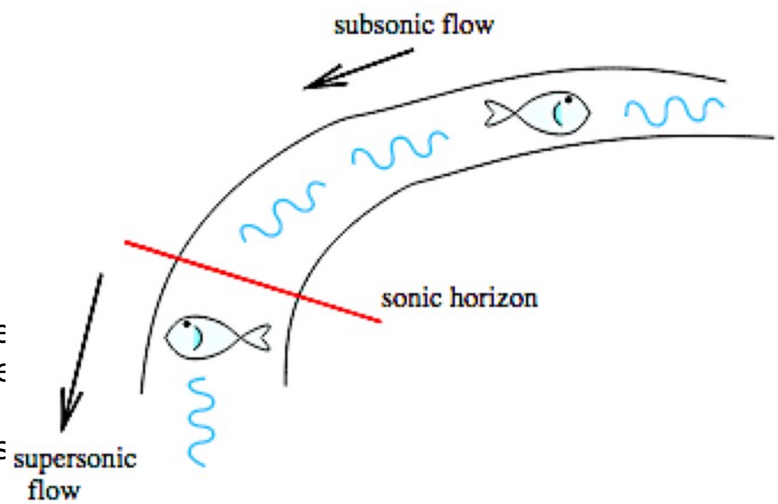
FIG. 1: (Color online) The fish upstream can make way against the current, but the fish downstream is swept to the bottom of the waterfall. F

River Model of a Black Hole

https://arxiv.org/pdf/cond-mat/0404480.pdf

The fish in the subsonic flow region cannot hear the screams of the fish in the supersonic region because the emitted sound travels too slowly to propagate upstream.

The fish upstream see a sonic horizon at the location where the fluid velocity becomes supersonic. (Unruh 1981)



Recap- Some Effects of Falling into a **Black Hole**

Light cannot escape from inside the horizon

All matter (particles, photons) must move towards the singularity

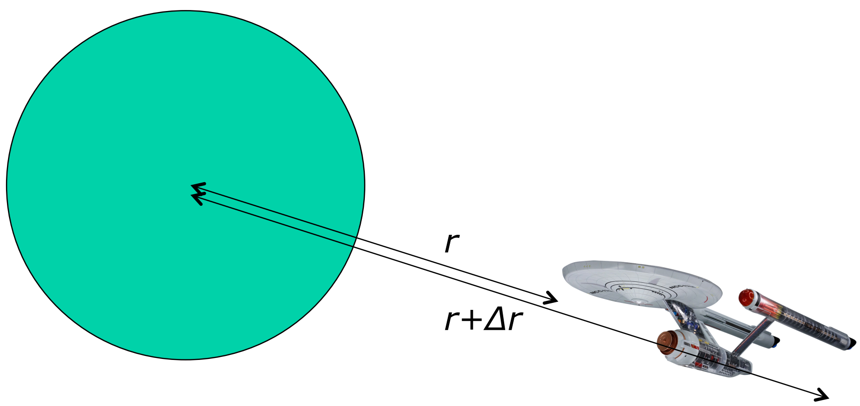
Objects are stretched radially and compressed transversely

Objects that falls through the horizon appears to an outside observer redshifted and frozen at the horizon

as the object approaches the horizon, light emitted by it takes an ever increasing time to reach an outside observer. e.g. If you fall into a black hole, you will actually cross an event horizon (finite τ) but your friend far away will never see you cross.

Spaghettification

- In fact, you would never make it to the center intact… the **gradient of gravity** would tear up an infalling observer in a low mass BH.



there is a "stretching force" known as a tidal force that is **proportional to M/r^3**. This will eventually rip the spaceship apart

- How strong is the tidal force you feel as you fall through the event horizon? Using a Newtonian approximation, we have

$$F_{tidal} \propto \frac{GM}{R_{evt}^3} \quad \textit{with} \quad R_{evt} = \frac{GM}{c^2}$$

- So,

$$F_{tidal,evt} \propto \frac{M}{M^3} = \frac{1}{M^2}$$

- Thus, the more massive the black hole, the weaker is the tidal force at the event horizon!

Tidal force math- Newtonian

$\Delta F = F_{feet} - F_{head} = GMm[1/r^2 - 1/(r+\Delta r)^2] = GMm/r^2[1-1/(1+\Delta r/r)^2]$

use binomial expansion $((1+x)^n \sim 1+nx+\ldots$ if $x<<1$

and you get
$GMm/r^2[1-1/(1+\Delta r/r)^2] = GMm/r^2[1-(2\Delta r/r\ldots)] = (2GMm/r^3)\,\Delta r$

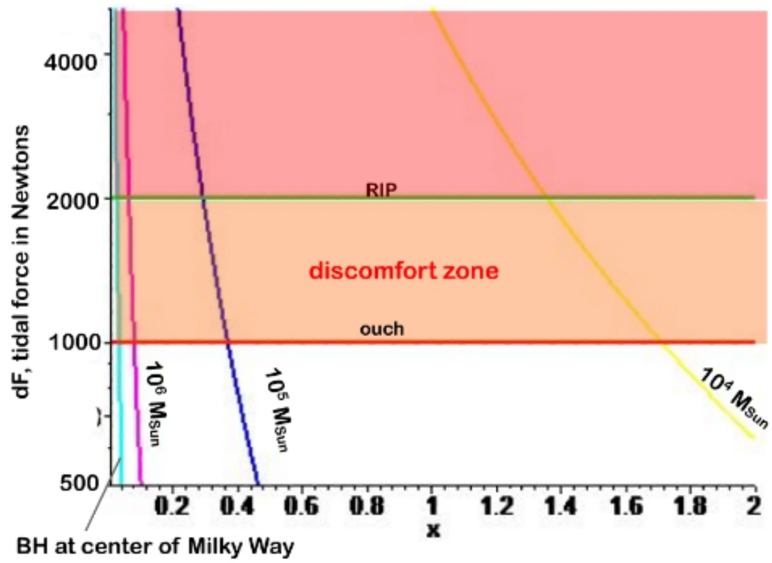
E.g. tidal force goes as $1/r^3$

So where do you fall apart?

- It depends on the mass of the black holes

x= 1 is the event horizon

https://web.stanford.edu/~oas/SI/SRGR/notes/FallingIntoBH.pdf

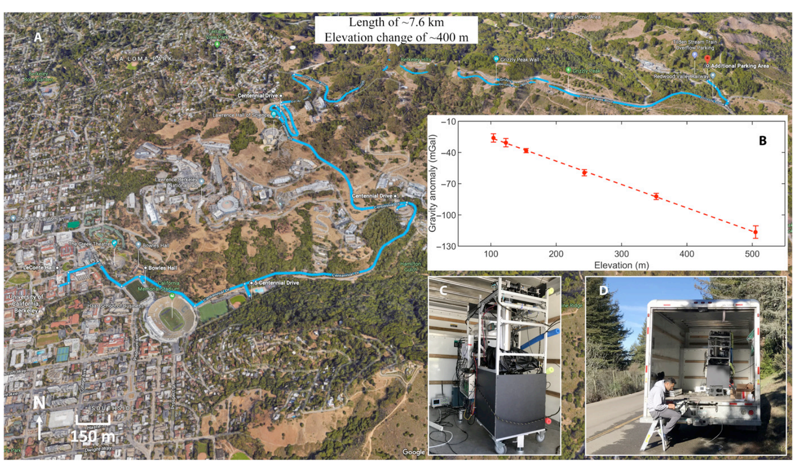


NOVA

Journey to a Black Hole-https://phys.org/news/2012-06-andrew-hamilton-black-holes.html

Measurement of GR Effect

Can now measure the GR effect on light with an accuracy of Feet



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