ASTR 421, Homework 3

March 26, 2017

Problem 1, S+G 2.8

1) 2pts

$$\Sigma = \int n\mathrm{d}z = \int ne^{-R/h_R}e^{-|z|/h_z}\mathrm{d}z = ne^{-R/h_R}\,2\int_0^{+\infty} e^{-z/h_z}\mathrm{d}z$$
$$= \boxed{2ne^{-R/h_R}h_z}$$

2) 3pts

$$\int \Sigma\mathrm{d}S = \int_0^{+\infty} \Sigma 2\pi R\mathrm{d}R = 2\pi\,2nh_z\int_0^{+\infty} Re^{-R/h_R}\mathrm{d}R$$
$$= 4\pi nh_z h_R^2 = 2\pi h_R^2\Sigma(0,S),$$

where $\Sigma(0,S) = 2\pi h_z$ has been used. Therefore

$$L_D = L(S)\int \Sigma\mathrm{d}S = 2\pi h_R^2 L(S)\Sigma(0,S) = \boxed{2\pi h_R^2 I(R=0)}. \tag{1}$$

3) 3pts Eq. (1) gives

$$I(R=0) = L_D/2\pi h_R^2 = 149L_\odot\mathrm{pc}^-2$$

Therefore

$$I(R=8\mathrm{kpc}) = I(R=0)e^{-8/4} = \boxed{20L_\odot\mathrm{pc}^{-2}}$$

4) 2pts Massive young stars are found only close to the midplane.

Problem 2, S+G 2.13

1) 3pts To do this we need to know the absolute magnitute of the Sun at V band: $M_{V,\odot} = 4.83$ (Table 1.4). Therefore, at a distance of 3pc we have (Eq. 1.15)

$$m_{V,\odot} = M_{V,\odot} + 5\log_{10}(3\text{pc}/10\text{pc}) = 2.216.$$

Using Eq. 1.10 and Eq. 1.1, we get

$$m_{V,\odot} - m_{V,\text{eye}} = -2.5\log_{10}\frac{L_\odot}{L_\text{eye}},$$

thus $L_\text{eye} = \boxed{0.077L_\odot}$.

2) 2pts

$$0.077 = \left(\frac{M}{M_\odot}\right)^5 \Rightarrow M = \boxed{0.60M_\odot}$$

3) 2pts $M_u = \boxed{1.5M_\odot}$ or $1.25M_\odot$.

4) 3pts

$$\xi \text{d}M = \boxed{\xi_0 M^{-2.35} M_\odot^{1.35} \text{d}M}$$

5) 4pts

$$N = \int_{0.2M_\odot}^{M_u} \xi \text{d}M = \int_{0.2}^{1.5} \xi_0 M^{-2.35} \text{d}M = 6.1\xi_0 \text{ or } \boxed{6.0\xi_0}$$

$$M = \int_{0.2M_\odot}^{M_u} \xi M \text{d}M = \boxed{2.54\xi_0 M_\odot}$$

6) 2pts Red giant phases do not last long.

7) 2pts

$$\xi_0 \approx 10^7 M_\odot / 2.54 M_\odot = \boxed{3.94 \times 10^6}$$

8) 2pts $M \geq 0.6M_\odot$. Doing the integration gives

$$N_\text{eye} = \int_{0.6M_\odot}^{M_u} \xi \text{d}M = \int_{0.6}^{1.5} \xi_0 M^{-2.35} \text{d}M \approx \boxed{4.1 \times 10^6}$$

3) 2.20 15'

$$M(<r_0) = \int_0^{r_0} \rho \cdot 4\pi r^2 dr = \int_0^{r_0} \frac{1}{4\pi G} \frac{V_H^2}{r^2+a_H^2} 4\pi r^2 dr$$

$$= \frac{V_H^2}{G} \int_0^{r_0} \frac{r^2}{r^2+a_H^2} dr$$

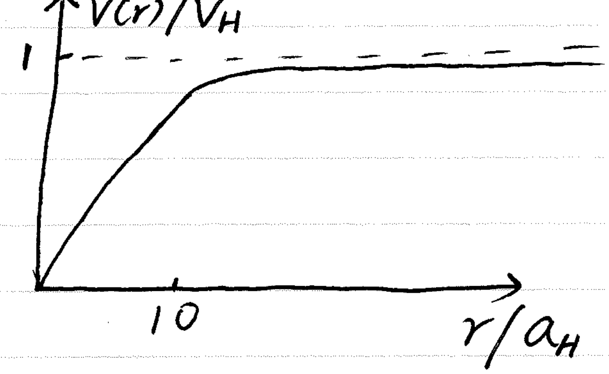
$$= \frac{V_H^2}{G} \left(r_0 - a_H \cdot \arctan \frac{r_0}{a_H} \right)$$ 5'

Eq. 2.18 →

$$V^2(r) = M(<r) \cdot G / r$$

$$= V_H^2 \left(r - a \cdot \arctan \frac{r}{a_H} \right) / r$$ 5'

See the figure.
Just like the rotations curves of galaxies, it becomes flat at large radii. 5'



4) 3.13 10'

$$\langle KE \rangle = \frac{1}{2} \langle v_r \rangle^2 M = \frac{3}{2} \sigma_r^2 M$$ 3'

Eq. 3.37 →

$$\langle PE \rangle = -\frac{3\pi}{32} \frac{GM^2}{a_p}$$ 3'

Using virical theorem,

$$2\langle KE \rangle + \langle PE \rangle = 0$$

we get

$$M = \frac{32 a_p \sigma_r^2}{\pi G} = 2.4 \times 10^6 M_\odot$$ 2'

We have assumed the star's motions are isotropic relative to the mean motion. 2'

5) 3.2

3' a) At $r \gg a_p$, $\phi_p(r) \to -\frac{GM}{r}$, so its total mass is M. (See Eq. 3.22). Yes, it is M! Not trivial!

5' b) The Laplace operator ∇^2 in radial ~~direction~~ component of spherical coordinate

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right). \quad \text{(IMPORTANT!)}$$

We have

$$\nabla^2 \phi_p(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\phi_p(r)\right)$$

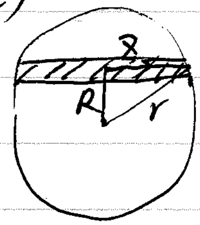
$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot GMr(r^2+a_p^2)^{-3/2}\right)$$

$$= 3Ga_p^2 \frac{M}{(r^2+a_p^2)^{5/2}}$$

There for the Poisson's equation $\nabla^2\phi = 4\pi G\rho$ gives

$$\rho = \frac{1}{4\pi G}\nabla^2\phi_p(r) = \frac{3a_p^2}{4\pi}\frac{M}{(r^2+a_p^2)^{5/2}}$$

5' c) $\rho_p(r) = \rho_p(\sqrt{R^2+z^2})$



So

$$\Sigma_p(R) = \int_{-\infty}^{\infty}\rho_p(r)\,dz = \int_{-\infty}^{\infty}\rho_p(\sqrt{R^2+z^2})\,dz$$

$$= \int_{-\infty}^{\infty}\frac{3a_p^2 M}{4\pi}\frac{1}{(R^2+z^2+a_p^2)^{5/2}}\,dz$$

$$= \frac{M\cdot a_p^2}{\pi(a_p^2+R^2)^2}$$

2' d) Solve $\Sigma_p(R) = \frac{1}{2}\Sigma_p(R=0) = \frac{1}{2}\cdot\frac{M}{\pi}a_p^{-2}$ and get $r_c \simeq 0.644\,a_p$.

Problem 6. What are the observational and theoretical difficulties in determining the IMF and how does one goes about trying to resolve them-so talk about things like converting light to mass (and the ucertainties), the effects of age and distance etc etc (15 pts)

To determine the IMF we have transform from observational values (brightness, color) to physical quantities (e.g. mass). Thus we need to convert apparent magnitude in some wavelength band into absolute magnitude (e.g. need to know the distance) and to transform this into mass. To go from one to the other we need to transform color into temperature via models correcting for age and metallicity. We also need to understand the selection effects since we want the number of stars per unit mass and need to figure out what fraction of which type of stars we are missing. We also need to correct for the effects of dust (reddening and extinction), age (since luminosity and color for a star of a given mass depends on age) and binarity. (see class notes which say) Determining the IMF is difficult

- Start with observed star counts
- Understand your selection effects, completeness
- Get the distances
- Correct for extinction
- Correct for unresolved binaries
- Take the data and determine the luminosity function (LF), Then apply: correction for main sequence lifetimes, and evolved stars no longer visible

Get the Present-Day Luminosity Function (PDLF)

- Assume a mass-luminosity relation which is a function of metallicity, bandpass
- Theoretical models tested by observations and the mass-luminosity (m-L) relation using stellar structure theory
- Convert to Present-Day Mass Function (PDMF)
- Use the evolutionary tracks from the same theoretical models
- Iterate over a star formation history
- Get the Initial Mass Function (IMF)

Problem 7. A dusty question: why does a fair fraction of a galaxies luminosity appear in the IR ? What is the source of energy and the source of IR photons. What does this tell us about star formation? Extra credit: why is observing in the far IR exciting/important for galaxy evolution studies? (15 pts)

Star formation is embedded in dusty molecular clouds. Young stars produce lots of luminosity in the UV where the opacity to dust is high (remember that opacity scales as wavelength to the -1 power). Since the covering fraction of the dust is high (e.g. most lines of sight from the star to the observer pass thru lots of dust) a large fraction (1/2) of the UV light is absorbed by dust and re-radiated in the IR; the reason the re-radiated light is in the IR is simple physics- the re-radiation is almost black body light (see previous homework problem where you calcuated the temperature).

Observing in the IR is important for galaxy evolution studies is due to the redshifting effect. As show in the class notes due to the combination of black body spectra and redshift at far IR wavelengths the observed flux of objects with low effective temperatures (20K) is constant as a function of redshift (e.g. distant objects with the same luminosity are almost as bright as nearby objects). This makes the study of distant objects much easier.