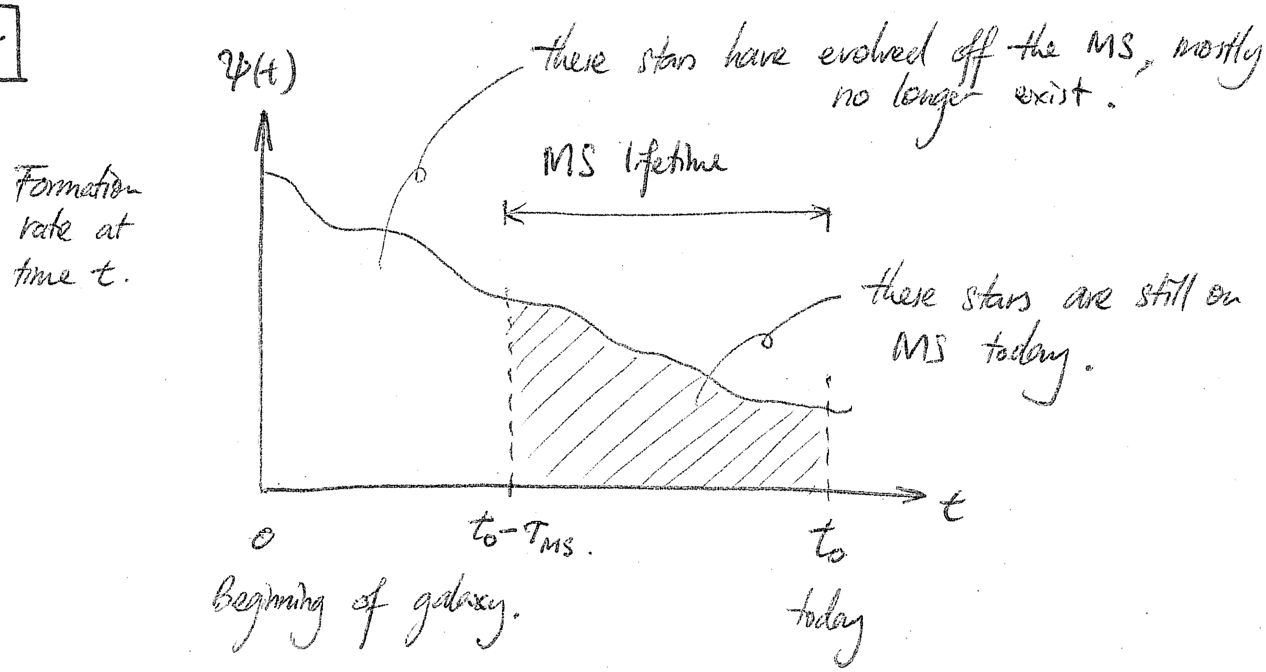
ASTR 421 Homework 2

Qian Wang

2.4



For stars on MS with a certain luminosity M_V

define $\Phi(M_V) = \int_{t_0-\tau_{MS}(M_V)}^{t_0} \Psi(t)\,dt$ — # of them still on MS today

and $\Psi(M_V) = \int_0^{t_0} \Psi(t)\,dt$. — # of them formed since the beginning.

Why are we doing this.

We want to know the IMF — probability density function for the mass of zero-age main sequence stars (ZAMS). If we assume that the IMF is time-independent, then it is simply the mass distribution of all stars ever formed. (IMF time-independent implies the SFR $\Psi = \Psi(t)$ ie. function of time only, not mass.) Ψ is needed for this (can't use Φ, since Ψ/Φ depends on τ_{MS} depends on the mass of the star.) Note, we'll also need to know how to transform M_V (observable) $\rightarrow$ mass.

For this problem just evaluate $\bar{\Phi}/\Phi$ for a few different masses and ψ.

Constant ψ: $\dfrac{\Phi}{\bar{\Phi}} = \dfrac{\psi\int_0^{t_0} dt}{\psi\int_{t_0-\tau_{MS}}^{t_0} dt} = \dfrac{t_0}{\tau_{MS}}$

$$\left\{\begin{array}{l} t_0 = \tau_{gal} = 10 \text{ Gyr.} \\ \tau_{MS}(2M_\odot) = 1.1 \text{ Gyr} \\ \tau_{MS}(3M_\odot) = 0.35 \text{ Gyr} \\ \tau_{MS}(0.5M_\odot) > 25 \text{ Gyr.} \end{array}\right.$$

a) $\dfrac{\bar{\Phi}}{\Phi}(2M_\odot) = \dfrac{1.1}{10} = \boxed{0.11}$

b) $\dfrac{\bar{\Phi}}{\Phi}(3M_\odot) = \dfrac{0.35}{10} = \boxed{0.035}$

c) $\tau_{MS}(0.5M_\odot) > \tau_{gal} \Rightarrow$ $\boxed{\text{all of them still on MS.}}$

Now suppose $\psi(t) \propto e^{-t/t_*}$, $t_* = 3$ Gyr.

$$\frac{\bar{\Phi}}{\Phi} = \frac{\int_{t_0-\tau_{MS}}^{t_0} e^{-t/t_*}\,dt}{\int_0^{t_0} e^{-t/t_*}\,dt} = \frac{\cancel{-\frac{1}{(1/t_*)}}\left[e^{-t/t_*}\right]_{t_0-\tau_{MS}}^{t_0}}{\cancel{-\frac{1}{(1/t_*)}}\left[e^{-t/t_*}\right]_0^{t_0}}$$

$$= \frac{\exp\left(\frac{-t_0+\tau_{MS}}{t_*}\right) - \exp\left(-\frac{t_0}{t_*}\right)}{1 - \exp\left(-\frac{t_0}{t_*}\right)}$$

d) $\underline{2 M_\odot}$ $\frac{\Phi}{\Psi} = \frac{e^{-8.9/3} - e^{-10/3}}{1 - e^{-10/3}} = \frac{0.0158}{0.964} = \boxed{0.016}$

e) $\underline{3 M_\odot}$ $\frac{\Phi}{\Psi} = \frac{e^{-9.65/3} - e^{-10/3}}{1 - e^{-10/3}} = \frac{0.00441}{0.964} = \boxed{0.0046}$

f) Intuitively, if SFR was higher at early times, then there must be more stars that have since burned out, leading to a larger value of Ψ/Φ. To show this, break up the integral of Ψ at time $t_0 - T_{ms}$:

$$\frac{\Psi}{\Phi} = \frac{\Phi + \int_0^{t_0 - T_{ms}} \psi(t)\, dt}{\Phi} = 1 + \frac{\int_0^{t_0 - T_{ms}} \psi(t)\, dt}{\int_{t_0 - T_{ms}}^{t_0} \psi(t)\, dt}$$

The fraction of ('extinct' stars) / (still on MS) increases if $\psi(t)$ is increased in earlier times or decreased in later times.

g) How much larger must Ψ/Φ become?

For $2 M_\odot$, $\psi = \text{constant} \Rightarrow \Psi/\Phi = 9.091$

$\psi \propto e^{-t/t_*} \Rightarrow \Psi/\Phi = 62.5$

so ~ 7 times larger.

h) Gradual slowdown would lead to a larger inferred Ψ. But $T_{ms}(< 1 M_\odot) \gtrsim T_{gal}$ so the inferred Ψ for stars $< 1 M_\odot$ is not greatly affected by $\psi(t)$.

$\boxed{2.5}$

$$\xi(\mathcal{M})\,\Delta\mathcal{M} = \xi\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-2.35}\left(\frac{\Delta\mathcal{M}}{\mathcal{M}_\odot}\right) \qquad (2.5)$$

$$\lim_{\Delta\mathcal{M}\to 0}\left\{\sum \xi(\mathcal{M})\,\Delta\mathcal{M}\right\} = \int \xi(\mathcal{M})\,d\mathcal{M}$$

$$= \int \xi_0\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-2.35} d\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)$$

$$N = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \xi(\mathcal{M})\,d\mathcal{M} = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \xi_0\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-2.35} d\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)$$

$$= \xi_0\left[\frac{1}{1.35}\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-1.35}\right]_{\mathcal{M}_u}^{\mathcal{M}_l}$$

$$= \frac{\xi_0}{1.35}\left[\left(\frac{\mathcal{M}_\odot}{\mathcal{M}_l}\right)^{1.35} - \left(\frac{\mathcal{M}_\odot}{\mathcal{M}_u}\right)^{1.35}\right]$$

Since $dN \propto \mathcal{M}^{-2.35}\,d\mathcal{M}$, smaller $\mathcal{M}$ contribute more.

In the limit $\mathcal{M}_u \gg \mathcal{M}_l$, $\frac{1}{\mathcal{M}_l} \gg \frac{1}{\mathcal{M}_u}$

$$\Rightarrow N \approx \frac{\xi_0}{1.35}\left(\frac{\mathcal{M}_\odot}{\mathcal{M}_l}\right)^{1.35}.$$

$$\frac{M}{\mathcal{M}_\odot} = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \underbrace{\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)\xi(\mathcal{M})\,d\mathcal{M}}_{\text{total mass between } \mathcal{M} \text{ and } \mathcal{M}+d\mathcal{M}.} = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \xi_0\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{-1.35} d\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)$$

$$= \frac{\xi_0}{0.35}\left[\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{0.35}\right]_{\mathcal{M}_u}^{\mathcal{M}_l}$$

$$= \frac{\xi_0}{0.35}\left[\left(\frac{\mathcal{M}_\odot}{\mathcal{M}_l}\right)^{0.35} - \left(\frac{\mathcal{M}_\odot}{\mathcal{M}_u}\right)^{0.35}\right]$$

$$\underset{\substack{\uparrow \\ d(\text{total mass})}}{dM} \propto \mathcal{M}^{-1.35} \underset{\substack{\uparrow \\ d(\text{stellar mass})}}{d\mathcal{M}}$$ so smaller masses contribute more.

$$\mathcal{M}_u \gg \mathcal{M}_l \Rightarrow \quad M \approx \frac{\xi_0}{0.35}\left(\frac{\mathcal{M}_\odot}{\mathcal{M}_l}\right)^{0.35} \mathcal{M}_\odot$$

$$\frac{L}{L_\odot} = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \underbrace{\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.5}}_{\frac{L}{L_\odot}} \xi(\mathcal{M})\, d\mathcal{M} = \int_{\mathcal{M}_l}^{\mathcal{M}_u} \xi_0 \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{2.15} d\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)$$

$$= \frac{\xi_0}{3.15}\left[\left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{3.15}\right]_{\mathcal{M}_l}^{\mathcal{M}_u}$$

$$= \frac{\xi_0}{3.15}\left[\left(\frac{\mathcal{M}_u}{\mathcal{M}_\odot}\right)^{3.15} - \left(\frac{\mathcal{M}_l}{\mathcal{M}_\odot}\right)^{3.15}\right].$$

$dL \propto \mathcal{M}^{2.15} d\mathcal{M}$ so larger masses contribute more.

$$\mathcal{M}_u \gg \mathcal{M}_l \Rightarrow \quad L \approx \frac{\xi_0 L_\odot}{3.15}\left(\frac{\mathcal{M}_u}{\mathcal{M}_\odot}\right)^{3.15}$$

Q3 See pages from Draine for ISM phases + obs signatures.

Heating mechanisms

Processes that contribute positively to the microscopic KE of the gas. Particles in the gas thermalise through elastic collisions (ions, free electrons, dust) So any process that give the particles some KE would add thermal energy to the system. Note: bulk KE does not contribute to thermal energy!

These are some such processes:

- Photoionisation

  A photon whose energy exceeds the binding energy of a bound electron can interact with the orbital and free it. Imparts to $\bar{e}$ amount of KE equal to $E_\gamma - E_{bind}$, which the $\bar{e}$ then brings into the system through thermalisation.

- Cosmic ray interaction

  Cosmic rays are not EM waves but atomic nuclei [redacted] at ~ ~~light~~ relativistic speeds. They can either ionise and contribute through the freed electron's KE, or interact elastically (coulomb interaction) with free electrons to increase the latter's KE.

- <u>Photoelectric heating of dust grains</u>

  In cooler phases, UV photons can strip electrons from dust molecules, and heat the dust with the remaining energy. The dust grain then thermalises with the gas particles.

- <u>Stellar winds</u>

  Hot gas outflows from certain luminous stages of stars heat the surrounding media.

- <u>Shock heating</u>

  Shocks develop where waves travel at faster than the Alfvén speed in plasmas. Possible through supernova or fast supersonic and super-Alfvén speed stellar winds. Because of the low densities involved this is actually a collisionless process (unlike shocks in denser media) heating occurs through plasma physics effects.

- <u>Grain-gas collisions</u>

  Grains can be heated by absorbing infrared radiation (not energetic enough to photoionise). They then thermalise with gas.

- <u>Gravitational collapse</u>

When self-gravitating gas clouds collapse, virialisation transforms GPE to KE. This is then transformed into internal (thermal) energy through shocks and viscous dissipation (gas collapse is turbulent). This is the process that heats the protostellar core, raising its temperature to H fusion level.

<u>Cooling mechanisms</u>

Most are radiative cooling processes through which the internal energy of the gas is radiated away. Note that because radiation interacts with the ISM, the radiation may be transformed back into the internal energy of the gas if it is energetic enough to photoionise. Only when the radiation escapes the gas can the energy it carries be considered lost.

- <u>Collisional excitation, followed by deexcitation</u>

Free electron can collide and interact with the orbital of a bound electron, and depart with lower KE while leaving the bound electron excited (higher energy orbital). The excited electron can then transition downwards to successively lower levels and emit photons carrying the energy difference (spontaneous or stimulated emissions) until it falls back to the ground level. The emitted photon may carry enough energy to cause an ionisation (eg. a Lyα photon can ionise an excited H electron)

So the overall cooling rate depends on the interaction crossections and lifetimes of the species involved, and is more complicated.

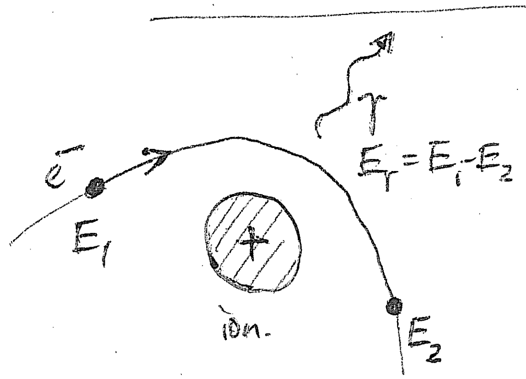
- <u>Collisional ionisation</u>

  If a bound electron is ionised through collision with a free electron, then overall there is loss in internal energy, to provide the binding energy.

- <u>Recombination</u>

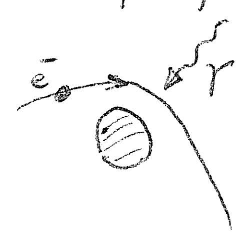
  Free electrons can recombine with ions and radiate a photon with $E_\gamma = E_{bnd} + KE$.

- <u>Thermal Bremsstrahlung</u> (braking-radiation)



$T > 10^7$ K, ~ X-rays.
In HII regions, dominates the radio emission..

N.B. Reverse can also happen, free-free absorption.



- <u>Adiabatic expansion</u>

  Happens to hot gas, thermodynamic effect.