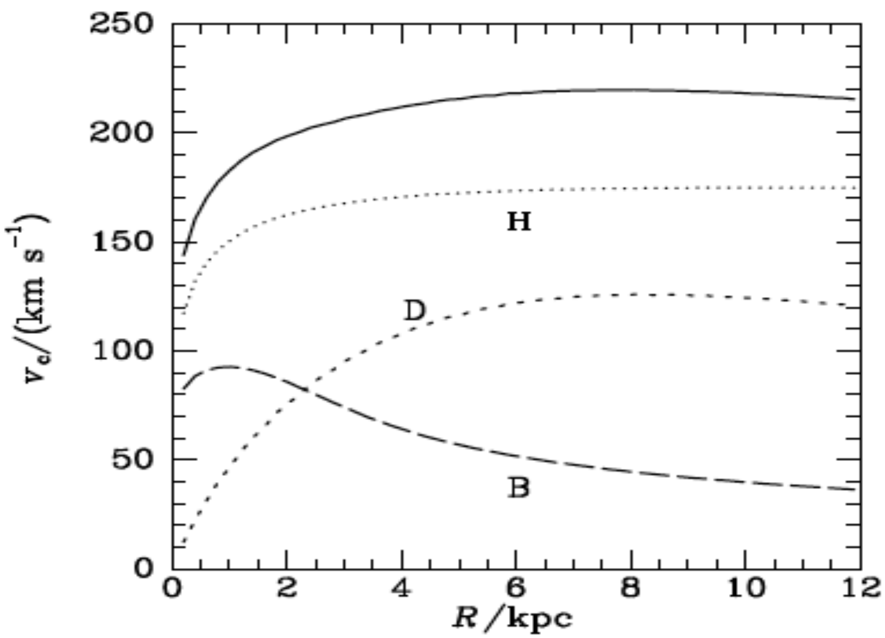
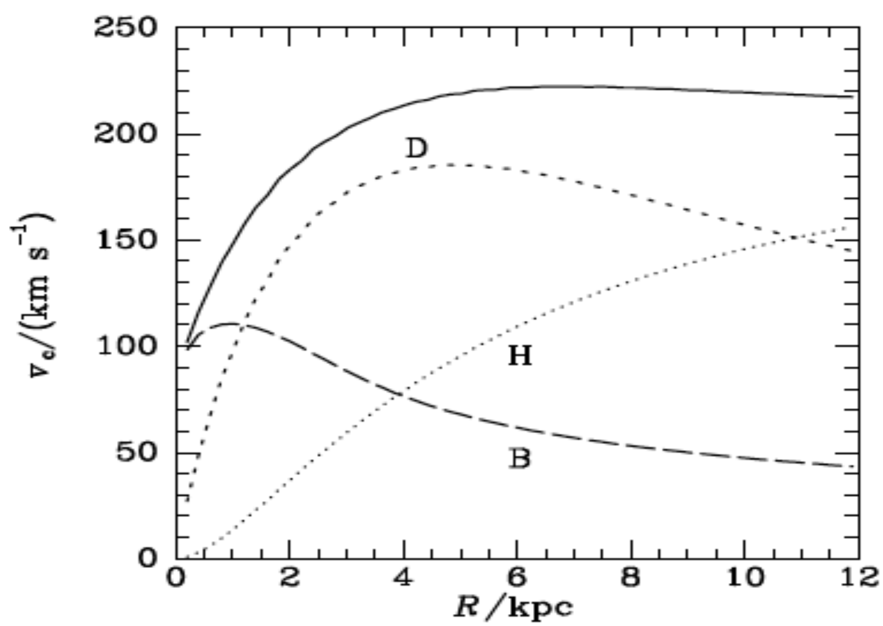
Modeling Spirals

- to fit the observed density and velocity distributions in the MW one needs a 3 component mass distribution
- Traditionally this is parameterized as the sum of
  - disk $\Sigma(R) = \Sigma_0[\exp -R/\mathbf{R_d}]$
  - spheroid (bulge) using $I(R)=I_0 R_s^2/[R+R_s]^2$ or similar forms
  - dark matter halo

    $\rho(r)=\rho(0)/[1+(r/a)^2]$
- See B&T sec 2.7 for more complex forms- 2 solutions in B&T- notice extreme difference in importance of halo (H) (table 2.3)

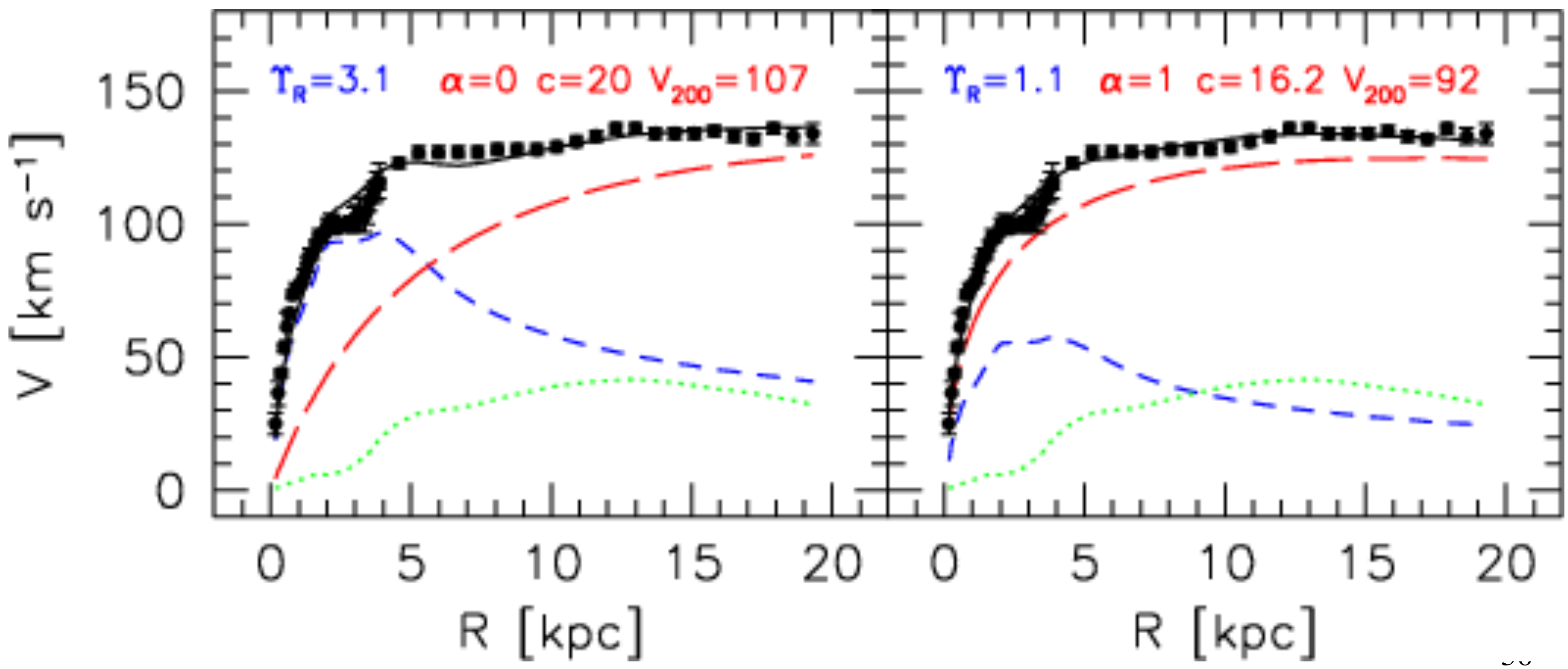




Disk Halo Degeneracy

- MBW fig 11.1: two solutions to rotation curve of NGC2403: stellar disk (blue lines), dark matter halo - red lines.

- Left panel is a 'maximal' disk, using the highest reasonable mass to light ratio for the stars, the right panel a lower value of M/L



Potential of Spiral Galaxies B&T 2.7

- The potential of spirals is most often modeled as a 3 component system
- Bulge
- Dark halo
- Disk

as stressed by B&T usually one **assumes** that the potential has a certain form and is well traced by stars/gas for analytic analysis

On pg 111 B&T give the observational constraints which models have to match.

Bulge; B&T assume $\rho(r)=\rho_B(0)(m/a_b)^{-\alpha_b} \exp(-m/r_b)^2{}_c)$; $m=\mathrm{sqrt}(R^2+z^2/q^2)$which for $q<1$ is an oblate spheroidal power-law model (no justification is given !)

They use IR star counts in the bulge (which is dominated by old stars) to get values for the parameters.

They use a similar form for the halo, but the parameters are much less well determined.

Disk: use a double exponential disk (thin and thick) for the stars and a somewhat more complex form for the gas (in the MW gas is ~25% of the mass of stars in the disk).

The most important parameter is the disk scale length :

Rotation Curve Mass Estimates sec 2.6 of B&T

- sec sec 11.1.2 in MBW
- Galaxy consists of a axisysmmetric disk and spherical dark matter halo
- Balance centrifugal force and gravity
- $V^2(R)=R\mathbf{F}(R)$; $\mathbf{F}(R)$ is the acceleration in the disk
- Split rotation into 2 parts due to disk and halo

  $V^2(R)=V^2_d(R)+V^2_h(R)$
- for a **spherical** system

  $V^2(R)=r d\phi/dr=GM(r)/r$
- Few analytic solutions: point mass $V_c(R)\sim r^{-1/2}$

  singular isothermal sphere $V_c(R)$=constant (see S+G eq 3.14)

  uniform sphere $V_c(R)\sim r$

for a pseudo-isothermal (S+G problem 2.20)

$\rho(r)=\rho(0)(R^2_c/R^2+R^2_c)$; $\rho(0)= V(\infty)^2/4\pi G R^2_c$ and the velocity profile is

$V(R)^2=V(\infty)^2(1-R_c/R \tan^{-1} R/R_c)$;for a **NFW** potential get a rather messy formula

MW Mass Model

- Notice that the mass of the bulge and the surafce mass density of the disk are highly uncertain

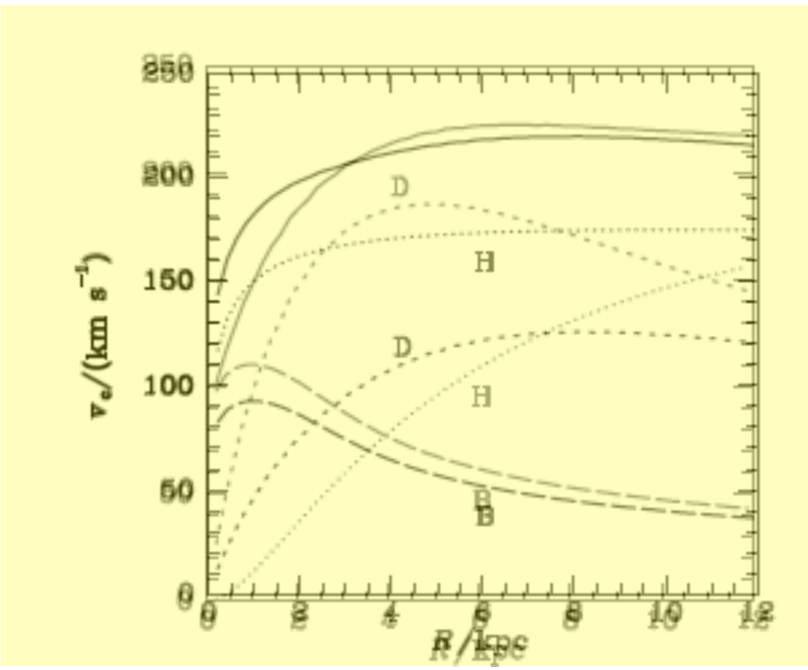


Table 2.3 Parameters of Galaxy models

Parameter	Model I	Model II
R_d/kpc	2	3.2
$(\Sigma_d + \Sigma_g)/\mathcal{M}_\odot\,pc^{-2}$	1905	536
$\rho_{b0}/\mathcal{M}_\odot\,pc^{-3}$	0.427	0.3
$\rho_{h0}/\mathcal{M}_\odot\,pc^{-3}$	0.711	0.266
α_h	−2	1.63
β_h	2.96	2.17
a_h/kpc	3.83	1.90
$M_d/10^{10}\,\mathcal{M}_\odot$	5.13	4.16
$M_b/10^{10}\,\mathcal{M}_\odot$	0.518	0.364
$M_{h,<10\,kpc}/10^{10}\,\mathcal{M}_\odot$	2.81	5.23
$M_{h,<100\,kpc}/10^{10}\,\mathcal{M}_\odot$	60.0	55.9
$v_e(R_0)$/km s^{-1}	520	494
f_b	0.05	0.04
f_d	0.60	0.33
f_h	0.35	0.63

Orbits in a **static** spherical potential B& T sec 3.1

angular momentum (**L)** is conserved

- $d^2r/dt^2=\phi(r)e_r$ e_r is the unit vector in radial direction; the radial acceleration $\phi=d^2r/dt^2$
- $d/dt(r \times dr/dt)=(dr/dt \times dr/dt)+r \times d^2r/dt^2=g(r)r \times e_r =0$;
- conservation of angular momentum L=rxdr/dt (eqs. 3.1-3.5)
- Define **L**=**r**xd**r**/dt; d**L**/drt=0

- Since this vector is constant, we conclude that the star moves in a plane, the orbital plane.
- This simplifies the determination of the star's orbit, since the star moves in a plane, we may use plane polar coordinates

for which the center is at r = 0 and φ is the azimuthal angle in the orbital plane

Stellar Dynamics B&T ch 3, S&G 3

- Orbits in a static spherical potential:

  angular momentum is conserved

  - $d^2r/dt^2=g(r)\mathbf{e}_r$ $\mathbf{e}_r$ is the unit vector in radial direction; the radial acceleration $g=d^2r/dt^2$
  - $d/dt(r \times dr/dt)=(dr/dt \times dr/dt)+r \times d^2r/dt^2=g(r)r \times \mathbf{e}_r=0$; conservation of angular momentum $L=r \times dr/dt$ around the z axis

Conservation of energy:

total energy =PE+KE or in above formalism

- stars move in a plane (orbital plane) $\theta=0$
- Use plane polar coordinates (R,φ,z) (Appendix B)
- eqs of orbits

R coordinate: $d^2R/dt^2-R(d^2\varphi/dt^2)=\phi(R)$

ψ coordinate :$2(dR/dt)(d\varphi/dt)+R(d^2\varphi/dt^2)=0$

equation of motion is $(d^2R/dt^2)-(L^2/R^3)=\phi(R)$

Orbits for a Constant Density Sphere

From before

Potential energy (B&T) eq 2.41, 2.32

$\phi(R)=-d/dr(M(R))$;

$R>a\ \phi(r)=4\pi G a^3\rho_0=-GM/r$

$R<a\ \phi(r)=-2\pi G\rho_0(a^2-1/3r^2))$;

Use Cartesian coordinates $x=r\cos\varphi$, $y=r\sin\varphi$

$F_x=-4\pi GR\rho_0\cos(\varphi)\mathbf{e}_x=-4\pi Gx\rho_0\mathbf{e}_x$

$F_y=-4\pi Gy\rho_0\mathbf{e}_y$

need to transform $d^2\mathbf{r}/dt^2=\mathbf{e}_x d^2\mathbf{x}/dt^2+\mathbf{e}_y d^2\mathbf{y}/dt^2$

define $\Omega^2=4\pi/3G\rho_0$;$d^2\mathbf{x}/dt^2=-\Omega^2 x$; $d^2\mathbf{y}/dt^2=-\Omega^2 y$

this the harmonic oscillator general solution

$x=A\cos(\Omega t+k_x)$; $y=B\cos(\Omega t+k_y)$;

A,B are amplitudes and k's the initial phase

going backwards to polar coordinates

$R=\text{sqrt}[A^2\cos^2(\Omega t+k_x)+B^2\cos^2(\Omega t+k_y)]$

$\psi=\tan^{-1}[B\cos(\Omega t+k_y)/A\cos(\Omega t+k_x);]$

The R and ψ define a closed ellipse on the center of the sphere; A and B are the major and semi-major axis.

Complete radial period in $\Delta\varphi=\pi$

Most mass distributions will lie between a pt mass and a uniform sphere

radial and azimuthal periods not the same

rosette pattern for orbits

'Real' Orbits

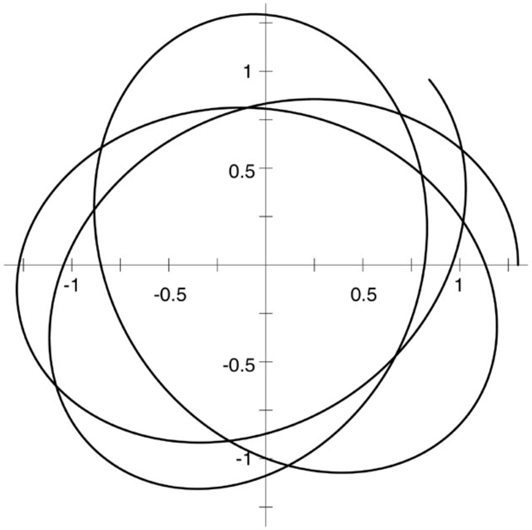
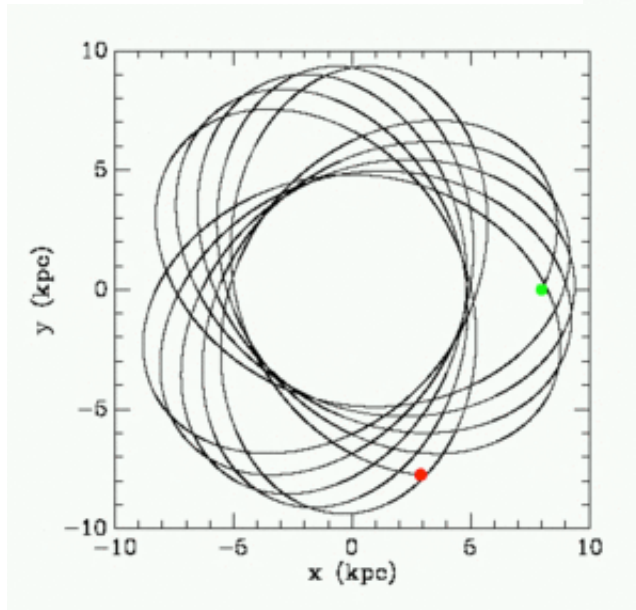
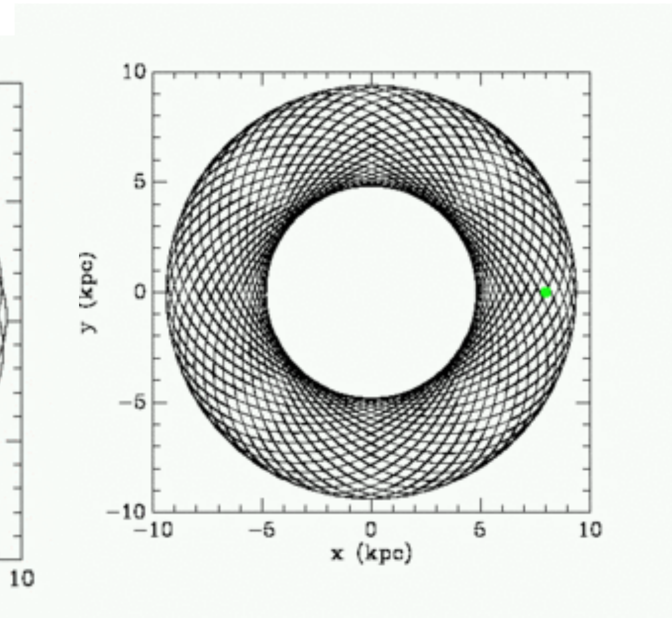


Fig 3.10 'Galaxies in the Universe' Sparke/Gallagher CUP 2007





- A few orbits, ~2 Gyr of orbits- 20 Gyrs from C. Flynn

Stellar Dynamics B&T ch 3; S&G 3.3

- Orbits of disk stars

only the component of angular momentum parallel to symmetry axis is constant.

- Since **L** is conserved, stars move in a plane - can use polar coordinates (R,φ) (do not need z, appendix B B&T B.24)
- R eq of motion $dR^2/dt^2 - Rd\varphi^2/dt^2 = \phi(r)$
- φ eq of motion $(2dR/dt*d\varphi/dt) + Rd\varphi^2/dt^2 = 0$; $\mathbf{L} = R^2 d\varphi/dt$ is a constant
- total equation of motion $dR^2/dt^2 - \mathbf{L}/R^2 = \phi(r)$

- Stars whose motions are confined to the equatorial plane of an axisymmetric galaxy 'feel' only an effectively spherically symmetric potential
- Therefore their orbits will be identical with those discussed previously
- The radial coordinate R of a star on such an orbit oscillates between the peri and apo-galacticon as the star revolves around the center, and the orbit forms a rosette figure.

Orbits in Axisymmetric Potentials- B&T 3.2, S&G 3.3

cylindrical coordinate system (R; φ; z) with origin at the galactic center, the z axis is the galaxy's symmetry axis.

- Stars in a axisymmetric galaxy 'see' a potential which is spherically symmetric. orbits will be identical to those in such a potential
- The situation is much more complex for stars whose motions carry them out of the equatorial plane of the system.
- orbits in axisymmetric galaxies can be reduced to a two-dimensional problem by exploiting the conservation of the z-component of angular momentum
- **S&G give nice physical description**

- $d^2\mathbf{r}/dt^2 = -\nabla\Phi\ (R,z)$; which can be written in cylindrical coordinates as
- $d^2R/dt^2 - Rd\varphi^2/dt^2 = -\partial\Phi/\partial R$
- Motion in the φ direction : $d/dt\ (R^2\ d\varphi/dt)=0$; $L_z = R^2(d\varphi/dt) = 0$ constant
- z direction : $d^2z/dt^2 = -\partial\Phi/\partial z$

Orbits in Axisymmetric Potentials- B&T 3.2

- Eliminating $d\varphi/dt$ and putting in angular momentum
- $d^2R/dt^2 - L^2_z/R^3 = -\partial\Phi/\partial R$ - if we define an effective potential $\Phi_{eff} = \Phi(R,z) + L^2_z/2R^2$
- $d^2R/dt^2 = -\partial\Phi_{eff}/\partial R$ (see B&T **eq 3.67-3.68**)

- Unless it has enough energy to escape from the Galaxy, each star must remain within some apogalactic outer limit.

Orbits in Axisymmetric Potentials- S&G 3.3

The three-dimensional motion of a star in an axisymmetric potential (R; z) can be reduced to the two dimensional motion of the star in the (R; z) plane (the meridional plane

- Since the change in ang mom in the z direction is zero (planar orbits)

$\partial/\partial z(L^2_z/2R^2) = 0$; $d^2z/dt^2 = -\partial\Phi_{eff}/\partial z$;

The effective potential is the sum of gravitational potential and KE in the φ direction.

and rises very steeply near the z axis

The minimum in Φ_{eff} has a "simple" physical meaning (see next page)

$0 = \partial\Phi_{eff}/\partial R = \partial\Phi/\partial R - L^2_z/R^3$; which is satisfied at a particular radius - the guiding center radius R_G where

$(\partial\Phi/\partial R)|_{R_G} = L^2_z/R^3 = R_G\,(d\varphi/dt)^2$

and $0 = \partial\Phi_{eff}/\partial z$ which is satisfied in the equatorial plane

these are the conditions for a circular orbit with angular speed $d\varphi/dt$

Orbits in Axisymmetric Potentials- S&G 3.3

- the minimum of Φ_{eff} occurs at the radius at which a circular orbit has angular momentum L_z, and the value of Φ_{eff} at the minimum is the energy of this circular orbit
- Unless Φ has a special form these eq's cannot be solved analytically

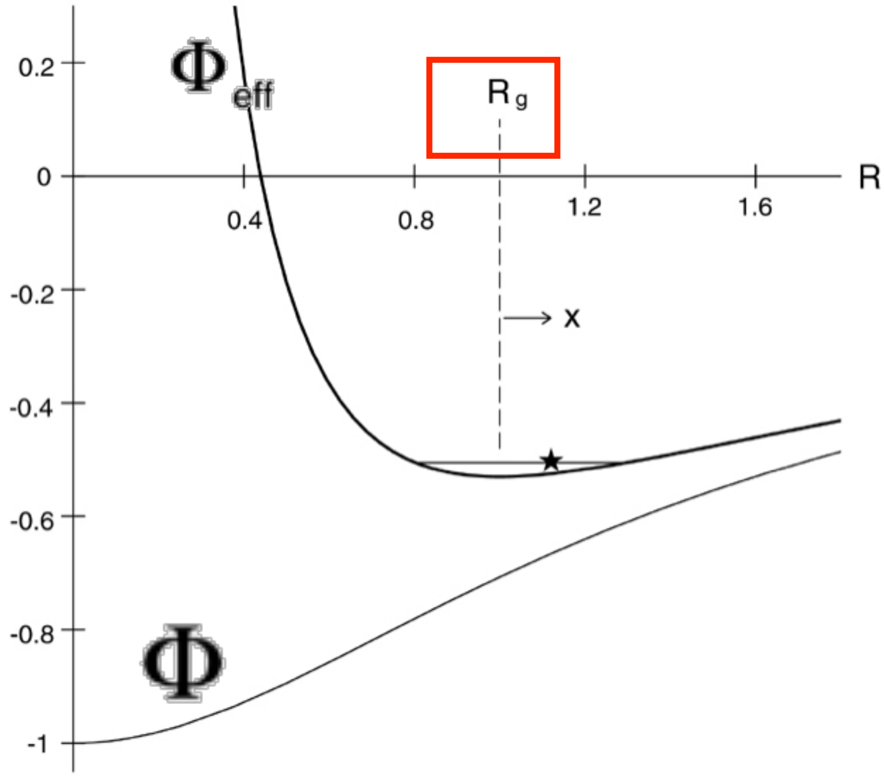


Fig 3.8 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Orbits in Axisymmetric Potentials- S&G 3.2.3

If assume in disk galaxies that the orbits are *nearly* circular
What approx can we make to the orbits??

let $x = R - R_g$; where $R_g(L_z)$ is the guiding-center radius for an orbit of angular momentum L_z (eq. 3.72).

Expand Φ_{eff} around x (see B&T eq 3.76) ; the epicycling approx ignores all terms of xz^2 or higher

Then define 2 new quantities:
$\kappa^2(R_G) = (\partial^2\Phi_{eff}/\partial R^2)$; $\nu^2(R_G) = (\partial^2\Phi_{eff}/\partial z^2)$; then keeping the lower orders $d^2x/dt^2 = -\kappa^2 x$; $d^2z/dz^2 = -\nu^2 z$; these are the harmonic oscillator eq's around x and z with frequencies κ and ν.
κ is the epicycle freq and ν the vertical frequency
this gives a vertical period $T = 2\pi/\nu \sim 6 \times 10^7$ yrs for the MW

EpiCycles B&T,S&G 3.3

- Remember the Oort constants??
- Well in the same limit (remember v_{circle} =$R\Omega(R)$)

Ω=A-B; κ^2 =-4B(A-B)=4BΩ ~ 2Ω^2 (eq 3.84); using the measured values of these constants one finds that near the sun κ_0^2= 37km/sec/kpc and the ratio of the freq of the suns orbit around the GC and the radial freq κ_0/Ω_0=sqrt(−B/(A−B))= 1.35

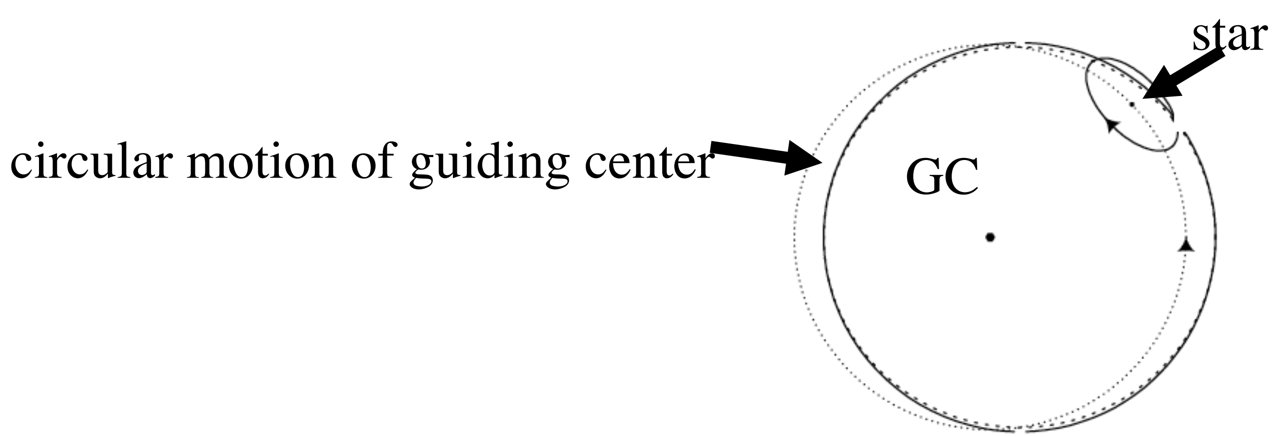
- Stellar orbits do not close on themselves in an inertial frame, but form a rosette figure like those discussed above for stars in spherically symmetric potentials

- The ratio ν^2/κ^2 ~3/2 $\rho/\langle\rho\rangle$ <u>a measure of how concentrated the mass is near the plane</u>
- The value of this approximation is in its ability to describe the motions of stars in the disk plane (does not work well for motion perpendicular to the plane) .
- The angular momentum on a circular orbit is $R^2\Omega(R)$;

  if it increases outward at radius R, the circular orbit is stable. This condition always holds for circular orbits in galaxy-like potentials.

Motion in Both Coordinates B&T 3.91-3.94

- $d^2x/dt^2=-\kappa^2x$; $d^2z/dz^2=-\nu^2z$; these are the harmonic oscillator eq's around x and z with frequencies κ and ν.
- and the general solution is
- $x(t)=C\cos(\kappa t+A)$; $C>0$ and A are arbitrary constants
- the solution for the φ direction is a bit messier and is
- $\varphi=(L_z/R^2_g)t-(\kappa/2B)(C/R_g)\sin(\kappa t+A)+\varphi_0$
- In the (x; y) plane the star moves on an ellipse called the epicycle around the guiding center



Epicycles

- Why did we go thru all that??
- Want to understand how to use stellar motions determine where the mass is.
- the orbits of stars take them through different regions of the galaxies -their motions at the time we observe them have been affected by the gravitational fields through which they have travelled earlier.
- use the equations for motion under gravity to infer from observed motions how mass is distributed in those parts of galaxies that we cannot see directly.

- The motions we have considered so far are the simplest !
- Using epicycles, we can explain the observed motions of disk stars near the Sun.

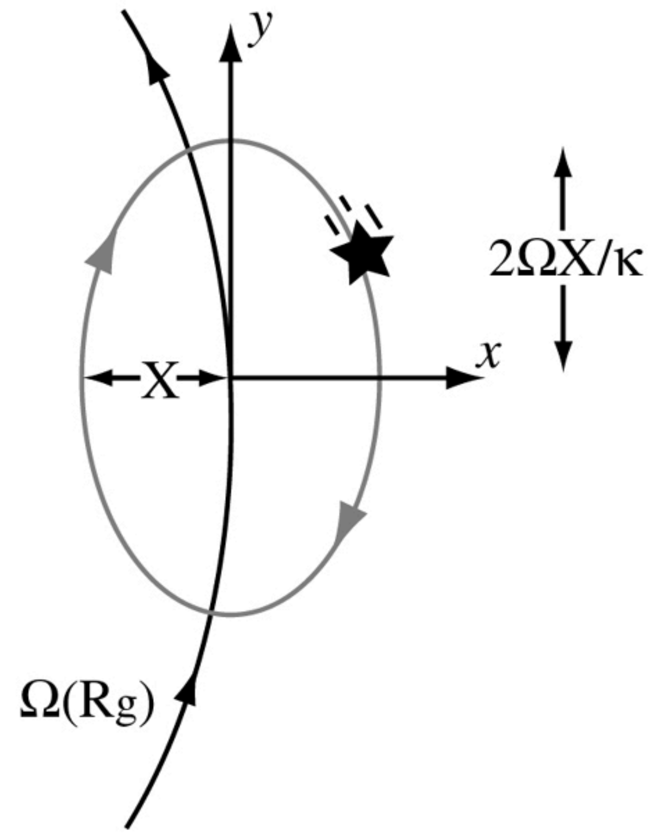


Fig 3.9 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Virial Theorem S&G pg 120-122

- S+G pg 120-121, MBW 5.4.4, B&T pg 360
- A rather different derivation (due to H Rix)
- Consider (for simplicity) the 1-D Jeans eq in steady state (more later)
- $\partial/\partial x[\rho v^2]+\rho\partial\phi/\partial x=o$
- Integrate over velocities and then over positions...
- $-2E_{kin}=E_{pot}$

- or restating in terms of forces
- if T= total KE of system of N particles < >= time average
- $2\langle T\rangle=-\Sigma(F_k \bullet r_k)$; summation over all particles k=1,N

call the 'virial 'Q

Q= $$\frac{1}{2}\frac{dI}{dt} = m\sum r.\frac{dr}{dt} = \sum p.r$$

dQ/dt= $$\sum F.r + 2T$$

Virial Theorem

- Another derivation following Bothun
  http://ned.ipac.caltech.edu/level5/Bothun2/Bothun4_1_1.html
- Moment of inertia, I, of a system of N particles
- $I=\Sigma m_i r_i^2$ sum over i=1,N (express r_i^2 as $(x_i^2+y_i^2+z_i^2)$)
- take the first and second time derivatives ; let dx^2/dt^2 be symbolized by x,y,z
- $dI^2/dt^2 = \Sigma m_i\,(dx_i^2/dt+dy_i^2/dt+dz_i^2/dt)+\Sigma m_i(x_i\,\mathrm{x}+y_i\,\mathrm{y}+z_i\,\mathrm{z})$

mv^2 (KE)+Potential energy (W) r •(m**a**)

after a few dynamical times, if unperturbed a system will come into Virial equilibrium–time averaged inertia will not change so 2<T>+W=0

For self gravitating systems $W=-GM^2/2R_H$; R_H is the harmonic radius- the sum of the distribution of particles appropriately weighted

$1/R_H = 1/N\ \Sigma_i\ 1/r_i$

The virial mass estimator is $M=2\sigma^2 R_H/G$; for many mass distributions $R_H \sim 1.25\ R_{eff}$

where R_{eff} is the half light radius σ is the 3-d velocity dispersion

Virial Thm MBW 5.4.4

- If I is the moment of inertia
- $1/2d^2I/t^2 = 2KE+W+\Sigma$
  - where Σ is the work done by external pressure
  - KE is the kinetic energy of the system
  - w is the potential energy (only if the mass outside some surface S can be ignored)
- For a static system ($d^2I/t^2 = 0$) 2KE +W+Σ =0

Virial Theorem B&T 4.8.3(a)

- This fundamental result describes how the total energy (E) of a self-gravitating system is shared between kinetic energy and potential energy .
- Go to one dimension and assume steady state

Integrate over velocity and space and one finds

$-2E_{kinetic}=PE_{potential}$ (W=PE)

see S&G pgs 120-121 for full derivation

This is important for find the masses of systems whose orbital distribution is unknown or very complex and more or less in steady state (so assumptions in derivation are ok)

In general $\langle v^2\rangle=W/M=GM/r_g$;$r_g$ the gravitational radius (which depends on the form of the potential)

Many of the forms of the potential have their scale parameter $\sim 1/2r_g$ (pg 361 B&T)

Virial Theorem - Simple Cases

- Circular orbit: $mV^2/r=GmM/r^2$
- Multiply both sides by r $mV^2=GmM/r$
- $mV^2=2KE$; $GmM/=-W$ so $2KE+W=0$

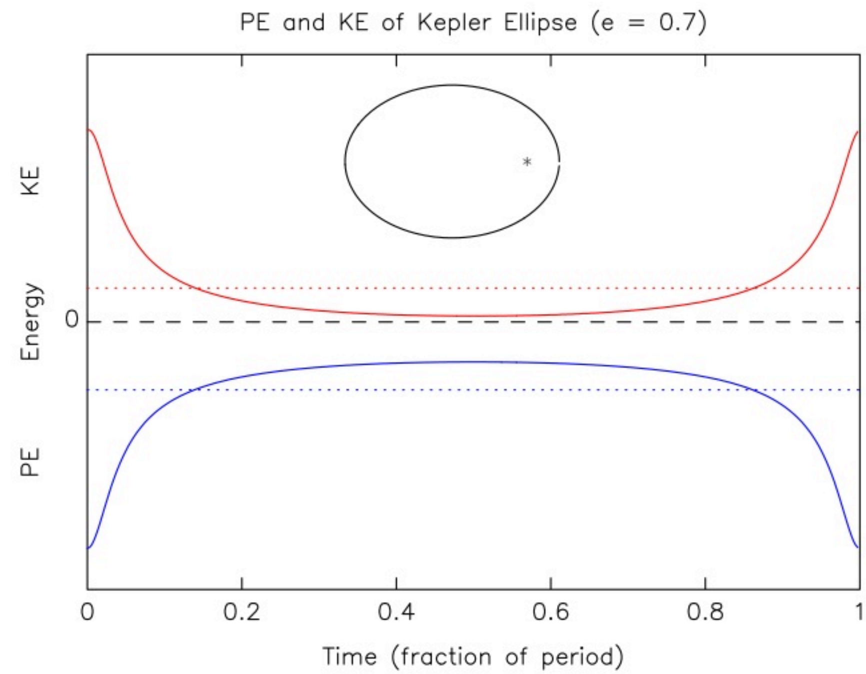
- Time averaged Keplerian orbit

define $U=KE/|W|$; as show in figure it clearly changes over the orbit; but

take averages

,$-W.=<GM/r>=GM<1/r>=GM(1/a)$

$KE=<1/2mV^2>=GM<1/r-1/2a>$
$=1/2GM(1/a)$ and again $2KE+W=0$



Red: kinetic energy (positive) starting at perigee
Blue: potential energy (negative)

Use of Virial Theorem

- Consider a statistically steady state, spherical, self gravitating system of N objects with average mass m and velocity dispersion σ.
- Total KE=(1/2)N$m\sigma^2$
- If average separation is r the PE of the system is $U=(-1/2)N(N-1)Gm^2/r$
- Virial theorem E=-U/2 so the total mass is $M=Nm=2\pi\sigma^2/G$ or using L as light and Σ as surface light density

  $\sigma^2 \sim (M/L)\Sigma R$-picking a scale (e.g. half light radius R_e)

  $R_e \sim \sigma^\alpha \Sigma^\beta$ $\alpha=2, \beta=1$ from viral theorem

  value of proportionality constants depends on shape of potential

  For clusters of galaxies and globular clusters often the observables are the light distribution and velocity dispersion. then one measures the ratio of mass to light as

  $M/L \sim 9\sigma^2/2\pi G I(0) r_c$ for spherical isothermal systems

Time Scales for Collisions S&G 3.2

- N particles of radius $r_{p;}$ Cross section for a direction collision $\sigma_d=\pi r^2_p$
- Definition of mean free path; if V is the volume of a particle $4/3\pi r^3_p$

  $\lambda=V/n\sigma_d$ where n is the number density of particles (particles per unit volume)

  $n=3N/4\pi r^3_p$

  and the characteristic time between collisions (Dimensional analysis)is

  $t_{collision}=\lambda/v\sim(\ell/r_p)^2 t_{cross}/N$ where v is the velocity of the particle.

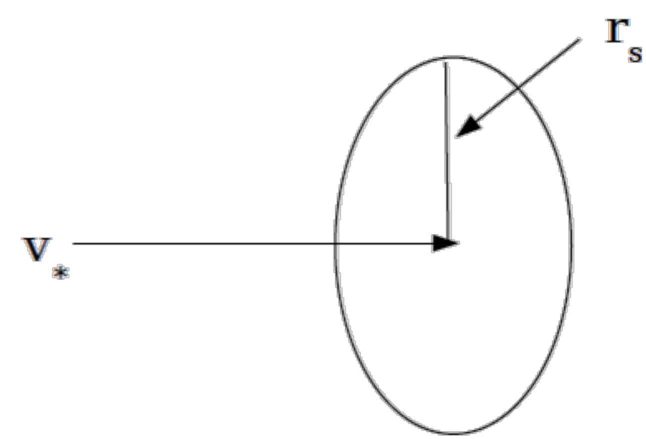
  for a body of size ℓ, $t_{cross}=\ell/v$

  So lets consider a galaxy with $\ell\sim$10kpc, $N=10^{10}$ stars and v~200km/sec

  if $r_p = R_{sun}$, $t_{collision}\sim10^{21}$ yrs
- For indirect collisions the argument is more complex (see S+G sec 3.2.2, MWB pg 231) but the answer is the same - it takes a very long time for star interactions to exchange energy (relaxation).
- $t_{relax}\sim Nt_{cross}/10\ln N$
- Its only in the centers of the densest globular clusters and galactic nuclei that this is important

How Often Do Stars Encounter Each Other

For a 'strong' encounter $GmM/r>1/2mv^2$ e.g. potential energy exceeds KE
So a critical radius is $r<r_s=2GM/v^2$



Putting in some typical numbers $m\sim1/2M_\odot$
$v=30km/sec$ $r_s=1AU$
So how often do stars get that close?

consider a cylinder $Vol=\pi r^2_s vt$; if have n stars per unit volume than on average the encounter occurs when
$n\pi r^2_s vt=1$, $t_s=v^3/ 4\pi nG^2m^3$
Putting in typical numbers $=4x10^{12}(v/10km/sec)^3(m/M_\odot)^{-2}(n/pc3)^{-1}$ yr- a very long time (universe is only 10^{10}yrs old- galaxies are essentially collisionless

What About Collective Effects ? sec S&G 3.2.2

For a weak encounter b >> r_s
Need to sum over individual interactions- effects are also small

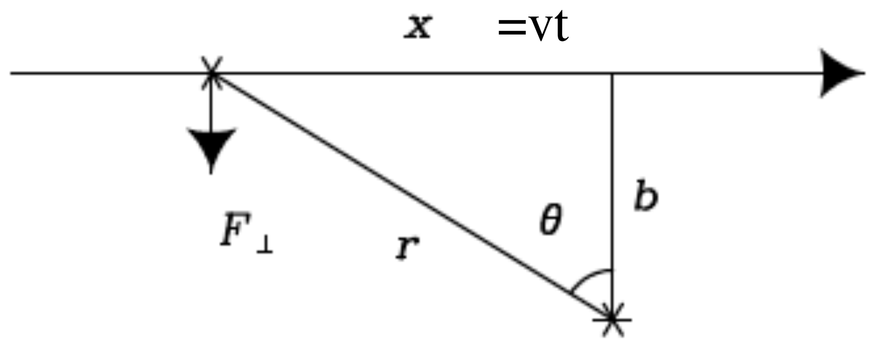
Relaxation Times

- Star passes by a system of N stars of mass m
- assume that the perturber is stationary during the encounter and that $\delta v/v \ll 1$- large b $> Gm/v^2$ (B&T pg 33-sec 1.2.1. sec 3.1 for exact calculation)
- Newton's Laws $m(dv/dt)=F$
- $(b^2+x^2)=r^2$
- $F=Gm^2\cos\theta/(b^2+x^2)=Gbm^2/(b^2+x^2)^{3/2}=$ $(Gm^2/b^2)(1+(vt/b)^2)^{-3/2}$ if v is constant
- Now integrate over time $\delta v = \int(F/m)dt=$

$(Gm/bv)\int(1+(vt/b)^2)^{-3/2}\,dt$ (change variables $s=(vt/b)$) ; $\delta v = 2Gm/bv$

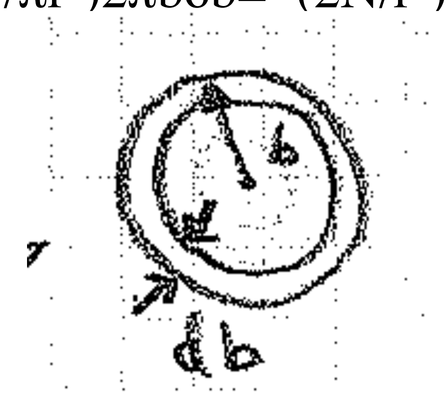
- **In words, δv is roughly equal to the acceleration at closest approach, Gm/b^2,**

times the duration of this acceleration $2b/v$.



The surface density of stars is $\sim N/\pi r^2$
N is the number of stars

let δn be the number of interactions a star encounters with impact parameter between b and δb crossing the galaxy once $\sim(N/\pi r^2)2\pi b\delta b = \sim(2N/r^2)b\delta b$



Relaxation...continued

- The net vectoral velocity due to these encounters is zero, but the mean square change is not

  $\delta v^2=(2Gm/bv)^2(2N/r^2)b\delta b$ (see B&T pg 34 eq. 1.3.2) - now integrating this over all impact parameters from b_{min} to b_{max}
- one gets $\delta v^2 \sim 8N(Gm/rv)^2 \ln \Lambda$; where r is the galaxy radius

  Λ is the Coulomb integral $\sim \ln(b_{max}/b_{min})$
- For gravitationally bound systems the typical speed of a star is roughly $v^2 \sim GNm/r$

  and thus $\delta v^2/v^2 \sim 8N \ln \Lambda/N$
- For each 'crossing' of a galaxy one gets the same δv so the number of crossing for a star to change its velocity by order of its own velocity is $n_{relax} \sim N/8\ln \Lambda$
- So how long is this?? well $t_{cross} \sim r/v$; and $\sim rv^2/(Gm)$ and thus
- $t_{relax} \sim (0.1N/\ln N)t_{cross}$; if we use $N \sim 10^{11}$; t_{relax} is very very long
- In all of these systems the dynamics over timescales $t < t_{relax}$ is that of a collisionless system in which the constituent particles move under the influence of the gravitational field generated by a smooth mass distribution,rather than a collection of mass points

Relaxation

- Values for some representative systems

	<m>	N	r(pc)	t_{relax}(yr)	age(yrs
Pleiades	1	120	4	1.7×10^7	$<10^7$
Hydaes	1	100	5	2.2×10^7	410^8
Glob cluster	0.6	10^6	5	2.9×10^9	10^9-10^{10}
E galaxy	0.6	10^{11}	3×10^4	4×10^{17}	10^{10}
Cluster of gals	10^{11}	10^3	10^7	10^9	10^9-10^{10}

scaling laws $t_{relax} \sim (R^3/Nm)^{1/2}$

- However numerical experiments (Michele Trenti and Roeland van der Marel 2013 astro-ph 1302.2152) show that even globular clusters never reach energy **equipartition** (!) to quote from this paper 'Gravitational encounters within stellar systems in virial equilibrium, such as globular clusters, drive evolution over the two-body relaxation timescale. The evolution is toward a thermal velocity distribution, in which stars of different mass have the same energy (Spitzer 1987). This thermalization also induces mass segregation. As the system evolves toward energy equipartition, high mass stars lose energy, decrease their velocity dispersion and tend to sink toward the central regions. The opposite happens for low mass stars, which gain kinetic energy, tend to migrate toward the outer parts of the system, and preferentially escape the system in the presence of a tidal field"

So Why Are Stars in Rough Equilibrium?

- It seems that another process 'violent relaxation' (MBW pg 251) is crucial.
- This is due to rapid change in the gravitational potential.
- Stellar dynamics describes in a statistical way the collective motions of stars subject to their mutual gravity-The essential difference from celestial mechanics is that each star contributes more or less equally to the total gravitational field, whereas in celestial mechanics the pull of a massive body dominates any satellite orbits
- The long range of gravity and the slow "relaxation" of stellar systems prevents the use of the methods of statistical physics as stellar dynamical orbits tend to be much more irregular and chaotic than celestial mechanical orbits-....woops.

How to Relax

- There are four different relaxation mechanisms at work in gravitational N-body systems:MBW sec 5.5.1-5.5.5
- phase mixing, chaotic mixing, Landau damping.
- Violent relaxation
  - time-dependent changes in the potential induce changes in the energies of the particles involved Exactly how the energy of a particle changes depends in a complex way on the initial position and energy of the particle (effects are independent of the mass of the particles)
  - Time scale is very fast ~free-fall time
- These processes are not well approximated by analytic calculations- need to resort to numerical simulations
  - simulations show that the final state depends strongly on the initial conditions, in particular on the initial virial ratio $|2T/W|$, collapse factor inversely related to virial ratio.
  - Since $T \sim M\sigma^2$ and $W \sim GM^2/r = MV^2_c$ with V_c the circular velocity at r, smaller values for the initial virial ratio $|2T/W| \sim (\sigma/V_c)^2$ indicate cold initial conditions - these come naturally out of CDM models (MBW pg 257)