**Spheroidal (Elliptical) Galaxies** MBW chap 13, S+G ch 6

- Visual Impression: smooth, roundish- *deceptively* simple appearing-
- **Collisionless systems**
- While visually 'similar', detailed analysis of spheroids shows 3 categories-with a smooth transition as a function of mass
  - Massive/luminous systems: little rotation or cool gas, flat central brightness distribution (cores), triaxial; lots of hot x-ray emitting gas, stars very old, lots of globular clusters, boxy **Low central surface brightness**
  - Intermediate mass/luminosity systems:
    - power law central brightness distribution
      - little cold gas; as mass drops effective rotation increases, oblate, 'disky'
  - Dwarf ellipticals: no rotation,
    - exponential surface brightness

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**Final Exam and Project**

**Final**

**Saturday Dec 16**  1:30 pm – 3:30 pm this room

**Deadline for project** Dec 4
Spheroidal (Elliptical) Galaxies MBW chap 13, S+G ch 6

At M>10^9M_☉ general properties fall on the 'fundamental plane' which includes metallicity, velocity dispersion, size, surface brightness (and some other properties)

- Spiral galaxies bulges, while visually similar are physically different in many ways from E galaxies
- For very extensive review on Morphology see Buta 2013 arxiv 1304.3529
Kormendy Relation

- Strong anti-correlation of scale length ($R_e$) and surface brightness at $R_e$

![Kormendy Relation Graph](image)

Faber Jackson Relation

- Like the Tully-Fisher relation, but for ellipticals
- Strong correlation of luminosity with velocity dispersion

![Faber Jackson Relation Graph](image)
• Roughly, \( L \sim \sigma^4 \)  
  
  \textbf{Faber-Jackson} 

• More luminous galaxies have deeper potentials 

• follows from the Virial Theorem (see derivation of Tuller Fisher, but now use \( \sigma \) instead of \( v_{\text{circular}} \)) 

• Recent scaling relations (Cappellari et al 2006) find \( M = 5R_e \sigma_e^2 / G \)

6 observables are all correlated via the \textbf{fundamental plane} 
Luminosity, Effective radius, Mean surface brightness, 
Velocity dispersion, metallicity, dominance of dispersion over rotation 

The F-P due principally to virial equilibrium 
To first order, the M/L ratios and dynamical structures of ellipticals are very similar : thus the populations, ages & dark matter properties are similar 
There is a weak trend for M/L to increase slightly with Mass 

fundamental plane : measurements of \( \sigma \) and surface brightness profile correlated with \( (M/L) \)

\[ \text{Figure 1. Classic scaling relations. The Faber-Jackson and the Kormendy relations are two special projection of a more fundamental one, aptly named the Fundamental Plane. The three figures are taken from Faber & Jackson (1976), Kormendy (1977) and Djorgovski & Davis (1987) respectively.} \]
Fundamental Plane

Motivations: circa 1985 (Djorgovski)

- How many statistically significant properties describe elliptical galaxies, and how are they related?
  - what is the “manifold of elliptical galaxies”?
- What is the “2nd parameter” in the F-J relation, so that it can be improved as a distance indicator for early-type galaxies (ala T-F relation)

Best Representation

- Remarkably small scatter de Carvalho & Djorgovski 1989

scale length vs combination of velocity dispersion and surface brightness
Meaning

- Derive Fundamental plane scaling exactly as for the T-F relation.
- Galaxies are on a “Virial Theorem Plane” in the space of mass, mean density, and kinetic temperature
  - If galaxies represent a homologous family of structures and had
    \[(M/L) = \text{const.}\]
    then they should follow the VTP:
    \[R \sim \sigma^2 \mu^{0.4}\]
    (\(\mu\) is the surface brightness)

Since they don’t, and the observed FP scaling is: \[R \sim \sigma^{1.4} I^{-0.8}\].
- either one or both of these assumptions must be broken

Virial Theorm and FJ relation

- Potential of a set of point masses, total mass \(M\), inside radius \(R\) is \(U=-3/5(GM^2/R)\)
- \(KE=3/2M\sigma^2\)
- use virial theorem \(2KE+U=0; \ \sigma^2=(1/5)GM/R\)
- if \(M/L\) is constant \(R \sim LG/\sigma^2\)
- \(L=4\pi R^2 I\) (assume for the moment that surface brightness \(I\) is constant)
- \(L \sim 4\pi I(LG/\sigma^2)^2\) and thus \(L \sim \sigma^4\)
- This is the Faber-Jackson relation
Physics of Fundamental Plane (M. Whittle)

\[ <I_e> = \frac{1}{2} L_{tot} / \pi R_e^2 \]
\[ M/R_e = c \sigma_e^2 \quad \text{(virial equilibrium)} \]

- these give:

\[ R_e = (c/2\pi) (M/L)^{-1} \sigma_e^2 <I_e>^{-1} \quad \text{or equivalently,} \]

\[
\log R_e = \log [(c/2\pi)(M/L)^{-1}] + 2 \log \sigma_e - \log <I_e>
\]

or

\[
\log R_e = \log [(c/2\pi)(M/L)^{-1}] + 2 \log \sigma_e + 0.4 <\mu_e>
\]

(since \( <\mu_e> = -2.5 \log <I_e> \))

if \( c \) and \( M/L \) are constants, then

\[ \log R_e = 2 \log \sigma_e + 0.4 <\mu_e> + \log [(c/2\pi)(M/L)^{-1}] \]

- **Which is close to, but not quite, the F-P relation:**

\[ \log R_e = 1.4 \log \sigma_e + 0.36 <\mu_e> \]

- To get agreement,

\[ (M/L) \sim M^{1/5} \sim L^{1/4} \]

The F-P is rooted principally in virial equilibrium

- To first order, the M/L ratios and dynamical structures of massive ellipticals are very similar
- the populations, ages & dark matter properties are highly uniform

There is a weak trend for \( M/L \) to increase slightly with mass (\( \times 3 \) across 5 magnitudes)

The narrow scatter on F-P and Mg relations place limits on the ranges of ages and metallicities:

- Ages \( \sim 10 - 13 \) Gyr; \( Z \sim 2-4 Z(\text{solar}) \)

What Does Fundamental Plane Tell US

- the existence of the FP is due to the galaxies being in virial equilibrium (e.g. Binney & Tremaine 2008) and that the deviation (tilt) of the coefficients from the virial predictions \( R_e = \sigma^2/\Sigma_e \) (\( \Sigma_e \) the stellar surface brightness at \( R_e \)) are due to a smooth variation of mass-to-light ratio M/L with mass
- The FP showed that galaxies assemble via regular processes and that their properties are closely related to their mass.
- The tightness of the plane gives constraints on the variation of stellar population among galaxies of similar characteristics and on their dark matter content
- The regularity also allows one to use the FP to study galaxy evolution, by tracing its variations with redshift.
Fundamental Plane - relate their structural/dynamical status to their stellar content.

Three key observables of elliptical galaxies, effective radius $R_e$, the central velocity dispersion $\sigma$, luminosity $L$ (or equivalently the effective surface brightness $I_e = L/2\pi R_e^2$)

elliptical galaxies are not randomly distributed within the 3D space ($R_e$, $\sigma$, $I_e$), but lie in a plane

The existence of the FP implies that ellipticals
- are virialised systems,
- their structures (e.g., the shape of the mass distribution) vary in a systematic fashion along the plane, and
- contain stellar populations which must fulfill tight age and metallicity constraints.

- Comparison of half light size $R_{1/2}$ to stellar mass for the range of spheroidal systems
  - Notice that properties of bulges of spirals and ellipticals overlap, but at the high mass end there are no bulges.

Remember $R_{1/2}$ from the Sersic model for the surface brightness distribution

see for more details astr553/Topic07/Lecture_7.html
Why Interesting

- The surface brightness profiles are a hint to the formation process.
- Hierarchical clustering implies that different galaxies are the products of different merger histories in which different progenitor morphologies and encounter geometries produced a variety of results.
- It is remarkable that the remnants of such varied mergers shows so much regularity (Kormendy 2009).
- At a given mass, smaller galaxies are redder in colour, have lower fractions of molecular gas, and are more rotationally dominated more massive systems are typically old, metal rich and $\alpha$ enhanced.

There are several simple types of mergers:
- Wet (lots of cold gas) - e.g. spiral x spiral
- Dry (little cold gas) - elliptical x elliptical
- Wet/dry - intermediate amount of gas spiral x elliptical
- Wide range of mass (dwarf into normal)

Fit of Sersic Profile

- Sersic profile for values of $n=0.5, 1, 2, 4, 10$.
- Fit of Sersic profile to 2 elliptical galaxies.
- (figures from Graham 2012)

For $n=4$ (the deV model the total luminosity (S+G problem 6.1) is $7.22pR_e^2 I(R_e)$ and half the light comes from within $R_e$.
Wide Range of Sizes- But Homologous

- the family of spheroids can usually be well fit by the Sersic model, but there are some deviations in the centers (cores and cusps)

More luminous galaxies tend to have cores, less luminous roughly power law shape in central regions

Surface Brightness Distribution of E Galaxies

why is a core a big deal?

- a core is a flattening of the surface brightness profile towards the center
- however theoretical "Cold dark matter only" profiles do not have a core.
  - either a core is a sign that the potential differs from CDM predictions OR
  - the sign of baryonic physics
Systematic Trend in Sersic Parameters with Luminosity

- Higher luminosity, larger "n" is – correlation between spatial structure and luminosity (mass)

Color-Luminosity

- there is a strong relation between the colors and luminosities of ellipticals
- This relation is so good it can be used to identify clusters of galaxies at high z via the 'red sequence'
- the correlation is due primarily to a trend of metallicity with luminosity.
- Small scatter argues for high z formation over a small $\delta z$

Renzini 2006 ARAA- Stellar population diagnostics of elliptical galaxy formation
Special Objects-The most massive systems

- 'cD' (central dominant) galaxies lie only at the centers of groups and clusters- not all brightest cluster galaxies (BCGs) are cDs.
- Their surface brightness profiles are very extended and they often have very rich populations of globular clusters. Quite spheroidal shape.
- X-ray emission in clusters is centered on them.
- Inner portions of the BCGs are formed outside the cluster, but interactions in the heart of the galaxy cluster grow and extend the envelopes of the BCGs.

Deviations from Sersic

- ~10-20% of ellipticals show 'ripples' - indicative of a merger
  (MBW 13.3.5 Merging Signatures)

But such fine structures form only when the merger involves at least one dynamically cold progenitor (disk or dwarf galaxy); mergers between two dynamically hot systems (i.e. between two ellipticals) do not produce shells and ripples, because the intrinsic velocity dispersion is too high.
Shell Formation

- Schweiver (1983) – small galaxy colliding with larger one
- Small galaxy completely tidally disassociated - the stars from that galaxy oscillate independently in the potential well of the new system (dominated by the elliptical) on more or less radial orbits
- They spend most of their time at the apocenters - the shells
- The wrapping in phase space (stars with smaller periods have more oscillations) give the multiple shells (Quinn 1984)

The "Complete" List of Parameters - Kormendy and Bender

- The physically important distinctions between the two varieties of ellipticals
- **Giant ellipticals** ($M_V < -21.5$)
  1. have cores, i.e., central missing light with respect to and inward extrapolation of the outer Sersic profile;
  2. rotate slowly, rotation is unimportant dynamically
  3. are moderately anisotropic and triaxial;
  4. low ellipticity
  5. have boxy-distorted isophotes;
  6. have Sersic (function outer profiles with $n \geq 4$;
  7. mostly are made of very old stars that are enhanced in $\alpha$ elements;
  8. often contain strong radio sources,
  9. have diffuse X-ray-emitting gas, more of it in bigger Es.
Normal and dwarf true ellipticals ($M_V> -21.5$)

(1) *coreless* and have central extra light with respect to an inward extrapolation of the outer Sersic profile (power law profile)
(2) rotate rapidly, rotation is dynamically important to their structure
(3) are nearly isotropic and oblate spheroidal,
(4) are flatter than giant ellipticals (ellipticity ~0.3);
(5) have diskly-distorted isophotes;
(6) have Sersic function outer profiles with $n < 4$;
(7) are made of younger (but still old) stars with only modest or no $\alpha$ element enhancement;
(8) rarely contain strong radio sources, and
(9) rarely contain X-ray-emitting gas.

Ellipticals - Shape

- What does 'roundish' mean
  - Oblate, prolate, triaxial
Old ideas: “Images have complete rotational symmetry – figures of revolution with two equal principal axes. The third, the axis of rotation, is smaller than the other two.” (Sandage) i.e. oblate spheroids, rotating about axis of symmetry

SURFACE PHOTOMETRY AND THE STRUCTURE OF ELLIPTICAL GALAXIES
John Kormendy,S. Djorgovski
Oblate, Prolate Triaxial

- **Oblate**: rotationally symmetric ellipsoid having a polar axis shorter than the diameter of the equatorial diameters-formed by rotating an ellipse about its minor axis.
- **Prolate**: a rotationally symmetric ellipsoid spheroid in which the polar axis is greater than the equatorial diameter.

Ellipticals - Shape

- Shape alone cannot tell us what is going on.
- Triaxial ellipsoids:
  \[
  \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1
  \]
- From morphology alone can't tell if elliptical galaxies are:
  1. spherical $a=b=c$
  2. prolate $a>b=c$ (rugby ball)
  3. oblate $a=b>c$ (smartie)
  4. triaxial $a>b>c$
Shapes

Oblate  Prolate  Triaxial

Ellipticals Shape

So an observer looking along the z axis would see an E0 (round) galaxy, when viewed at an angle you would see an elliptical shape with apparent axis ratio q = b/a. Looking at the tangent point to the elliptical surface (T) the coordinates of this point are

\[ \tan i = \frac{\Delta z}{\Delta x} = -\left(\frac{\Delta z}{\Delta x}\right) \frac{\Delta y}{\Delta z} \]

If elliptical galaxies are oblate spheroids then

\[ \rho(A) = \rho(m^2) \text{ where } m^2 = \frac{x^2 + y^2}{B^2} = \frac{z^2}{B^2} \text{ with } A \geq B > 0 \]

Triaxiality \( r(m) = x^2 + y^2/p^2 + z^2/q^2 \)

D. Davis

Distribution of B/A

Looking from a random direction what fraction of galaxies do we see between \( i \) and \( i + \Delta i \)? It’s just \( \sin(i) \) \( \Delta i \)

So if all galaxies have an axial ratio of \( B/A \), then the fraction with apparent ratios between \( q \) and \( q + \Delta q \) is

\[ f_{\Delta q}(q) = \frac{\sin(i) \Delta q}{dq/di} = \frac{q \Delta q}{\sqrt{1 - (B/A)^2} \sqrt{q^2 - (B/A)^2}} \]

For very flattened systems, \( B << A \) the distribution is almost uniform.
If $q$ is the ratio of the minor to the major axis then

$$q_{obl} = \frac{b}{a} = OQ \sin(i) = \frac{B^2 m}{m A} \sin(i) = \left[ \frac{B^2}{A^2} + \cot^2(i) \right]^{1/2} \sin(i)$$

Using our definition of $m$ for the last step, finally we can rewrite this as

$$q_{obl}^2 = (b/a)^2 = (B/A)^2 \sin^2(i) + \cos^2(i)$$

For an oblate spheroid we can do all this again and get

$$q_{prol}^2 = (b/a)^2 = \left[ (B/A)^2 \sin^2(i) + \cos^2(i) \right]^{-1}$$

**Ellipticals are Triaxial**

- No selection of oblate spheroids can give the observed distribution
- These galaxies must be triaxial

Shape could also be due to rotation around $z$ axis.

Axial ratios for galaxies fit with de Vaucouleurs profiles (Khairul Alam & Ryden 2002).
The shapes of Quiescent Galaxies

SDSS study of shape distribution of ‘passive’ (~early type) galaxies: (van der Wel et al 2009)

At $M<10^{11}M_{\odot}$ there is a wide range of axial ratios (disks/highly flattened systems)

At high mass systems more uniform

The gray scale represents, normalized to the total number of galaxies in narrow bins of stellar mass, the fraction of galaxies with axial ratio $b/a$

- $I(R)$ is the projected luminosity surface brightness, $j(r)$ is the 3-D luminosity density (circular images- if image is elliptical no general solution)

$$I(R) = \int_{-\infty}^{\infty} j(r) \, dz = 2 \int_{R}^{\infty} \frac{j(r) \, r \, dr}{\sqrt{r^2 - R^2}}$$

$$j(r) = -\frac{1}{\pi} \int_{R}^{\infty} (dI/dR) dR/sqrt(R^2 - r^2)$$

this is an Abel integral which has only a few analytic solutions for $j(r)/I(r)$ pairs

Simple power law models $I(R) = r^{-\alpha}$ then $j(r) = r^{-\alpha - 1}$

While the Sersic model is a better fit to the surface brightness profiles it is not easily invertable to density-often use a generalized King profile with surface brightness $I(r) = I(0)(1+(r/r_c)^2)^{-3/2}$ which gives a density law $\rho(r) = \rho(0)(1+\frac{r}{r_c})^{3/2}$ where $r_c = 3\sigma/sqrt(4\pi G \rho_c)$
Ages of Elliptical Galaxies

- Using optical spectra there is an age-metallicity degeneracy in old pops
- This can be broken (to some extent) via use of IR data and by measuring galaxies at higher redshifts
- Analysis (van Dokkum and van der Maaerl 2007) indicates consistency with 'passive' evolution (no star formation for a long time) and a formation redshift ~2 (depends on the IMF) for the stars - not clear when the galaxies formed
  - theory/observations indicate that ellipticals probably formed from mergers and thus the age of the galaxy and the stars may differ.

Solid line is Saltpeter-IMF passive evolution formation at z=2
Dotted line- top heavy IMF $z_{\text{form}}=4$

Evolution of Elliptical Galaxies

- 'age date' the galaxies with higher redshift observations
- The evolution with redshift of the $M_*/L_B$ ratio of simple stellar populations of solar metallicity and various initial mass function slopes and formation redshifts:

![Graph showing evolution of $M_*/L_B$ ratio with redshift for different initial mass function slopes and formation redshifts.](image)
Higher z observations constraint on origin

- At higher z massive elliptical galaxies in clusters have colors and luminosities (at z<1.2) consistent with 'passive' evolution e.g. galaxy forms at higher z and does not change with time and stars 'just evolve'- a SSP (!)

- Consistency of the colors of these galaxies with 'passive' evolution the ages of massive ellipticals in clusters is ~10-13Gyr (!).


Growth of Elliptical Galaxies

- Massive elliptical galaxies had lots of star formation at high (z>1.5) redshift but ~ stopped forming stars at more recent times

- Growth in E galaxy mass z<2 has been primarily via mergers- this is also consistent with chemical abundance gradients

van Dokkum et al 2010
• Renzini 2006 arxiv 0603479, theoretical change in $M/L_B$ vs formation redshift and IMF slope vs data- notice small change in the main sequence turn off – black points are data.

Elliptical Galaxies So Far

• Visual Impression: smooth, roundish-deceptively simple appearing- collisionless systems
• Galaxies are very old
• Strong correlations of many properties: size, surface brightness, metallicity, velocity dispersion, color, luminosity
• Effect of viewing geometry on shape, projection effect - inversion of surface brightness profiles to density (Abel integral, in general non-analytic)
• Surface brightness profiles fit by 'Sersic' law, 3 free parameters (n, I(0), $R_e$)
• See chapter 13 in MBW for lots of information!