1.3

Betelgeuse

T_{eff} 3500K

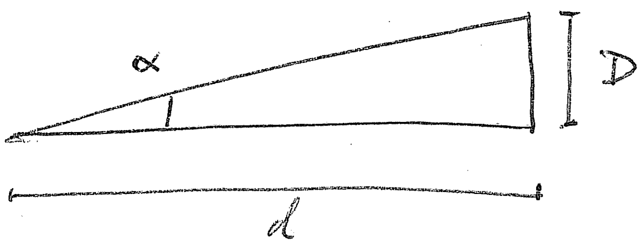
α 0.045''

d 140 pc

Sun

$R_\odot = 6.955 \times 10^8$ m

$T_\odot = 5780$ K.



Small angle approx.:

$\sin\alpha = \tan\alpha = \alpha = \frac{D}{d}$.

Definition of the parsec:

$$1'' = \frac{1\ \text{AU}}{1\ \text{pc}}$$

$$\rightarrow \quad \frac{D/\text{AU}}{d/\text{pc}} = \alpha / \text{arcsec}.$$

(a) $R_{Betelgeuse} = \frac{1}{2}(140)(0.045)$ AU

$= 4.7 \times 10^{11}$ m $\approx 680 R_\odot$.

(b) $L = 4\pi R^2 \sigma T_{eff}^4$ Stefan-Boltzmann.

$$\frac{L_{Betelgeuse}}{L_\odot} = \left(\frac{R_{Betelgeuse}}{R_\odot}\right)^2 \left(\frac{T_{Betelgeuse}}{T_\odot}\right)^4 = 680^2 \cdot \left(\frac{3500}{5780}\right)^4$$

$$= 6.2 \times 10^4$$

$\log(62000) = 4.8 \approx 5$.

1.8

$$m - M = 5\log_{10}\left(\frac{d}{10pc}\right)$$

distance modulus.

comes from

$$\begin{cases} m = -2.5\log_{10}(\text{Flux}) \\ M = -2.5\log_{10}(\text{Flux at } 10pc) \end{cases}$$

a) $M_V = V - 5\log_{10}\left(\frac{140}{10}\right)$

$= 0 - 5 \times 1.146 = \underline{-5.731}$

b) $V - K \simeq 5 \Rightarrow M_V - M_K \simeq 5$ as well.

$\Rightarrow \underline{M_K = -10.731}$

(straightforward result from distance modulus definition)

c) Comparing magnitudes of objects at the <u>same</u> distance:

$M_1 - M_2 = -2.5\log_{10}\left(\frac{L_1}{L_2}\right)$ (distance cancel) both 10pc.

$\Rightarrow \frac{L_1}{L_2} = 10^{(M_2 - M_1)/2.5}$ use this for the next few parts.

Solar $\begin{cases} M_{V\odot} = 4.83 \\ M_{K\odot} = 3.31 \end{cases}$

$\frac{L_V}{L_{V\odot}} = 10^{(M_{V\odot} - M_V)/2.5}$

$= 10^{(4.83 + 5.73)/2.5}$ $\therefore \underline{\frac{L_V}{L_{V\odot}} = 1.7 \times 10^4}$

d) $$\frac{L_K}{L_{K\odot}} = 10^{(M_{K\odot} - M_K)/2.5}$$

$$= 10^{(3.31 + (0.731))/2.5}$$

$$\therefore \frac{L_K}{L_{K\odot}} = 4.1 \times 10^5$$

e) Look up a bolometric correction for Betelgeuse (spectral type M2)

$BC \approx 2$

$$M_{bol} = M_V - BC$$

$$= -5.73 - 2.7$$

$$\approx -8.$$

If you use any value between 1 and 3 and justified it somehow (and followed through in next part) you are okay.

f) $M_{bol,\odot} = 4.75.$

$$\frac{L_{bol}}{L_{bol,\odot}} = 10^{(M_{bol,\odot} - M_{bol})/2.5}$$

$$= 10^{(4.75+8)/2.5} = 1.3 \times 10^5$$

g) L_{bol} is the total emitted power.

$$L_\odot = 3.84 \times 10^{26}\ W$$

$$\Rightarrow \quad L_{bol} = 4.96 \times 10^{31}\ W$$

1.4

$L_\odot = 3.839 \times 10^{26}$ W.

This is to be supplied by nuclear burning:

$E = mc^2 \Rightarrow$ energy release $= c^2$ J/kg.

$\Rightarrow$ mass conversion rate $= \dfrac{L_\odot}{c^2 \text{ J/kg}}$

$= \dfrac{3.839 \times 10^{26}}{(3 \times 10^8)^2}$ kg/s

$(= 4.3 \times 10^9$ kg/s$)$

H burning efficiency $= 0.007$

$\Rightarrow \dfrac{4.3 \times 10^9}{0.007}$ kg/s $= 6.1 \times 10^{11}$ kg/s.

Initial mass of H in Sun $\approx$ same composition as surface now, where fusion does not take place.

$X \simeq 0.72$.

$\Rightarrow \tau \approx \dfrac{X M_\odot}{\text{rate}} = \dfrac{0.72 \times 2 \times 10^{30} \text{ kg}}{6.1 \times 10^{11} \text{ kg/s}} = 2.4 \times 10^{18}$ s

≈ 75 Gyr

1.12

Use $L = 4\pi R^2 \sigma T^4$ Stefan-Boltzmann.

Given
$$\begin{cases} L_{mw} = 2\times10^{10} L_\odot \\ R_{mw} = 5 \text{ kpc} \end{cases}$$

Plug into S-B and compare to $T_\odot$:

$$\frac{T_{mw}}{T_\odot} = \left[\frac{L_{mw}/R_{mw}^2}{L_\odot/R_\odot^2}\right]^{1/4} = \left(\frac{L_{mw}}{L_\odot}\right)^{1/4}\left(\frac{R_\odot}{R_{mw}}\right)^{1/2}$$

$$= (2\times10^{10})^{1/4}(4.51\times10^{-12})^{1/2}$$

$$= 0.0008$$

$$T_\odot = 5780 \text{ K}$$

$$\Rightarrow \underline{T_{mw} = 4.6 \text{ K}}$$

(4)

- Stellar contribution to galaxy's light depends on its luminosity. The most luminous stars dominate.

- Age of a stellar population is a luminosity weighted average. Young stellar pop., ongoing star formation and presence of massive, hot, luminous O, B stars go hand in hand (since they have short lives).
- In a young population, light from the hot O, B stars dominate as they are very luminous. The galaxy is relatively blue.
- In an aged population, massive stars have burnt out. Starting from the high mass end of MS, stars turn off the ZAMS by moving to the red on the HR diagram, also ascending in luminosity. The stars are mainly low mass stars still on the MS + more massive ones advancing toward RG. The latter now dominate. Galaxy appears more red. (Gained luminous cool stars, lost luminous hot stars.)
- Special exceptions such as blue stragglers are rare.

(5)

	Ellipticals	Spirals
Morphological:	Ellipsoidal. Smooth projected intensity profile.	Flat disk + central bulge. Discernable arms, has halo.
Observational:	Redder, old stellar pop. Metal-enriched pop II stars. Very little star formation (quiescent). Very little cold gas; has hot and warm gas not suitable for star formation.	Central bulge similar to ellipticals but spirals have dense regions, cold gas, star formation. Looks more blue. Pop. I metal rich.
	Typically found in dense regions of universe eg. centre of galaxy clusters.	Found in less dense regions.
Dynamical:	Stars typically on circular orbits, random thermalised motion, small amount of bulk rotation.	Stars mostly in plane of disk, bulk rotation, smaller velocity dispersion.

Not ~~exates~~ exhaustive!

Min. one comparison from each category.

The more valid points you have the better.