Homework 2 Astro421 Due Thursday Sept 27
S+G

1) The first half of problem 2.4 (e.g. Do NOT do the part after the sentence starting with "For an accelerating..."

2) The first half of problem 2.5 stop at the part of the problem starting with 'Taking M_l=0.3..."

3) In a short essay describe the 'ecology' of the ISM in spirals. E.g.
What processes heat the gas and cool the gas.
What are the observable signatures of the phases of the ISM

4) What is the fundamental difference twixt the ISM in spirals and ellipticals? How do we know?

Problem 2.4 Suppose that stars are born at a constant rate. Assuming $\tau_{gal} =$ 10 Gyr and using Table 1.1 for stellar lifetimes, show that only 11% of all the $2\mathcal{M}_\odot$ stars ever made are still on the main sequence today. What fraction of all the $3\mathcal{M}_\odot$ stars are still there? What fraction of all the $0.5\mathcal{M}_\odot$ stars? Now suppose that star formation slows with time t as $e^{-t/t_\star}$, with $t_\star = 3$ Gyr. Show that now only 1.6% of all $2\mathcal{M}_\odot$ stars survive, and merely 0.46% of stars of $3\mathcal{M}_\odot$.

For these stars, explain why $\Psi(M_V)$ is larger for a given observed $\Phi(M_V)$ when starbirth declines with time (the ★ and • points in Figure 2.4 must be further apart) than if it stays constant. How much larger must $\Psi(M_V)/\Phi(M_V)$ become for stars of $2\mathcal{M}_\odot$? How would a gradual slowdown change the inferred $\Psi(M_V)$ for stars longer-lived than the Sun?

Problem 2.5 Suppose that Equation 2.5 describes stars formed within a 100 pc cube with masses between $\mathcal{M}_\ell$ and an upper limit $\mathcal{M}_u \gg \mathcal{M}_\ell$. Write down and solve the integrals that give (a) the number of stars, (b) their total mass, and (c) the total luminosity, assuming that Equation 1.6 holds with $\alpha \approx 3.5$. Show that the number and mass of stars depend mainly on the mass $\mathcal{M}_\ell$ of the smallest stars, while the luminosity depends on $\mathcal{M}_u$, the mass of the largest stars.

$$R \sim R_\odot \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{0.7}, \quad L \sim L_\odot \left(\frac{\mathcal{M}}{\mathcal{M}_\odot}\right)^{\alpha} \qquad (1.6)$$

Salpeter initial mass function: number of stars born with mass between $\mathcal{M}$ and $\mathcal{M} + \Delta\mathcal{M}$:

$$\xi(\mathcal{M})\Delta\mathcal{M} = \xi_0 (\mathcal{M}/\mathcal{M}_\odot)^{-2.35} (\Delta\mathcal{M}/\mathcal{M}_\odot) \qquad (2.5)$$

where ξ_0 is the local stellar density.

Table 1.1 Stellar models with solar abundance, from Figure 1.4

Mass ($\mathcal{M}_\odot$)	L_{ZAMS} ($L_\odot$)	T_{eff} (K)	*Spectral type*	τ_{MS} (Myr)	τ_{red} (Myr)	$\int(L\,d\tau)_{MS}$ (Gyr $\times L_\odot$)	$\int(L\,d\tau)_{pMS}$ (Gyr $\times L_\odot$)
0.8	0.24	4860	K2	25 000		10	
1.0	0.69	5640	G5	9800	3200	10.8	24
1.25	2.1	6430		3900	1650	11.7	38
1.5	4.7	7110	F3	2700	900	16.2	13
2	16	9080	A2	1100	320	22.0	18
3	81	12 250	B7	350	86	38.5	19
5	550	17 180	B4	94	14	75.2	23
9	4100	25 150		26	1.7	169	40
15	20 000	31 050		12	1.1	360	67
25	79 000	37 930		6.4	0.64	768	145
40	240 000	43 650	O5	4.3	0.47	1500	112
60	530 000	48 190		3.4	0.43	2550	9
85	1 000 000	50 700		2.8		3900	
120	1 800 000	53 330		2.6		5200	

Note: L and T_{eff} are for the zero-age main sequence; spectral types are from Table 1.3; τ_{MS} is main-sequence life; τ_{red} is time spent later as a red star ($T_{eff} \lesssim 6000$ K); integrals give energy output on the main sequence (MS), and in later stages (pMS).

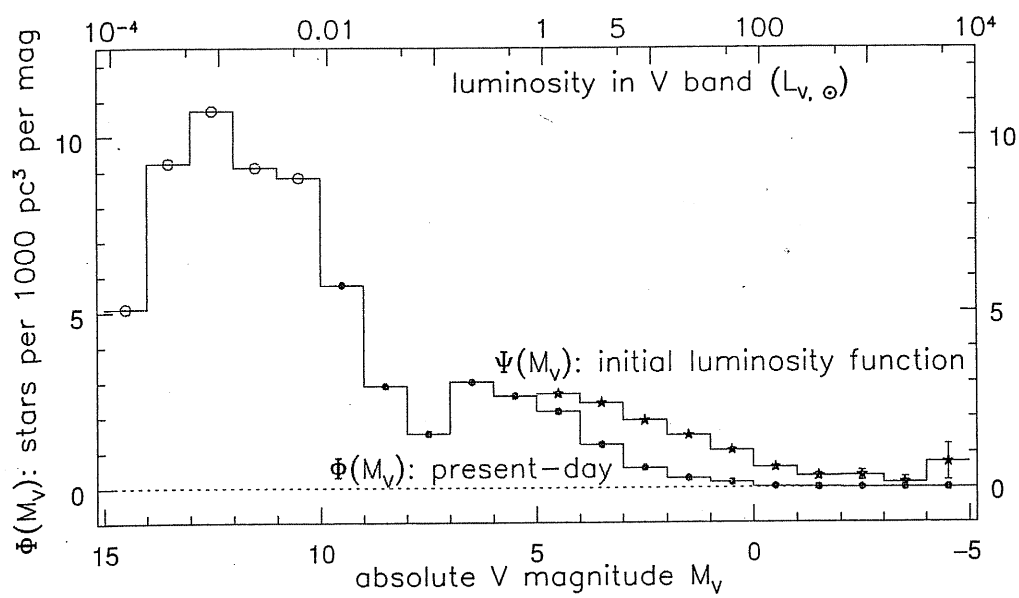
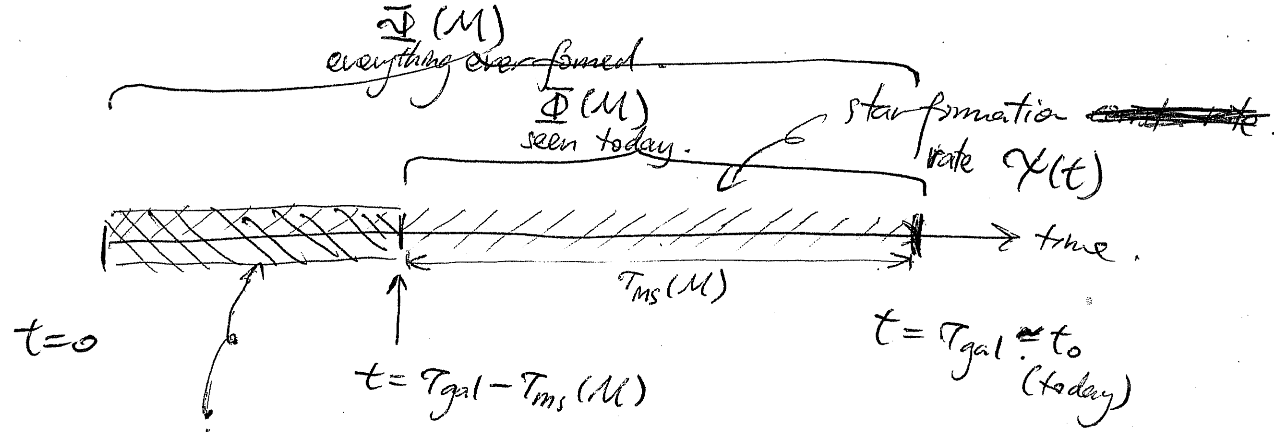


Fig. 2.4. Circles show the luminosity function $\Phi_{MS}(M_V)$ for main-sequence stars as in Figure 2.3. The histogram gives the initial luminosity function $\Psi(M_V)$, assuming that stars were born at a constant rate over the past 10 Gyr. Both functions have a minimum, the *Wielen dip*, at $M_V \approx 8$. This V-band luminosity corresponds to only a tiny range of stellar mass $\mathcal{M}$. The mass function $\xi(\mathcal{M})$ probably has no dip or inflection at this mass.



stars formed longer than $\tau_{ms}(\mathcal{M})$ ago can no longer be seen.

To get $\Psi(\mathcal{M})$ from $\Phi(\mathcal{M})$ ⊛ need to know the SF history $\psi(t)$.

[MBW §9.6.1]

$$\Phi(\mathcal{M}) = \int_{t_0-\tau_{ms}(\mathcal{M})}^{t_0} \psi(t)\,dt$$ → can be measured w/o knowing $\psi(t)$ since we see these today.

$$\Psi(\mathcal{M}) = \int_0^{t_0} \psi(t)\,dt.$$

$$= \Phi(\mathcal{M}) + \underbrace{\int_0^{t_0-\tau_{ms}(\mathcal{M})} \psi(t)\,dt}$$

off MS, probably can't see them, as stars evolve quickly after MS. This part can't be observed directly, so must guess $\psi(t)$.

- If $t_0 < \tau_{ms}$ then we can see all stars that ever formed, in which case $\Psi(\mathcal{M}) = \Phi(\mathcal{M})$.

- Suppose we know $\psi(t)$ then it is easy to relate the initial luminosity function to what we see today (just divide the two)

$$\Psi(\mathcal{M}) = \Phi(\mathcal{M}) \frac{\int_0^{t_0} \psi(t)\,dt}{\int_{t_0-\tau_{ms}(\mathcal{M})}^{t_0} \psi(t)\,dt}.$$

- Note that initial luminosity function means the <u>total initial luminosity (ie. as MS stars) of all stars formed</u> (as function of $\mathcal{M}$, say). It sounds like initial total luminosity, but it is not to be confused with that.

⊛ Caveat: <u>IMF must be time-independent</u>, or the logic would not be sound. Think about it!