Galaxies | ASTR421
Prof. Richard Mushotzky | Fall 2012

Homework 3
Due Oct 11

1. *S&G Problem 2.8*

The surface brightness is as seen by an external observer located far from the disk perpendicular to the suns position (e.g. ignore galactic reddening). [10 marks]

2. *S&G Problem 2.13*

“Uniform sphere of stars” means a constant density sphere of stars. For the IMF (eq 2.5) the problem asks you to consider the case for which there are no stars below 0.2 solar masses. The last section (it says 'for advanced students') is for extra credit. [20 + 5 marks]

3. *S&G Problem 2.20* [15 marks]

4. *S&G Problem 3.19*

Comment on the physical significance of the result. Does it make sense? [15 marks]

5. *Obtaining the Initial Mass Function*

Write an essay about the IMF. What are the observational and theoretical difficulties in determining the IMF, and how does one go about trying to resolve them? Include discussion of converting light to mass (and uncertainties), effects of age and distance, etc. [15 marks]

6. *A dusty question*

Write an essay answering the following questions. Why does a fair fraction of a galaxy’s luminosity appear in the IR? What is the source of IR photons? What does this tell us about star formation? Extra credit: why is observing in the far IR exciting/important for galaxy evolution studies? [15 + 5 marks]

Problem 2.8 By integrating Equation 2.8, show that at radius R the number of stars per unit area (the surface density) of type S is $\Sigma(R,S) = 2n(0,0,S)h_z(S) \exp[-R/h_R(S)]$. If each has luminosity $L(S)$, the surface brightness $I(R,S) = L(S)\Sigma(R,S)$. Assuming that h_R and h_z are the same for all types of star, show that the disk's total luminosity $L_D = 2\pi I(R=0)h_R^2$.

For the Milky Way, taking $L_D = 1.5 \times 10^{10} L_\odot$ in the V band and $h_R =$ 4 kpc, show that the disk's surface brightness at the Sun's position 8 kpc from the center is $\sim 20 L_\odot\,\mathrm{pc}^{-2}$. We will see in Section 3.4 that the mass density in the disk is about $(40\text{–}60)\mathcal{M}_\odot\,\mathrm{pc}^{-2}$, so we have $\mathcal{M}/L_V \sim 2\text{–}3$. Why is this larger than $\mathcal{M}/L_V$ for stars within 100 pc of the Sun? (Which stars are found only close to the midplane?)

Problem 2.13 Here we make a crude model to estimate how many stars you could see with your unaided eye, if you observed from the center of the Galaxy. Naked-eye stars are those brighter than apparent magnitude $m_V \approx 5$; from Earth, we see about 7000 of them. Assume that the Milky Way's nucleus is a uniform sphere of stars with radius 3 pc, and ignore the dimming effects of dust. What is the luminosity L_{eye} of a star that is seen 3 pc away with $m_V = 5$? For a main-sequence star, use Equation 1.6 to show that L_{eye} corresponds to $\mathcal{M} \approx 0.6\mathcal{M}_\odot$.

In our simple model, almost all stars that spend less than 3 Gyr on the main sequence have now died; according to Table 1.1, what stellar mass $\mathcal{M}_u$ does this correspond to? Approximate the number $\xi(\mathcal{M})\Delta\mathcal{M}$ of main-sequence stars with masses between $\mathcal{M}$ and $\mathcal{M} + \Delta\mathcal{M}$ by Equation 2.5: $\xi(\mathcal{M}) \propto \mathcal{M}^{-2.35}$ for $\mathcal{M} \gtrsim 0.2\mathcal{M}_\odot$, with few stars of lower mass. Find the total number and total mass of main-sequence stars with $\mathcal{M} < \mathcal{M}_u$, in terms of the parameter ξ_0. How do we know that red giants will contribute little mass? Taking the total mass as $10^7\mathcal{M}_\odot$, find ξ_0; show that the nucleus contains $N_{\text{eye}} \sim 4 \times 10^6$ main-sequence stars with $L \geq L_{\text{eye}}$. How do we know that many fewer red giants will be visible? (For advanced students: stars with $L < L_{\text{eye}}$ will be seen as naked-eye stars if they are close enough to the observer. Show that these make little difference to the total.)

Problem 2.20 Consider the spherical density distribution $\rho_H(r)$ with

$$4\pi G\rho_H(r) = \frac{V_H^2}{r^2 + a_H^2}, \tag{2.19}$$

where V_H and a_H are constants; what is the mass $\mathcal{M}(<r)$ contained within radius r? Use Equation 2.18 to show that the speed $V(r)$ of a circular orbit at radius r is given by

$$V^2(r) = V_H^2[1 - (a_H/r)\arctan(r/a_H)], \tag{2.20}$$

and sketch $V(r)$ as a function of radius. This density law is often used to represent the mass of a galaxy's dark halo – why?

Problem 3.19 With the temperature T defined in Equation 3.59, find the kinetic energy of a system with N stars each of mass m, and use the virial theorem to show that its energy $\mathcal{E}$ satisfies

$$\frac{d\mathcal{E}}{dT} = -\frac{3}{2}Nk_B < 0\ (!) \tag{3.63}$$

The specific heat of a gravitating system is negative – removing energy makes it hotter. (As a mundane example, think of an orbiting satellite subject to the frictional drag of the Earth's atmosphere; as it loses energy, the orbit shrinks, and its speed increases.)

$$n(R,z,S) = n(0,0,S)\exp\left(-\frac{R}{h_R(S)}\right)\exp\left(-\frac{|z|}{h_z(S)}\right) \tag{2.8}$$

$$\mathcal{M}(<R) = \frac{RV^2}{G} \tag{2.18}$$

$$\frac{1}{2}m\langle v^2(x)\rangle = \frac{3}{2}k_B T \tag{3.59}$$