Qian Wang.

Galaxies Homework 4

ASTR 421 — Fall 2012

Prof Richard Mushotzky

$\boxed{1}$ S&G Problem 4.2.

Virial theorem : $2T + V = 0$

$$\Rightarrow 2\left\langle \sum_i \frac{1}{2} m_i v_i^2 \right\rangle - \frac{GM^2}{R} = 0.$$

Clusters and dwarf galaxies most stars quite old and about the same mass; supposing that then

$$M\langle v^2\rangle - \frac{GM^2}{R} = 0$$

$$\langle v^2\rangle = \frac{GM}{R}$$

$$\Rightarrow M \propto \langle v^2\rangle R$$

If $\langle v^2\rangle$ is only $\left(\frac{1}{3}\right)^2$ times, R is 50 times, then

$$M_{Carina} = \frac{1}{9} \times 50\ M_{\omega Centauri}$$

$$5.6 \approx 6.$$

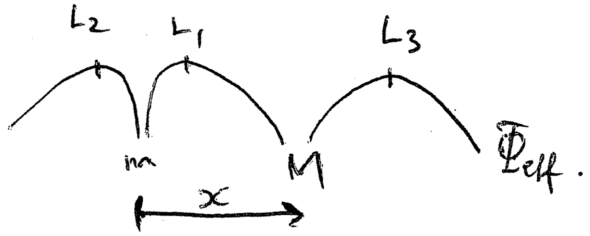
Table 4.2 $\quad \omega$ Cen gc $\quad L_V = 0.1 \times 10^7 L_\odot$

Carina dSph $\quad L_V = 0.04 \times 10^7 L_\odot$

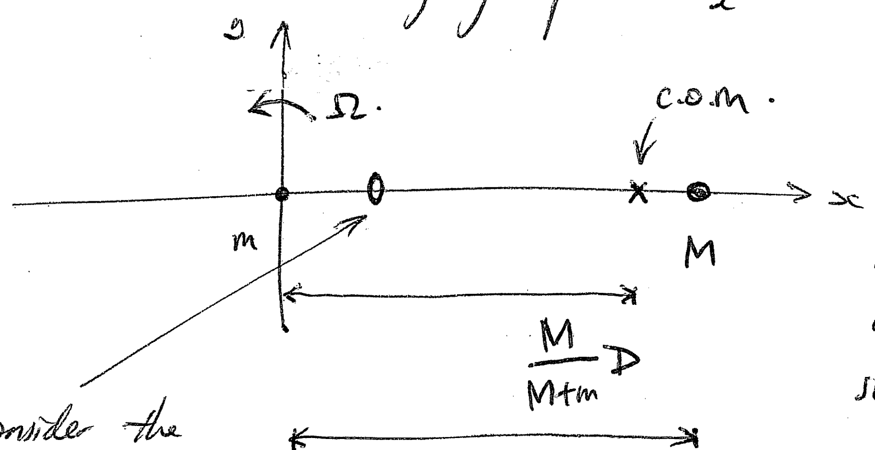
$$\frac{M/L\ (\text{Carina})}{M/L\ (\omega\text{Cen})} = \frac{5.6}{0.04/0.1} = 14.$$

[2] S&G Problem 4.5.

Seeking $\frac{\partial \Phi_{eff}}{\partial x} = 0$ is the same as requiring zero net force in the x direction in the rotating frame. The potential looks like



so at those three Lagrange points $F_x = 0$.



In the two body problem the mutual force between m and M determine the dynamics; so here $M = M(<D)$.

consider the forces at a position x here. (x is small).

$$F_x = \underbrace{\mp \frac{Gm}{x^2}}_{\text{attracted to } m} - \underbrace{\Omega^2\left(\frac{DM}{M+m} - x\right)}_{\text{centrifugal force}} + \underbrace{\frac{GM(D-x)}{(D-x)^2}}_{\text{attracted to } M.}$$

attracted to m: $-$ve if $x>0$, $+$ve if $x<0$.

centrifugal force acts to repel it from the c.o.m.

attracted to M.

Ω^2 is the rotation speed of the two-body system so it is just given by Kepler's 3rd laws.

$$\Omega^2 = \frac{G(M+m)}{D^3}.$$

The mass that attracts towards the halo changes with x, because it is an extended halo. Eq. 2.20 gives

$$V^2 = V_H^2\left(1 - \frac{a_H}{r}\arctan\left(\frac{r}{a_H}\right)\right) = \ddot{r}r = \frac{GM(<r)}{r^2}r$$

$$\Rightarrow M(<r) = \frac{rV_H^2}{G}\left(1 - \frac{a_H}{r}\arctan\left(\frac{r}{a_H}\right)\right).$$

$$\approx \frac{rV_H^2}{G}, \quad r \gg a_H.$$

For this problem, $r = D - x$. Hence the last term in F_x becomes

$$\frac{G}{(D-x)^2}\frac{(D-x)}{G}V_H^2.$$

V_H can be replaced by M and D by solving for

$$M = M(<D) = \frac{DV_H^2}{G} \Rightarrow V_H^2 = \frac{MG}{D}.$$

$$\Rightarrow F_x = \mp\frac{Gm}{x^2} - \frac{G(M+m)}{D^3}\left(\frac{DM}{M+m} - x\right) + \frac{GM}{D(D-x)}$$

$$= \mp\frac{Gm}{x^2} - \frac{GM}{D^2} + \frac{G(M+m)x}{D^3} + \frac{GM}{D^2}\left(1-\frac{x}{D}\right)^{-1}$$

Expand last term for small $\frac{x}{D} \ll 1$, and do garbage cleaning.

$$F_x = \mp \frac{Gm}{x^2} - \cancel{\frac{GM}{D^2}} + \underbrace{\frac{GM}{D^2}\left(\frac{x}{D}\right) + \frac{Gm}{D^2}\left(\frac{x}{D}\right) + \cancel{\frac{GM}{D^2}} + \frac{GM}{D^2}\left(\frac{x}{D}\right)}_{\frac{G}{D^2}(2M+m)\left(\frac{x}{D}\right)} = 0 \,.$$

$$\times \frac{x^2}{G} \Rightarrow \mp m + \left(\frac{x}{D}\right)^3 (2M+m) = 0 \,.$$

$$\Rightarrow \left(\frac{x}{D}\right) = \pm\left(\frac{m}{2M+m}\right)^{\frac{1}{3}} \doteq \pm r_J / D.$$

For $m \ll M$, $r_J = D\left(\frac{m}{2M}\right)^{\frac{1}{3}}$.

$\boxed{3}$

— nearby galaxies we can resolve stars.

— study individual stellar spectra

— find local IMF; chemical evolution studies.

— distant objects we can't resolve individual stars for and must rely on M/L scaling laws learned locally.

— must consider universality of things we learn locally.

— parallax first step of cosmic distance ladder.

$\boxed{4}$ Compare properties such as morphological differences, (spiral arm structure, disk scale height, disk size) dynamics (rotation), ISM distribution, mass, luminosity, dynamical interactions in the past (merger, tidal disruption) etc.

$\boxed{5}$ (a) disk is rotationally supported, multiphase ISM, bulk motion; bulge supported by dynamical potential (similar to ellipticals)

(b) halo dominated by dark matter; old stars found here; huge extent, physically.

(6)

6 Going from S0 → Sd the trends are:

- decrease in mass, decrease in luminosity
- more blue; B−V decreases.
- M/L decreases — stars are younger
- spiral arms wound less tightly.
- bulge / disk light ratio decreases.
- HI / total mass [redacted] increases

7 (a) $\Phi(L)\,dL$ gives number density of galaxies between $L-\frac{dL}{2}$ and $L+\frac{dL}{2}$.

(b) Schechter luminosity function ← power law index shapes curve at faint end.

$$\Phi(L)\,dL = \underbrace{\frac{n_*}{L_*}}_{\text{normalisation constant.}}\left(\frac{L}{L_*}\right)^{\alpha}\exp\left(-\frac{L}{L_*}\right)dL .$$

characteristic luminosity.

(c) Ellipticals dominate at high luminosity while spirals dominate at lower luminosities. Their sum is described by the Schechter luminosity function.

8 (a) Sersic profile.

$$\Sigma(R) = \Sigma_0 \exp\left[-K\left(\left(\frac{R}{R_0}\right)^{\frac{1}{n}} - 1\right)\right].$$

(b) Disk — exponential profile.

$$\Sigma \propto \exp(-R/R_d) \quad , \quad R_d \sim \text{disk radius.}$$

Spheroids (ellipticals and bulge)

$$\Sigma \propto \exp\left(-7.67\left[\left(\frac{R}{R_0}\right)^{\frac{1}{4}} - 1\right]\right) \quad \text{deVaucouleur's profile.}$$

(c) The scale length is a parameter in the exponential function above which is ~ size of the system, since Σ drops by a factor of e every multiple of the scale length.

9 (a) Spider diagram is a contour map of the measured radial velocity (relative to observer, not radial to the galaxy). If we assume the disk is circular and appears elliptical because of projection, we can deduce the inclination and velocity field in the plane.

(b) Tully-Fisher relation is an empirical relation between the intrinsic luminosity and the amplitude of the rotation curve of spiral galaxies.
Since the rotation curve is determined by gravitating mass, most of which made up of non-luminous matter, and the luminosity is due to stars, the relationship suggests some correlation between them.

$\boxed{10}$ If stars were rotating in the plane of the disk in Keplerian orbits, differential rotation will cause the inner portion to rotate faster, making the arms more tightly wound. The characteristic timescale for this to destroy structures is much shorter than the lifetime of the galaxies we see these features in. So the spiral arms cannot be static features; they appear to be dynamically formed over-dense regions.