# Modeling the Panchromatic Spectral Energy Distributions of Galaxies

CHARLIE CONROY

Department of Astronomy and Astrophysics, University of California, Santa Cruz, California, 95064; email: cconroy@ucolick.org

## **Key Words**

stars, abundances, dust, stellar populations, galaxies: stellar content, galaxy evolution

#### Abstract

The spectral energy distributions (SEDs) of galaxies are shaped by nearly every physical property of the system, including the star formation history, metal content, abundance pattern, dust mass, grain size distribution, star-dust geometry, and interstellar radiation field. The principal goal of stellar population synthesis (SPS) is to extract these variables from observed SEDs. In this review I provide an overview of the SPS technique and discuss what can be reliably measured from galaxy SEDs. Topics include stellar masses, star formation rates and histories, metallicities and abundance patterns, dust properties, and the stellar initial mass function.

# 1 INTRODUCTION

Many of the fundamental properties of unresolved stellar populations are encoded in their SEDs. These properties include the star formation history (SFH), stellar metallicity and abundance pattern, stellar initial mass function (IMF), total mass in stars, and the physical state and quantity of dust and gas. Some of these properties are easier to measure than others, and each provides important clues regarding the formation and evolution of galaxies. It is precisely these quantities, measured from the SEDs of galaxies, that have provided the foundation for our modern understanding of galaxy formation and evolution.

Over the past several decades considerable effort has been devoted to extracting information from the SEDs of galaxies, exploiting information from the FUV to the FIR. Early attempts at understanding the visible and NIR spectral windows approached the problem by combining mixtures of stars in ad hoc ways until a match was achieved with observations (e.g., Spinrad & Taylor 1971). More sophisticated versions of this technique were developed that incorporated physical constraints and automated fitting techniques (Faber 1972). At about the same time, synthesis models were being developed that relied on stellar evolution theory to constrain the range of possible stellar types at a given age and metallicity (e.g., Tinsley 1968, Searle, Sargent & Bagnuolo 1973, Tinsley & Gunn 1976, Bruzual 1983). The

substantial progress made in stellar evolution theory in the 1980s and 1990s paved the way for the latter approach to become the de facto standard in modeling the SEDs of galaxies (e.g., Charlot & Bruzual 1991, Bruzual & Charlot 1993, Bressan, Chiosi & Fagotto 1994, Worthey 1994, Fioc & Rocca-Volmerange 1997, Leitherer et al. 1999, Vazdekis 1999). This modeling technique, which will be described in detail in the next section, is sometimes referred to as 'evolutionary population synthesis' (e.g., Maraston 1998), although the term 'stellar population synthesis' (SPS) has garnered wider use. The latter term will be used throughout this review.

The UV and IR spectral windows are rather more difficult to probe owing to the obscuring effects of the atmosphere. Nonetheless, numerous balloon and space-based observatories have opened up the ultraviolet and infrared to detailed investigations. In these spectral regions dust plays a major role; it absorbs and scatters much of the UV light emitted by stars and re-radiates that energy in the IR. In young stellar populations the UV is dominated by hot massive stars, while in old stellar populations the UV can be influenced by hot evolved stellar types such as post-AGB and extreme HB stars (see the review by O'Connell 1999, for details).

The development of models for the IR SEDs of galaxies (e.g., Draine & Lee 1984, Zubko, Dwek & Arendt 2004) has proceeded in parallel with the development of models for UV, optical, and NIR SEDs, and it is only recently that models have been developed to simultaneously and self-consistently predict the FUV through FIR SEDs (e.g., Devriendt, Guiderdoni & Sadat 1999; Silva et al. 1998; da Cunha, Charlot & Elbaz 2008; Groves et al. 2008; Noll et al. 2009a).

Several broad questions will serve to focus this review. They are:

- What have we learned about the physical properties of galaxies from their observed SEDs?
- How reliable are the quantities thus derived?
- What can be learned, in principle, from the SEDs of galaxies?

The first question is relatively straightforward to address, while the second and third are necessarily more complicated. Attention will be given to cases where answers to these questions are presently unknown, but knowable (the 'unknown unknowns', in Rumsfeld's sense, are of course the most interesting, but most difficult to discuss).

There are surprisingly few thorough reviews of stellar population synthesis and its application to modeling SEDs. The foundational review by Tinsley (1980) is highly recommended to anyone seeking a broad yet intuitive understanding of stellar populations and galaxy formation. The reviews by Faber (1977) and Frogel (1988) are more narrowly focused on old stellar populations but they are informative because many of the issues raised therein are still relevant today. Recently, Walcher et al. (2011) has presented an excellent review of many aspects related to the modeling of galaxy SEDs.

The topic of modeling galaxy SEDs is vast, and it would be impossible to provide a thorough review of the entire field. Hard decisions therefore had to be made. With regard to wavelength, energies higher than the FUV (roughly the Lyman limit), and lower than the submillimeter (roughly 1mm) will not be discussed. Nebular emission lines will also be neglected, except in a few cases. The measurement of and uncertainty induced by photometric redshifts will not be discussed and active galactic nuclei (AGN) will be ignored. The technique of fitting models to data was reviewed recently by Walcher et al. (2011) and so will not be discussed here. A whole review could (and should) be written on the evaluation and comparison of existing SPS models. This will not be undertaken herein, except in cases

where different models, when applied to data, produce starkly different results. The reader is referred to Conroy & Gunn (2010) for a recent comparison of several popular models. Finally, it is worth emphasizing what this review is not: it is not a summary of science results derived with SPS models. There is an extensive literature devoted to applying SPS models to data in order to derive insights into the formation and evolution of galaxies. Results of this nature, though fascinating in their own right, will not feature prominently in this review. Readers are referred to Renzini (2006) and Blanton & Moustakas (2009) for recent reviews along these lines.

This review is organized as follows. An overview of SPS model construction and application is provided in Section 2. We then turn to the topics of mass-to-light ratios and stellar masses (Section 3), star formation rates, histories, and stellar ages (Section 4), stellar metallicities and abundances patterns (Section 5), dust (Section 6), and the initial mass function (Section 7). Several concluding remarks are provided in Section 8.

# 2 OVERVIEW OF STELLAR POPULATION SYNTHESIS

The construction of models for simple and composite stellar populations is conceptually straightforward. There are however certain constraints that make the creation of such models rather difficult in practice (incomplete isochrone tables, incomplete empirical stellar libraries, poorly calibrated physics, etc.). In this section the ingredients necessary for constructing model SEDs will be discussed. Areas of particularly large uncertainties will be highlighted, but it is not the goal of this review to provide an exhaustive inter-comparison of various possible model choices. An overview of the entire process of constructing composite stellar populations is given in Figure 1.

# 2.1 The Simple Stellar Population

The starting point of any SPS model is the simple stellar population (SSP), which describes the evolution in time of the SED of a single, coeval stellar population at a single metallicity and abundance pattern. An SSP therefore requires three basic inputs: stellar evolution theory in the form of isochrones, stellar spectral libraries, and an IMF, each of which may in principle be a function of metallicity and/or elemental abundance pattern. These components are typically combined in the following way:

$$f_{\rm SSP}(t,Z) = \int_{m_{\rm lo}}^{m_{\rm up}(t)} f_{\rm star}[T_{\rm eff}(M), \log g(M)|t, Z] \,\Phi(M) \,\mathrm{d}M,\tag{1}$$

where M is the initial (zero-age main sequence) stellar mass,  $\Phi(M)$  is the initial mass function,  $f_{\rm star}$  is a stellar spectrum, and  $f_{\rm SSP}$  is the resulting time and metallicity-dependent SSP spectrum. The lower limit of integration,  $m_{\rm lo}$ , is typically taken to by the hydrogen burning limit (either 0.08 or  $0.1 M_{\odot}$  depending on the SPS code), and the upper limit is dictated by stellar evolution. The isochrones determine the relation between  $T_{\rm eff}$ ,  $\log g$ , and M for a given t and Z. This approach to constructing SSPs is common but not universal. Alternatives include the fuel consumption theorem (Renzini & Buzzoni 1986, Maraston 1998), or the use of empirical spectra of star clusters as templates for SSPs (Bica & Alloin 1986).

Nearly all SPS models provide SSPs as a 'black-box' (e.g., Leitherer et al. 1999, Bruzual & Charlot 2003, Maraston 2005, Vazdekis et al. 2010). The user therefore has little working knowl-

edge of how SSPs are built and where major issues lie. The following discussion is therefore largely pedagogical and geared toward users of SPS models, rather than model builders.

2.1.1 Stellar Evolution & Isochrones An isochrone specifies the location in the Hertzsprung-Russell (HR) diagram of stars with a common age and metallicity. Isochrones are constructed from stellar evolution calculations for stars from the hydrogen burning limit ( $\approx 0.1 \, M_{\odot}$ ) to the maximum stellar mass ( $\approx 100 \, M_{\odot}$ ). The construction of isochrones is straightforward for stellar evolution tracks that are infinitely well-sampled in mass and time. In practice, evolutionary tracks are discretely sampled and this can lead to issues in isochrone construction for fast evolutionary phases. Modern sets of isochrones have constructed specifically to ensure that the models are relatively immune to these effects (Charlot & Bruzual 1991).

A number of widely-used isochrone tables exist in the literature. The most popular models span a wide range in ages (masses), chemical compositions, and cover most relevant evolutionary phases. Models in this category include the Padova models (Bertelli et al. 1994, Girardi et al. 2000, Marigo et al. 2008), and the BaSTI models (Pietrinferni et al. 2004, Cordier et al. 2007). The Geneva models (Schaller et al. 1992, Meynet & Maeder 2000) are tailored to follow high-mass stars through advanced evolutionary phases, including the Wolf-Rayet (WR) phase, but they do not model low mass stars. Other models have focused on the main sequence, red giant branch (RGB), and horizontal branch (HB) evolution of low-mass stars ( $M < 3 M_{\odot}$ ), including the  $Y^2$  models (Yi et al. 2001, Yi, Kim & Demarque 2003), the Dartmouth models (Dotter et al. 2008), and the Victoria-Regina models (Vandenberg & Bell 1985, VandenBerg, Bergbusch & Dowler 2006). Finally, there are isochrones tailored to very low-mass stars and brown dwarfs. The Lyon models are the most widely used in this regime (Chabrier & Baraffe 1997, Baraffe et al. 1998). It is noteworthy that none of the isochrones listed here cover the post-AGB evolutionary phase. Remarkably, the post-AGB isochrones computed by Schoenberner (1983), Vassiliadis & Wood (1994), and Bloecker (1995) are still widely used in modern SPS codes.

Implementing isochrones in an SPS model is challenging because no single set spans the necessary range of ages, metallicities and evolutionary phases. It is common to use the Padova isochrones for the bulk of the age and metallicity range and to supplement with the Geneva models at young ages. Little attention is normally paid to the lowest mass portion of the isochrones, since low mass stars contribute only  $\sim 1\%$  of the light of an old stellar population. An exception in this regard is the model of Conroy & van Dokkum (2012a), who paid special care to the modeling of low-mass stars for SPS. The splicing together of various isochrone sets can be difficult. For example, different codes make different assumptions regarding convection, rotation, etc., and so the age at which certain stars evolve off the main sequence varies between codes.

Modules for Experiments in Stellar Astrophysics (MESA) is a new, highly modular and sophisticated stellar evolution code that includes the latest stellar interior ingredients including opacity tables, equations of state, nuclear reaction networks, and surface boundary conditions (Paxton et al. 2011). There is great hope that MESA will be employed to produce high-quality isochrones over the full age and metallicity range and for all evolutionary phases.

In addition to the practical difficulty of implementing current isochrone libraries, stellar evolution calculations contain a number of uncertainties relevant to SPS. For example, all of the stellar models mentioned above are based on one dimensional codes. As such, they require approximations to inherently three dimensional processes such as convection,

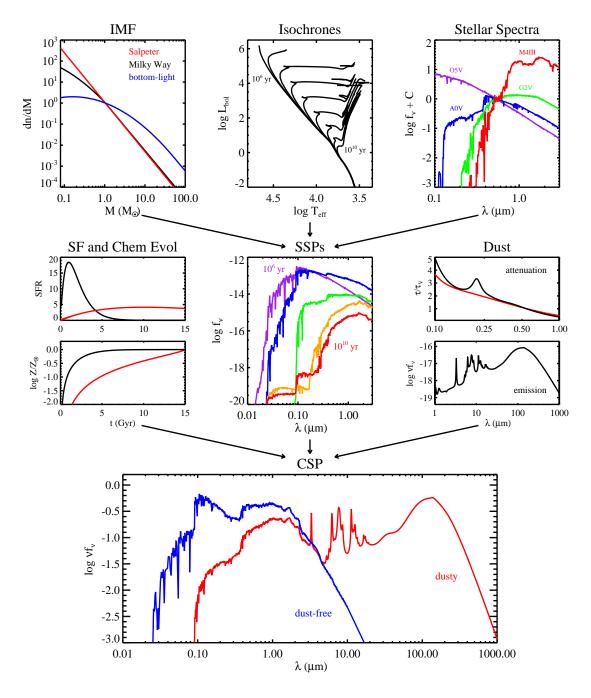


Figure 1:

Overview of the stellar population synthesis technique. The upper panels highlight the ingredients necessary for constructing simple stellar populations (SSPs): an IMF, isochrones for a range of ages and metallicities, and stellar spectra spanning a range of  $T_{\rm eff}$ ,  $L_{\rm bol}$ , and metallicity. The middle panels highlight the ingredients necessary for constructing composite stellar populations (CSPs): star formation histories and chemical evolution, SSPs, and a model for dust attenuation and emission. The bottom row shows the final CSPs both before and after a dust model is applied.

rotation, mass-loss, close binary interactions, and thermal pulses during AGB evolution. These processes lead to major uncertainties in the isochrones that impact the resulting SPS model predictions (e.g., Charlot, Worthey & Bressan 1996; Charlot 1996; Yi 2003; Lee et al. 2007; Conroy, Gunn & White 2009).

The cores of stars more massive than  $M \sim 1.1 M_{\odot}$  are convective, as are the envelopes of evolved giants and low mass stars. Classically, the boundary between convective and radiative regions is specified by the Schwarzschild criterion. However, this criterion is effectively a requirement that the acceleration of a convective fluid element be zero. The fluid element will likely have a non-zero velocity as it crosses this boundary, and so some amount of 'overshooting' is expected. This will result in a larger convective region than would be expected from the Schwarzschild criterion alone. With regards to the convective core, a wide body of observational evidence favors the existence of a moderate amount of overshooting in the mass range  $1.1 < M/M_{\odot} < 7$  (Stothers 1991, Nordstroem, Andersen & Andersen 1997, VandenBerg & Stetson 2004, Keller & Wood 2006). The amount of core overshooting required to fit the data results in a  $\sim 25\%$  increase in the main sequence lifetime compared to models that do not include overshooting. Nearly all SPS models adopt isochrones with a modest amount of overshooting in the convective core, as supported by observations. The exception to this is the SPS model of Maraston (1998, 2005), which uses isochrones without core convective overshooting. The treatment of core overshooting has a noticeable effect on the color evolution of SSPs in the age range of  $\sim 0.1-1$  Gyr, by as much as  $\approx 0.1$  mag (Yi 2003, Conroy & Gunn 2010) and can therefore be an important source of systematic uncertainty when modeling SEDs.

As discussed in Cassisi (2004), the amount of overshooting in the convective envelopes of evolved giants is less constrained. This is unfortunate because the treatment of envelope convection affects, among other observables, the ratio of red to blue helium-burning giants (Renzini et al. 1992). These stars can contribute several tens of percent to the integrated light, depending on the star formation history (Melbourne et al. 2012), and so uncertainty in the amount of envelope overshooting should affect the integrated light predictions at a significant level. Overshooting in the convective envelope also affects the location of the luminosity bump in the RGB luminosity function. This fact was used by Alongi et al. (1991) and more recently by Cassisi et al. (2011) to argue for a modest amount of overshooting in the convective envelope.

The importance of stellar rotation has been investigated by the Geneva group, amongst others, over the past two decades (see Maeder & Meynet 2000, 2012, for a review). Perhaps most importantly, rotation increases the main sequence lifetimes (by  $\sim 25\%$ ) due to rotation-induced mixing bringing fresh fuel to the convective core. Rotation also lowers the effective surface gravity, lowers the opacity in the radiative envelope, increases the luminosity, and changes the ratio of red-to-blue supergiants. The mixing to the surface of H-burning products caused by rotation will also affect the number and type of Wolf-Rayet stars. Vázquez et al. (2007) and Levesque et al. (2012) investigated the impact of rotating massive star models on integrated light properties by comparing to non-rotating models and found that the number of ionizing photons increased by as much as an order of magnitude and colors became bluer by as much as 0.1-1 mag, depending on wavelength and age. The signatures of WR stars as a function of time and metallicity in integrated spectra are also different between rotating and non-rotating models.

Most massive stars are in binary systems, and there is some evidence that massive star binaries preferentially have comparable masses (Kobulnicky & Fryer 2007, Sana & Evans 2011). The interaction of close binary stars via mass-transfer and common envelope evolu-

tion will affect the evolution of such stars and bring about further changes to their observable properties. Eldridge, Izzard & Tout (2008) demonstrated that in many respects the effects of binary star models without rotation are similar to single star models with rotation. Eldridge & Stanway (2012) argued that SPS models including binary star evolution produced a better fit to UV spectra of star-forming galaxies. Regardless of the details, it is clear that where massive stars matter in galaxy SEDs, the effects of both rotation and binary evolution will play an important role. In fact, binary star evolution may also create blue straggler stars and extreme HB stars, which would suggest that binary evolution can affect older stellar populations as well (Han et al. 2002, 2003, Zhang et al. 2005). No popular SPS model includes the effects of binary star evolution.

The potential importance of thermally-pulsating AGB (TP-AGB) stars in the context of SPS models was emphasized by Maraston et al. (2006), and has since become a controversial topic. This phase occurs for stars in the mass range  $\approx 1 < M/M_{\odot} < 8$  (depending on metallicity) and is difficult to model for several reasons, including the fact that nuclear burning occurs in alternate hydrogen-rich and helium-rich shells. When the helium shell burns it does so explosively because of the thin shell instability (Schwarzschild & Harm 1968), which gives rise to the thermal pulses. Mass-loss becomes catastrophic during this phase, thereby terminating the life of the star. Recently the Padova group has developed a new suite of isochrones with updated TP-AGB models calibrated against observations in the Large Magellanic Cloud (LMC Marigo & Girardi 2007, Marigo et al. 2008). However, Conroy & Gunn (2010) found that the updated Padova models failed to reproduce the colors of intermediate-age Magellanic Cloud star clusters, by as much as 0.5 mag in some cases. These authors provided recalibrated SPS models in which the weight given to TP-AGB stars was reduced in order to match the LMC data. More recently, Melbourne et al. (2012) analyzed resolved color-magnitude diagrams in nearby dwarf galaxies and concluded that these updated models produce twice as much luminosity in the TP-AGB phase as observed, roughly independent of the inferred mass fraction in young stars. In addition, Melbourne et al. (2012) found that the latest Padova model predictions for the luminosity contributed by red core helium burning stars is a factor of two higher than observed in dwarf galaxies. This problem in the models may be related to the treatment of convection in the envelopes of evolved giants.

Mass-loss is another critical parameter in stellar evolution models. At  $M \lesssim 8\,M_\odot$ , it determines when a star will end its life as a white dwarf, and how massive the white dwarf will be. At higher masses mass-loss can significantly alter the course of advanced evolutionary phases, especially at  $M > 40\,M_\odot$ . In high mass stars the mass loss mechanism is thought to be via line-driven winds, while in lower mass stars ( $\lesssim 8\,M_\odot$ ) it is believed to be due to pulsation-induced dust-driven winds (Willson 2000). In any event, the mass-loss prescription is another free parameter in these models, and it is a critical one because it strongly affects the lifetimes of advanced (and luminous) evolutionary phases. For example, the lifetime of TP-AGB stars and the number of thermal pulses they undergo depend strongly on the mass-loss prescription (Ventura & Marigo 2010).

In summary, the computation of isochrones for use in SPS depends on many uncertain aspects of stellar evolution including the treatment of convection, close binary evolution, rotation effects, and mass-loss, amongst many others. The importance of these uncertainties on derived SPS results will be highlighted throughout this review.

2.1.2 Stellar Remnants Stars eventually die, usually leaving behind stellar remnants in the form of white dwarfs, neutron stars, or black holes (theory predicts that a cer-

tain class of very massive metal-poor stars undergoes pair-instability supernovae that leave behind no stellar remnant; Heger et al. 2003). The relation between the initial, zero-age main sequence stellar mass and the final remnant mass is not well-constrained observationally, especially for massive stars. The initial-final mass relation for white dwarfs can be reasonably well constrained by measuring white dwarf masses in open clusters with known ages (e.g., Kalirai et al. 2008). The initial-final mass relation is predicted to be a function of metallicity (Marigo 2001, Heger et al. 2003), further complicating the situation.

In SPS models stellar remnants are usually included in the total stellar mass budget, and their contribution can be significant. For example, if a certain amount of mass is formed into stars instantly, then after 13 Gyr only 60% of the mass remains in stars or stellar remnants; the other 40% has returned to the interstellar medium. Of the remaining stellar mass, 25% is locked in stellar remnants. Of the total remnant mass, 73% is comprised of white dwarfs, 7% is in neutron stars, and 20% is in black holes. These numbers were computed assuming solar metallicity and a Kroupa (2001) IMF, with the initial-final mass relations from Renzini & Ciotti (1993). The Maraston (2005) SPS model predicts similar numbers (largely because the same initial-final mass relation was used). The point is that stellar remnants can make an important contribution to the total stellar mass of a system.

2.1.3 STELLAR SPECTRAL LIBRARIES Stellar spectral libraries are required to convert the outputs of stellar evolution calculations — surface gravities, g, and effective temperatures,  $T_{\rm eff}$  — as a function of metallicity, Z, into observable SEDs. There is however no single spectral library, whether theoretical or empirical, that covers the entire range of parameter space necessary for constructing SPS models. Stitching together various libraries, often of widely varying quality, is therefore necessary. Some modelers take the approach of constructing spectral libraries entirely from theoretical calculations, while others use purely empirical libraries. The benefits and drawbacks of these two approaches will be discussed in this section.

Theoretical Libraries Theoretical libraries offer the great advantage of densely covering parameter space, including spectral resolution, and of producing spectra that are not subject to observational issues such as flux calibration and atmospheric absorption. The clear disadvantage is that the libraries are only as good as the input atomic and molecular parameters and the approximations made in the computation of the models, as discussed below.

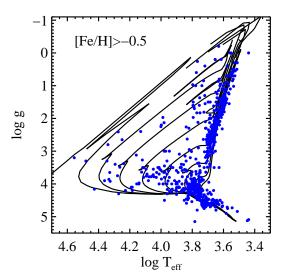
There are a number of decisions that must be made when computing synthetic spectral libraries including how to treat convection and the microturbulent velocity profile, and whether or not to model departures from LTE and plane-parallel geometries. Additional important limitations include the incomplete and often inaccurate atomic and molecular line lists. As emphasized by Kurucz (2011), models still fail to reproduce all the observed features in ultra high-resolution spectra of the Sun due to incomplete atomic line lists. The situation is even more serious for cooler stars because the molecular line lists can carry fairly large uncertainties, in particular TiO (Allard, Homeier & Freytag 2011). A related problem is that a significant fraction of the atomic and molecular lines are derived from theoretical calculations, rather than measured in the laboratory, and so they have uncertain strengths and central wavelengths. Unfortunately, these 'predicted lines' can be important for determining the overall SED shape of stars and hence of galaxies. Worse still, the partition function for many molecules is poorly known, so even the total abundance of particular molecules is uncertain.

The quality and state of the art of the theoretical models varies considerably as a function of  $T_{\rm eff}$ . At the high temperature end, i.e., Wolf-Rayet and O-type main sequence stars, the state-of-the-art libraries are from Smith, Norris & Crowther (2002) and Lanz & Hubeny (2003), while for hotter compact objects (e.g., post-AGB stars) the most up-to-date models are from Rauch (2003). By far the widest range in parameter space is covered by the BaSeL atlas (Lejeune, Cuisinier & Buser 1997, 1998; Westera et al. 2002). This library is comprised of a variety of theoretical models from Kurucz (1995), Fluks et al. (1994), Bessell et al. (1989, 1991), and Allard & Hauschildt (1995). The broadband SEDs of the stars in this library have been modified to agree with observed UBVRIJHKL colortemperature relations for individual stars. The BaSeL atlas is almost universally used in modern SPS codes, despite the fact that the input theoretical spectra are now almost 20 years old. At the very coolest temperatures the MARCS and NEXTGEN/PHOENIX grids are the state-of-the-art because of their comprehensive molecular line lists and treatment of spherical effects on the atmospheres and spectra of cool giants. Finally, the Aringer et al. (2009) models fill out the low temperature end by providing carbon star spectra over a range of parameter space.

There are several modern spectral libraries computed by Munari et al. (2005), Martins et al. (2005), and Coelho et al. (2005) that offer fairly wide coverage in parameter space and are at high spectral resolution. The Martins et al. (2005) library has even been incorporated into a fully theoretical high resolution SPS model (González Delgado et al. 2005). However, these libraries appear to be geared toward high resolution spectroscopic analyses because they do not include the predicted lines. They therefore cannot be used to model broadband SEDs, or any spectral region that contains a significant contribution from predicted lines. However, progress can be made by making corrections to these high resolution libraries for the effect of predicted lines on the broadband SED, as for example done in Coelho et al. (2007).

How Accurate are the Theoretical Libraries? Normally, observed stars are assigned physical parameters based on comparison to models. However, if one wants to constrain the models with observed stellar data then one requires an independent estimate of the physical properties of stars. This is a significant obstacle to assessing the accuracy of the models. One can obtain essentially model-independent estimates of  $T_{\rm eff}$  via angular diameter measurements, but these are very difficult to obtain for giants, let alone main sequence stars (Perrin et al. 1998, Boyajian et al. 2012a). A more widely used technique is the infrared flux method (IRFM), first proposed by Blackwell & Shallis (1977). The basic idea is that the flux of a star depends only weakly on  $T_{\rm eff}$  in the IR, and so one can deduce angular diameters from fluxes alone with only a weak model dependence. This technique underpins nearly all existing color- $T_{\rm eff}$  relations (e.g., Alonso, Arribas & Martinez-Roger 1996; Ramírez & Meléndez 2005; Casagrande et al. 2010).

The latest generation of model spectra succeed in reproducing the broadband colors for FGK dwarfs and warm giants (Bertone et al. 2004, Kučinskas et al. 2005, Martins & Coelho 2007, Boyajian et al. 2012b). Not surprisingly, the models continue to have difficultly fitting the SEDs of the coolest stars and the flux in the Wien tail of the flux distribution (typically  $\lesssim 5000\,\text{Å}$  for types G and later). The latter difficulty arises because the Wien tail is extremely sensitive to  $T_{\rm eff}$  and so even minor changes to the model assumptions lead to large changes in that spectral region. In addition, metal line blanketing becomes very strong in the blue/UV, and so the requirements on the line lists are demanding. Martins & Coelho (2007) explored the ability of three modern synthetic libraries to reproduce a variety of spectral



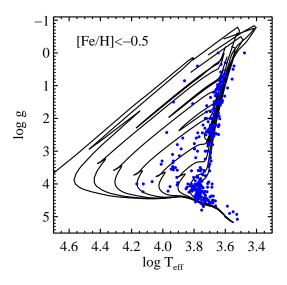


Figure 2:

Physical properties of the MILES empirical spectral library, separated by [Fe/H] (solid symbols). Isochrones from the Padova group are overplotted for  $t=3\times 10^6-10^{10}$  yr. Typical errors on  $T_{\rm eff}$  and  $\log g$  are 60-100 K and 0.2 dex, respectively (Cenarro et al. 2007). Notice that the MILES library covers the lower main sequence and RGB both at high and low metallicity, but it only sparsely covers the upper main sequence and supergiants. This figure highlights the difficulty in constructing SPS models based on empirical stellar libraries, especially at low metallicity.

features of observed stars. Overall the agreement was found to be satisfactory, although there were notable deficiencies. Some models were unable to reproduce the observed CH features in cool stars, while other models failed to match the observed Mgb and MgH features. The majority of the model shortcomings were restricted to stars with  $T_{\rm eff} < 4500$  K. Modern models also have difficulty reproducing the strength of the TiO bands in cool stars (Kučinskas et al. 2005, Allard, Homeier & Freytag 2011).

The situation regarding the  $H_2O$  band strengths in the NIR should serve as a cautionary tale: models have for decades predicted water bands in cool stars that are stronger than observations. Part of the problem seems to have been the molecular line lists, which improved steadily over the years, culminating in the Barber et al. (2006) line list, which contains 500 million transitions. However, Allard, Homeier & Freytag (2011) demonstrated that the recent revision of the solar abundance scale by Asplund et al. (2009), which entails a factor of two reduction in the oxygen abundance, has a much larger effect on the predicted strength of the water bands. Their new models with updated solar abundances reproduce remarkably well the NIR colors of M dwarfs, which have strong water band features. Thus, not only the atomic and molecular parameters but also the abundance patterns can have a very significant effect on theoretical spectra.

**Empirical Libraries** The strengths and weaknesses of the theoretical libraries are the weaknesses and strengths of the empirical libraries. Empirical spectra of course do not suffer from issues with line lists, treatment of convection, etc., but they are plagued by standard

observational constraints such as correction for atmospheric absorption, flux calibration, and limited wavelength coverage and spectral resolution. Worse, the empirical libraries are woefully incomplete in their coverage of parameter space. This is a long-standing issue that is difficult to address owing to the fact that empirical libraries are drawn from samples of stars in the solar neighborhood. For example, hot main sequence stars at low metallicity are very rare, as are stars in rapid phases of stellar evolution such as WR and TP-AGB stars.

One of the first comprehensive empirical stellar spectral libraries was constructed by Gunn & Stryker (1983). Later optical/NIR libraries included Pickles (1998), Jones (1999), ELODIE (Prugniel & Soubiran 2001), STELIB (Le Borgne et al. 2003), Indo-US (Valdes et al. 2004), NGSL (Gregg et al. 2006, Heap & Lindler 2011), MILES (Sánchez-Blázquez et al. 2006), IRTF (Rayner, Cushing & Vacca 2009), and the X-shooter library (Chen et al. 2011). Other, specialized libraries include an ultraviolet atlas compiled from IUE data (Fanelli et al. 1992), a TP-AGB library (Lançon & Mouhcine 2000), and the Spitzer Space Telescope IR stellar library (Ardila et al. 2010). The Sloan Digital Sky Survey (SDSS) has obtained spectra of several hundred thousand stars and has devoted considerable effort to measuring stellar parameters from the spectra (Lee et al. 2008). As yet there is no official stellar spectral library based on the SDSS stars, but it would clearly provide a great leap forward both in terms of coverage in parameter space and uniformity in spectral quality.

An example of the difficulties posed by empirical libraries for SPS model construction is shown in Figure 2. This figure shows the location in the HR diagram of all stars from the MILES spectral library, with stellar parameters determined by Cenarro et al. (2007). MILES is the empirical library today that provides the greatest coverage in terms of  $\log g$ ,  $T_{\rm eff}$ , and [Fe/H] with a total of 985 stars. In this figure the metal-rich and metal-poor stars are split into two panels. Overplotted are isochrones from the Padova group for ages ranging from  $10^{6.6}-10^{10.1}$  yr. While the lower main sequence and red giant branch are well covered by the empirical library over a wide range in metallicity, hotter stars are rare, especially at lower metallicity. Because the HR diagram is covered so sparsely in this regime, constructing SPS models based on empirical libraries at young ages ( $\lesssim 1$  Gyr) is clearly challenging.

A second, related problem with empirical libraries is irregular coverage in the HR diagram. Interpolation within the library, necessary for SPS model construction, can be difficult. Vazdekis et al. (2010) has developed a sophisticated algorithm to deal with this issue. These authors assign quality numbers to the resulting SPS models based on the number of empirical spectra comprising each model. This is a profitable approach that should be standard in the field because it allows users to assess the reliability of various regions of parameter space.

A third problem associated with empirical libraries is the assignment of physical parameters to stars, including  $\log g$ ,  $T_{\rm eff}$ , and [Fe/H]. Of particular note is the  $T_{\rm eff}$  determination, which carries an uncertainty of order 100 K (Alonso, Arribas & Martinez-Roger 1996; Ramírez & Meléndez 2005; Casagrande et al. 2010). Using theoretical models, Percival et al. (2009) explored the impact of uncertainties in  $T_{\rm eff}$ ,  $\log g$ , and [Fe/H] for individual stars on SPS models. They found that for models with old ages (> 4 Gyr), the effect of changing the temperature scale by 100 K had significant effects (0.1 – 0.4 Åin EW in the most extreme cases) on a variety of spectral absorption features, including the hydrogen Balmer lines and several commonly employed iron and magnesium lines used to interpret the stellar populations of early-type galaxies. The work of Percival & Salaris was mostly exploratory, and it would be desirable to consider the full propagation of these sorts of uncertainties into the

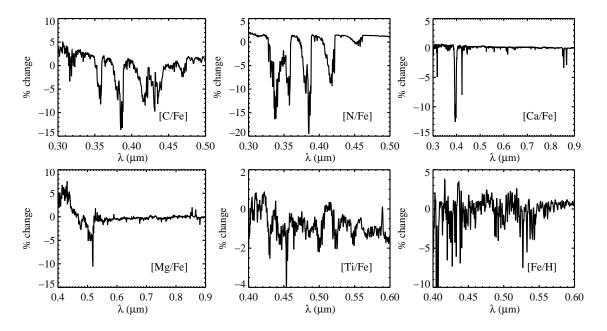


Figure 3:

Variation in the spectrum of a 13 Gyr SSP due to changes in individual elemental abundances. All abundance changes are +0.3 dex except for C and Ca which are varied by +0.15 dex. Spectra have been broadened to a velocity dispersion of  $150~{\rm km\,s^{-1}}$ . Notice the different x and y-axes in each panel. Prominent spectral features include the CH, CN, NH, C<sub>2</sub>, Ca I, Ca II H&K, Ca II triplet, MgH, and Mg b (Mg I) features, along with numerous atomic Fe and Ti features. Computed from the models of Conroy & van Dokkum (2012a).

derived properties of galaxies.

Yet another issue with empirical libraries is the abundance patterns of the stars. It is well-known that low metallicity stars in the Galaxy tend to be  $\alpha$ -enhanced, such that  $[Mg/Fe] \approx 0.0$  at  $[Fe/H] \approx 0.0$  and  $[Mg/Fe] \approx 0.4$  at [Fe/H] < -1.0 (Edvardsson et al. 1993). Thus, any model based on empirical stars at low metallicity must somehow correct for this  $[\alpha/Fe]$  'bias' in the models (see e.g., Thomas, Maraston & Bender 2003b; Schiavon 2007).

Variable Abundance Patterns Moderate resolution spectra (i.e.,  $R \sim 1000-5000$ ) contain a wealth of information on the detailed abundance patterns of the stellar populations. In the context of SPS models, abundance ratios have historically been estimated through the analysis of spectral indices. Ideally, an index is sensitive to a single feature (such as an atomic absorption line or a molecular band head). One usually defines a feature bandpass and one or two pseudocontinua, and then an equivalent width (EW), or in some cases a magnitude, can be measured. The Lick/IDS system is the most popular index system, defining 21 indices in the wavelength range  $4000-6500\,\text{Å}$  (Burstein et al. 1984, Worthey et al. 1994). Other index systems have been defined at other wavelengths (e.g., Fanelli et al. 1992, Alvarez et al. 2000, Cenarro et al. 2001, Serven, Worthey & Briley 2005). In practice, the use of indices is greatly complicated by the fact that there are rarely

if ever clean regions of the spectrum from which one can estimate the continuum level. The strength of each index thus depends not only on the feature of interest but also the features in the pseudocontinua.

Tripicco & Bell (1995) were the first to assess the sensitivity of the Lick/IDS indices to variation in the abundances of 10 elements with theoretical spectral models. Korn, Maraston & Thomas (2005) provided an update to these 'response functions' with updated line lists and transition probabilities, and for a range in metallicity. Serven, Worthey & Briley (2005) considered variation in 23 elements, including several neutron-capture elements. These authors defined new indices sensitive to a host of elements not considered in the earlier work. The Serven models were based on theoretical spectra of only two stars per abundance pattern, a turnoff star and a luminous giant, and the model effectively had a fixed age of  $\sim 5$  Gyr. Lee et al. (2009a) provided response functions for Lick indices by employing a larger number of synthetic spectra. Conroy & van Dokkum (2012a) produced theoretical stellar spectra with variation in 11 elements. These models contained theoretical spectra at 20 points along the isochrone, for ages from 3-13.5 Gyr. Rather than focusing on spectral indices, these models made predictions for the response of the full spectrum from  $3500 \text{ Å} - 2.4 \,\mu\text{m}$  to abundance variations. Sample response spectra from that work are shown in Figure 3. In this figure the relative response of the spectrum to an increase in the abundance of a particular element is shown. The full spectrum is rich in diagnostic features beyond what any index system can capture.

To construct models with arbitrary abundance patterns one would want to create synthetic spectra for each possible abundance pattern. When considering large numbers of elements, such an approach becomes computationally infeasible. Instead, the standard technique is to create arbitrary abundance patterns by treating the effect of each element on the spectrum as being independent of the other elements (e.g., Thomas, Maraston & Bender 2003b; Schiavon 2007; Lee et al. 2009a; Conroy & van Dokkum 2012a). This is a reasonable assumption for trace elements, but it is less realistic for elements such as C, N, O, Na, Ti, and Fe, which affect the opacity, free electron density, and/or molecular equilibrium. There has been surprisingly little work in the literature testing this assumption.

2.1.4 The IMF The initial distribution of stellar masses along the main sequence, known as the stellar initial mass function (IMF), has been studied extensively for decades (e.g., Salpeter 1955, Scalo 1986, Scalo et al. 1998, Kroupa 2001, Chabrier 2003). As emphasized in a recent review by Bastian, Covey & Meyer (2010), there is no compelling evidence for variation in the IMF from direct probes, e.g., star counts. The canonical Salpeter IMF has the form  $dN/dM \propto M^{-x}$  with x=2.35. The IMF measured in the solar neighborhood deviates from the Salpeter form only at  $M<1\,M_\odot$ , where x becomes shallower (Kroupa 2001, Chabrier 2003).

From the perspective of SPS, the IMF (1) determines the overall normalization of the stellar mass-to-light ratio, M/L; (2) determines the rate of luminosity evolution for a passively evolving population; (3) affects the SED of composite stellar populations; (4) has a small effect on the shape of the SED of single stellar populations. The last point is due to the fact that the integrated light of a coeval population is overwhelmingly dominated by stars at approximately the same mass, i.e., the turnoff mass. (4) will not hold for IMFs that depart dramatically from the Salpeter IMF. (3) arises because composite populations have a range of turnoff masses that contribute to the integrated light. The fractional contribution of stars of various masses to the total number of stars, stellar mass, and bolometric luminosity is shown in Figure 4. This figure demonstrates quantitatively that low mass stars dominate

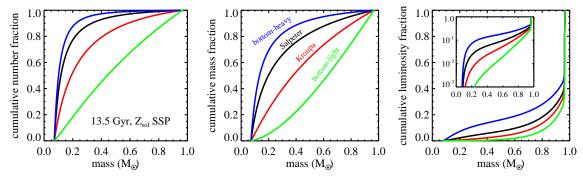


Figure 4:

Fractional contribution to the total number, mass, and bolometric luminosity as a function of stellar mass for a 13.5 Gyr solar metallicity model. Lines correspond to different IMFs: a bottom-heavy with logarithmic slope x=3.0 (blue line); Salpeter (x=2.35; black line); MW IMF (specifically a Kroupa IMF; red line); a bottom-light IMF (specifically of the form advocated by van Dokkum (2008); green line). The inset in the right panel shows the cumulative luminosity fraction in logarithmic units. Low mass stars dominate the total number and mass in stars, but contribute a tiny fraction of the luminosity of old stellar populations.

the stellar mass and number of stars in a galaxy, but contribute only a few percent to the bolometric light of an old stellar population. At younger ages the light contribution from low mass stars is even less.

Tinsley (1980) demonstrated that the evolution in M/L for a passively evolving stellar population is sensitive to the logarithmic slope of the IMF, x, at the main sequence turnoff point. The logarithmic evolution of the luminosity per logarithmic time  $(\mathrm{dln}L/\mathrm{dln}t)$  scales linearly with x, at least for plausible values of x (see also van Dokkum 2008, Conroy, Gunn & White 2009). This dependency arises because the giant branch dominates the luminosity for all plausible values of x, and so the IMF determines the rate at which the giant branch is fed by turnoff stars. Steeper IMFs imply that the giant branch is more richly populated with time, and therefore the natural luminosity dimming is reduced. For sufficiently steep IMFs (e.g.,  $x \gtrsim 5$ ), the unevolving dwarfs would dominate the light, and so the integrated luminosity would be approximately constant over time.

It is somewhat less-well appreciated that the IMF above  $1 M_{\odot}$  also strongly affects the shape of the SED of composite stellar populations. In composite populations the SED is influenced by stars with a range of masses (see Section 2.3), and so the IMF must play an important role (see e.g., Pforr, Maraston & Tonini 2012).

# 2.2 Dust

Interstellar dust is a component of nearly all galaxies, especially those that are actively star-forming. Dust plays a dual role in SPS, both as an obscurer of light in the UV through NIR and as an emitter of light in the IR. For both historical and theoretical reasons these two aspects are often modeled independently of one another. Indeed, these two aspects are sensitive to different properties of a galaxy (e.g., dust obscuration is highly sensitive to geometry while dust emission is more sensitive to the interstellar radiation field), and so it is not unreasonable to decouple the modeling of these two components.

Dust grains obscure light by both absorbing and scattering starlight. ATTENUATION From observations of individual stars one can infer the total extinction along the line of sight by comparing an observed spectrum to the expected spectrum of the star in the absence of dust (the latter is typically obtained from models in conjunction with an estimated  $T_{\rm eff}$  and  $\log g$  of the source). Thus, extinction measures the total loss of light along a single line of sight. Observations in the Milky Way (MW) and Large and Small Magellanic Clouds (LMC and SMC, respectively) have been obtained for a number of sightlines, enabling construction of average extinction curves for these galaxies (Cardelli, Clayton & Mathis 1989; Pei 1992; Gordon & Clayton 1998). In the MW and LMC the only striking feature in the extinction curve is the broad absorption at 2175 Å. This feature is absent in three out of four sightlines to the SMC, and it is weaker in the LMC than in the MW. The grain population responsible for this feature is not known, but polycyclic aromatic hydrocarbons (PAHs) are a leading candidate (Draine 2003). In general, dust extinction is a consequence of the optical properties of the grains and the grain size and shape distribution (Weingartner & Draine 2001).

When modeling SEDs of galaxies, the relevant concept is dust attenuation, which differs from extinction in two important respects: (1) light can be scattered both out of and into a given line of sight; (2) the geometrical distribution of dust with respect to the stars strongly affects the resulting SED (see Calzetti 2001, for an extensive discussion of these issues). The total dust attenuation in a galaxy can be estimated by analogy with how one estimates dust extinction: a spectrum of a galaxy is obtained and compared with the expected spectrum of the same galaxy in the absence of dust. For obvious reasons, estimating dust attenuation is considerably more complex than estimating dust extinction.

Although the shape of the dust attenuation curve depends on the star-dust geometry, grain size distribution, etc., in a complex manner, several general rules of thumb can be stated (see e.g., Witt & Gordon 2000, for a more thorough discussion). The simplest dust geometry, that of a homogenous foreground screen, yields an attenuation curve whose shape depends only weakly on the total dust column density (the weak dependence arises from the scattered light component). More complex geometries generally yield attenuation curves that become greyer (i.e., shallower) as the column density increases. Clumpy interstellar media also result in greyer attenuation curves than their homogenous counterparts. In all cases, the total attenuation optical depth is less than the amount of extinction that would be produced by the same column density because some scattered light is always returned to the line-of-sight. Finally, because the 2175 Å dust feature is believed to be due to pure absorption, the effects of radiative transfer will cause this feature to respond differently to geometrical effects compared to the rest of the attenuation curve. The use of a single dust attenuation curve for analyzing a wide range of SED types is therefore without theoretical justification.

In practice, most SPS modelers include dust attenuation by fixing the shape of the attenuation curve and fitting for the normalization. Popular attenuation curves include the Calzetti law, the MW, LMC, and SMC extinction curves, or the time-dependent attenuation model from Charlot & Fall (2000). The impact of the chosen attenuation curve on derived properties of galaxies will be discussed in later sections.

2.2.2 EMISSION Beyond  $\lambda \sim 10 \,\mu m$  the SEDs of normal galaxies are dominated by emission from dust grains. Mathis, Rumpl & Nordsieck (1977) were the first to postulate that the dust grain population is comprised of both silicate and carbonaceous grains. In modern theories, the carbonaceous grains are assumed to be PAHs when they are small,

and graphite when they are large (Draine 2003). The observed dust emission spectrum results from exposing these grains to a range of interstellar radiation field strengths.

A variety of models exist that combine grain size distributions and grain optical properties with models for starlight (or simply the radiation field) to predict IR emission (Desert, Boulanger & Puget 1990; Silva et al. 1998; Devriendt, Guiderdoni & Sadat 1999; Popescu et al. 2000; Dale et al. 2001; Piovan, Tantalo & Chiosi 2006; Jonsson 2006; Draine & Li 2007; Groves et al. 2008; Popescu et al. 2011). The modeling of PAH emission features (the most prominent being at  $3.6\,\mu m$ ,  $6.2\,\mu m$ ,  $7.7\,\mu m$ ,  $8.6\,\mu m$ , and  $11.3\,\mu m$ ) has become dramatically more sophisticated since the early attempts by Desert, Boulanger & Puget (1990), culminating in state-of-the-art models by Draine & Li (2007). At long wavelengths ( $\lambda > 50\,\mu m$ ) the emission is dominated by grains at a nearly steady temperature of  $\sim 15-20$  K and contributes  $\sim 2/3$  of the total IR luminosity. At shorter wavelengths the IR emission arises from single photon heating of dust grains (including PAHs) and accounts for the remaining  $\sim 1/3$  of the total IR emission (see the review by Draine 2003, for details). The IR emission at the shortest wavelengths ( $\lambda < 12\,\mu m$ ) is supplied almost entirely by PAHs. In detail these relative contributions will depend on the grain composition and size distribution and the interstellar radiation field.

The models listed above are not always well-suited for interpreting large numbers of observed IR SEDs because they contain a large number of parameters, require knowledge of the star-dust geometry, and/or require radiative transfer calculations. Simpler models for dust emission have therefore been developed in parallel to the more complex ones. The IR templates of Chary & Elbaz (2001) and Dale et al. (2001) are widely used for estimating bolometric luminosities and k-corrections. These templates are based on sophisticated models, but are constrained to match observations of normal star-forming and starburst (i.e., ULIRG) galaxies. The resulting templates are functions of only one variable. For example, the Chary & Elbaz (2001) templates are a function of the bolometric IR luminosity. da Cunha, Charlot & Elbaz (2008) have developed a simple phenomenological model for dust emission that consists of a series of modified blackbodies for the thermal dust emission and for the emission from stochastically heated dust grains. In addition, they include an empirical spectrum of M17 to represent the PAH emission.

2.2.3 Dust Around AGB Stars Stars experience very high mass-loss rates as they climb up the AGB (as high as  $10^{-4} M_{\odot} \ \rm yr^{-1}$  in the superwind phase). The mass lost is often observed to be dust rich (Bedijn 1987). These stars are often heavily dust obscured in the optical and emit copiously in the IR; they are often so dusty that their SEDs peak at  $\sim 10 \, \mu m$  (Bedijn 1987). Dust around AGB stars is important from the standpoint of SPS for two reasons: it will diminish the importance of AGB light in the optical and NIR, and it will contribute additional flux in the mid-IR beyond what would be expected from standard dust models (see e.g., Kelson & Holden 2010).

Despite the obvious importance of AGB dust, this aspect is included in few SPS models. Notable exceptions include the models of Bressan, Granato & Silva (1998), Silva et al. (1998), Piovan, Tantalo & Chiosi (2003), and González-Lópezlira et al. (2010), which are all based on theoretical models for AGB stars, their mass-loss rates, and dust formation and composition. Even the use of currently available empirical libraries of AGB stellar spectra require reddening the spectra to account for circumstellar dust (Lançon & Mouhcine 2002). Increasingly sophisticated radiative transfer models of dusty circumstellar envelopes are being developed (e.g., Groenewegen 2012, Sargent, Srinivasan & Meixner 2011, Srinivasan, Sargent & Meixner 2011). The outputs from these models should be incorporated into SPS codes as a standard

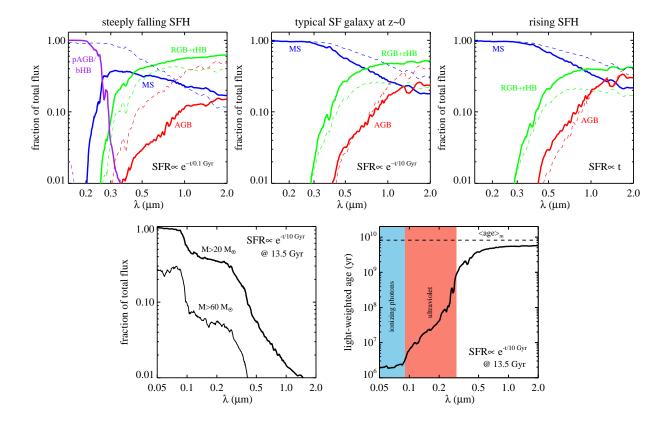


Figure 5:

Top Panels: Fractional contribution to the total flux from stars in various evolutionary phases, for three different SFHs. The left panel is representative of a galaxy that formed nearly all of its stars very rapidly at early times, the middle panel is representative of a typical star-forming galaxy at  $z \sim 0$ , and the right panel may be representative of the typical galaxy at high redshift. Flux contributions are at 13 Gyr (solid lines) and 1 Gyr (dashed lines) after the commencement of star formation; all models are solar metallicity, dust-free, and are from FSPS (v2.3; Conroy, Gunn & White 2009). Labeled phases include the main sequence (MS), red giant branch (RGB), asymptotic giant branch (AGB, including the TP-AGB), post-AGB (pAGB), and the blue and red horizontal branch (bHB and rHB). Bottom Left Panel: Fractional flux contributions for stars more massive than  $20\,M_\odot$  and  $60\,M_\odot$  for the SFH in the middle panel of the top row. Bottom Right Panel: Light-weighted age as a function of wavelength for the same SFH. The dashed line indicates the corresponding mass-weighted age.

practice.

# 2.3 Composite Stellar Populations

The simple stellar populations discussed in Section 2.1 are the building blocks for more complex stellar systems. Composite stellar populations (CSPs) differ from simple ones in three respects: (1) they contain stars with a range of ages given by their SFH; (2) they

contain stars with a range in metallicities as given by their time-dependent metallicity distribution function, P(Z,t); (3) they contain dust. These components are combined in the following way:

$$f_{\text{CSP}}(t) = \int_{t'=0}^{t'=t} \int_{Z=0}^{Z_{\text{max}}} \left( \text{SFR}(t-t') P(Z, t-t') f_{\text{SSP}}(t', Z) e^{-\tau_d(t')} + A f_{\text{dust}}(t', Z) \right) dt' dZ$$
(2)

where the integration variables are the stellar population age, t', and metallicity, Z. Time-dependent dust attenuation is modeled via the dust optical depth,  $\tau_d(t')$  and dust emission is incorporated in the parameter  $f_{\text{dust}}$ . The normalization constant A is set by balancing the luminosity absorbed by dust with the total luminosity re-radiated by dust.

The SFH can in principle be arbitrarily complex, although simple forms are usually adopted. By far the most popular is the exponential, or  $\tau$ -model, where SFR  $\propto e^{-t/\tau}$ . This form arises naturally in scenarios where the SFR depends linearly on the gas density in a closed-box model (Schmidt 1959). Recently, rising SFHs have become popular to explain the SEDs of high-redshift galaxies (Maraston et al. 2010, Papovich et al. 2011). Rising SFHs at early times seem to be a natural consequence of galaxy evolution in a hierarchical universe (Finlator, Davé & Oppenheimer 2007; Lee et al. 2010). Functional forms that incorporate an early phase of rising SFRs with late-time decay, such as SFR  $\propto t^{\beta} \, e^{-t/\tau}$ , may therefore become more popular amongst modelers.

The treatment of metallicity in composite stellar populations is usually even more simplistic than the treatment of SFRs. The widely adopted simplification is to replace P(Z,t) in Equation 2 with a  $\delta$ -function. In other words, a single metallicity is assumed for the entire composite population. The impact of this simplification on the SPS modeling procedure has not been extensively explored.

The approach to Equation 2 outlined above is standard but not universal. A notable exception is the technique of fitting non-parametric SFHs and metallicity histories in either a pre-defined or adaptive set of age bins (Cid Fernandes et al. 2005, Ocvirk et al. 2006, Tojeiro et al. 2009). The latter approach is computationally expensive since as many as thirty parameters are simultaneously fit to the data. Very high quality data are also a prerequisite for non-parametric techniques. Nonetheless, they offer the promise of less biased reconstruction of the SFH and metallicity history of galaxies based solely on their SEDs.

Figure 5 presents some basic properties of CSPs. The top panels show the fractional flux contribution from stars in different evolutionary phases for three representative SFHs. At late times the RGB and red HB dominate the red and NIR flux of  $\tau$ -model SFHs, as is well-known. However, at young ages and/or for rising SFHs, the red and NIR are also influenced strongly by AGB stars, suggesting that the NIR can be susceptible to large uncertainties (due to uncertainties in the modeling of the AGB, as discussed in detail in later sections). The lower left panel shows the fractional flux contribution from stars more massive than  $20\,M_\odot$  and  $60\,M_\odot$  for a  $\tau=10$  Gyr SFH. Massive star evolution is uncertain due to the complicating effects of rotation and binarity, and so this panel provides a rough sense of the extent to which uncertainties in massive star evolution will affect SED modeling. Finally, the lower right panel shows the light-weighted age as a function of wavelength, again for a  $\tau=10$  Gyr SFH. The dashed line indicates the mass-weighted mean age. Already by  $0.5\,\mu m$  (approximately the V-band), the light-weighted age reaches its maximal value, again suggesting that going to redder restframe wavelengths is not providing substantial new information on the SFH. Notice also that the maximum light-weighted age never reaches

the mass-weighted age. We return to this point in Section 4.

# 2.4 Nebular Emission

Although the effects of nebular emission on SEDs will not be discussed in detail in this review, a brief overview of this component is given for completeness.

Nebular emission is comprised of two components: continuum emission consisting of free-free, free-bound, and two photon emission, and recombination line emission. Several photoionization codes exist that make predictions for the nebular emission as a function of the physical state of the gas, including CLOUDY (Ferland et al. 1998) and MAPPINGSIII (Groves, Dopita & Sutherland 2004). Other approaches can be taken to the modeling of nebular emission lines. For example, Anders & Fritze-v. Alvensleben (2003) implement non-hydrogen emission lines based on observed line ratios as a function of metallicity. The nebular emission model must then be self-consistently coupled to a model for the starlight. Several groups have done this, with varying degrees of complexity, and with some only including nebular continuum, others only line emission, and others including both (e.g., Leitherer et al. 1999, Charlot & Longhetti 2001, Panuzzo et al. 2003, Groves et al. 2008, Mollá, García-Vargas & Bressan 2009, Schaerer & de Barros 2010).

The effect of nebular emission on the SED is complex, especially when line emission is considered. As a general rule, nebular emission is more important at low metallicity and at young ages. In such cases the contribution of nebular emission to broadband fluxes can be as high as 20-60% (Anders & Fritze-v. Alvensleben 2003). Nebular emission will also be more important at high redshift because a feature with a fixed restframe EW will occupy a larger fraction of the filter bandpass due to the redshifting of the spectrum. The effect of nebular emission therefore cannot be ignored at high redshift, where high SFR, low metallicity galaxies are common (Schaerer & de Barros 2010, Atek et al. 2011).

It is noteworthy that amongst the most widely used SPS codes, including Bruzual & Charlot (2003), Maraston (2005), PEGASE (Fioc & Rocca-Volmerange 1997), STARBURST99 (Leitherer et al. 1999), and FSPS (Conroy, Gunn & White 2009), only PEGASE and STARBURST99 include nebular continuum emission, and only PEGASE includes both nebular continuum and line emission. Since nebular emission is relatively straightforward to implement (notwithstanding assumptions about the physical state of the gas), it should be a standard component of all SPS codes.

#### 2.5 Fitting Models to Data

The SPS models described in this section are most frequently used to measure physical parameters of stellar populations. This is achieved by fitting the models to data, either in the form of broadband SEDs, moderate-resolution optical/NIR spectra, or spectral indices. The SSPs are usually taken as given, and the user then fits for a variety of parameters including metallicity, dust attenuation, and one or more parameters for the SFH. If IR data are available, then additional variables must be considered. The fitting techniques vary but are generally limited to grid-based  $\chi^2$  minimization techniques. As the number of parameters increases Markov Chain Monte Carlo techniques become increasingly more efficient (Conroy, Gunn & White 2009; Acquaviva, Gawiser & Guaita 2011). One must also be aware of the influence of the chosen priors on the derived parameters; in cases where parameters are poorly constrained the prior can have a significant effect on the best-fit values (see e.g., Kauffmann et al. 2003, Salim et al. 2007, Taylor et al. 2011, for discussion).

Moreover, it is highly advisable to derive the 'best-fit' parameters from the marginalized posterior distribution, rather than from the minimum of  $\chi^2$ , since the likelihood surface can often be highly irregular (see e.g., Bundy, Ellis & Conselice 2005; Taylor et al. 2011).

When fitting SEDs it is important to remember that only the shape is being used to constrain the model parameters. In other words, the SED shape (or, more crudely, broadband colors) constrains parameters such as M/L, specific SFR (SSFR $\equiv$ SFR/M), dust attenuation and metallicity. To obtain stellar masses one needs to multiply M/L by the observed luminosity and to obtain SFRs one then multiplies SSFR by M. This also holds true for M/L ratios and SSFRs inferred from EWs of emission lines, spectral indices, and full spectral fitting.

The reader is referred to Walcher et al. (2011) for further details on SED fitting techniques.

# 3 MASS-TO-LIGHT RATIOS & STELLAR MASSES

#### 3.1 Techniques & Uncertainties

There are three basic techniques for estimating the stellar mass-to-light ratio, M/L, of a galaxy: (1) using tabulated relations between color and M/L; (2) modeling broadband photometry; (3) modeling moderate resolution spectra. The first technique is the simplest to use as it requires photometry in only two bands and no explicit modeling. The other techniques require construction of a library of models and a means to fit those models to the data. How do these different techniques compare?

3.1.1 Color-based M/L Ratios The color-based M/L estimators have their origin in the pioneering work of Bell & de Jong (2001). These authors used a preliminary version of the Bruzual & Charlot (2003) SPS model to chart out the relation between M/L and color as a function of metallicity and SFH. Remarkably, they found that variation in metallicity and SFH (parameterized as a  $\tau$  model) moved galaxies along a well-defined locus in the space of  $M/L_B$  vs. B-R, suggesting that the B-R color could be a useful proxy for  $M/L_B$ . Perhaps most importantly, they demonstrated that the dust reddening vector was approximately parallel to the inferred color-M/L relation, implying that dust should have only a second-order effect on derived M/L values. They also investigated NIR colors and found much larger variation in M/L at fixed I-K, due mostly to variation in the SFH. Late bursts of SF complicate the interpretation of color-M/L relations by driving the M/L ratios lower at fixed color compared to smoothly declining SFHs. Their analysis was for a fixed IMF; allowing for IMF variation will change the M/L while leaving the color basically unchanged. These authors concluded that for a fixed IMF and assuming that large starbursts and low-metallicity systems are not common properties of galaxies, and neglecting uncertainties in the SPS models, one could estimate M/L from a single color to an accuracy of  $\sim 0.1 - 0.2$  dex.

In subsequent work, Bell et al. (2003) analyzed the optical-NIR SEDs of  $\sim 12,000$  galaxies with SDSS and 2MASS photometry. They constructed a grid of SPS models with varying SFHs, including bursts, and metallicity. They did not allow for reddening due to dust. They derived best-fit M/L values from SED fitting and created new, observationally-constrained color-M/L relations. The resulting B-R vs.  $M/L_B$  relation was similar to Bell & de Jong (2001), but the NIR relation was considerably more shallow than the earlier work. At the bluest colors, the  $M/L_K$  ratios differed by  $\approx 0.3$  dex between the old and new relations. The

difference arose due to a population of galaxies with blue observed colors and high inferred M/L ratios, which Bell et al. (2003) interpreted as being due to the lower metallicities allowed in their fitting procedure compared to the analytic models of Bell & de Jong. These differences are of some concern because the Bell et al. (2003) color-M/L relations are very widely used to derive 'cheap' stellar mass estimates of galaxies.

Zibetti, Charlot & Rix (2009) revisited this issue with new SPS models from Charlot & Bruzual (in prep). They followed a somewhat different approach compared to Bell et al. (2003) in that they constructed color-M/L relations directly from a library of model galaxies. This approach is sensitive to how one chooses to populate parameter space, since the resulting color-M/L relations are simple averages over the model space. In contrast, Bell et al. (2003) constructed color-M/L relations only from those models that provided good fits to observed optical-NIR photometry. Zibetti et al. allowed for dust attenuation, and they also allowed for larger mass fractions of stars formed in recent bursts compared to Bell et al. Zibetti, Charlot & Rix (2009) found substantial differences in the q-i color-M/Lrelation compared to Bell et al. ( $\sim 0.5$  dex at the bluest colors), which they attributed to their allowance for much stronger bursts of star formation than allowed in Bell et al. (2003). Gallazzi & Bell (2009) also found a very different color-M/L relation compared to Bell et al. (2003), which they attributed to the different age distributions in their model library. Taylor et al. (2011) followed the philosophy of Bell et al. (2003) in fitting models to observed SEDs in order to derive color-M/L relations. They found systematic differences of order 0.1-0.2 dex compared to the Zibetti et al. relations, which they attributed mostly to their decision to select only those models that fit the photometric data, and also the treatment of dust reddening.

Zibetti, Charlot & Rix (2009) found a large difference in  $M/L_H$  ratios estimated from i-H colors between the new Charlot & Bruzual models and the previous version (Bruzual & Charlot 2003), with typical offsets of 0.2-0.4 dex. This difference is largely due to the different treatment of the TP-AGB evolutionary phase (see Section 3.2 for details). In contrast, optical color-M/L relations showed little difference between the two SPS models, and also showed slightly less discrepancy with Bell et al. (2003). These authors argued that optical color-M/L relations should generally be preferred to NIR-based ones owing to the larger impact of the uncertain TP-AGB phase in the NIR. Exceptions to this rule could be made for galaxies with very large amounts of dust reddening and/or very strong starbursts, in which cases the optical SED becomes a poor indicator of the SFH and hence M/L. The upshot is that NIR color-based M/L estimates should be treated with caution, since they are subject to large stellar evolution-based systematics. However, both optical and NIR-based M/L estimates will depend on the allowed range of SFHs and metallicities in the model library, with large systematics appearing for the bluest colors.

3.1.2 M/L From Broadband and Spectral Fitting Techniques The modern era of SED fitting with SPS models was ushered in by Sawicki & Yee (1998) and Giallongo et al. (1998), who analyzed optical-NIR broadband photometry of high-redshift galaxies. The model grid of Sawicki & Yee (1998) consisted of a  $\tau$ -model SFH with varying  $\tau$  and start time for the SFH, and variation in metallicity and reddening. Subsequent modeling of broadband data has largely followed this approach (e.g., Brinchmann & Ellis 2000, Papovich, Dickinson & Ferguson 2001, Shapley et al. 2001, Salim et al. 2007). Generically, when fitting broadband SEDs one finds that stellar masses for 'normal' galaxies (i.e., not including pathological SFHs) can be recovered at the  $\approx 0.3$  dex level (1 $\sigma$  uncertainty). This uncertainty does not include potential systematics in the underlying SPS models;

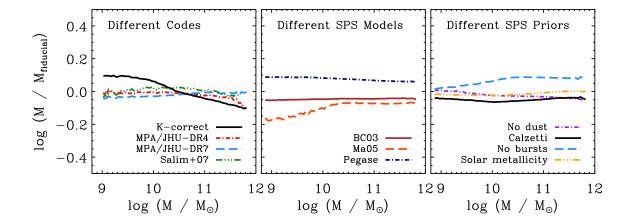


Figure 6:

Comparison of different fitting codes, SPS models, and priors on derived stellar masses (from Moustakas et al. 2012). The fiducial masses are based on fitting SDSS and GALEX photometry of  $z\sim 0$  galaxies using the iSEDfit code (Moustakas et al. 2012), with SSPs from FSPS (Conroy, Gunn & White 2009), including dust attenuation, a range in metallicities, and SFHs with both smooth and bursty components. The left panel compares stellar mass catalogs produced by different groups/codes. K-correct and MPA/JHU-DR7 are based on SDSS photometry; MPA/JHU-DR4 is based on SDSS spectral indices, and Salim+07 is based on SDSS and GALEX photometry. The middle panel shows the effect of different SPS models (i.e., different SSPs), and the right panel shows the effect of varying the priors on the model library. The mean systematic differences between mass estimators is less than  $\pm 0.2$  dex. Figure courtesy of J. Moustakas.

see Section 3.2. Stellar masses appear to be the most robust parameter estimated from SED fitting (e.g., Papovich, Dickinson & Ferguson 2001; Shapley et al. 2001; Wuyts et al. 2009; Muzzin et al. 2009; Lee et al. 2009b). The reason for this appears to not be fully understood, although it probably is at least partly due to the fact that the dust reddening vector is approximately parallel to SFH and metallicity variations in color-M/L diagrams (Bell & de Jong 2001). The choice of the SFH, in particular whether it is rising, declining, or bursty, can significantly change the best-fit stellar mass, by perhaps as much as 0.6 dex in extreme cases (Pforr, Maraston & Tonini 2012). A general rule of thumb is that M/L ratios estimated via simple SFHs (or single-age models) will be lower limits to the true M/L ratios (Papovich, Dickinson & Ferguson 2001; Shapley et al. 2001; Trager, Faber & Dressler 2008; Graves & Faber 2010; Pforr, Maraston & Tonini 2012). This is a consequence of the fact that young stars outshine older ones, making it relatively easy to 'hide' old stellar populations in galaxies with a large number of young stars. This also explains why color-M/L relations are so uncertain for very blue colors. This general rule is not universal: the modeling of certain galaxy types may lead to systematic biases in the opposite direction (Gallazzi & Bell 2009). Partly for these reasons, stellar masses estimated for quiescent systems should be more reliable than for star-forming ones. Relatedly, Zibetti, Charlot & Rix (2009) demonstrated that stellar masses estimated for individual nearby galaxies based on a pixel-by-pixel analysis of colors are generally larger than those estimated from integrated colors. The differences are typically  $\sim 0.05-0.15$  dex, depending on galaxy type. This should not be surprising, especially for systems with star-forming disks and quiescent bulges, since the M/L ratio estimated from the integrated colors will tend to be biased low by the more luminous young stars in the star-forming component. Wuyts et al. (2012) performed a similar analysis on galaxies at 0.5 < z < 2.5 with WFC3 Hubble Space Telescope (HST) photometry and found no systematic difference in stellar masses derived from resolved vs. integrated light, although the fact that their galaxies were at high redshift means that they were probing spatial scales an order of magnitude larger than in Zibetti et al.

There has been some confusion in the literature regarding the importance of restframe NIR photometry for estimating stellar masses. First, as indicated in Figure 5, for smoothly varying SFHs, photometry at and redward of the V-band is sensitive to the same lightweighted age, and so redder bands do not provide stronger constraints on the mean stellar age. Taylor et al. (2011) analyzed mock galaxies and concluded that the addition of NIR data did not yield more accurate masses. Taylor et al. also found that different SPS models produced good agreement in derived properties when NIR data was excluded from their fits, but poor agreement when NIR was included. These authors also found much larger residuals in their SED fits when NIR data were included, suggesting that the models are still poorly calibrated in this regime. As discussed in Section 5.2, the NIR is at present probably most useful for constraining metallicities (within the context of a particular SPS model), and so NIR data may be useful in cases where there is a degeneracy between M/L and Z. In general however stellar mass estimates do not appear to be strongly improved with the addition of NIR data, at least with currently available models. Exceptions to this rule may be made for galaxies with very high dust opacities.

As first emphasized by Bell & de Jong (2001) and Bell et al. (2003), some of the largest uncertainties in derived M/L ratios stem from uncertainties in the assumed SFHs, in particular the presence of bursty SF episodes. The consideration of the Balmer lines with other age and metallicity-sensitive features, such as the 4000 Å break  $(D_n4000)$ , can constrain the burstiness of the SFH. Optical spectra therefore offers the possibility of providing stronger constraints on the M/L ratio. Kauffmann et al. (2003) modeled the H $\delta$  and  $D_n4000$  spectral features measured from SDSS spectra in order to constrain SFHs and M/L ratios. They obtained 95% confidence limits on stellar masses of  $\sim 0.2$  dex and  $\sim 0.3$  dex for quiescent and star-forming galaxies, respectively. Again, these are statistical uncertainties because the underlying SPS model and other aspects such as the adopted parameterization of the SFH were held fixed. Chen et al. (2012) employed principle component analysis (PCA) to model the optical spectra of massive galaxies in SDSS. These authors found systematic uncertainties in the recovered stellar masses of order  $\sim 0.1$  dex depending on the assumed metallicity, dust model, and SFH. They also demonstrated that their PCA technique was capable of measuring parameters at much lower S/N than direct fitting to selected spectral indices. They determined statistical uncertainties on their mass measurements to be  $\sim 0.2$ dex, which they showed to be comparable to the formal errors estimated from modeling broadband photometry. This result is in agreement with Gallazzi & Bell (2009), who argued that for galaxies with simple SFHs and lacking dust, the uncertainty on the derived masses are not much larger when using color-based estimators compared to spectroscopicbased estimators. With regards to estimating M/L, the real value of spectra appears to be restricted to galaxies with unusual SFHs.

The comparison between spectroscopically-based and photometrically-based stellar masses is informative because the approaches suffer from different, though by no means orthogonal systematics. The most obvious difference is with regards to dust — spectroscopic masses are much less sensitive to dust attenuation than photometric masses. Drory et al.

(2004) compared their own photometrically-derived masses to the spectroscopic masses from Kauffmann et al. (2003). They found an rms scatter of  $\sim 0.2$  dex between the two estimates, with a modest systematic trend that correlated with H $\alpha$  EW. Blanton & Roweis (2007) estimated photometric stellar masses via a technique that is similar to PCA except that the templates are constrained to be non-negative and are based on an SPS model. They compared their derived stellar masses to those of Kauffmann et al. and found good agreement, with systematic trends between the two restricted to  $\lesssim 0.2$  dex.

A comparison between stellar masses for the same galaxies estimated with different SPS models, priors, and fitting techniques is shown in Figure 6 (from Moustakas et al. 2012). The galaxies included in this figure are predominantly 'normal' star-forming and quiescent  $z\sim 0$  galaxies. For these galaxies the mean absolute differences between various mass estimators is less than 0.2 dex, in agreement with other work on the systematic uncertainties in stellar masses of normal galaxies.

## 3.2 Uncertainties in M/L due to Stellar Evolution Uncertainties

Maraston et al. (2006) was the first to draw attention to the sensitivity of derived stellar masses to uncertain stellar evolutionary phases, in particular the TP-AGB. The Maraston (2005) SPS model predicts much more luminosity arising from TP-AGB stars than in previous models (e.g., Fioc & Rocca-Volmerange 1997, Bruzual & Charlot 2003). At ages where the TP-AGB phase is most prominent ( $\sim 3 \times 10^8 - 2 \times 10^9$  yr), Maraston's model predicts roughly a factor of two more flux at  $> 1\,\mu m$  compared to earlier work. Maraston et al. (2006) found that her SPS model implied a factor of two smaller stellar mass, on average, compared to masses derived with the Bruzual & Charlot (2003) SPS model for galaxies at  $z \sim 2$ . When the fits excluded the possibility of dust reddening, Maraston's model provided a better fit to the optical-NIR SEDs than previous models, whereas when dust was allowed, the quality of the fits became indistinguishable, although the offsets in best-fit masses remained.

Subsequent work has largely confirmed the sensitivity of estimated stellar masses to the adopted SPS model (Wuyts et al. 2007, Kannappan & Gawiser 2007, Cimatti et al. 2008, Muzzin et al. 2009, Longhetti & Saracco 2009, Conroy, Gunn & White 2009). Essentially all work on this topic has focused on comparing Maraston's model to Bruzual & Charlot's, where the difference in TP-AGB treatment it probably the most significant, though not the only difference (other differences include different RGB temperatures and different treatments for core convective overshooting). Many authors have found a maximum factor of  $\sim 2-3$  difference in derived M/L ratios between the two models, especially when NIR data were included. Kannappan & Gawiser (2007) showed that the differences between Maraston's and Bruzual & Charlot's models are relatively modest ( $\lesssim 1.3$ ) when NIR data are excluded. Conroy, Gunn & White (2009) constructed a new SPS model in which the luminosity contribution from the TP-AGB phase could be arbitrarily varied. These authors isolated the importance of the TP-AGB phase and confirmed previous work indicating that the adopted weight given to this phase in the models can have a large modulating effect on the stellar mass.

The contribution of TP-AGB stars to the integrated light peaks at  $\sim 3 \times 10^8 - 2 \times 10^9$  yr, depending on metallicity, and so the importance of this phase to modeling SEDs will depend on the SFH of the galaxy. It was for this reason that Maraston et al. (2006) focused their efforts on quiescent galaxies at  $z \sim 2$ ; at this epoch even a quiescent galaxy will have a typical stellar age not older than several Gyr. In addition to high-redshift quiescent galaxies,

Lançon et al. (1999) suggested that post-starburst galaxies should harbor large numbers of TP-AGB stars because their optical spectra show strong Balmer lines, indicative of large numbers of  $10^8-10^9$  yr old stars. This fact was exploited by Conroy & Gunn (2010), Kriek et al. (2010), and Zibetti et al. (2012) to constrain the TP-AGB contribution to the integrated light. These authors all find evidence for a low contribution from TP-AGB stars, and in particular they argue that Maraston's models predict significantly too much flux in the NIR for these objects.

The situation with TP-AGB stars is extremely complex owing to the fact that this phase is so sensitive to age and metallicity. Moreover, using galaxy SEDs to constrain the importance of this phase is difficult because many parameters must be simultaneously constrained (metallicity, SFH, dust, etc.). A case in point is the analysis of high-redshift galaxies, where Maraston et al. (2006) found large differences in the quality of the fit of Bruzual & Charlot models depending on whether or not dust was included in the fits (as an aside, if these galaxies do have copious numbers of TP-AGB stars then one may expect them to also contain dust, since these stars are believed to be efficient dust factories). It is also worth stressing that the conclusion to the TP-AGB controversy may not be an 'either-or' situation in the sense that some models may perform better for some ranges in age and metallicity while other models may perform better in different regions of parameter space. Ultimately of course we desire models that perform equally well over the full range of parameter space, and this requires a continual evolution and improvement of SPS models.

Of course, there are other aspects of stellar evolution that are poorly constrained, including convective overshooting, blue stragglers and the HB, and even the temperature of the RGB. The propagation of these uncertainties into derived properties such as M/L ratios is only just beginning (e.g., Conroy, Gunn & White 2009). Melbourne et al. (2012) has for example demonstrated that the latest Padova isochrones fail to capture the observed flux originating from massive core He burning stars as observed in nearby galaxies. These stars are very luminous and can dominate the NIR flux of young stellar populations ( $\lesssim 300 \text{ Myr}$ ).

Finally, it is worth stressing that the effect of TP-AGB stars is limited to galaxies of a particular type, in particular those that are dominated by stars with ages in the range  $\sim 3\times 10^8-2\times 10^9$  yr. Examples include post-starburst galaxies, which are rare at most epochs, or quiescent galaxies at  $z\gtrsim 2$ . For typical galaxies at  $z\sim 0$ , the treatment of this phase seems to be of little relevance for estimating stellar masses, at least on average, as is evident in Figure 6.

#### 3.3 Stellar Masses at High Redshift

As first emphasized by Papovich, Dickinson & Ferguson (2001), galaxies with high SFRs can in principle contain a large population of 'hidden' older stars, since these stars have high M/L ratios. Papovich et al. found that the data allowed for significantly larger stellar masses when two-component SFH models (young+old) were compared to their fiducial single component models (a median difference of a factor of  $\approx 3$  with extreme cases differing by an order of magnitude). This issue becomes more severe at higher redshifts because the typical SFRs are higher and because of the increasing possibility of rising SFHs (to be discussed in detail in Section 4). This makes the analysis of high redshift galaxy SEDs much more complicated.

However, at the highest redshifts ( $z \gtrsim 6$ ), the analysis of SEDs may actually become simpler. At z=8 the age of the universe is only  $\approx 640$  Myr, and if one presumes that galaxy formation commences after  $z\approx 20$ , then even the oldest stars at z=8 will be no

more than  $\approx 460$  Myr old. The oldest possible main sequence turnoff stars will thus be A type stars. Put another way, the M/L ratio in the blue (U through V bands) at 13 Gyr is 4.5-15 times higher for an instantaneous burst of SF compared to a constant SFH, whereas the M/L ratio for these two SFHs differs by the more modest 2.5-5 at 500 Myr. So there is some expectation that 'hidden mass' will be less hidden when modeling SEDs at the highest redshifts. This expectation appears to be borne out by detailed modeling. Finkelstein et al. (2010) analyzed the SEDs of  $z\sim7-8$  galaxies and concluded that even allowing for the possibility of extreme amounts of old stars (90% by mass), the best-fit stellar masses increased by no more than a factor of two compared to a fiducial single-component SFH (see also Curtis-Lake et al. 2012).

At even higher redshifts ( $z \gtrsim 10$ ), the analysis of galaxy SEDs may become even simpler, at least with regards to estimating stellar masses. At sufficiently high redshifts the age of the oldest possible stars will eventually become comparable to the SF timescale probed by the restframe UV ( $\sim 10^8$  yr at  $\sim 2500$  Å; see Figure 5). The measured SFR will thus be averaged over roughly the entire age of the galaxy, implying that a reasonable estimate of the stellar mass can be obtained by multiplying the UV-derived SFR with the age of the universe. This may in fact explain the unexpectedly strong correlation between stellar mass and UV luminosity at z > 4 reported by González et al. (2011). Of course, at the highest redshifts other difficulties arise, including accounting for the effects of very high EW emission lines and the reliability of the underlying models at very low metallicities.

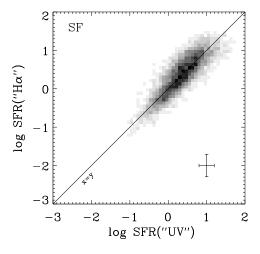
#### 3.4 Summary

M/L ratios for most galaxies with normal SEDs are probably accurate at the  $\sim 0.3$  dex level, for a fixed IMF, with the majority of the uncertainty dominated by systematics (depending on the type of data used). Galaxies with light-weighted ages in the range of  $\sim 0.1-1$  Gyr will have more uncertain M/L ratios, with errors as high as factors of several, owing to the uncertain effect of TP-AGB stars and perhaps other stars in advanced evolutionary phases. Galaxies that have very young light-weighted ages (e.g., high-redshift and starburst galaxies) will also have very uncertain M/L ratios because of the difficulty in constraining their past SFHs. Uncertainties due to dust seem to be subdominant to other uncertainties, at least for standard reddening laws and modest amounts of total attenuation. Essentially all stellar masses are subject to an overall normalization offset due both to the IMF and the uncertain contribution from stellar remnants. Owing to the lingering model uncertainties in the NIR (e.g., from TP-AGB and cool core He burning stars), it may be advisable to derive physical parameters by modeling the restframe UV-optical (see e.g., the discussion in Taylor et al. 2011).

# 4 STAR FORMATION RATES, HISTORIES, & STELLAR AGES

#### 4.1 Measuring SFRs

The practice of measuring SFRs from monochromatic indicators (and combinations thereof) is well established (see e.g., the reviews of Kennicutt 1998, Kennicutt & Evans 2012). A wide variety of relations have been established between SFRs and the UV,  ${\rm H}\alpha$ ,  ${\rm P}\alpha$ ,  ${\rm 24}\,\mu m$  luminosity, total IR luminosity, radio continuum luminosity, and even the X-ray flux. As they are not the focus of the present review, only a few brief remarks will be given regarding monochromatic SFR indicators. Each indicator is sensitive to a different SFR timescale,



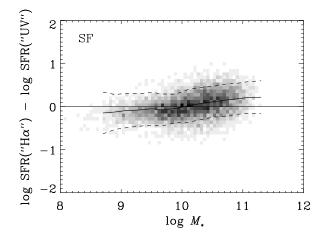


Figure 7:

Comparison between SFRs estimated from emission lines ("H $\alpha$ "; Brinchmann et al. 2004) and from modeling the UV-optical SEDs ("UV"; Salim et al. 2007) of star-forming galaxies at  $z \sim 0$ . The right panel shows the difference between SFR estimators as a function of stellar mass. There is a systematic trend between the two estimators, which may be due to modeling differences and/or the uncertainty in extrapolating the fiber-based H $\alpha$  SFRs to the total galaxy. From Salim et al. (2007).

with ionizing radiation (as probed by e.g., the recombination lines) being sensitive to the shortest timescale,  $\sim 10^6$  yr, and the UV and total IR being sensitive to longer timescales, e.g.,  $\sim 10^8$  yr. In detail, the UV and IR are sensitive to a range of SF timescales depending on wavelength and SFH. In the IR this sensitivity arises because shorter wavelengths are probing dust heated mostly by H II regions, while FIR photons are sensitive to dust heated both by old and young stars. Amongst the recombination lines, 'redder is better' since dust attenuation will be less severe at longer wavelengths, and so  $P\alpha$  should be preferred over  $H\alpha$ . For reference, an attenuation of 1 mag in the V-band implies an attenuation of only 0.15 mag at  $P\alpha$ , for standard attenuation curves. The optical and UV SFR indicators require correction for dust when used on their own. For this reason, mixed SFR indicators have become popular, usually via the combination of UV and IR or  $H\alpha$  and IR indicators (Calzetti et al. 2007, Kennicutt et al. 2007).

Estimating SFRs from SED fitting is very challenging for several reasons: (1) the age-dust-metallicity degeneracy makes it difficult to reliably measure ages and hence SFRs unless high quality data are available (see Section 6); (2) the choice of model priors on the dust model and SFH library imposes often severe biases on the resulting ages and SFRs. These difficulties were recognized early on in the study of high-redshift galaxies (Papovich, Dickinson & Ferguson 2001) and are still relevant today.

Brinchmann et al. (2004) measured SFRs for SDSS galaxies by modeling a suite of optical emission lines. Their model included variation in the total metallicity, dust attenuation, ionization parameter, and dust-to-metals ratio. As the SDSS is a fiber-based survey, these authors had to make a correction for the fact that the spectra sample only a fraction of the galaxy light. In addition, they modeled the stellar continuum in order to subtract off any absorption features under the emission lines. Salim et al. (2007) analyzed a comparably

sized sample of  $z\sim 0$  galaxies, focusing on UV photometry from the Galaxy Evolution Explorer (GALEX) and optical photometry from SDSS. These authors fit the data to SPS models with a range of SFHs (including exponentially declining SFHs and superimposed starbursts), metallicities, and dust attenuation. They presented a direct comparison between their SED-based SFRs and the emission line-based SFRs from Brinchmann et al. The result is shown in Figure 7. While the overall agreement is good there is a systematic trend with stellar mass resulting in offsets of  $\sim 0.2$  dex at the low and high mass ends. Salim et al. explored the origin of this offset and concluded that the different derived dust attenuation optical depths between the two methods were the source of the discrepancy. They further speculated that in most cases it was the emission line-based dust attenuation values that were more accurate, and therefore also the SFRs. Salim et al. also demonstrated that for galaxies with low SFRs and/or galaxies with emission line ratios indicative of AGN activity, the UV-based SFRs tended to be more reliable than those estimated by Brinchmann et al. These results suggest that SFRs based on modeling UV-optical SEDs carry systematic uncertainties at the  $\lesssim 0.3$  dex level.

Wuyts et al. (2011) compared the SFRs of galaxies at  $z\sim 3$  computed with three different techniques: H\$\alpha\$-based SFRs, SED-fitting based SFRs, and a mixed SFR indicator based on the UV and IR emission. For SFRs  $\lesssim 100\,M_\odot\,\mathrm{yr}^{-1}$ , these authors found that the three indicators agreed reasonably well (no systematic offsets were detected), provided that extra attenuation was included toward H II regions when modeling the H\$\alpha\$-based SFRs. They also found that at the highest SFRs  $\gtrsim 100\,M_\odot\,\mathrm{yr}^{-1}$  the SED-based values tended to underpredict the SFR, compared to the mixed indicator (see also Santini et al. 2009). They argued that this was due to their assumption of a uniform dust screen, and at high column density this becomes an increasingly poor representation of reality.

At high redshift the effect of nebular emission lines becomes an additional source of systematic uncertainty in the modeling, and can significantly affect the best-fit SFRs. The impact of emission lines is larger at higher redshift because of three effects: (1) specific SFRs are higher; (2) metallicities are lower; and (3) the redshifting of the spectrum implies that a line of given EW comprises a larger fraction of the flux through a filter. In Figure 8 the specific SFR for galaxies with a mass of  $10^{9.5} M_{\odot}$  is shown as a function of redshift for various choices regarding the contribution of emission lines to the observed broadband fluxes. Evidently the inclusion or not of emission lines in the modeling can change the derived specific SFRs in a redshift-dependent manner, by a factor of roughly two in the most extreme cases (see also Pacifici et al. 2012, Labbe et al. 2012).

#### 4.2 Light-Weighted Mean Ages

The mean stellar ages of galaxies are another important probe of the SFH. Of course, mass-weighted ages offer a direct probe of the integrated SFH, while light-weighted ages are more directly measurable from SEDs. Mass-weighted ages will always be larger than light-weighted ones, owing to the fact that young stars outshine older stars. As discussed further in the next section, the mass-weighted ages of actively star-forming galaxies turn out to be highly sensitive to the assumed model SFH used in the SED fitting. Due to the fact that light-weighting is highly non-linear, light-weighted ages computed from spatially resolved photometry will tend to be older than ages computed from integrated photometry, as demonstrated by Wuyts et al. (2011).

The mean ages of quiescent galaxies have been estimated largely via the modeling of spectral features, including  $D_n4000$  and the Lick indices. A widely adopted procedure is

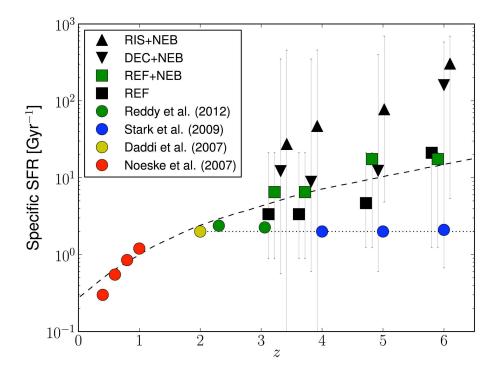


Figure 8:

Specific SFRs for galaxies with stellar masses of  $10^{9.5}\,M_{\odot}$  as function of redshift. The black and green symbols reflect different assumptions regarding the SFH ('REF', 'RIS', 'DEC' for constant, rising, and declining) and the correction for emission lines ('NEB'). Error bars reflect 68% confidence limits. From de Barros, Schaerer & Stark (2012)

to fit single-age SSPs to the Lick indices in order to estimate ages. In this approach it is the hydrogen Balmer lines that provide the primary constraint on the age, as these lines are strongest in A type stars and thus for populations with ages of  $\approx 10^8-10^9~\rm yr.$  The derived ages are sometimes referred to as 'SSP-equivalent ages' in order to highlight the fact that no attempt is made to model composite stellar populations. A number of authors have demonstrated that these derived ages are lower limits to the true mass-weighted and light-weighted ages (Trager et al. 2000a, Serra & Trager 2007, Trager & Somerville 2009, MacArthur, González & Courteau 2009). As emphasized by Trager & Somerville (2009), this fact implies that any observed trend of SSP-equivalent ages with other parameters such as velocity dispersion will be stronger than the true underlying mass-weighted trend. The SSP-equivalent ages are younger even than light-weighted ages largely because of the sensitivity of the Balmer lines to  $\sim 1~\rm Gyr$  old stars. Trager & Somerville (2009) concluded that SSP-equivalent ages are mostly measuring residual star formation over  $\sim 1~\rm Gyr$  timescales.

These conclusions are not restricted to the analysis of spectral indices. Ferreras et al. (2009) estimated mean ages of early-type galaxies at  $z \sim 1$  by fitting models to HST grism spectra. The ages estimated from single-age models showed almost no correlation with

ages estimated with composite models. Likewise, MacArthur, González & Courteau (2009) performed full spectrum fitting on a sample of nearby galaxies and found only a modest correlation between ages derived from single-age models and light-weighted ages derived from composite models.

It is widely believed that the H $\beta$  index is the ideal age indicator because it is largely insensitive to metallicity and abundance patterns (Worthey 1994, Tripicco & Bell 1995, Korn, Maraston & Thomas 2005). However, as noted by Worthey (1994) and later by de Freitas Pacheco & Barbuy (1995), Maraston & Thomas (2000), and Lee, Yoon & Lee (2000), the Balmer lines are also sensitive to the presence of blue HB stars, and there will therefore be some ambiguity in determining ages from  $H\beta$  without additional constraints on the presence of such stars. However, Trager et al. (2005) demonstrated that, while the inferred ages of the oldest stellar populations ( $\gtrsim 10 \text{ Gyr}$ ) are susceptible to uncertainties of 2-5 Gyr due to the presence of blue HB stars, the ages of intermediate-age populations  $(\sim 1-10 \text{ Gyr})$  are rather less affected by blue HB stars. The reason for this is that the Balmer lines increase so strongly with decreasing age that very large quantities of blue HB stars would be required to be present in old stellar populations to reproduce the high observed H $\beta$  EWs. As an example, Thomas et al. (2005) found that they could reproduce the highest H $\beta$  EWs in their sample with a model in which 50% of the old metal-rich stars had blue HB stars. In this two-component model, the component with blue HB stars needed to have a metallicity 0.2 - 0.4 dex higher than the other component. Percival & Salaris (2011) also found that models with very extreme HB populations and old ages could masquerade as a relatively young population with ages as young as  $\sim 3$ Gyr, based on  $H\beta$  EWs. There is no compelling evidence supporting the existence of a population of blue HB stars in metal-rich quiescent galaxies large enough to explain the full range of  $H\beta$  EWs. However, it is reasonable to assume that there will be a population of blue HB stars in such systems, and they will affect the derived stellar ages.

The higher order Balmer lines are promising because they will be even more sensitive to warm/hot stars than H $\beta$ . In principle then, the consideration of multiple Balmer lines should provide constraints on multiple age components, and/or allow for the separation of age and blue HB effects (Schiavon et al. 2004, Schiavon 2007, Serra & Trager 2007). However, the potential power of the higher-order Balmer lines is tempered by the fact that these features are more sensitive to metallicity and abundance ratios than H $\beta$  (Thomas, Maraston & Korn 2004; Korn, Maraston & Thomas 2005). Schiavon (2007) analyzed the H $\beta$ , H $\gamma$ , and H $\delta$  indices measured from stacked SDSS early-type galaxy spectra with a variety of composite models including varying blue HB stars, blue stragglers, and multi-age populations. He concluded that small fractions of young/intermediate-age stellar populations, perhaps in the form of extended SFHs, was the most likely scenario to explain the observed trends. This proposal is similar, though not identical to the 'frosting' of young stars proposed by Trager et al. (2000a) in order to explain the observed correlations between velocity dispersion, age, metallicity, and abundance pattern.

There exist other proposed methods for constraining multi-age components based on spectral indices. Rose (1984) proposed a Ca II index that is sensitive to the H $\epsilon$  Balmer line. Leonardi & Rose (1996) demonstrated that this index in conjunction with H $\delta$  can break the 'age-strength degeneracy', referring to the age and strength of a recent burst of star formation. Smith, Lucey & Hudson (2009) used this diagnostic to argue against a frosting of small fractions of young stars (< 1 Gyr) to explain the young apparent ages for quiescent galaxies in the Shapley supercluster. Kauffmann et al. (2003) considered both the H $\delta$  and  $D_n4000$  spectral features in an attempt to measure two-component SFHs (in

particular, a burst of SF and an underlying  $\tau$  model SFH). Galaxies with a sizable mass fraction formed in a recent burst of SF are significant outliers in the H $\delta$ - $D_n$ 4000 plane, and so strong constraints on the burst fraction can be obtained in this way. Percival & Salaris (2011) argued that the Ca II Rose index and the Mg II feature near 2800 Å could also be used to separate age from blue HB effects.

Renzini & Buzzoni (1986) suggested that the onset of the AGB in  $\sim 0.1-1$  Gyr old populations could be used to age-date galaxies in the NIR. Subsequent modeling efforts by Maraston (2005) confirmed that NIR spectral features attributable to oxygen-rich and carbon-rich AGB stars vary strongly over the age range  $\sim 0.1-1$  Gyr, and may be used as an independent constraint on the presence of intermediate-age populations in galaxies. In principle, the very strong sensitivity of AGB evolution to mass (age) and metallicity suggests that they should be good clocks. However, their sensitivity to mass and metallicity is due to poorly understood physics, including convection and mass-loss, and so the calibration of an AGB-based clock is quite uncertain. Miner, Rose & Cecil (2011) attempted to use NIR spectral features to place constraints on the ages of M32 and NGC 5102 based on Maraston's models. They found broad qualitative agreement between NIR-based ages and CMD-based ages. Future quantitative modeling of the optical-NIR spectra will be required to assess the utility of AGB-sensitive spectral indices as age indicators.

Finally, it is also worth emphasizing that it is fundamentally very difficult to measure ages for systems older than  $\gtrsim 9$  Gyr because evolution in the isochrones is so slow at late times. For example, at solar metallicity between 9-13.5 Gyr, the RGB changes by only  $\approx 50$  K while the main sequence turnoff point changes by  $\approx 230$  K. Very accurate models and very high quality data are required to be able to distinguish such small temperature variations. In theory the UV should be much more sensitive to age at late times due to the onset of an extended (blue) HB (Yi et al. 1999). In the context of a particular model, the UV colors become very sensitive to age for ages  $\gtrsim 7$  Gyr. The problem with this chronometer is that it is very sensitive to the uncertain physical inputs, such as the mass-loss prescription and the helium abundance, and also to the underlying distribution of stellar metallicities (Yi, Demarque & Oemler 1997). These uncertainties will render absolute ages unreliable, although relative ages (or even simply the rank-ordering of galaxies by age) are probably more robust.

# 4.3 Which Model SFHs to Use?

Papovich, Dickinson & Ferguson (2001) recognized that SED modeling suffered from a fundamental limitation, namely that the young stars, being so bright, tend to outshine the older, less luminous stars. This outshining effect was known to result in underestimated stellar masses when single-component SFH models are used (e.g., Papovich, Dickinson & Ferguson 2001; Daddi et al. 2004; Shapley et al. 2005; Lee et al. 2009b; Pforr, Maraston & Tonini 2012), but its effect on recovered SFRs and ages remained relatively less well-explored until recently. Lee et al. (2009b) analyzed the SEDs of high-redshift mock galaxies and concluded that the use of single-component exponentially-decreasing SFHs ( $\tau$  models) resulted in substantially underestimated SFRs and overestimated mean ages (by factors of two for both parameters). They attributed this shortcoming to the fact that the mock galaxies tended to have rising SFHs in contrast to the declining model SFHs. Subsequent work by Lee et al. (2010), Maraston et al. (2010), Wuyts et al. (2011), and Pforr, Maraston & Tonini (2012) largely confirmed these findings, and further concluded that models with rising SFHs tended to provide a better fit to observed high redshift SEDs and produced SFRs in better agree-

ment with other indicators. Relatedly, the choice of the allowed range of each SFH parameter (i.e., the prior) can significantly affect the inferred best-fit SFRs and ages (e.g., Lee et al. 2010, Wuyts et al. 2011).

A specific example of the effect of the adopted SFH parameterization on SFRs is shown in Figure 8. In this figure the specific SFR is shown as a function of redshift at a reference stellar mass of  $10^{9.5} M_{\odot}$ . At high redshift the scatter between the "RIS", "DEC", and "REF" models is entirely due to the choice of the assumed SFH (rising, declining, or constant, respectively). At the highest redshifts these different choices result in a change in specific SFRs of more than an order of magnitude.

There has been some difference in opinion regarding which functional form one should adopt for the rising SFHs. Maraston et al. (2010), and Pforr, Maraston & Tonini (2012) advocated exponentially increasing SFHs while Lee et al. (2010) advocated delayed  $\tau$  models of the form SFR $\propto t\,e^{-t/\tau}$ . The data do not presently favor one functional form over the other. The basic conclusion to draw from these analyses is that the model SFH library must be sufficiently diverse to allow for a wide range in SFH types. Model SFHs that are too restricted will inevitably lead to biased results. Of course, the criterion for 'sufficiently diverse' SFHs will depend on the galaxy type in question and has not been thoroughly examined in a general sense. At present, all that can be stated with confidence is that for certain SED types, especially highly star-forming galaxies, great care must be taken when constructing the model library of SFHs.

Another option is to use a library of SFHs drawn from semi-analytic or hydrodynamic models of galaxy formation. This technique was introduced by Finlator, Davé & Oppenheimer (2007), who analyzed SEDs of six high redshift galaxies with SFHs drawn from their hydrodynamic simulations. The advantage of this approach is that the model SFH library is more likely to contain realistic SFHs (with rising and falling SFHs, multiple bursts, etc.) than simple analytic models. Recently, Pacifici et al. (2012) explored a similar technique using SFHs from a semi-analytic model rather than a hydrodynamic simulation. The advantage of using a semi-analytic model is that a much larger range of SFHs can be computed in a given amount of computational time. As with any other SFH library, the results can be sensitive to the choice of priors. In this case, the 'prior' is in some sense the reliability of the model in producing a realistic range of SFHs. A possible disadvantage of this approach is that it can be much more difficult to quantify the influence of this prior on the derived physical properties than when using analytic SFH libraries.

#### 4.4 Non-Parametric Star Formation Histories

When modeling SEDs one typically must adopt some parameterization of the SFH, and it is this step that clearly introduces a number of poorly quantified systematics. An appealing solution to this problem is to model SEDs with non-parametric SFHs. This is precisely the approach taken by several groups, including MOPED (Heavens, Jimenez & Lahav 2000), STARLIGHT (Cid Fernandes et al. 2005), STECMAP (Ocvirk et al. 2006), VESPA (Tojeiro et al. 2007), ULySS (Koleva et al. 2009), and MacArthur, González & Courteau (2009). In the simplest implementation one specifies a fixed set of age bins and then fits for the fraction of mass formed within each bin. More sophisticated algorithms utilize adaptive age binning that depends on the SED type and S/N (e.g., Tojeiro et al. 2007). In all cases one is engaging in full spectrum fitting (in the restframe optical; for an example of the technique see Figure 9). In principle one could use such codes to fit broadband SEDs, but the fits would likely be seriously under-constrained. These methods have been tested

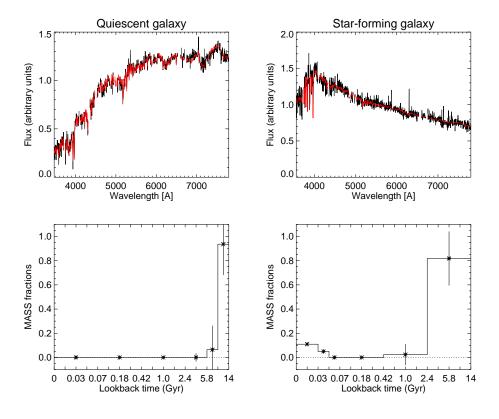


Figure 9:

Example fits to the optical spectrum of a quiescent and a star-forming galaxy with VESPA (Tojeiro et al. 2009). The SFHs are estimated non-parametrically with an age binning that is adaptive depending on the S/N and information content of the spectrum. The upper panels show the best-fits (red line is the model, black line is the data; emission lines have been excluded from the fits); lower panels show the recovered SFHs plotted as the mass fraction formed within each age bin. Figure courtesy of R. Tojeiro.

in a variety of ways, and it appears that they are capable of recovering complex SFHs remarkably well, at least when the mock galaxies are built with the same SPS models as used in the fitting routines (Ocvirk et al. 2006, Tojeiro et al. 2007, Koleva et al. 2009). Not surprisingly, the requirements on the data quality are demanding: a wide wavelength coverage, high spectral resolution, and high S/N (typically > 50/Å) are required in order to robustly recover complex SFHs (Ocvirk et al. 2006, Tojeiro et al. 2007). In addition, there are clear systematics in the recovered SFHs due to different SPS models, particularly for the SFHs at ages of  $\sim 0.1-1$  Gyr (Tojeiro et al. 2009, 2011). This is perhaps related to the different treatment of AGB stars and/or core convective overshooting amongst SPS models. Moreover, current implementations of non-parametric SFH recovery utilize SPS models that only allow variation in metallicity and age — the abundance pattern is fixed to the solar value. When fitting to spectra this practice can introduce additional systematics since many of the features that are being fit are sensitive to the detailed abundance pattern of the system.

There have been few comparisons between the SFHs derived via non-parametric methods

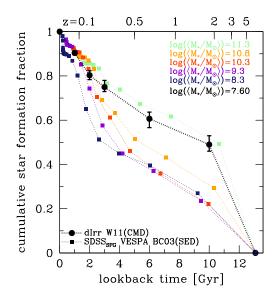


Figure 10:

Comparison between SFHs estimated from integrated light SEDs to SFHs estimated from CMDs. The SED-based SFHs are based on SDSS spectra, as derived in Tojeiro et al. (2009), and binned by stellar mass. The CMD-based SFHs are from Weisz et al. (2011) based on data from the ANGST survey and are for low-mass dwarf irregulars. It is striking that the CMD-based SFHs for low-mass dwarfs look closer to the SED-based SFHs for the most massive galaxies. From Leitner (2012).

with more conventional techniques. To jeiro et al. (2009) compared SFRs estimated with VESPA to emission line-based SFRs from Brinchmann et al. (2004). No obvious discrepancies were found, but further detailed analyses would be desirable. Moreover, there appears to be no published work comparing the mean stellar ages (or some other moment of the SFH) between these various techniques.

SFHs from CMDs versus SFHs from Integrated Light Ideally one would like an independent constraint on the SFH to compare against the SFH derived from integrated light spectra. Unfortunately no direct tests of this sort have been performed to date. The most robust method for determing SFHs is via the modeling of resolved color magnitude diagrams of the stars within galaxies. This technique is well-developed, and has recently been applied by Weisz et al. (2011) to a large sample of dwarf galaxies in the local volume compiled by the ACS Nearby Galaxy Treasury Survey (ANGST; Dalcanton et al. 2009). Leitner (2012) compared the SFHs derived by Weisz et al. (2011) to the SFHs derived from integrated light spectra by Tojeiro et al. (2009). The result is shown in Figure 10. The dwarf galaxies from the ANGST Survey have masses slightly lower than the lowest mass bin in the integrated light sample, and it is worth emphasizing that there is no overlap between the two samples. And yet, the dwarfs have SFHs comparable to the highest mass bin in the integrated light sample. There are many complications in making this comparison, not the least of which is the question of whether or not nearby dwarfs are representative of the cosmological average. Nonetheless, the difference in the qualitative trend derived from CMD-based and integrated light-based SFHs is striking and worthy of further study.

#### 4.5 Low-Level Star Formation

Probing low levels of star formation in galaxies is challenging because at low levels classic SF indicators can become contaminated by other processes. At low luminosities, emission lines can be influenced by low-level AGN activity and they become difficult to measure on top of the strong absorption features in the continuum, dust emission can be due to heating from old stars (rather than tracing the formation of new stars), and ultraviolet light can be attributed to hot evolved low-mass stars. The former two indicators have been effectively abandoned as low-level SF indicators, while the third, UV light, still holds some promise.

It is beyond the scope of this review to discuss in detail the complications in interpreting UV (especially FUV) emission from predominantly quiescent systems, but a few remarks will serve to round out this section. The reader is referred to O'Connell (1999) for a thorough review, and Yi (2008) for a recent update. Low-level UV emission from galaxies can be due either to young massive stars or to hot evolved low-mass stars. In the latter category includes extreme HB stars, post-AGB stars, AGB-Manqué, etc. Extreme HB stars are known to exist in essentially all low metallicity and some high metallicity globular clusters. Post-AGB stars should exist at all metallicities, and AGB-Manqué are thought to occur only in special circumstances. It is very difficult to predict the specific numbers of these stars as a function of metallicity and time, and it is for this reason that low-levels of UV flux are so difficult to interpret. The recent catalog of evolved hot stars in Galactic globular clusters by Schiavon et al. (2012) may help to calibrate the models for these advanced evolutionary phases.

There have been several important developments since the review of O'Connell (1999), which are mainly due to progress provided by HST and GALEX. It is now clear that galaxies on the optically-defined red sequence (e.g., in B-V or g-r color space) frequently harbor low levels of star formation, at the rate of  $\sim 0.1-1\,M_\odot\,\mathrm{yr}^{-1}$  (Yi et al. 2005, Kaviraj et al. 2007, Salim & Rich 2010, Salim et al. 2012, Fang et al. 2012), although for the most massive galaxies SF levels appear to be even lower. The implication is that optical SEDs are not sensitive to specific SF at levels of  $\lesssim 10^{-11} \text{ yr}^{-1}$ . It is also now quite clear that opticallydefined red sequence galaxies have a wide range of UV colors, from those with low levels of star formation to those that are extremely red. It has become common practice to define an arbitrary UV-optical color boundary in order to separate star forming from quiescent galaxies (Yi et al. 2005, Jeong et al. 2009). Of course, such a practice does not provide an answer to the question of how low star formation rates actually are in the reddest galaxies. Instead it signals the difficulty in measuring SFRs for such red systems because any UV flux becomes easily confused with hot evolved low-mass stars. Further progress can be made by directly resolving luminous young stars and star clusters in nearby galaxies. This was precisely the strategy taken by Ford & Bregman (2012), who observed four nearby elliptical galaxies with HST. These authors argued that the galaxies in their sample have current specific SFRs in the range of  $10^{-14} - 10^{-16} \text{ yr}^{-1}$ , far below what can be reliably probed via SED modeling. UV spectroscopic features may also aide in distinguishing hot evolved low-mass stars from hot upper main sequence stars (O'Connell 1999).

# 4.6 Summary

Measuring the detailed SFH of a galaxy from its observed SED is one of the holy grails of SPS modeling. Unfortunately, two unrelated physical effects make this task very difficult in practice: (1) high mass stars are so luminous that they dominate the SEDs of star-

forming galaxies, easily outshining the more numerous, older, lower mass stars; (2) at late times isochrones evolve very little, so that it is difficult in practice to distinguish an  $\sim 8$  Gyr population from a  $\sim 12$  Gyr one. The first effect implies that light-weighted ages will always underestimate the true mass-weighted age. It also implies that the priors on the model SFHs can strongly influence the best-fit parameters. Non-parametric SFH reconstruction methods offer the best hope of providing unbiased results, but they require data of very high quality. As a general rule of thumb, optical SEDs are sensitive to specific SFRs as low as  $10^{-11}$  yr<sup>-1</sup>, UV fluxes are sensitive to levels as low as  $10^{-12}$  yr<sup>-1</sup>, while resolved UV photometry is in principle sensitive to specific SFRs down to at least  $\lesssim 10^{-14}$  yr<sup>-1</sup>. SED-based SFRs are probably accurate at the factor of two level (at a fixed IMF), except for extreme systems where uncertainties can be much larger. There appears to be little consensus on the accuracy of light-weighted and mass-weighted stellar ages derived from SED fitting.

# 5 STELLAR METALLICITIES & ABUNDANCE PATTERNS

# 5.1 The Age-Metallicity Degeneracy

The metallicity, Z, affects the SED in two distinct ways: (1) an increase in metallicity results in lower effective temperatures, including a cooler main sequence and giant branch; (2) at fixed  $T_{\rm eff}$ , an increase in metallicity results in stronger spectroscopic absorption features and generally redder colors. Both of these effects contribute to the overall reddening of an SED with increasing metallicity. In addition to the overall metal content, the detailed elemental abundance pattern is imprinted on the SED, especially in the strength of spectral absorption features.

Age will also tend to redden the SED through the effect of age on the isochrones. Worthey (1994) studied the degeneracy between age and metallicity in detail using his SPS models and concluded that at ages > 5 Gyr neither broadband colors nor most spectral indices were able to reliably separate age from metallicity. Worthey introduced his "3/2 rule", whereby an increase/decrease in the population age by a factor of three is almost perfectly degenerate with an increase/decrease in metallicity by a factor of two. He identified a small number of spectral indices that were unusually sensitive to age, including the hydrogen Balmer lines, and several indices that were unusually sensitive to metallicity, including the Fe4668 and Fe5270 indices (see also O'Connell 1980, Rabin 1982, Rose 1985, for early attempts at separating age and metallicity). Worthey therefore demonstrated conclusively that the age-metallicity degeneracy could be broken by jointly considering a handful of carefully chosen indices. He also pointed out that the age-metallicity degeneracy is less severe at low metallicity, where the turnoff point and giant branch have very different age sensitivity at fixed metallicity.

It has long been appreciated that a clean separation of age and metallicity can be achieved with Lick indices only for data of very high S/N ( $\gtrsim 100/\text{Å}$ ; Trager et al. 2000b, Kuntschner 2000). Figure 11 demonstrates that an age-metallicity degeneracy persists when exploiting Lick indices at lower S/N. In contrast, full spectral fitting offers a much more robust separation of age and metallicity at moderate to low S/N (at least within the context of an underlying set of model SSPs; see MacArthur, González & Courteau 2009; Sánchez-Blázquez et al. 2011). This is simply a consequence of the fact that much more information is available in the full optical spectrum than can be captured by a limited number of spectral indices.

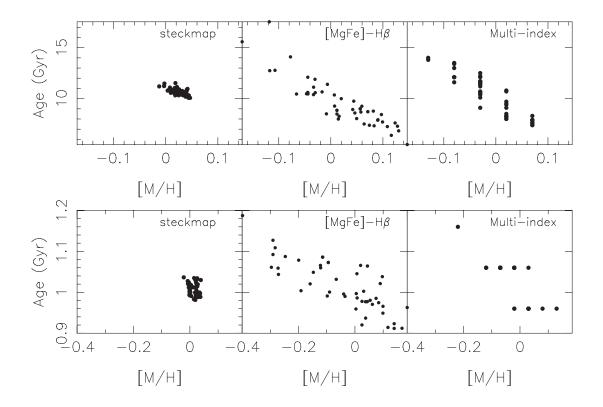


Figure 11:

Illustration of the age-metallicity degeneracy in spectra of modest S/N. Three techniques are compared, the full spectrum fitting technique of Ocvirk et al. (2006, left panel), classic two-index fitting (middle panel), and multi-index fitting (right panel). Results are shown for a mock galaxy with S/N= 50/Å, [M/H]= 0.0, and an age of 10 Gyr (top panels) and 1 Gyr (bottom panels). Each point represents the best-fit for a single realization of the mock galaxy spectrum. From Sánchez-Blázquez et al. (2011).

## 5.2 Photometric Metallicities

The age-metallicity degeneracy discussed in Worthey (1994) strictly applies to systems that are coeval with ages > 5 Gyr; models at younger ages were not considered by Worthey. In subsequent work it became clear that the broadband optical-NIR colors of composite stellar populations could separate age and metallicity effects. The physics underlying the idea is that blue colors probe the age-sensitive main sequence turnoff point while red/NIR colors probe the metallicity-sensitive giant branches. Bell & de Jong (2000) and MacArthur et al. (2004), amongst others, showed that age and metallicity vectors were nearly orthogonal in optical-NIR colors (e.g., B-R vs. R-H). These authors therefore concluded that estimates of stellar metallicities were possible from photometric data alone, for systems whose light-weighted ages were not too old. Eminian et al. (2008) showed that galaxies grouped by spectroscopically-derived ages and metallicities separate relatively well in g-r vs. Y-K color-color space. Notice that this statement does not depend on the predicted model colors, thereby providing strong evidence that separation of age and metallicity is at

least in principle possible based on broadband optical-NIR data alone.

Lee et al. (2007) investigated the model uncertainties associated with separating age and metallicity based on broadband colors. They echoed previous work in that the young populations in composite stellar models dominate the light and therefore allow a separation of age and metallicity effects when optical-NIR colors are employed. However, they also emphasized that different SPS models produce very different age-metallicity vectors in colorcolor space, a point also emphasized by Eminian et al. (2008). The result is that age and metallicity can be separately constrained within the context of a particular SPS model, but the derived ages and metallicities will vary from model to model. The underlying cause of this problem is that the NIR light is heavily influenced by AGB stars for populations of moderage age, and so different treatments of this uncertain evolutionary phase cause substantial differences in the age-metallicity grids. Because of the difficulty in measuring metallicities from photometric data, many SPS studies simply treat metallicity as a nuisance parameter to be marginalized over. Other studies take the even simpler approach of fixing the metallicity to the solar value. The danger in making the latter simplification is that assumptions in the assumed metallicity will generally affect other parameters of interest, such as the stellar mass, dust opacity, and star formation rate (see e.g., Wuyts et al. 2009, Marchesini et al. 2009, Muzzin et al. 2009, Pforr, Maraston & Tonini 2012).

## 5.3 Spectroscopic Metallicities

The more robust approach to estimating stellar metallicities is to employ spectroscopic features. Optical spectra of galaxies are rich in atomic and molecular absorption features that are readily apparent even at low spectral resolution ( $R \sim 1000$ ). For a fixed population age, the strengths of the absorption features will depend not only on the overall metallicity, Z, but also on the detailed elemental abundance pattern. In general, this greatly complicates interpretation of absorption features and spectral indices. However, with the aide of models that allow for variation in both metallicity and abundance pattern, one can search for combinations of features that are relatively robust against abundance pattern variations. Thomas, Maraston & Bender (2003b) refined a proposal by González (1993) of an index comprised of magnesium and iron lines that is almost entirely insensitive to the level of  $\alpha$  enhancement,  $[\alpha/\text{Fe}]$ , and is therefore a relatively robust tracer of Z. The index is defined as  $[MgFe]' \equiv \sqrt{Mgb} (0.72 \text{ Fe} 5270 + 0.28 \text{ Fe} 5335)$ , where Mgb, Fe5270, and Fe5335 are Lick indices defined in Worthey et al. (1994). The power of this index, and others like it (see e.g., Bruzual & Charlot 2003), is that variable abundance pattern models are then not required to measure total metallicities from galaxy spectra. However, a significant caveat to this approach is the assumption that abundance patterns in galaxies can be described by two parameters: [Fe/H], and  $[\alpha/\text{Fe}]$ . In reality, analyses of high resolution stellar spectra in the Galaxy consistently demonstrate that the  $\alpha$  elements do not vary in lock-step (Edvardsson et al. 1993, Venn et al. 2004, Fulbright, McWilliam & Rich 2007). This means that the true total metallicity, Z, can only be accurately estimated if the detailed abundance patterns are known.

Gallazzi et al. (2005) applied the idea that [MgFe]' is insensitive to  $[\alpha/\text{Fe}]$  (assuming that the  $\alpha$  elements, C and N all vary in lock-step) to the analysis of 175,000 galaxies from the SDSS with the aide of the Bruzual & Charlot (2003) SPS models. Several age-sensitive and Z-sensitive spectral indices were used in their analysis. They derived light-weighted stellar metallicities for individual galaxies with statistical errors as low as 0.1 dex for the highest S/N spectra. These authors also explored a variety of systematic effects and concluded that

these could add  $\sim 0.1$  dex systematic uncertainty to the error budget on Z. Probably the dominant systematic in the results of Gallazzi et al., and others based on similar techniques, is the assumption that the  $\alpha$  elements and C and N all vary in lock-step.

Another technique is to fit the entire optical spectrum with models that allow for variation in age and total metallicity, and to simply ignore the possibility that the observed spectrum may have non-solar abundance patterns. This approach has become increasingly common as automated spectral fitting codes garner wider use. To jeiro et al. (2011) fit stacked spectra of luminous red galaxies from SDSS as a function of luminosity and redshift with the VESPA code. Remarkably, they found essentially no variation in Z with galaxy luminosity (though the sample was restricted to very luminous galaxies), and for the Conroy, Gunn & White (2009) and Maraston (2005) SPS models there was no evolution in Z with redshift over the interval 0.1 < z < 0.5. When the Bruzual & Charlot (2003) models are employed, VESPA favored higher metallicities at higher redshift. Variation in the best-fit metallicity between SPS models points toward systematic uncertainties in the models at the  $\sim 0.2$  dex level.

Koleva et al. (2008) investigated the accuracy of full spectrum fitting codes at recovering metallicities. They considered both the ability of one SPS model to recover the known metallicity of a second model and the ability of an SPS model to recover the known metallicities of Galactic globular clusters. They concluded that the high-resolution version of the Bruzual & Charlot (2003) model carries large systematic uncertainties due to its use of the STELIB stellar library. STELIB does not sample a wide enough range in metallicity, and so metallicities derived from the Bruzual & Charlot models will carry large uncertainties. The other models tested, including the Vazdekis/MILES and Pegase-HR models, reproduced the known globular cluster metallicities to an accuracy of  $\approx 0.14$  dex.

Of course, the most robust method for estimating stellar metallicities is with models that self-consistently include abundance pattern variation. Such models have existed for the Lick indices since the late 1990s while models for the full optical-NIR spectrum have only recently become available (Coelho et al. 2007, Walcher et al. 2009, Conroy & van Dokkum 2012a).

Trager et al. (2000b,a) modeled three spectral indices with SPS models that allowed for variation in abundance patterns. Trager and colleagues focused on fitting for the age, total metallicity, Z, and enhancement ratio, [ $\alpha$ /Fe]. In practice, they grouped elements into two types, the 'depressed' and the 'enhanced' group, with the latter comprised of  $\alpha$  elements except Ca, plus C and N, while the former comprised the iron-peak elements and Ca. In subsequent work, Thomas et al. (2005) modeled the same set of spectral indices as in Trager et al., though with their own SPS models and with an expanded dataset. Their conclusions largely echoed those of Trager and collaborators, with the novel finding that at fixed velocity dispersion galaxies in denser environments appeared to be more metal-rich than galaxies in less dense environments.

As discussed in Section 4, most analyses of Lick indices fit single-age models to the data. Serra & Trager (2007) and Trager & Somerville (2009) considered the effect of multiple age components in the fitting of the Lick indices of mock galaxies and found that the derived metallicities agreed well with the light-weighted mean metallicities when the youngest component was older than  $\sim 3$  Gyr. For young components with ages 1-3 Gyr there was a slight bias in the sense that the derived metallicities were  $\sim 0.1$  dex higher than the true light-weighted mean metallicities. These authors did not consider young components younger than 1 Gyr.

The estimation of total metallicity with models that include abundance pattern variation are the most robust but also the most restricted. Models such as those by Thomas et al.,

Conroy & van Dokkum, Worthey, or Schiavon are limited to ages > 1 Gyr, and therefore are routinely applied only to early-type galaxies. A major area for future growth in this field is the extension of these models to younger ages in order to model galaxies of all types.

#### 5.4 Abundance Patterns

Wallerstein (1962) was the first to show that non-solar abundance patterns were common amongst metal-poor stars in the Galaxy. It was therefore natural to wonder if other galaxies harbored non-solar abundance patterns. Early data suggested that magnesium may vary more than iron amongst the early-type galaxies (O'Connell 1976, Peterson 1976, Peletier 1989), but it was with the availability of new, relatively high-quality spectral synthesis models that Worthey, Faber & Gonzalez (1992) were able to firmly establish that the [Mg/Fe] ratio varied within the early-type galaxy population. The discovery of non-solar abundance patterns has been one of the most significant discoveries afforded by population synthesis models. The ratio between the abundance of  $\alpha$ -elements and iron-peak elements, [ $\alpha$ /Fe], is considered to be particularly valuable, as it provides insight into the SFH of galaxies (Tinsley 1979). Following the development of models that allowed for arbitrary variation in abundance patterns, the measurement of abundance ratios became routine. Particular attention has been paid to [Mg/Fe], in large part because it is the most readily measurable abundance ratio in moderate resolution spectra, thanks to the numerous strong Fe I features and the Mgb feature at 5200 Å (see Figure 3).

As discussed above, Trager et al. (2000b,a) presented the first systematic investigation of abundance patterns in early-type galaxies. They found enhancement ratios,  $[\alpha/\text{Fe}]$  generally in excess of zero, with a strong correlation with velocity dispersion. As Trager and collaborators only fit H $\beta$ , Mgb, and two iron indices, their  $[\alpha/\text{Fe}]$  is in reality a statement about [Mg/Fe]. Since then, numerous authors, using different SPS models, have all reached the same conclusion that [Mg/Fe] exceeds zero and increases with galaxy velocity dispersion (Thomas et al. 2005, Schiavon 2007, Smith et al. 2009, Graves, Faber & Schiavon 2009, Johansson, Thomas & Maraston 2012).

As noted in the previous section, early studies grouped elements into two classes, 'enhanced' and 'depressed', and so only the total metallicity and the enhancement factor could be measured from the data (Trager et al. 2000a, Thomas et al. 2005). More recent work has allowed for the abundance of a larger number of elements to vary, including C, N, Ca, and Ti (see Schiavon 2010, for a complementary review to the one presented below).

The calcium abundance is most commonly measured from the Ca I line at 4227 Å, as probed by the Ca4227 Lick index (see Figure 3). Other regions of the spectrum with strong sensitivity to calcium abundance include the Ca II H&K lines and the triplet of Ca II lines at 8600 Å (CaT). The calcium abundance in early-type galaxies has been described as a 'puzzle' because its abundance does not seem to track magnesium, despite the fact that they are both  $\alpha$  elements. This was noted qualitatively in the early work of Vazdekis et al. (1997) and Trager et al. (1998) and quantified with variable abundance models by Thomas, Maraston & Bender (2003a). The CaT feature is weaker than even solar-scaled models, for reasons that will be discussed in Section 7. Schiavon (2007) and Johansson, Thomas & Maraston (2012) fit the Ca4227 index (along with others) of early-type galaxies from SDSS and find [Ca/Fe] values close to zero.

Carbon and nitrogen abundances can be measured from the NH, CH, C<sub>2</sub>, and CN molecular absorption features in the blue spectral region (see Figure 3). Kelson et al. (2006) were the first to probe the carbon and nitrogen content of early-type galaxies

based on Lick indices. Schiavon (2007) measured [C/Fe] and [N/Fe] from Lick indices as a function of early-type galaxy luminosity based on stacked spectra from SDSS. He found that the abundance of these elements increased significantly with increasing luminosity. Johansson, Thomas & Maraston (2012) reached similar conclusions based on analysis of individual early-type galaxies in SDSS. The NH feature at  $\approx 3360$  Å is in principle a powerful probe of [N/Fe] because it is sensitive only to N (unlike CN). To date this feature has not been extensively utilized, both because it is near the atmospheric transmission cutoff and thus difficult to observe and because variable abundance models are not well-developed in this spectral region. Nonetheless, several authors have investigated empirical trends of this feature with other properties of early-type galaxies, with mixed results (Davidge & Clark 1994, Ponder et al. 1998, Serven et al. 2011).

Moderate resolution spectra of early-type galaxies respond strongly to changes in the abundances of Fe, Mg, C, N, Ca, and Ti, and thus these elements are the most easily measured. However, the optical and NIR spectrum of an old stellar population contains information on many more elements. In a forward-looking study, Serven, Worthey & Briley (2005) considered the spectral response due to variation in the abundance of 23 separate elements and concluded that for S/N=100 spectra and a galaxy velocity dispersion of  $\sigma=200~{\rm km\,s^{-1}}$ , one could in principle measure the abundance of C, N, O, Na, Mg, Al, Si, Ca, Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Sr, and Ba. Of course, as the effects are often quite subtle, all relevant systematics must be well-controlled. As a first step toward extracting these more subtle features, Conroy, van Dokkum & Graves (2013) reported measurements of the abundances of the neutron-capture elements Sr and Ba from analysis of moderate-resolution spectra of nearby early-type galaxies. Studies such as these should provide important new clues to the formation histories of old stellar systems.

Attention here has focused exclusively on early-type galaxies with little or no ongoing star formation, and for good reason. The hot massive stars that dominate the SEDs of actively star-forming galaxies have generally weak metal absorption lines, especially in the optical-NIR, and so their presence acts primarily to dilute the strength of the metal lines of the older, cooler stars. More fundamentally, models simply do not currently exist to study the abundance patterns of actively star-forming galaxies.

What About Oxygen? Oxygen is the most abundant element in the universe after hydrogen and helium. Its abundance relative to iron, [O/Fe], is therefore a critical variable in both stellar interior models (VandenBerg & Bell 2001) and stellar atmospheres. There are no transitions of atomic oxygen visible in moderate resolution spectra, but molecules involving oxygen (including TiO, H<sub>2</sub>O, CO, and OH) do create significant features in the optical-NIR spectra of cool stars. TiO is the only molecule with transitions in the optical (beginning at  $\approx 4500 \,\text{Å}$ ), and unfortunately the work on response functions of the Lick indices to abundance changes have neglected TiO line lists from their computations (Tripicco & Bell 1995, Korn, Maraston & Thomas 2005). The situation with oxygen is quite complex because CO has the highest dissociation potential of any molecule, and thus increasing the abundance of oxygen has a cascading effect on other species. For example, increasing oxygen causes more carbon to be consumed by the formation of CO, which in turn causes a decrease in the concentration of CN and C<sub>2</sub>. The oxygen abundance must therefore be known, or assumed, before one can interpret other spectral features. The OH lines in the H-band probably afford the most direct constraint on the oxygen abundance, but these features have not been exploited because of a lack of available models. This limitation has now been overcome with the variable abundance ratio models of Conroy & van Dokkum (2012a) that extend to the K-band. The upshot is that direct constraints on the photospheric abundance of oxygen in other galaxies currently do not exist, but they are possible with newly available models. This presents a significant limitation to current models because of the strong effect of oxygen on the effective temperatures, etc., of stars. More troubling is the observation that in the Galactic bulge the abundances of oxygen and magnesium do not track each other (Fulbright, McWilliam & Rich 2007). In fact, bulge stars with [Fe/H]>0, have [O/Fe] < 0.2, [Si/Fe] < 0.2, [Ca/Fe] < 0.2, and [Ti/Fe] < 0.2, while  $[Mg/Fe] \sim 0.3$  for the same stars. The lesson to draw from the Galactic bulge is that one should be very cautious in using [Mg/Fe] as a tracer of  $[\alpha/\text{Fe}]$ . Because of these uncertainties surrounding oxygen, Thomas et al. (2005) investigated its effect on their analysis of Lick indices. They concluded that if oxygen is in reality decoupled from the other  $\alpha$ -elements (e.g., [O/Mg]is non-zero) then the ages, [Fe/H], and [Mg/Fe] abundances are largely unchanged with respect to their standard model where oxygen tracks the other  $\alpha$ -elements. However, these authors only considered the effect of oxygen on the stellar spectra; they did not consider the non-trivial effect of oxygen on the isochrones (Dotter et al. 2007). Further work is clearly needed regarding the important role of oxygen in SPS models.

Z vs. [Fe/H] The iron abundance, [Fe/H], is often regarded as a proxy for the total metallicity, Z, but, as discussed above, the correspondence between these two quantities breaks down for non-solar abundance patterns. As shown in Trager et al. (2000b), the relation between [Z/H] and [Fe/H] can be parameterized by  $[Z/H]=[Fe/H]+A[\alpha/Fe]$ . The constant A depends on the detailed abundance pattern; for models in which all  $\alpha$  elements and C and N are grouped together, A = 0.93, while for models with [O/Fe]=[C/Fe]=0, A = 0.77. It needs to be emphasized here that formulae such as the one presented above only apply in the simple case in which all  $\alpha$  elements vary in lock-step and C and N either are solar-scaled or vary with  $\alpha$ . This is probably not a good approximation to reality. From the modeling perspective, some have chosen to enhance or depress element groups at fixed Z (e.g., Trager et al. 2000b, Thomas, Maraston & Bender 2003b), while others have chosen to fix [Fe/H] as other elements vary (e.g., Schiavon 2007, Conroy & van Dokkum 2012a). The choice would be rather arbitrary and inconsequential were it not for the fact that most models adopt a fixed set of isochrones as the abundances are varied. The important question is then whether isochrones vary more at fixed Z or fixed [Fe/H]. At least for variation in [O/Fe], Dotter et al. (2007) have demonstrated that the isochrones vary in  $T_{\rm eff}$  by approximately the same absolute amount (though with a different sign) when the variation is computed at fixed Z or fixed [Fe/H]. The disadvantage, at least conceptually, of computing models at fixed Z is that a variation in one element forces variation in another so that Z is held constant.

# 5.5 The Low-Metallicity Tail

Stars within the Galaxy have a wide range of metallicities, and so one might expect other massive galaxies to harbor stars of varying metallicity. This expectation was confirmed with resolved color magnitude diagrams of nearby early-type galaxies (e.g., Grillmair et al. 1996, Harris & Harris 2000, Monachesi et al. 2011). The difference between a monometallic metal-rich stellar population and one that includes even a small fraction of metal-poor stars will be most apparent in the UV. This occurs for two reasons: at low metallicity the main sequence turn-off point is considerably hotter than at solar metallicity, and at low metallicity stellar populations are observed to contain significant numbers of blue HB stars.

The closed box model of galactic chemical evolution predicts that  $\approx 10\%$  of the stars in an evolved galaxy will have [Fe/H]< -1. Old stellar populations with [Fe/H] $\lesssim -1$  can be expected to harbor blue HB stars (by analogy with the Galactic globular clusters), and thus there will be a large difference in the UV between a closed box metallicity distribution function and one that is monometallic.

Bressan, Chiosi & Fagotto (1994) constructed spectrophotometric evolutionary models for elliptical galaxies and found that closed box models produced an excess of UV flux compared to observations. They argued that the failure of the model was due to the excess of metal-poor stars and concluded that real early-type galaxies contain a more narrowly peaked metallicity distribution function. This conclusion mirrors the observation in the solar neighborhood, where fewer low-metallicity stars are observed than predicted by the closed box model (this is known as the 'G-dwarf problem'; Tinsley 1980). Worthey, Dorman & Jones (1996) arrived at a similar conclusion from analysis of a UV-optical spectrum of M31 and spectral indices for a sample of early-type and S0 galaxies. These authors argued for a metal-poor component a factor of 2-3 lower than predicted by the closed box model. From the analysis of Lick indices of a sample of early-type galaxies, Greggio (1997) also concluded that a closed box model for the metallicity distribution failed to reproduce the data.

While it is reasonably clear that early-type galaxies contain fewer low-metallicity stars than predicted by the closed box model, the more general task of measuring the detailed metallicity distribution function, or even the fraction of low-metallicity stars from integrated light measurements is much more complex. As discussed in Section 4.5, the UV spectral region is sensitive to minority populations of hot stars of many types, including metal-poor main sequence stars, young stars, and exotic and uncertain evolutionary phases such as blue stragglers, extreme HB stars, and post-AGB stars. The latter two are very hot, and so are more influential at  $\lambda < 2000 \,\text{Å}$ , where the UV upturn phenomenon in early-type galaxies exists. From consideration of the mid-UV (2000 Å $< \lambda < 4000$  Å) spectra of M31 and other nearby galaxies, Worthey, Dorman & Jones (1996) argued for metal-poor fractions  $\leq 5\%$ . Maraston & Thomas (2000) modeled the mid-UV spectra of four nearby galaxies and found metal-poor fractions  $\leq 6\%$ , and Lotz, Ferguson & Bohlin (2000) argued in favor of small, but non-zero fractions of metal-poor stars. Maraston & Thomas (2000) also highlight the point, first raised by Greggio (1997), that known metallicity gradients within galaxies, when combined with projection effects, will 'contaminate' nuclear spectra of galaxies with metal-poor stars. It is notable that Maraston & Thomas (2000) find evidence for a spread in metal-poor fractions, ranging from 0-6%. This suggests that one should include the metalpoor fraction as an additional free parameter when fitting the mid-UV of early-type galaxies. Care must be taken when interpreting these fractions because the definition of 'metal-poor' varies, although it generally refers to metallicities at least as low as  $[Fe/H] \lesssim -1.5$ . The problem boils down to the fact that the hot star fraction can be measured from UV (and perhaps blue spectral) data, but it is very challenging to convert this hot star population into a fraction of metal-poor stars owing to uncertainties in advanced phases of stellar evolution.

For actively star-forming galaxies essentially nothing is known regarding their internal distribution of metallicities, as the young stars overwhelm the subtle UV features signaling the presence or absence of low-metallicity stars.

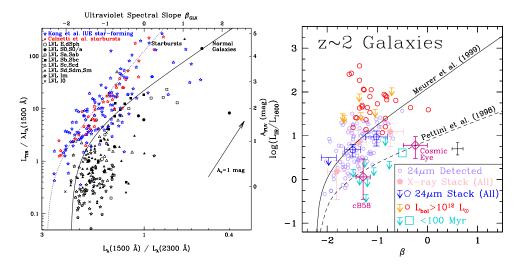


Figure 12:

The IRX- $\beta$  relation at z=0 from the Local Volume Legacy Survey (LVL; Dale et al. 2009, left panel) and at  $z\sim 2$  from Reddy et al. (2010, right panel). The starburst (Meurer) relation is shown as a dotted (solid) line in the left (right) panel. In the left panel, symbols indicate starburst and star-forming galaxies, along with galaxies split by morphological type. In the right panel, symbols indicate galaxies selected in several different ways, either as having a young light-weighted age, high bolometric luminosity, or on the basis of x-ray or  $24\,\mu m$  fluxes. There is an overall trend that redder UV slopes correspond to higher IRX values. However, the more striking impression is the tremendous scatter in IRX at fixed  $\beta$ , especially in the region of the diagram where most galaxies reside, at  $-2 < \beta < -1$ .

## 5.6 Summary

The technique of measuring metallicities and abundance patterns from old stellar systems is well-established. The age-metallicity degeneracy can be robustly broken by considering the hydrogen Balmer lines in conjunction with iron features in moderate resolution optical spectra. Abundance ratios including [Mg/Fe], [C/Fe], [N/Fe], and [Ca/Fe] are now routinely measured from optical spectra. Other elements including O, Na, Si, Sc, Ti, V, Cr, Mn, Co, Ni, Sr, and Ba should also be measurable from high S/N optical-NIR spectra of old stellar systems, and models are currently being developed to extract this information. It is possible to measure metallicities based on photometric data, but the derived values depend on the underlying SPS model and on various assumptions like the form of the SFH. Photometric metallicities are therefore probably accurate only in a relative sense. Spectroscopic metallicities seem to be accurate at the  $\sim 0.2$  dex level, with model systematics again being the dominant source of error. There is clearly information on the distribution of stellar metallicities within galaxies, and previous work suggests that it is possible to measure at least the low metallicity tail of the distribution. Further modeling is required to understand precisely how much information on the metallicity distribution function is available in the SEDs of quiescent galaxies.

## 6 DUST

#### 6.1 Total Dust Attenuation

There exist four primary techniques for measuring dust attenuation in galaxies. (1) As the obscuring effects of dust increase with decreasing wavelength, multi-wavelength analysis (including moderate-resolution spectra) of the UV-NIR SED can yield constraints on the total dust attenuation. (2) One can exploit the fact that the Balmer line ratio  ${\rm H}\alpha/{\rm H}\beta$  (sometimes referred to as the Balmer decrement) in the absence of dust can be computed from first principles and depends only weakly on the temperature of the gas (Osterbrock 1989). Comparison of the observed to the intrinsic ratio provides a measurement of the attenuation toward H II regions. (3) Energy conservation demands that whatever photons are absorbed in the UV/optical must be re-radiated in the IR. The ratio of IR to UV luminosity (often referred to as 'IRX') therefore provides a measure of the total dust absorption. (4) A sufficiently luminous background source with a known intrinsic spectrum can provide a constraint on the attenuation in the foreground galaxy. This last technique will not be discussed herein, as it falls outside the scope of what one can learn from the SEDs of galaxies.

Techniques (1)-(3) provide complementary constraints on the dust within galaxies. This was most clearly demonstrated in the work of Calzetti, Kinney & Storchi-Bergmann (1994), who analyzed the reddening in a sample of 39 starburst and blue compact galaxies from both the Balmer line ratio  $H\alpha/H\beta$  and the slope of the UV continuum. Calzetti and collaborators found that the dust optical depth measured from the Balmer lines was nearly twice that estimated from the continuum (sometimes referred to as the '1:2' rule). They speculated that this could occur if young hot stars are embedded in dusty natal clouds while older stars are subject to attenuation only from the diffuse ISM. Charlot & Fall (2000) later provided a detailed model for dust absorption which adopted this physical picture wherein young stars are embedded in their natal dusty clouds and, in addition, all stars experience dust attenuation due to the diffuse ISM. In this two-component model one must specify a transition time, after which the natal cloud is dispersed (typically taken to be 10<sup>7</sup> yr), and attenuation curves for both the natal cloud and the diffuse ISM. In the standard implementation, both curves are taken to be power-laws with the same exponent  $(\tau_d \propto \lambda^{-0.7})$  and the normalization of the natal cloud attenuation curve is taken to be twice that of the diffuse ISM. The Charlot & Fall dust model is widely employed when modeling dusty SEDs.

In subsequent work, Meurer, Heckman & Calzetti (1999) compared the UV continuum slope,  $\beta$ , to the ratio between FIR and UV flux,  $\log(L_{IR}/L_{UV}) \equiv \text{IRX}$ , and concluded that the starburst galaxies formed a well-defined sequence in the IRX- $\beta$  plane (see Figure 12). With additional assumptions about the underlying stellar populations and dust attenuation curve, these authors were able to relate IRX (and hence  $\beta$ ) to the total FUV dust attenuation. The resulting 'Meurer relation' has experienced widespread use for estimating the dust attenuation in distant galaxies for which only measurements of  $\beta$  are available. However, many subsequent studies have cast doubt on the practice of using  $\beta$  to estimate UV dust attenuation. Bell et al. (2002) noted that the relationship between  $\beta$  and UV attenuation measured in H II regions in the LMC differed from the Meurer relation. Kong et al. (2004) demonstrated that, while dust content was the main variable responsible for the Meurer relation, other stellar population parameters, particularly the recent SFH, drive galaxies to other locations in the IRX- $\beta$  plane (see also Boquien et al. 2012). The underlying extinction curve and star-dust geometry also play an important role in determining

where galaxies lie in this plane (Charlot & Fall 2000, Kong et al. 2004, Panuzzo et al. 2007, Conroy, Schiminovich & Blanton 2010).

Based on UV-IR photometry of 1000 galaxies, Johnson et al. (2007) cast further doubt on the utility of the IRX- $\beta$  relation for estimating dust content. Their conclusion rested partly on the observation that the relation defined by normal star-forming galaxies is so steep that even small errors in  $\beta$  will translate into large errors on IRX. Figure 12 shows the IRX- $\beta$  relation for galaxies from the Local Volume Legacy Survey (Dale et al. 2009), and the IRX- $\beta$  relation for  $z \sim 2$  galaxies from Reddy et al. (2010). It is clear that, while galaxies of a particular type define their own IRX- $\beta$  relations, the overall galaxy population presents a complicated picture. Fortunately, as multi-wavelength datasets become increasingly common, the estimation of dust content from crude measures such as the IRX- $\beta$  relation will give way to more robust techniques.

SED fitting of UV-NIR data has become remarkably common in the past decade. In nearly all cases the dust content is a parameter included in the fit, although the derived dust opacities are rarely discussed. The reason for this seems to be the fact that the dust opacity is a relatively poorly constrained quantity when data only reach the NIR. As an example of the difficulty in constraining the dust opacity from broadband data, Salim et al. (2007) fit UV-NIR photometry to models and were able to recover V-band dust opacities with an uncertainty of  $\sim 0.4$  mag. Furthermore, there was only a weak correlation between the dust opacity derived from broadband data and the opacity derived from emission lines via Brinchmann et al. (2004). Some of the difference can be attributed to aperture effects and real differences in the opacity measured from emission lines and stellar continua, but overall the weak correlation confirms that dust content measured from broadband data is quite uncertain. Poorly constrained dust opacities appears to be a generic result of the modeling of UV-NIR broadband SEDs (e.g., Papovich, Dickinson & Ferguson 2001; Shapley et al. 2006; Kriek et al. 2008; Taylor et al. 2011).

Broadband UV-NIR data do not provide strong constraints on dust attenuation because of the well-known degeneracy between age and dust (e.g., Papovich, Dickinson & Ferguson 2001). This degeneracy can be broken, and stronger constraints on the dust opacity can be obtained, when either spectroscopic or FIR data are available. Spectroscopic data offer an estimate of the mean stellar age and metallicity independent of dust content because narrow spectroscopic features are relatively immune to overall changes in the continuum shape. Kauffmann et al. (2003) exploited this technique to measure mean ages of  $\sim 10^5$  SDSS galaxies from H $\delta$  and the 4000 Å break. With the ages thus constrained, an estimate of the dust attenuation could be obtained by comparing the model colors to the observed ones. This study was limited primarily by the fact that the SDSS spectra sample a modest fraction of the total galaxy, and so a correction must be made for this aperture effect (ongoing and future planned IFU surveys such as CALIFA and MaNGA will remedy this limitation). Narrowband photometric surveys also offer the promise of separately constraining age and dust attenuation, primarily because of the strongly age-sensitive  $D_n4000$  spectral feature, which is so strong that it can be probed with low resolution data (e.g., Whitaker et al. 2010, Perez-Gonzalez et al. 2012). FIR data can also provide a robust measurement of the total dust attenuation because they significantly reduce the degeneracy between age and dust (e.g., Burgarella, Buat & Iglesias-Páramo 2005; Noll et al. 2009a).

Wuyts et al. (2007) and Williams et al. (2009) pioneered the use of restframe optical-NIR color-color diagrams (specifically U - V vs. V - J; commonly referred to as the UVJ diagram) to efficiently separate quiescent and star-forming galaxies. The UVJ diagram has since become a popular tool for separating these two galaxy types, especially at highredshift where data are limited. In a broad sense, the UVJ diagram allows one to break the age-dust degeneracy, but only in the sense of separating quiescent and dusty star-forming galaxies. Within the star-forming sequence, the age-dust degeneracy is still manifest when only broadband information is available.

Extremely blue UV continua can place strong constraints on the amount of dust in the system. Bouwens et al. (2012) and Finkelstein et al. (2012) measured the UV continuum slopes of high-redshift galaxies and found that  $\beta$  systematically decreases toward higher redshift and fainter galaxy luminosities. At the highest redshifts and faintest luminosities they find  $-2.0 < \beta < -2.5$ , which not only suggests that these galaxies are dust-free but also begins to place interesting constraints on their stellar populations. The models of Schaerer (2003) reach such blue slopes only at very young ages ( $\sim 10^6$  yr) and/or very low metallicities ( $< 10^{-3} Z_{\odot}$ ), and the inclusion of nebular continuum emission, which presumably must be present whenever stars of these young ages are still alive, sets a lower limit to the UV continuum slope of  $-2.5 < \beta < -3.0$ , depending on the details of the SFH. Clearly more data probing the restframe UV at very high redshifts would be valuable.

## 6.2 Constraints on the Attenuation Curve

Calzetti, Kinney & Storchi-Bergmann (1994) estimated a mean attenuation curve from 39 starburst and blue compact galaxies based on UV-optical spectra. By assuming that their sample was comprised of galaxies with the same underlying stellar populations, they were able to construct average attenuation curves by comparing galaxies with high and low Balmer line ratios, i.e., by comparing dusty to dust-free galaxies. The resulting average attenuation curve, measured over  $\approx 1200\,\text{Å}-8000\,\text{Å}$ , was greyer (shallower) than either the MW or LMC extinction curve and lacked the broad 2175 Å dust feature visible in the MW and LMC. The derived curve is now referred to as the "Calzetti attenuation law" and is widely used when modeling the broadband SEDs of galaxies.

The Calzetti law was estimated from a relatively small sample of unusual (i.e., starburst and blue compact) galaxies at z=0 and so one may wonder to what extent it is applicable to other systems. As noted in the previous section, the IRX- $\beta$  relation is sensitive to the attenuation curve. The large range in  $\beta$  at a fixed IRX, evident in Figure 12, suggests considerable variation in the attenuation curve, but interpretation of the IRX- $\beta$  plane is complicated by the additional variables affecting IRX and  $\beta$ .

Progress can be made by estimating stellar population parameters such as the SFH and metallicity independently of the dust content (e.g., via narrow spectral features). With such quantities in hand, one can exploit the technique pioneered by Calzetti and collaborators by computing ratios of SEDs of more to less dusty galaxies with the same underlying stellar populations. Johnson et al. (2007) estimated mean attenuation curves in this way for galaxies as a function of the 4000 Å break strength and stellar mass. The curves were constrained from broadband FUV-NIR photometry and were consistent both with a simple power-law,  $\tau_d \propto \lambda^{-0.7}$ , and with the Calzetti law. Wild et al. (2011) exploited a similar technique using pairs of galaxies with similar gas-phase metallicities, specific star formation rates, and inclinations, and significantly different Balmer decrements to construct attenuation curves from broadband FUV-NIR data. These authors found significant variation in the mean attenuation curve as a function of specific SFR, inclination, and stellar mass surface density, based on a sample of 20,000  $z \sim 0$  star-forming galaxies. They also found variation in the ratio between dust opacity in the stellar continuum and in the nebular line emission, implying that the '1/2' rule of Calzetti, Kinney & Storchi-Bergmann (1994) — one unit of

continuum opacity for ever two units of line opacity — is not universal. Buat et al. (2012) modeled the UV-FIR SEDs of galaxies at 1 < z < 2, allowing for variation in the dust attenuation curve. They found evidence for a steeper attenuation curve (i.e., faster rise in the UV) than the Calzetti law in 20 - 40% of their sample.

6.2.1 The 2175 Å DUST FEATURE The absence of the 2175 Å dust feature from the Calzetti attenuation law is striking given its ubiquity in the MW and LMC. Witt & Gordon (2000) were able to reproduce the Calzetti law in their radiative transfer models only by assuming SMC-type dust, i.e., by adopting an underlying extinction curve that lacked the 2175 Å dust feature. In contrast, Granato et al. (2000) were able to reproduce the Calzetti law in their radiative transfer models with MW-type dust. In their model the star-dust geometry plays a central role in shaping the starburst attenuation curve. Their molecular clouds are optically thick and so the attenuation curve is determined primarily by the fraction of stars inside molecular clouds as a function of age (see also Panuzzo et al. 2007). Fischera & Dopita (2011) were able to reproduce a Calzetti-like attenuation curve if the carrier responsible for the 2175 Å feature (e.g., PAHs) was destroyed at high column density.

In all radiative transfer dust models the expectation is that normal star-forming galaxies should show evidence for the 2175 Å dust feature provided that the underlying grain population is similar to the MW or LMC. Owing to a paucity of restframe UV spectra of starforming galaxies, it has proven difficult to verify this expectation. Burgarella, Buat & Iglesias-Páramo (2005) analyzed the UV-FIR SEDs of 180 star-forming galaxies and concluded that the data required an attenuation curve with a 2175 Å dust feature that was on average weaker than observed in the MW extinction curve. Noll et al. (2009b) presented stacked restframe UV spectra of  $z \sim 2$  star-forming galaxies and found strong evidence for the presence of the 2175 Å dust feature in a subsample of their objects. Conroy, Schiminovich & Blanton (2010) analyzed UV-NIR photometry of  $z \sim 0$  star-forming galaxies as a function of inclination and found evidence for a 2175 Å dust feature with a strength slightly weaker than observed in the MW extinction curve. Hoversten et al. (2011) analyzed medium band UV photometry from the Swift UV/Optical Telescope, and found evidence for a prominent 2175 Å bump in M81. Wild et al. (2011) found evidence in their derived attenuation curves for the presence of the 2175 Å dust feature, with a slight tendency for galaxies with higher specific SFRs to have lower bump strengths. Wijesinghe et al. (2011) compared dust-corrected SFRs estimated from  $H\alpha$  and the UV and found agreement only when the 2175 Å feature was removed from the attenuation curve, but these authors did not consider variation in the shape of the attenuation curve. Buat et al. (2012) found evidence that the 2175 Å dust feature is present in 20% of their sample, with an amplitude that is half the strength found in the MW extinction curve. Similar to Wild et al., they found an anti-correlation between bump strength and specific SFR. The emerging consensus appears to be that the 2175 Å dust feature is present in the typical star-forming galaxy over the redshift range 0 < z < 2, with a strength that is generally lower than in the MW extinction curve, and that increases with increasing inclination and decreasing specific SFR. The absence of the dust feature in the Calzetti law may then simply be a reflection of the fact that the galaxies in the Calzetti sample have unusually high specific SFRs. An outstanding question is whether these trends are due to varying star-dust geometries or changes in the underlying grain population. As emphasized in Conroy, Schiminovich & Blanton (2010) and elsewhere, these results have consequences for interpreting the IRX- $\beta$  relation, since  $\beta$  is frequently measured over a wavelength range that includes the 2175 Å feature.

## 6.3 Physical Dust Properties

The IR emission by dust grains provides strong constraints on a variety of physical properties of a galaxy including the bolometric luminosity, dust temperature(s), dust mass, and relative abundance of PAH molecules (or, more generally, the grain size distribution). A constraint on the bolometric luminosity helps to break the age-dust degeneracy in the UV-NIR, as discussed in the previous section. The other parameters provide clues to the nature of dust in galaxies and its spatial relation to the stars and gas.

It has long been recognized that constraints on the dust mass and dust temperatures required data beyond the peak in the thermal dust emission spectrum at  $\sim 100\,\mu m$ . The SCUBA bolometer array at the James Clerk Maxwell Telescope, operating at  $450\,\mu m$  and  $850\,\mu m$ , has been extremely influential in this field, and now the SPIRE instrument onboard the Herschel Space Observatory, operating at  $250-500\,\mu m$ , promises to revolutionize the field by dramatically increasing sample sizes over a wide range in redshifts. With data obtained for the SCUBA Local Universe Galaxy Survey, Dunne et al. (2000) and Dunne & Eales (2001) constrained dust masses and temperatures for 104 nearby galaxies. The data favored models that contained both cold and warm dust components (at  $\sim 20 \rm K$  and  $\sim 40 \rm K$ , respectively). They also found that dust masses estimated with single-temperature models were on average lower by a factor of  $\sim 2$  compared to two-temperature models, and the two-temperature dust masses yielded dust-to-gas ratios in agreement with measurements in the Galaxy.

Draine et al. (2007) presented a thorough analysis of the IR SEDs of galaxies from the SINGS survey of 65 nearby galaxies. These authors analyzed data from Spitzer, IRAS, and SCUBA with the physical dust models of Draine & Li (2007). The model considers grain populations exposed to a variety of starlight intensities and includes both thermal emission and single photon heating of dust particles. In addition, the fraction of dust mass contained in PAH molecules is a free variable. Draine et al. confirmed the results of previous work that the sub-mm data, provided by SCUBA, were essential for deriving dust masses with an uncertainty of < 50%. They also derived strong constraints on the fraction of dust mass in PAHs, the interstellar radiation field strength in the diffuse ISM, and the flux-weighted mean radiation field strength averaged over the galaxy. The well-sampled spectral coverage from the mid-IR through the sub-mm was essential in order to separately constrain each of these components.

Dale et al. (2012) analyzed FIR and sub-mm photometry for SINGS galaxies from Herschel in conjunction with previously available 2MASS, Spitzer, IRAS, ISO, and SCUBA data. These authors modeled the SEDs with the Draine & Li (2007) dust model, both with and without the Herschel data. They found modest differences (factors of  $\approx 1.6$  on average) in the best-fit dust masses, PAH mass fractions, and fraction of dust emission arising from the diffuse ISM when the FIR and sub-mm data were either included or excluded. They also demonstrated that dust masses estimated via simple modified blackbody fits significantly underestimated the dust masses (by factors of  $\approx 3$  in the worst cases), with the underestimation becoming more severe as bluer wavelengths were included in the fits. The Herschel data are of such high quality that simplistic, modified blackbody dust models are no longer capable of providing adequate fits to the data.

da Cunha, Charlot & Elbaz (2008) developed a phenomenological model for the UV-FIR SEDs of galaxies that is based on the Bruzual & Charlot (2003) SPS models for the starlight, the Charlot & Fall (2000) dust attenuation model, and dust emission comprised of multiple components including an empirical spectrum for the PAH emission, warm and cold thermal

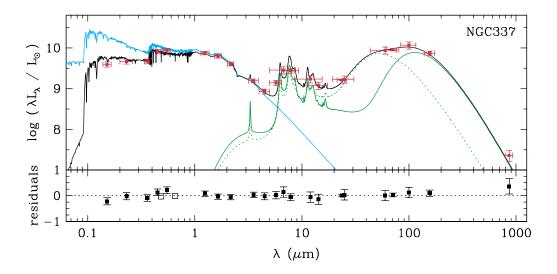


Figure 13:

Model fit to the FUV through FIR SED of NGC 337, from da Cunha, Charlot & Elbaz (2008). The black line is the best-fit model. Also shown is the unattenuated model starlight (blue line) and the dust emission separated into dust in the diffuse ISM (solid green line) and dust in birth clouds (dotted green line). Figure courtesy of E. da Cunha.

dust emission, and emission from stochastically heated grains (see Figure 13). Based on tests with mock galaxies, they concluded that a variety of parameters could be constrained within the context of their model, including the temperature of the cold dust component, the fractional contribution of the cold dust to the total FIR emission, and the total dust mass. They analyzed the same SINGS galaxies as in Draine et al. and derived dust masses that agreed to within 50%, demonstrating that dust masses can be reliably measured with reasonably well-sampled mid-IR, FIR, and sub-mm data.

Galliano, Dwek & Chanial (2008) employed the dust model of Zubko, Dwek & Arendt (2004) to constrain a variety of parameters including the total dust mass, fraction of dust mass in PAHs, fraction of dust luminosity originating from photodissociation regions (PDRs), and the gas-phase metallicity. These authors measured these and other parameters for 35 galaxies with optical, IR, and sub-mm photometry and mid-IR spectroscopy. Although no direct comparison was made with the results from Draine et al. (2007), there appears to be a quantitative difference in the derived PAH mass fractions. As discussed extensively in Draine & Li (2007), the optical properties of the PAH molecules are uncertain and have in many cases been tuned to fit extragalactic data. It would therefore be valuable to compare the results obtained from dust models that assume different treatments of the uncertain PAH properties.

## 6.4 Cosmic Evolution of IR SEDs

Measuring the cosmic evolution of IR SEDs is important not only for constraining models of galaxy evolution but also because local IR SED templates are frequently employed to interpret high-redshift data. For example, the local templates are often used to convert  $24\,\mu m$  observations of high-redshift galaxies into SFRs. If IR SEDs at fixed  $L_{\rm bol}$  evolve with time then it would considerably complicate the interpretation of higher redshift data.

Pope et al. (2006) presented evidence that the SEDs of ULIRGs at  $z\sim 2$  peak at longer wavelengths than local ULIRGs, implying that the dust temperature in high-redshift ULIRGs is on average  $\sim 5$ K cooler than local ULIRGs. Muzzin et al. (2010) analyzed highquality data for two  $z \sim 2$  ULIRGs and found that these galaxies contained colder dust than local ULIRGs, in agreement with Pope et al. (2006). Muzzin et al. showed that the SED shapes of their ULIRGs were well-fit by the z=0 Chary & Elbaz (2001) templates that were an order of magnitude less luminous than their  $z \sim 2$  galaxies. In other words, the  $z\sim 2$  ULIRGs had SED shapes that were similar to local LIRGs. Exploiting the power of Herschel data, Hwang et al. (2010) estimated dust temperatures by fitting modified blackbodies to the data, and found evidence for slightly cooler dust temperatures at  $z \sim 1$ , by 2-5 K on average, for galaxies with  $L_{\rm IR} > 10^{11} L_{\odot}$ . Daddi et al. (2007) compared restframe  $8 \,\mu m$  luminosities of  $z \sim 2$  galaxies to local templates and found evidence for excess mid-IR emission for galaxies with  $L_{8\mu m} > 10^{11} L_{\odot}$ . This mid-IR excess problem was confirmed by Papovich et al. (2007) and later by Elbaz et al. (2010) with Herschel data. It is noteworthy that the problem only appears at z > 1.5. Bolometrically luminous high-redshift galaxies thus have colder dust and more flux in the mid-IR compared to galaxies at the same  $L_{\rm bol}$ at z=0. The mid-IR excess problem has important implications for the common practice of estimating SFRs for galaxies at z > 2 with Spitzer 24  $\mu m$  observations (i.e., restframe  $\sim 8 \,\mu m$ ). The sign of the excess is such that SFRs estimated by extrapolating observed  $24 \,\mu m$  flux with local templates will tend to overestimate the true SFRs, by factors of several in the most extreme cases (e.g., Papovich et al. 2007).

Elbaz et al. (2011) investigated the origin of the differences between high and low redshift IR SEDs with very deep *Herschel* observations. They argued that it is the distribution of SED types that is changing at high luminosity as a function of redshift. At low redshift, high luminosity systems (i.e., ULIRGs) are predominantly star-bursting and compact, while at higher redshift ULIRGs are mostly the high-luminosity extension of normal star-forming galaxies. Local ULIRGs therefore have hotter dust temperatures and a depressed emission component from PAHs in the mid-IR compared to distant ULIRGs because the former are compact starbursts while the latter are normal, extended star-forming galaxies (see also Rujopakarn et al. 2012, who reached similar conclusions). The physical origin of this result is not yet fully understood, but a plausible explanation is that compact starbursts have harder and more intense radiation fields. The radiation field obviously has a direct influence on the dust temperature, and it may also modulate the dust mass fraction in PAHs and therefore the mid-IR luminosity (Voit 1992, Madden et al. 2006, Draine et al. 2007).

## 6.5 Summary

The Calzetti attenuation curve works remarkably well at describing the mean attenuation properties of star-forming galaxies over most of cosmic history. However, closer scrutiny of the data reveals variation in the attenuation curve of galaxies as a function of galaxy

properties, as expected on theoretical grounds. In contrast to the Calzetti attenuation curve for starburst galaxies, the 2175 Å dust feature is apparently common in normal star-forming galaxies, with a strength that is weaker than observed in the MW extinction curve. The  $IRX-\beta$  relation should be used with caution, as there is evidently no universal relation for all galaxies. In addition, the relation is so steep over the range in  $\beta$  where most galaxies reside that estimating dust opacity in this way is highly unreliable. Dust masses can be measured to an accuracy of  $\sim 50\%$  when FIR and sub-mm data are available. Physical dust models, such as the model of Draine & Li (2007), are capable of extracting a variety of parameters from the global SEDs of galaxies, including the fraction of dust mass in PAHs, the fraction of dust emission due to PDRs vs. the diffuse ISM, and the typical interstellar radiation field strength heating the dust grains (or, alternatively, the typical dust temperature). Further work appears to be needed to sort out which derived parameters are robust, and which are dependent on uncertain components of the model. Below  $L_{\rm bol} \sim 10^{11} L_{\odot}$ , the IR SEDs of galaxies evolve little over the interval 0 < z < 2. At higher luminosities there is a marked shift such that high redshift ULIRGs have more mid-IR flux and a colder dust component that peaks at longer wavelengths. A plausible explanation for this trend is that ULIRGs at high-redshift are at the massive end of the normal star-forming galaxy sequence, while ULIRGs in the local universe are unusual, compact, star-bursting systems.

## 7 THE INITIAL MASS FUNCTION

The stellar IMF is of fundamental importance for many areas of astrophysics. The form of the IMF is well-constrained in the Galactic disk (Salpeter 1955, Miller & Scalo 1979, Kroupa 2001, Chabrier 2003). However, it is not clear whether the IMF has had the same form over all of cosmic time and in all environments. The low mass end is of particular importance, as  $\sim 60-80\,\%$  of the stellar mass density in the Universe is in the form of stars with masses  $< 0.5\,\mathrm{M}_{\odot}$ . Low mass stars are very faint and so contribute only a few percent to the integrated light of an old population. For this reason, when fitting the SEDs of galaxies, an IMF is typically assumed, and so the IMF is one of the largest sources of systematic uncertainty in studies of extragalactic stellar populations.

It has been known since at least the 1960s that dwarf-sensitive and, to a lesser degree, giant-sensitive spectral features could be used to count the number of low-mass stars in the integrated light of old stellar populations (e.g., Spinrad 1962, Spinrad & Taylor 1971, Whitford 1977, Frogel et al. 1978, Frogel, Persson & Cohen 1980, Vazdekis et al. 1996, Schiavon, Barbuy & Sin 1997, Schiavon et al. 1997, Schiavon, Barbuy & Bruzual A. 2000). This idea is limited to quiescent systems because the massive stars associated with ongoing star formation would dramatically outshine the faint low mass stars. The fundamental difficulty then, as now, is the separation of abundance effects from giant-to-dwarf ratio effects. Both effects can change the strength of the gravity-sensitive lines.

Spinrad & Taylor (1971) concluded that the nuclei of the nearby galaxies M31, M32, and M81 are dominated by dwarfs, with an implied M/L=44. These conclusions were based on photoelectric scanner observations reaching to  $1\,\mu m$ . Subsequently, Cohen (1978) measured the IMF-sensitive Na I, CaT, FeH and TiO features in the nucleus of M31 and M32 and concluded that an IMF comparable to that of the solar neighborhood was favored, along with metallicity enhancement in M31. Faber & French (1980) also measured the strength of the Na I doublet in M31 and M32 and instead argued for a dwarf-rich IMF, along the lines of Spinrad & Taylor (1971), with  $M/L_B=28$ . Frogel et al. (1978) analyzed optical-NIR colors and the CO bandhead at  $2.3\,\mu m$  for 51 early-type galaxies and concluded that dwarf-rich

IMFs were inconsistent with the data and favored models with  $M/L_V < 10$ . With updated detector technology and more sophisticated models, Carter, Visvanathan & Pickles (1986) argued against dwarf-rich IMFs on the basis of the Na I, CaT, TiO and FeH features in 14 elliptical and lenticular galaxies. Their results implied mass-to-light ratios of  $M/L \sim 6$ . In a series of papers, Hardy, Couture and collaborators (Hardy & Couture 1988, Delisle & Hardy 1992, Couture & Hardy 1993) analyzed the IMF features Na I and FeH for of order a dozen galaxies and concluded that dwarf-rich IMFs were strongly disfavored. Measurements of the FeH band at  $1 \mu m$  (also known as the Wing-Ford band) were particularly important, as this feature becomes very strong for IMF exponents steeper than Salpeter (Whitford 1977).

Early work aimed at measuring the IMF from integrated light spectra suffered from serious limitations, including (1) the lack of accurate stellar evolution calculations across the main sequence and through advanced evolutionary phases; (2) the use of empirical stellar spectra collected from the solar neighborhood, which implied that the effect of non-solar abundance ratios could not be adequately assessed; (3) poor NIR detector technology, which made it very difficult to measure red spectra at the sub-percent level precision necessary to measure the low-mass IMF in integrated light. These limitations contributed to the wide range of reported IMF exponents and mass-to-light ratios noted above. In the past two decades, each of these limitations has been significantly reduced, though not eliminated. Stellar evolution calculations have improved dramatically since the 1970s, the response of stellar spectra to elemental abundance variations has been studied in detail (e.g., Tripicco & Bell 1995, Korn, Maraston & Thomas 2005, Serven, Worthey & Briley 2005, Conroy & van Dokkum 2012a), and NIR detector technology has steadily improved.

The debate over IMF variation was renewed by Saglia et al. (2002) and Cenarro et al. (2003), who independently reported an anti-correlation between the strength of the IMF-sensitive CaT index and velocity dispersion for elliptical galaxies. Comparison to SPS models with solar-scaled abundance ratios required dwarf-rich IMFs, with an exponent of  $\alpha \approx 4$  at the highest dispersions (compared to the Salpeter value of 2.35). Saglia et al. (2002) argued against this interpretation, as such a steep IMF would imply  $M/L \approx 40$ , well in excess of newly available dynamical constraints. Worthey, Ingermann & Serven (2011) investigated several calcium-sensitive spectral features, and concluded that a modest decrease in [Ca/Fe] with velocity dispersion could also explain the observed trends. Worthey, Ingermann & Serven (2011) also noted that such dwarf-rich IMFs would yield optical-NIR colors (e.g., V-K) much redder than even the reddest observed early-type galaxies.

An IMF different from the Galactic disk (often referred to as a non-universal IMF) was also reported by van Dokkum & Conroy (2010) based on the analysis of high S/N spectra extending to  $1\,\mu m$  for eight massive galaxies in the Coma and Virgo galaxies. These data were of extremely high quality thanks to new detector technology installed on the Low Resolution Imaging Spectrometer (LRIS) instrument at the Keck Telescope. The data were interpreted with new models limited to solar-scaled abundance patterns. Dwarf-rich IMFs were favored, with an IMF exponent in the range 2.3 < x < 3. It is worth pausing here to note that the meaning of 'dwarf-rich' has evolved over the years. In the 1970s dwarf-rich IMFs indicated mass-to-light ratios exceeding 20, often reaching as high as 40. With the IMF in the solar neighborhood now well-established (Kroupa 2001, Chabrier 2003), the reference point has shifted. Now, 'dwarf-rich' IMFs are those that contain relatively more low-mass stars than found in the solar neighborhood and would indicate, amongst other implications, a non-universal IMF. In this sense, the Salpeter IMF is 'dwarf-rich', as it contains more low-mass stars than the Kroupa (2001) IMF for the solar neighborhood.

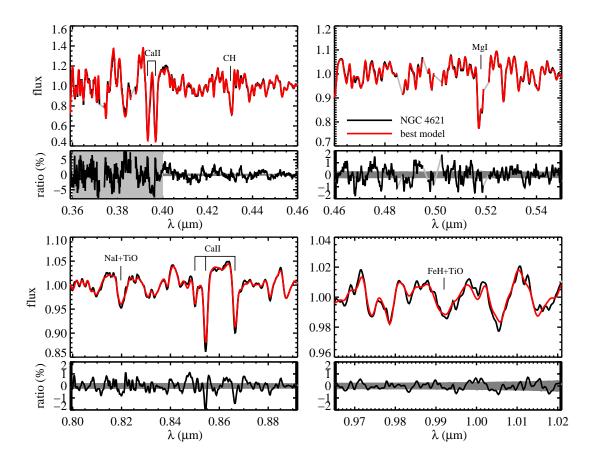


Figure 14:

Comparison between the best-fit model and observed spectrum for NGC 4621, a massive early-type galaxy. Both data and model have been continuum normalized. The S/N is indicated as a grey shaded band; the data at < 4000 Å were not included in the fit but are included here for completeness. The model has 21 free parameters including the abundance of 11 elements and the shape of the low-mass IMF. The rms difference between the model and data is 0.8%. IMF-sensitive features include the Na I doublet at  $0.82\,\mu\text{m}$ , the Ca II triplet at  $\approx 0.86\,\mu\text{m}$ , and the FeH bandhead (i.e., the Wing-Ford band) at  $0.99\,\mu\text{m}$ . The best-fit M/L is  $\approx 2\times$  as large as would be expected for a MW IMF, suggesting that the IMF is not universal. From Conroy & van Dokkum (2012b).

Conroy & van Dokkum (2012b) were the first to analyze high-quality absorption line spectra of 34 early-type galaxies and the nuclear bulge of M31 with a model that allowed for both IMF variation and abundance pattern variation (in addition to variation in age and several nuisance parameters). The model, presented in Conroy & van Dokkum (2012a), allows for variation in the abundance pattern of 11 elements, and is based on the latest isochrone tables and the MILES and IRTF stellar libraries. Conroy & van Dokkum (2012b) simultaneously fit the full blue and red galaxy spectra to their models, including

the classic IMF-sensitive features Na I, CaT, and FeH, as well as IMF-sensitive features in the blue spectral region. An example fit to a massive early-type galaxy is shown in Figure 14. They argued that the models favor IMFs that become progressively more dwarf-rich (i.e., more bottom heavy) for more massive and more  $\alpha$ —enhanced galaxies. Over the full sample, the mass-to-light ratios predicted by their best-fit IMFs vary by only a factor of 3 at fixed age and metallicity. The IMF variation advocated by Conroy & van Dokkum (2012b) is thus much subtler than previous claims for dwarf-rich IMFs. The best-fit mass-to-light ratios do not violate dynamical constraints. Similar conclusions were reached by Smith, Lucey & Carter (2012) based on the analysis of  $1 \mu m$  spectroscopy of 92 galaxies in the Coma cluster, and by Spiniello et al. (2012), who analyzed indices sensitive to Na I and TiO for a sample of early-type galaxies.

Meanwhile, there have been several less-direct arguments favoring top-heavy (or bottomlight) IMFs in some types of galaxies and in some phases of a galaxy's evolution. Tinsley (1980) suggested that a constraint on the IMF near the turn-off mass could be obtained by comparing the evolution of the M/L ratio and color of passively evolving galaxies. van Dokkum (2008) applied this idea to the evolution of galaxies on the fundamental plane, and found evidence that the IMF is flatter than the MW IMF at  $\sim 1 M_{\odot}$  in early-type galaxies. However, van Dokkum & Conroy (2012) demonstrated that when galaxies are compared at fixed velocity dispersion, as opposed to fixed stellar mass, the evolution in the fundamental plane is consistent with the standard Salpeter slope at  $\sim 1 M_{\odot}$  (see also Holden et al. 2010). Numerous authors have demonstrated that the evolution of the cosmic star formation rate and cosmic stellar mass density are mutually inconsistent but can be brought into agreement by adopting a top-heavy IMF (Hopkins & Beacom 2006, Davé 2008, Wilkins, Trentham & Hopkins 2008). However, recent improvement in the modeling of the data, including luminosity-dependent dust corrections, rising SFHs, and revised values of the faint end slope of the mass function have reconciled the cosmic star formation and mass densities derived with standard IMFs (Reddy & Steidel 2009, Papovich et al. 2011, Behroozi, Wechsler & Conroy 2012). A third argument in favor of top-heavy IMFs is the apparent inability of galaxy formation models to reproduce the observed number of luminous submillimeter galaxies (Baugh et al. 2005). Here too recent observations and more sophisticated models have significantly reduced the tension between models and data (Hayward et al. 2012). It thus appears that much of the evidence originally favoring a topheavy IMF at certain epochs and under certain conditions can now be explained by more mundane effects.

#### 7.1 Summary

Extremely dwarf-rich IMFs are now routinely ruled out both by sophisticated SPS models and dynamical constraints, and earlier evidence for top-heavy IMFs has not held up to further scrutiny. However, more modest IMF variations appear to be supported by the data, at the level of a factor of 2-3 in M/L. It is intriguing that constraints on the IMF based solely on dynamical modeling also favor a mild steepening (i.e., relatively more low-mass stars) with increasing velocity dispersion (Treu et al. 2010, Auger et al. 2010, Graves & Faber 2010, Spiniello et al. 2012, Thomas et al. 2011, Sonnenfeld et al. 2012, Cappellari et al. 2012, Dutton, Mendel & Simard 2012, Dutton et al. 2012a,b). As both dynamical and stellar population-based techniques suffer from non-negligible but largely orthogonal systematics, the most promising direction for future constraints will come from joint analyses of the same systems with SPS and dynamical techniques.

## 8 CONCLUDING REMARKS

The goal of this review has been to summarize what we have learned about galaxies from their panchromatic SEDs using the tools of stellar population synthesis. By way of concluding, I would like to turn toward the future and highlight areas where additional work is needed to make SED modeling both a more firmly quantitative science and also an effective engine for new discoveries. In short, and not surprisingly, the future requires better data, better models, better comparison of models to data, and a better understanding of what is in principle knowable from the analysis of galaxy SEDs.

A clear theme of this review has been the power of combining broadband data with moderate resolution spectra. The SDSS has been a revolutionizing force in this regard, as it has provided high-quality photometry and optical spectra for over one million objects. The great drawback of the SDSS is that it is fiber-based and thus the spectra only sample the central 3" (in diameter) of galaxies. This drawback will be alleviated with IFU spectroscopic surveys of nearby galaxies, including the recently completed SAURON survey (Bacon et al. 2001), the ongoing ATLAS3D survey (Cappellari et al. 2011) of 260 early-type galaxies, the CALIFA survey of ~ 600 galaxies (Sánchez et al. 2012), and the proposed MaNGA survey of a mass-limited sample of  $\sim 10,000$  galaxies. Such surveys will provide high-quality spectra that are well-matched to the broadband data. Restframe NIR spectra will be a new frontier for SPS studies as next generation NIR facilities become operational, including ground-based spectrographs (FIRE, FMOS, KMOS, MOSFIRE, FLAMINGOS-2, etc.), and the eventual launch of the James Webb Space Telescope. High-quality models are only now being developed to interpret such data. In addition to spectra, narrow-band photometry will also be a valuable addition to the landscape, as pioneered by the COMBO-17 survey (Wolf et al. 2004) and now utilized by surveys such as ALHAMBRA (Moles et al. 2008), the NEWFIRM Medium-Band Survey (Whitaker et al. 2011), and SHARDS (Perez-Gonzalez et al. 2012). Grism data will also help bridge the gap between broadband photometry and moderate resolution spectra, as demonstrated by the 3D-HST survey (Brammer et al. 2012). In addition to these object-by-object surveys, the construction of composite SEDs from galaxies spanning a range of redshifts allows for the creation of very high-quality and well-sampled SEDs that will be invaluable for SPS studies (Assef et al. 2008, Kriek et al. 2011).

Another theme of this review has been the growing realization that uncertainties in the SPS models are becoming a critical limiting factor to the interpretation of galaxy SEDs. The challenge here is not simply to enumerate the uncertainties but rather to identify areas where clear progress can be made. As a first step, all SPS models should include contributions from nebular emission and dust around AGB stars, as these processes are known to occur and the incorporation of such effects into the models is reasonably straightforward, even if the details are uncertain. Panchromatic models (i.e., FUV-FIR coverage) should also become standard both because IR data are now widely available and because sophisticated dust emission models are well-developed (e.g., Draine & Li 2007). The stellar atmospheric and synthetic spectral models will benefit from new asteroseismology measurements from the Kepler mission, interferometric observations (e.g., by CHARA), and new very high resolution UV-NIR spectral atlases of nearby stars across the HR diagram (Bagnulo et al. 2003, Lebzelter et al. 2012). Perhaps the most vexing issues lie with the stellar evolution uncertainties, as obvious calibrating data are lacking. The well-known problem is that, while globular clusters are the canonical testing ground for stellar evolution, metal-rich clusters are rare, and so the models tend to be poorly constrained precisely in the metallicity range most relevant for modeling galaxies. Moreover, the largest uncertainties are associated with fast evolutionary phases and so stars in such phases will be rare in all but the most massive star clusters. Efforts to constrain uncertain stellar evolutionary phases from the SEDs of galaxies is promising because the right metallicity ranges are probed and there are sufficient numbers of stars to overcome Poisson noise, but the obvious complexity of dealing with composite stellar populations makes this approach challenging (e.g., MacArthur et al. 2010, Kriek et al. 2010, Zibetti et al. 2012). The Panchromatic Hubble Andromeda Treasury survey (Dalcanton et al. 2012) is an HST program covering  $\sim 1/3$  of M31's star-forming disk in six filters. It promises to provide new and powerful constraints on luminous and advanced stellar evolutionary phases at moderately high metallicities.

More accurate models and higher-quality data will necessitate a more sophisticated approach to comparing the two. Presently, model fitting is something of an art, owing to the fact that large regions of parameter space are often severely under-constrained, which implies that the choice of priors on the model parameters can have a large impact on the derived results. A few basic guidelines should be followed to ensure that results are robust. For example, one should not simply fix a parameter to a particular value because it is under-constrained. Moreover, because the likelihood surface often contains multiple peaks and valleys and is frequently computed on a coarse grid, the best-fit parameters ought not be chosen based on the minimum of  $\chi^2$  (see e.g., Taylor et al. 2011). Rather, the full posterior distributions should be used to derive best-fit values and associated uncertainties. The choice of priors needs to be considered carefully, and in fact the model space should probably depend on the type of data being fit, the redshift of the object, and even its spectral type (quiescent vs. star-forming vs. peculiar). As the number of parameters increases, Markov Chain Monte Carlo techniques will see more widespread use owing to their efficient exploration of parameter space. With regards to the modeling of moderate resolution spectra, the analysis of spectral indices should eventually give way to full spectral fitting as the latter not only allows for the extraction of more information but also allows the modeler to visually inspect the fits in a way that is not possible when only EWs are extracted from the data. This is important for identifying model systematics and areas for future improvement.

Finally, further work is needed to understand what is knowable, in principle, from the modeling of galaxy SEDs. Questions such as 'how many discrete SF episodes can be measured from high quality optical spectra?', or 'how many moments of the metallicity distribution function can be extracted from SEDs?', or 'can the detailed dust attenuation curve be measured with sufficiently high quality data on an object-by-object basis?' have yet to be thoroughly explored. Addressing these questions will be difficult because they depend sensitively on the quality of the data, the SED type, and the reliability of the models. Fitting routines such as STARLIGHT, STECMAP, and VESPA that attempt a non-parametric recovery of the SFH and metallicity distribution function offer probably the most reliable tools to explore these questions. Theoretical studies aimed at understanding 'what is knowable' will help guide the next generation of surveys aimed at studying the detailed physical properties of galaxies.

# 9 DISCLOSURE STATEMENT

The author is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

## 10 ACKNOWLEDGMENTS

I would like to thank my collaborators for the continuous lively conversations that have helped form my views on this topic. I would also like to thank the authors who generously shared their figures for this review, and especially Elisabete da Cunha, John Moustakas, Naveen Reddy and Rita Tojeiro for providing new or modified figures. Stéphane Charlot, Daniel Dale, Sandy Faber, Jerome Fang, Ricardo Schiavon, Rita Tojeiro and Scott Trager are thanked for very useful comments on an earlier version of this manuscript.

#### LITERATURE CITED

Acquaviva V, Gawiser E, Guaita L. 2011. Ap. J. 737:47

Allard F, Hauschildt PH. 1995. Ap. J. 445:433-450

Allard F, Homeier D, Freytag B. 2011. In 16th Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun, eds. C Johns-Krull, MK Browning, AA West, vol. 448 of Astronomical Society of the Pacific Conference Series

Alongi M, Bertelli G, Bressan A, Chiosi C. 1991. Astron. Astrophys. 244:95-106

Alonso A, Arribas S, Martinez-Roger C. 1996. Astron. Astrophys. 313:873–890

Alvarez R, Lançon A, Plez B, Wood PR. 2000. Astron. Astrophys. 353:322-334

Anders P, Fritze-v. Alvensleben U. 2003. Astron. Astrophys. 401:1063-1070

Ardila DR, Van Dyk SD, Makowiecki W, Stauffer J, Song I, et al. 2010. Ap. J. Suppl. 191:301–339

Aringer B, Girardi L, Nowotny W, Marigo P, Lederer MT. 2009. Astron. Astrophys. 503:913–928

Asplund M, Grevesse N, Sauval AJ, Scott P. 2009. Annu. Rev. Astron. Astrophy. 47:481–522

Assef RJ, Kochanek CS, Brodwin M, Brown MJI, Caldwell N, et al. 2008.  $Ap.\ J.\ 676:286-303$  Atek H, et al. 2011.  $Ap.\ J.\ 743:121$ 

Auger MW, Treu T, Gavazzi R, Bolton AS, Koopmans LVE, Marshall PJ. 2010. Ap. J. Let. 721:L163–L167

Bacon R, et al. 2001. MNRAS 326:23–35

Bagnulo S, Jehin E, Ledoux C, Cabanac R, Melo C, et al. 2003. The Messenger 114:10-14

Baraffe I, Chabrier G, Allard F, Hauschildt PH. 1998. Astron. Astrophys. 337:403–412

Barber RJ, Tennyson J, Harris GJ, Tolchenov RN. 2006. MNRAS 368:1087-1094

Bastian N, Covey KR, Meyer MR. 2010. Annu. Rev. Astron. Astrophy. 48:339-389

Baugh CM, Lacey CG, Frenk CS, Granato GL, Silva L, et al. 2005. MNRAS 356:1191–1200

Bedijn PJ. 1987. Astron. Astrophys. 186:136–152

Behroozi PS, Wechsler RH, Conroy C. 2012. ArXiv e-prints

Bell EF, de Jong RS. 2000. MNRAS 312:497-520

Bell EF, de Jong RS. 2001. Ap. J. 550:212–229

Bell EF, Gordon KD, Kennicutt Jr. RC, Zaritsky D. 2002. Ap. J. 565:994-1010

Bell EF, McIntosh DH, Katz N, Weinberg MD. 2003. Ap. J. Suppl. 149:289-312

Bertelli G, Bressan A, Chiosi C, Fagotto F, Nasi E. 1994. Astron. Astrophys. Suppl. 106:275–302

Bertone E, Buzzoni A, Chávez M, Rodríguez-Merino LH. 2004. Astron. J. 128:829–841

Bessell MS, Brett JM, Scholz M, Wood PR. 1991. Astron. Astrophys. Suppl. 89:335–366

Bessell MS, Brett JM, Wood PR, Scholz M. 1989. Astron. Astrophys. Suppl. 77:1–30

Bica E, Alloin D. 1986. Astron. Astrophys. 162:21-31

Blackwell DE, Shallis MJ. 1977. MNRAS 180:177-191

Blanton MR, Moustakas J. 2009. Annu. Rev. Astron. Astrophy. 47:159-210

Blanton MR, Roweis S. 2007. Astron. J. 133:734-754

Bloecker T. 1995. Astron. Astrophys. 299:755

Boquien M, et al. 2012. Astron. Astrophys. 539:A145

Bouwens RJ, et al. 2012.  $Ap.\ J.\ 754:83$ 

Boyajian TS, et al. 2012a. Ap. J. 746:101

Boyajian TS, et al. 2012b. Ap. J. 757:112

Brammer GB, et al. 2012. Ap. J. Suppl. 200:13

Bressan A, Chiosi C, Fagotto F. 1994. Ap. J. Suppl. 94:63-115

Bressan A, Granato GL, Silva L. 1998. Astron. Astrophys. 332:135-148

Brinchmann J, Charlot S, White SDM, Tremonti C, Kauffmann G, et al. 2004. MNRAS 351:1151–1179

Brinchmann J, Ellis RS. 2000. Ap. J. Let. 536:L77-L80

Bruzual G. 1983. Ap. J. 273:105-127

Bruzual G, Charlot S. 1993. Ap. J. 405:538-553

Bruzual G, Charlot S. 2003. MNRAS 344:1000–1028

Buat V, Noll S, Burgarella D, Giovannoli E, Charmandaris V, et al. 2012. Astron. Astrophys. 545:A141

Bundy K, Ellis RS, Conselice CJ. 2005. Ap. J. 625:621-632

Burgarella D, Buat V, Iglesias-Páramo J. 2005. MNRAS 360:1413–1425

Burstein D, Faber SM, Gaskell CM, Krumm N. 1984. Ap. J. 287:586-609

Calzetti D. 2001. Pub. Astron. Soc. Pac. 113:1449-1485

Calzetti D, Kinney AL, Storchi-Bergmann T. 1994. Ap. J. 429:582-601

Calzetti D, et al. 2007. Ap. J. 666:870-895

Cappellari M, et al. 2011. MNRAS 413:813-836

Cappellari M, et al. 2012. Nature 484:485-488

Cardelli JA, Clayton GC, Mathis JS. 1989. Ap. J. 345:245-256

Carter D, Visvanathan N, Pickles AJ. 1986. Ap. J. 311:637-650

Casagrande L, Ramírez I, Meléndez J, Bessell M, Asplund M. 2010. Astron. Astrophys. 512:A54

Cassisi S. 2004. In IAU Colloq. 193: Variable Stars in the Local Group, eds. DW Kurtz, KR Pollard, vol. 310 of Astronomical Society of the Pacific Conference Series

Cassisi S, Marín-Franch A, Salaris M, Aparicio A, Monelli M, Pietrinferni A. 2011. Astron. Astrophys. 527:A59

Cenarro AJ, Cardiel N, Gorgas J, Peletier RF, Vazdekis A, Prada F. 2001. MNRAS 326:959-980

Cenarro AJ, Gorgas J, Vazdekis A, Cardiel N, Peletier RF. 2003. MNRAS 339:L12-L16

Cenarro AJ, et al. 2007. MNRAS 374:664-690

Chabrier G. 2003. Pub. Astron. Soc. Pac. 115:763-795

Chabrier G, Baraffe I. 1997. Astron. Astrophys. 327:1039-1053

Charlot S. 1996. In From Stars to Galaxies: the Impact of Stellar Physics on Galaxy Evolution, vol. 98 of Astronomical Society of the Pacific Conference Series

Charlot S, Bruzual AG. 1991. Ap. J. 367:126-140

Charlot S, Fall SM. 2000. Ap. J. 539:718–731

Charlot S, Longhetti M. 2001. MNRAS 323:887-903

Charlot S, Worthey G, Bressan A. 1996. Ap. J. 457:625

Chary R, Elbaz D. 2001. Ap. J. 556:562-581

Chen Y, Trager S, Peletier R, Lançon A. 2011. Journal of Physics Conference Series 328:012023

Chen YM, et al. 2012. MNRAS 421:314-332

Cid Fernandes R, Mateus A, Sodré L, Stasińska G, Gomes JM. 2005. MNRAS 358:363-378

Cimatti A, et al. 2008. Astron. Astrophys. 482:21–42

Coelho P, Barbuy B, Meléndez J, Schiavon RP, Castilho BV. 2005. Astron. Astrophys. 443:735–746

Coelho P, Bruzual G, Charlot S, Weiss A, Barbuy B, Ferguson JW. 2007. MNRAS 382:498–514

Cohen JG. 1978. Ap. J. 221:788-796

Conroy C, Gunn JE. 2010. Ap. J. 712:833-857

Conroy C, Gunn JE, White M. 2009. Ap. J. 699:486–506

Conroy C, Schiminovich D, Blanton MR. 2010. Ap. J. 718:184-198

Conroy C, van Dokkum PG. 2012a. Ap. J. 747:69

Conroy C, van Dokkum PG. 2012b. Ap. J. 760:71

Conroy C., van Dokkum PG., Graves GJ. 2013. Ap. J. Ap. J., submitted

Cordier D, Pietrinferni A, Cassisi S, Salaris M. 2007. Astron. J. 133:468-478

Couture J, Hardy E. 1993. Ap. J. 406:142-157

Curtis-Lake E, et al. 2012. ArXiv e-prints

da Cunha E, Charlot S, Elbaz D. 2008. MNRAS 388:1595-1617

Daddi E, et al. 2004. Ap. J. Let. 600:L127-L130

Daddi E, et al. 2007.  $Ap.\ J.\ 670:156-172$ 

Dalcanton JJ, et al. 2009. Ap. J. Suppl. 183:67–108

Dalcanton JJ, et al. 2012. Ap. J. Suppl. 200:18

Dale DA, Helou G, Contursi A, Silbermann NA, Kolhatkar S. 2001. Ap. J. 549:215–227

Dale DA, et al. 2009. Ap. J. 703:517-556

Dale DA, et al. 2012. Ap. J. 745:95

Davé R. 2008. MNRAS 385:147–160

Davidge TJ, Clark CC. 1994. Astron. J. 107:946-957

de Barros S, Schaerer D, Stark DP. 2012. ArXiv e-prints

de Freitas Pacheco JA, Barbuy B. 1995. Astron. Astrophys. 302:718

Delisle S, Hardy E. 1992. Astron. J. 103:711-727

Desert F, Boulanger F, Puget JL. 1990. Astron. Astrophys. 237:215-236

Devriendt JEG, Guiderdoni B, Sadat R. 1999. Astron. Astrophys. 350:381-398

Dotter A, Chaboyer B, Ferguson JW, Lee Hc, Worthey G, et al. 2007. Ap. J. 666:403-412

Dotter A, Chaboyer B, Jevremović D, Kostov V, Baron E, Ferguson JW. 2008. Ap. J. Suppl. 178:89–101

Draine BT. 2003. Annu. Rev. Astron. Astrophy. 41:241–289

Draine BT, Lee HM. 1984. Ap. J. 285:89-108

Draine BT, Li A. 2007. Ap. J. 657:810-837

Draine BT, et al. 2007. Ap. J. 663:866-894

Drory N, Bender R, Feulner G, Hopp U, Maraston C, et al. 2004. Ap. J. 608:742-751

Dunne L, Eales S, Edmunds M, Ivison R, Alexander P, Clements DL. 2000. MNRAS 315:115–139

Dunne L, Eales SA. 2001. MNRAS 327:697-714

Dutton AA, Maccio' AV, Mendel JT, Simard L. 2012a. ArXiv:1204.2825

Dutton AA, Mendel JT, Simard L. 2012. MNRAS 422:L33

Dutton AA, Treu T, Brewer BJ, Marshall PJ, Auger MW, et al. 2012b. MNRAS:187

Edvardsson B, Andersen J, Gustafsson B, Lambert DL, Nissen PE, Tomkin J. 1993. Astron. Astrophys. 275:101

Elbaz D, et al. 2010. Astron. Astrophys. 518:L29

Elbaz D, et al. 2011. Astron. Astrophys. 533:A119

Eldridge JJ, Izzard RG, Tout CA. 2008. MNRAS 384:1109–1118

Eldridge JJ, Stanway ER. 2012. MNRAS 419:479-489

Eminian C, Kauffmann G, Charlot S, Wild V, Bruzual G, et al. 2008. MNRAS 384:930–942

Faber SM. 1972. Astron. Astrophys. 20:361-374

Faber SM. 1977. In Evolution of Galaxies and Stellar Populations, ed. B. M. Tinsley & R. B. Larson

Faber SM, French HB. 1980.  $Ap.\ J.\ 235:405-412$ 

Fanelli MN, O'Connell RW, Burstein D, Wu CC. 1992. Ap. J. Suppl. 82:197–245

Fang JJ, Faber SM, Salim S, Graves GJ, Rich RM. 2012. Ap. J. 761:23

Ferland GJ, Korista KT, Verner DA, Ferguson JW, Kingdon JB, Verner EM. 1998. Pub. Astron. Soc. Pac. 110:761–778

Ferreras I, et al. 2009. Ap. J. 706:158-169

Finkelstein SL, Papovich C, Giavalisco M, Reddy NA, Ferguson HC, et al. 2010. Ap. J. 719:1250–1273

Finkelstein SL, et al. 2012. Ap. J. 756:164

Finlator K, Davé R, Oppenheimer BD. 2007. MNRAS 376:1861-1878

Fioc M, Rocca-Volmerange B. 1997. Astron. Astrophys. 326:950–962

Fischera J, Dopita M. 2011. Astron. Astrophys. 533:A117

Fluks MA, Plez B, The PS, de Winter D, Westerlund BE, Steenman HC. 1994. Astron. Astrophys.

Suppl. 105:311-336

Ford HA, Bregman JN. 2012. ArXiv e-prints

Frogel JA. 1988. Annu. Rev. Astron. Astrophy. 26:51-92

Frogel JA, Persson SE, Cohen JG. 1980. Ap. J. 240:785-802

Frogel JA, Persson SE, Matthews K, Aaronson M. 1978. Ap. J. 220:75-97

Fulbright JP, McWilliam A, Rich RM. 2007. Ap. J. 661:1152-1179

Gallazzi A, Bell EF. 2009. Ap. J. Suppl. 185:253-272

Gallazzi A, Charlot S, Brinchmann J, White SDM, Tremonti CA. 2005. MNRAS 362:41-58

Galliano F, Dwek E, Chanial P. 2008. Ap. J. 672:214–243

Giallongo E, D'Odorico S, Fontana A, Cristiani S, Egami E, et al. 1998. Astron. J. 115:2169-2183

Girardi L, Bressan A, Bertelli G, Chiosi C. 2000. Astron. Astrophys. Suppl. 141:371-383

González JJ. 1993. Line strength gradients and kinematic profiles in elliptical galaxies. Ph.D. thesis, Thesis (PH.D.)—UNIVERSITY OF CALIFORNIA, SANTA CRUZ, 1993. Source: Dissertation Abstracts International, Volume: 54-05, Section: B, page: 2551.

González V, Labbé I, Bouwens RJ, Illingworth G, Franx M, Kriek M. 2011. Ap. J. Let. 735:L34

González Delgado RM, Cerviño M, Martins LP, Leitherer C, Hauschildt PH. 2005. MNRAS 357:945–960

González-Lópezlira RA, Bruzual-A. G, Charlot S, Ballesteros-Paredes J, Loinard L. 2010. MNRAS 403:1213–1238

Gordon KD, Clayton GC. 1998. Ap. J. 500:816

Granato GL, Lacey CG, Silva L, Bressan A, Baugh CM, et al. 2000. Ap. J. 542:710-730

Graves GJ, Faber SM. 2010. Ap. J. 717:803-824

Graves GJ, Faber SM, Schiavon RP. 2009. Ap. J. 693:486-506

Gregg MD, Silva D, Rayner J, Worthey G, Valdes F, et al. 2006. In The 2005 HST Calibration Workshop: Hubble After the Transition to Two-Gyro Mode, eds. AM Koekemoer, P Goudfrooij, LL Dressel

Greggio L. 1997. MNRAS 285:151-166

Grillmair CJ, et al. 1996. Astron. J. 112:1975

Groenewegen MAT. 2012. Astron. Astrophys. 543:A36

Groves B, Dopita MA, Sutherland RS, Kewley LJ, Fischera J, et al. 2008. Ap. J. Suppl. 176:438-456

Groves BA, Dopita MA, Sutherland RS. 2004. Ap. J. Suppl. 153:9-73

Gunn JE, Stryker LL. 1983. Ap. J. Suppl. 52:121-153

Han Z, Podsiadlowski P, Maxted PFL, Marsh TR. 2003. MNRAS 341:669-691

Han Z, Podsiadlowski P, Maxted PFL, Marsh TR, Ivanova N. 2002. MNRAS 336:449-466

Hardy E, Couture J. 1988. Ap. J. Let. 325:L29–L31

Harris GLH, Harris WE. 2000. Astron. J. 120:2423-2436

Hayward CC, Narayanan D, Kereš D, Jonsson P, Hopkins PF, et al. 2012. ArXiv e-prints

Heap SR, Lindler D. 2011. In Astronomical Society of the Pacific Conference Series, eds. C Johns-Krull, MK Browning, AA West, vol. 448 of Astronomical Society of the Pacific Conference Series

Heavens AF, Jimenez R, Lahav O. 2000. MNRAS 317:965–972

Heger A, Fryer CL, Woosley SE, Langer N, Hartmann DH. 2003. Ap. J. 591:288–300

Holden BP, van der Wel A, Kelson DD, Franx M, Illingworth GD. 2010. Ap. J. 724:714–729

Hopkins AM, Beacom JF. 2006. Ap. J. 651:142-154

Hoversten EA, Gronwall C, Vanden Berk DE, Basu-Zych AR, Breeveld AA, et al. 2011. Astron. J. 141:205

Hwang HS, et al. 2010. MNRAS 409:75–82

Jeong H, et al. 2009. MNRAS 398:2028–2048

Johansson J, Thomas D, Maraston C. 2012. MNRAS 421:1908-1926

Johnson BD, et al. 2007. Ap. J. Suppl. 173:392-403

Jonsson P. 2006. MNRAS 372:2-20

Kalirai JS, Hansen BMS, Kelson DD, Reitzel DB, Rich RM, Richer HB. 2008. Ap. J. 676:594-609

Kannappan SJ, Gawiser E. 2007. Ap. J. Let. 657:L5-L8

Kauffmann G, et al. 2003. MNRAS 341:33-53

Kaviraj S, et al. 2007. Ap. J. Suppl. 173:619-642

Keller SC, Wood PR. 2006. Ap. J. 642:834-841

Kelson DD, Holden BP. 2010. Ap. J. Let. 713:L28-L32

Kelson DD, Illingworth GD, Franx M, van Dokkum PG. 2006. Ap. J. 653:159-183

Kennicutt RC, Evans NJ. 2012. Annu. Rev. Astron. Astrophy. 50:531-608

Kennicutt Jr. RC. 1998. Annu. Rev. Astron. Astrophy. 36:189-232

Kennicutt Jr. RC, et al. 2007. Ap. J. 671:333-348

Kobulnicky HA, Fryer CL. 2007. Ap. J. 670:747-765

Koleva M, Prugniel P, Bouchard A, Wu Y. 2009. Astron. Astrophys. 501:1269–1279

Koleva M, Prugniel P, Ocvirk P, Le Borgne D, Soubiran C. 2008. MNRAS 385:1998–2010

Kong X, Charlot S, Brinchmann J, Fall SM. 2004. MNRAS 349:769-778

Korn AJ, Maraston C, Thomas D. 2005. Astron. Astrophys. 438:685-704

Kriek M, Labbé I, Conroy C, Whitaker KE, van Dokkum PG, et al. 2010. Ap. J. Let. 722:L64-L69

Kriek M, van Dokkum PG, Whitaker KE, Labbé I, Franx M, Brammer GB. 2011. Ap. J. 743:168

Kriek M, et al. 2008.  $Ap.\ J.\ 677:219-237$ 

Kroupa P. 2001. MNRAS 322:231–246

Kuntschner H. 2000. MNRAS 315:184–208

Kurucz RL. 2011. Canadian Journal of Physics 89:417-428

Kučinskas A, Hauschildt PH, Ludwig HG, Brott I, Vansevičius V, et al. 2005. Astron. Astrophys. 442:281–308

Labbe I, Oesch PA, Bouwens RJ, Illingworth GD, Magee D, et al. 2012. ArXiv:1209.3037

Lançon A, Mouhcine M. 2000. In Massive Stellar Clusters, eds. A Lançon, CM Boily, vol. 211 of Astronomical Society of the Pacific Conference Series

Lançon A, Mouhcine M. 2002. Astron. Astrophys. 393:167-181

Lançon A, Mouhcine M, Fioc M, Silva D. 1999. Astron. Astrophys. 344:L21-L24

Lanz T, Hubeny I. 2003. Ap. J. Suppl. 146:417-441

Le Borgne JF, Bruzual G, Pelló R, Lançon A, Rocca-Volmerange B, et al. 2003. Astron. Astrophys. 402:433–442

Lebzelter T, et al. 2012. Astron. Astrophys. 539:A109

Lee Hc, Worthey G, Dotter A, Chaboyer B, Jevremović D, et al. 2009a. Ap. J. 694:902–923

Lee Hc, Worthey G, Trager SC, Faber SM. 2007. Ap. J. 664:215–225

Lee Hc, Yoon SJ, Lee YW. 2000. Astron. J. 120:998-1005

Lee SK, Ferguson HC, Somerville RS, Wiklind T, Giavalisco M. 2010. Ap. J. 725:1644–1651

Lee SK, Idzi R, Ferguson HC, Somerville RS, Wiklind T, Giavalisco M. 2009b. Ap. J. Suppl. 184:100–132

Lee YS, et al. 2008. Astron. J. 136:2022-2049

Leitherer C, et al. 1999. Ap. J. Suppl. 123:3-40

Leitner SN. 2012. Ap. J. 745:149

Lejeune T, Cuisinier F, Buser R. 1997. Astron. Astrophys. Suppl. 125:229–246

Lejeune T, Cuisinier F, Buser R. 1998. Astron. Astrophys. Suppl. 130:65-75

Leonardi AJ, Rose JA. 1996. Astron. J. 111:182

Levesque EM, Leitherer C, Ekstrom S, Meynet G, Schaerer D. 2012. Ap. J. 751:67

Longhetti M, Saracco P. 2009. MNRAS 394:774-794

Lotz JM, Ferguson HC, Bohlin RC. 2000. Ap. J. 532:830–844

MacArthur LA, Courteau S, Bell E, Holtzman JA. 2004. Ap. J. Suppl. 152:175-199

MacArthur LA, González JJ, Courteau S. 2009. MNRAS395:28–63

MacArthur LA, McDonald M, Courteau S, Jesús González J. 2010. Ap. J. 718:768-773

Madden SC, Galliano F, Jones AP, Sauvage M. 2006. Astron. Astrophys. 446:877-896

Maeder A, Meynet G. 2000. Annu. Rev. Astron. Astrophy. 38:143-190

Maeder A, Meynet G. 2012. Reviews of Modern Physics 84:25-63

Maraston C. 1998. MNRAS 300:872-892

Maraston C. 2005. MNRAS 362:799–825

Maraston C, Daddi E, Renzini A, Cimatti A, Dickinson M, et al. 2006. Ap. J. 652:85-96

Maraston C, Pforr J, Renzini A, Daddi E, Dickinson M, et al. 2010. MNRAS 407:830-845

Maraston C, Thomas D. 2000. Ap. J. 541:126-133

Marchesini D, van Dokkum PG, Förster Schreiber NM, Franx M, Labbé I, Wuyts S. 2009. Ap. J. 701:1765–1796

Marigo P. 2001. Astron. Astrophys. 370:194-217

Marigo P, Girardi L. 2007. Astron. Astrophys. 469:239–263

Marigo P, Girardi L, Bressan A, Groenewegen MAT, Silva L, Granato GL. 2008. Astron. Astrophys. 482:883–905

Martins LP, Coelho P. 2007. MNRAS 381:1329-1346

Martins LP, González Delgado RM, Leitherer C, Cerviño M, Hauschildt P. 2005. MNRAS 358:49-65

Mathis JS, Rumpl W, Nordsieck KH. 1977. Ap. J. 217:425–433

Melbourne J, et al. 2012. Ap. J. 748:47

Meurer GR, Heckman TM, Calzetti D. 1999. Ap. J. 521:64-80

Meynet G, Maeder A. 2000. Astron. Astrophys. 361:101-120

Miller GE, Scalo JM. 1979. Ap. J. Suppl. 41:513-547

Miner J, Rose JA, Cecil G. 2011. Ap. J. Let. 727:L15

Moles M, et al. 2008. Astron. J. 136:1325–1339

Mollá M, García-Vargas ML, Bressan A. 2009. MNRAS 398:451–470

Monachesi A, Trager SC, Lauer TR, Freedman W, Dressler A, et al. 2011. Ap. J. 727:55

Moustakas J, et al. 2012. Ap. J. Ap. J., submitted

Munari U, Sordo R, Castelli F, Zwitter T. 2005. Astron. Astrophys. 442:1127-1134

Muzzin A, Marchesini D, van Dokkum PG, Labbé I, Kriek M, Franx M. 2009. Ap. J. 701:1839–1864

Muzzin A, van Dokkum P, Kriek M, Labbé I, Cury I, et al. 2010. Ap. J. 725:742-749

Noll S, Burgarella D, Giovannoli E, Buat V, Marcillac D, Muñoz-Mateos JC. 2009a. Astron. Astrophys. 507:1793–1813

Noll S, et al. 2009b. Astron. Astrophys. 499:69-85

Nordstroem B, Andersen J, Andersen MI. 1997. Astron. Astrophys. 322:460-476

O'Connell RW. 1976. Ap. J. 206:370-390

O'Connell RW. 1980. Ap. J. 236:430-440

O'Connell RW. 1999. Annu. Rev. Astron. Astrophy. 37:603–648

Ocvirk P, Pichon C, Lançon A, Thiébaut E. 2006. MNRAS 365:46-73

Osterbrock DE. 1989. Astrophysics of gaseous nebulae and active galactic nuclei

Pacifici C, Charlot S, Blaizot J, Brinchmann J. 2012. MNRAS 421:2002-2024

Panuzzo P, Bressan A, Granato GL, Silva L, Danese L. 2003. Astron. Astrophys. 409:99-114

Panuzzo P, Granato GL, Buat V, Inoue AK, Silva L, et al. 2007. MNRAS 375:640-648

Papovich C, Dickinson M, Ferguson HC. 2001. Ap. J. 559:620-653

Papovich C, Finkelstein SL, Ferguson HC, Lotz JM, Giavalisco M. 2011. MNRAS 412:1123-1136

Papovich C, Rudnick G, Le Floc'h E, van Dokkum PG, Rieke GH, et al. 2007. Ap. J. 668:45-61

Paxton B, Bildsten L, Dotter A, Herwig F, Lesaffre P, Timmes F. 2011. Ap. J. Suppl. 192:3 Pei YC. 1992. Ap. J. 395:130–139

Peletier RF. 1989. Elliptical Galaxies - Structure and Stellar Content. Ph.D. thesis, , University of Groningen, The Netherlands, (1989)

Percival SM, Salaris M. 2011. MNRAS 412:2445–2453

Percival SM, Salaris M, Cassisi S, Pietrinferni A. 2009. Ap. J. 690:427-439

Perez-Gonzalez PG, et al. 2012. ArXiv e-prints

Perrin G, Coudé du Foresto V, Ridgway ST, Mariotti JM, Traub WA, et al. 1998. Astron. Astrophys. 331:619–626

Peterson RC. 1976. Ap. J. Let. 210:L123-L126

Pforr J, Maraston C, Tonini C. 2012. MNRAS 422:3285–3326

Pickles AJ. 1998. Pub. Astron. Soc. Pac. 110:863-878

Pietrinferni A, Cassisi S, Salaris M, Castelli F. 2004. Ap. J. 612:168–190

Piovan L, Tantalo R, Chiosi C. 2003. Astron. Astrophys. 408:559-579

Piovan L, Tantalo R, Chiosi C. 2006. MNRAS 366:923-944

Ponder JM, Burstein D, O'Connell RW, Rose JA, Frogel JA, et al. 1998. Astron. J. 116:2297–2314 Pope A, et al. 2006. MNRAS 370:1185–1207

Popescu CC, Misiriotis A, Kylafis ND, Tuffs RJ, Fischera J. 2000. Astron. Astrophys. 362:138–150

Popescu CC, Tuffs RJ, Dopita MA, Fischera J, Kylafis ND, Madore BF. 2011. Astron. Astrophys. 527:A109

Prugniel P, Soubiran C. 2001. Astron. Astrophys. 369:1048-1057

Rabin D. 1982. Ap. J. 261:85-101

Ramírez I, Meléndez J. 2005. Ap. J. 626:465-485

Rauch T. 2003. Astron. Astrophys. 403:709-714

Rayner JT, Cushing MC, Vacca WD. 2009. Ap. J. Suppl. 185:289-432

Reddy NA, Erb DK, Pettini M, Steidel CC, Shapley AE. 2010. Ap. J. 712:1070-1091

Reddy NA, Steidel CC. 2009. Ap. J. 692:778–803

Renzini A. 2006. Annu. Rev. Astron. Astrophy. 44:141-192

Renzini A, Buzzoni A. 1986. In Spectral Evolution of Galaxies, eds. C Chiosi, A Renzini, vol. 122 of Astrophysics and Space Science Library

Renzini A, Ciotti L. 1993. Ap. J. Let. 416:L49

Renzini A, Greggio L, Ritossa C, Ferrario L. 1992. Ap. J. 400:280-303

Rose JA. 1984. Astron. J. 89:1238-1251

Rose JA. 1985. Astron. J. 90:1927-1956

Rujopakarn W, Rieke GH, Weiner BJ, Rex M, Walth GL, Kartaltepe JS. 2012. ArXiv:1107.2921

Saglia RP, Maraston C, Thomas D, Bender R, Colless M. 2002. Ap. J. Let. 579:L13-L16

Salim S, Fang JJ, Rich RM, Faber SM, Thilker DA. 2012. Ap. J. 755:105

Salim S, Rich RM. 2010. Ap. J. Let. 714:L290-L294

Salim S, et al. 2007. Ap. J. Suppl. 173:267-292

Salpeter EE. 1955. Ap. J. 121:161

Sana H, Evans CJ. 2011. In *IAU Symposium*, ed. C. Neiner, G. Wade, G. Meynet, & G. Peters, vol. 272 of *IAU Symposium* 

Sánchez SF, et al. 2012. Astron. Astrophys. 538:A8

Sánchez-Blázquez P, Ocvirk P, Gibson BK, Pérez I, Peletier RF. 2011. MNRAS 415:709-731

Sánchez-Blázquez P, Peletier RF, Jiménez-Vicente J, Cardiel N, Cenarro AJ, et al. 2006. MNRAS 371:703–718

Santini P, et al. 2009. Astron. Astrophys. 504:751-767

Sargent BA, Srinivasan S, Meixner M. 2011. Ap. J. 728:93

Sawicki M, Yee HKC. 1998. Astron. J. 115:1329-1339

Scalo J, Vazquez-Semadeni E, Chappell D, Passot T. 1998. Ap. J. 504:835

Scalo JM. 1986. Fundamentals of Cosmic Physics 11:1-278

Schaerer D. 2003. Astron. Astrophys. 397:527-538

Schaerer D, de Barros S. 2010. Astron. Astrophys. 515:A73

Schaller G, Schaerer D, Meynet G, Maeder A. 1992. Astron. Astrophys. Suppl. 96:269-331

Schiavon RP. 2007. Ap. J. Suppl. 171:146-205

Schiavon RP. 2010. Publication of Korean Astronomical Society 25:83-90

Schiavon RP, Barbuy B, Bruzual A. G. 2000. Ap. J. 532:453-460

Schiavon RP, Barbuy B, Rossi SCF, Milone A. 1997. Ap. J. 479:902

Schiavon RP, Barbuy B, Singh PD. 1997. Ap. J. 484:499

Schiavon RP, Rose JA, Courteau S, MacArthur LA. 2004. Ap. J. Let. 608:L33-L36

Schiavon RP, et al. 2012. Astron. J. 143:121

Schmidt M. 1959. Ap. J. 129:243

Schoenberner D. 1983. Ap. J. 272:708-714

Schwarzschild M, Harm R. 1968. The Astronomical Journal Supplement 73:35

Searle L, Sargent WLW, Bagnuolo WG. 1973. Ap. J. 179:427-438

Serra P, Trager SC. 2007. MNRAS 374:769-774

Serven J, Worthey G, Briley MM. 2005. Ap. J. 627:754-766

Serven J, Worthey G, Toloba E, Sánchez-Blázquez P. 2011. Astron. J. 141:184

Shapley AE, Steidel CC, Adelberger KL, Dickinson M, Giavalisco M, Pettini M. 2001.  $Ap.\ J.$  562:95–123

Shapley AE, Steidel CC, Erb DK, Reddy NA, Adelberger KL, et al. 2005. Ap. J. 626:698-722

Shapley AE, Steidel CC, Pettini M, Adelberger KL, Erb DK. 2006. Ap. J. 651:688-703

Silva L, Granato GL, Bressan A, Danese L. 1998. Ap. J. 509:103-117

Smith LJ, Norris RPF, Crowther PA. 2002. MNRAS 337:1309-1328

Smith RJ, Lucey JR, Carter D. 2012. MNRAS 421:2982–2997

Smith RJ, Lucey JR, Hudson MJ. 2009. MNRAS 400:1690–1705

Smith RJ, Lucey JR, Hudson MJ, Bridges TJ. 2009. MNRAS 398:119-132

Sonnenfeld A, Treu T, Gavazzi R, Marshall PJ, Auger MW, et al. 2012. Ap. J. 752:163

Spiniello C, Trager SC, Koopmans LVE, Chen YP. 2012. Ap. J. Let. 753:L32

Spinrad H. 1962. Ap. J. 135:715

Spinrad H, Taylor BJ. 1971. Ap. J. Suppl. 22:445

Srinivasan S, Sargent BA, Meixner M. 2011. Astron. Astrophys. 532:A54

Stothers RB. 1991. Ap. J. 383:820-836

Taylor EN, et al. 2011. MNRAS 418:1587–1620

Thomas D, Maraston C, Bender R. 2003a. MNRAS 343:279-283

Thomas D, Maraston C, Bender R. 2003b. MNRAS 339:897-911

Thomas D, Maraston C, Bender R, Mendes de Oliveira C. 2005. Ap. J. 621:673-694

Thomas D, Maraston C, Korn A. 2004. MNRAS 351:L19-L23

Thomas J, Saglia RP, Bender R, Thomas D, Gebhardt K, et al. 2011. MNRAS 415:545-562

Tinsley BM. 1968. Ap. J. 151:547

Tinsley BM. 1979.  $Ap.\ J.\ 229:1046-1056$ 

Tinsley BM. 1980. Fundamentals of Cosmic Physics 5:287-388

Tinsley BM, Gunn JE. 1976.  $Ap.\ J.\ 203:52-62$ 

Tojeiro R, Heavens AF, Jimenez R, Panter B. 2007. MNRAS 381:1252-1266

Tojeiro R, Percival WJ, Heavens AF, Jimenez R. 2011. MNRAS 413:434-460

Tojeiro R, Wilkins S, Heavens AF, Panter B, Jimenez R. 2009. Ap. J. Suppl. 185:1–19

Trager SC, Faber SM, Dressler A. 2008. MNRAS 386:715-747

Trager SC, Faber SM, Worthey G, González JJ. 2000a. Astron. J. 120:165–188

Trager SC, Faber SM, Worthey G, González JJ. 2000b. Astron. J. 119:1645-1676

Trager SC, Somerville RS. 2009. MNRAS 395:608–624

Trager SC, Worthey G, Faber SM, Burstein D, Gonzalez JJ. 1998. Ap. J. Suppl. 116:1

Trager SC, Worthey G, Faber SM, Dressler A. 2005. MNRAS 362:2–8

Treu T, Auger MW, Koopmans LVE, Gavazzi R, Marshall PJ, Bolton AS. 2010. Ap. J. 709:1195–1202

Tripicco MJ, Bell RA. 1995. Astron. J. 110:3035

Valdes F, Gupta R, Rose JA, Singh HP, Bell DJ. 2004. Ap. J. Suppl. 152:251-259

van Dokkum PG. 2008. Ap. J. 674:29-50

van Dokkum PG, Conroy C. 2010. Nature 468:940-942

van Dokkum PG, Conroy C. 2012. Ap. J. 760:70

Vandenberg DA, Bell RA. 1985. Ap. J. Suppl. 58:561–621

VandenBerg DA, Bell RA. 2001. New Astron. Rev. 45:577-582

VandenBerg DA, Bergbusch PA, Dowler PD. 2006. Ap. J. Suppl. 162:375-387

VandenBerg DA, Stetson PB. 2004. Pub. Astron. Soc. Pac. 116:997-1011

Vassiliadis E, Wood PR. 1994. Ap. J. Suppl. 92:125-144

Vazdekis A. 1999. Ap. J. 513:224-241

Vazdekis A, Casuso E, Peletier RF, Beckman JE. 1996. Ap. J. Suppl. 106:307

Vazdekis A, Peletier RF, Beckman JE, Casuso E. 1997. Ap. J. Suppl. 111:203

Vazdekis A, Sánchez-Blázquez P, Falcón-Barroso J, Cenarro AJ, Beasley MA, et al. 2010. MNRAS 404:1639–1671

Vázquez GA, Leitherer C, Schaerer D, Meynet G, Maeder A. 2007. Ap. J. 663:995–1020

Venn KA, Irwin M, Shetrone MD, Tout CA, Hill V, Tolstoy E. 2004. Astron. J. 128:1177–1195

Ventura P, Marigo P. 2010. MNRAS 408:2476-2486

Voit GM. 1992. MNRAS 258:841-848

Walcher CJ, Coelho P, Gallazzi A, Charlot S. 2009. MNRAS 398:L44–L48

Walcher J, Groves B, Budavári T, Dale D. 2011. Astrophy. Space Sci 331:1-52

Wallerstein G. 1962. Ap. J. Suppl. 6:407

Weingartner JC, Draine BT. 2001. Ap. J. 548:296-309

Weisz DR, et al. 2011.  $Ap.\ J.\ 739:5$ 

Westera P, Lejeune T, Buser R, Cuisinier F, Bruzual G. 2002. Astron. Astrophys. 381:524-538

Whitaker KE, et al. 2010. Ap. J. 719:1715-1732

Whitaker KE, et al. 2011. Ap. J. 735:86

Whitford AE. 1977. Ap. J. 211:527-538

Wijesinghe DB, et al. 2011. MNRAS 410:2291-2301

Wild V, Charlot S, Brinchmann J, Heckman T, Vince O, et al. 2011. MNRAS 417:1760-1786

Wilkins SM, Trentham N, Hopkins AM. 2008. MNRAS 385:687-694

Williams RJ, Quadri RF, Franx M, van Dokkum P, Labbé I. 2009. Ap. J. 691:1879–1895

Willson LA. 2000. Annu. Rev. Astron. Astrophy. 38:573-611

Witt AN, Gordon KD. 2000. Ap. J. 528:799-816

Wolf C, Meisenheimer K, Kleinheinrich M, Borch A, Dye S, et al. 2004. Astron. Astrophys. 421:913–936

Worthey G. 1994. Ap. J. Suppl. 95:107-149

Worthey G, Dorman B, Jones LA. 1996. Astron. J. 112:948

Worthey G, Faber SM, Gonzalez JJ. 1992. Ap. J. 398:69-73

Worthey G, Faber SM, Gonzalez JJ, Burstein D. 1994. Ap. J. Suppl. 94:687–722

Worthey G, Ingermann BA, Serven J. 2011. Ap. J. 729:148

Wuyts S, Franx M, Cox TJ, Hernquist L, Hopkins PF, et al. 2009. Ap. J. 696:348-369

Wuyts S, Labbé I, Franx M, Rudnick G, van Dokkum PG, et al. 2007. Ap. J. 655:51-65

Wuyts S, et al. 2011. Ap. J. 738:106

Wuyts S, et al. 2012. Ap. J. 753:114

Yi S, Demarque P, Kim YC, Lee YW, Ree CH, et al. 2001. Ap. J. Suppl. 136:417-437

Yi S, Demarque P, Oemler Jr. A. 1997. Ap. J. 486:201

Yi S, Lee YW, Woo JH, Park JH, Demarque P, Oemler Jr. A. 1999. Ap. J. 513:128-141

Yi SK. 2003. Ap. J. 582:202-214

Yi SK. 2008. In Hot Subdwarf Stars and Related Objects, eds. U Heber, CS Jeffery, R Napiwotzki, vol. 392 of Astronomical Society of the Pacific Conference Series

Yi SK, Kim YC, Demarque P. 2003. Ap. J. Suppl. 144:259–261

Yi SK, et al. 2005.  $Ap.\ J.\ Let.\ 619:L111-L114$ 

Zhang F, Han Z, Li L, Hurley JR. 2005. MNRAS 357:1088-1103

Zibetti S, Charlot S, Rix HW. 2009. MNRAS 400:1181-1198

Zibetti S, Gallazzi A, Charlot S, Pierini D, Pasquali A. 2012. MNRAS:120

Zubko V, Dwek E, Arendt RG. 2004. Ap. J. Suppl. 152:211-249